CS610-Computer Network

SOLVED MCQS QUIZ 1

LEACTURE (1 to 10)

Solved by JUNAID MALIK

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1. In heap sort algorithm, the total running time for heavily procedure is _.

- Big-oh(log n)
- O (1) i.e. Constant time
- Theta (log n)
- Omega (log n)

2. Quick sort algorithm is required a lot of comparison in the _____ condition.

- Worse case
- Best and average case
- Average case
- Best case

3. In heap sort algorithm (using max heap). When every time maximum element is removed from top.

- Divide and conquer strategy helps us
- We are left with a lot
- We call merge sort algorithm
- It becomes order n2 algorithm

4. In average-case time analysis of quick sort algorithm,
The most balanced case for partition is where we divide the list of element into _____

- Three nearly equal pieces
- Single piece exactly
- Two nearly piece
- Equal no. of piece as of input element

5. Consider three matrices X,Y,Z dimensions 1×2.2×.3×4 respectively. The number of multiplication of (XYZ) is:

- 32
- 30
- 24
- 18

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6. Quicksort is a/an _____ and_____ sorting algorithm.

- In-place, not stable one
- Not in-place, stable one
- In-place, stable one
- Not in-place, not stable one

7. _____items are not allowed in the 0/1 knapack.

- Lighter
- Whole
- Weighty
- Fractional

8. The main purpose of mathematical analysis is measuring the _____ required by the algorithm.

- Space
- Execution time and memory
- Input & output
- Execution time

9. Execution time of an algorithm can be measured by _____.

- Divide and conquer approach
- Both brute force and divide and conquer approach
- Mathematical analysis
- Brute force approach

10. Quick sort is based on _____ strategy.

- Graph theory
- Divide-and-conquer
- Dynamic programming
- Greedy approach

11. A sorting algorithm is called as _____ if duplicate element remain in the same relative position after sorting.

- O(n) algorithm
- Stable
- Parallel

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- Complex

12. Which one sorting algorithm is best suited to sort an array of 2 million elements?

- Insert sort
- Quick sort
- Merge sort
- Bubble sort

13. We can use the _____ property to devise a recursive formulation of the edit distance problem.

- Algorithm
- Small substructure
- Optimal substructure
- Real

14. While sorting. The ordered domain means for any two input elements x and y _____ satisfies only.

- All of the above
- x > y
- x < y
- x = y

15. 8n2 + 2n -3 will eventually exceed c2*(n) no matter how large we make _____.

- 2n
- n
- this equation
- c2

16. _____ is a method of solving a problem in which we check all possible solution to the problem to find the solution we need.

- Sorting algorithm
- Greedy approach
- Plane-sweep algorithm
- Brute-force algorithm

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17. In quick sort algorithm, provost form _____.

- Graph
- Stack
- Binary search tree
- Queue

18. In asymptotical analysis of n(n -3) and 4n*n, as n becomes large, the dominant (fastest growing) term is some constant time_____

- n +1
- n*n
- n
- n-1

19. If Matrix-A has dimensions "3×2" and Matrix-B has dimensions "2×3", then multiplication of Matrix –A and Matrix-B will result a new Matrix-C having dimensions

- 2×3
- 2×2
- 3×2
- 3×3

20. Boolean operation is a _____ operation on an idealized RAM model of computation.

- Advance
- Normal
- Basic
- Starting

21. There are _____ entries in the Edit Distance Matrix.

- Θ (n^2)
- Θ (n+100)
- Θ (n)
- Θ (n+2)

22. Counting sort is suitable for sorting the elements within range 1 to P. where_____

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- P is undetermined
- P is small
- P is very large
- P is large

23. Suppose we have 4 matrices A,B,C,D. what is correct expansion of m[1,2] in chain matrix multiplication?

- m[1,2] = m[1,1] + m[2,2] +p0 . p1. p3
- m[1,2] = m[1,1] + m[2,2] +p0 . p1. p3
- m[1,2] = m[1,1] + m[2,2] +p0 . p1. p3
- m[1,2] = m[1,1] + m[2,2] +p0 . p1. p3

24. Which one is not passed as parameter in Quick Sort algorithm?

- Array (containing input elements)
- Middle of the array
- Start of the array
- End of the array

25. In asymptotical analysis of n*(5+2)-3. As n becomes large, the dominant (fastest growing) term is some constant times_____

- n+1
- n*n
- n
- n_1

26. For _____ values of n, any algorithm is fast enough.

- Medium
- Small
- Infinity
- Large

27. Dynamic programming algorithms often use some kind of _____ to store the result of intermediate sub-problems.

- Stack
- Loop
- Table

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- Variable

28. In selection problem, the Sieve technique works in_____

- One complete go
- Constant time
- Non-recursive manner
- Phases

29. In heap Sort algorithm, the maximum levels an element can move upward is _____.

- Theta (log n)
- O (1) i.e. Constant time
- Omega (log n)
- Big-oh (log n)

30. While analysis of the brute-force maxima algorithm, an array storted in the reverse order is the type of _____ case input.

- Worst
- Best
- Somewhat
- Average

31.1Divide-and-Conquer is as breaking the problem into small number of

- **Smaller Sub Problems**
- Pivot
- Sieve

32. Analysis of Selection Sort ends up with

- **T(n)**
- T(1/1+n)
- T(n/2)
- T((n/2) +n)

33. For a digraph G = (V, E), Sum of in-degree(v) _____

- Not equal to Sum of out-degree(v)
- = Sum of out-degree(v)
- < Sum of out-degree(v)
- Sum of out-degree(v)

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34. In _____ problem, we want to find the best solution.

- Minimization
- Averaging
- Optimization
- Maximization

35. Timestamp structure of _____ is used in determining the strong components of a digraph.

- DFS
- BFS
- Both DFS & BFS
- None

36. The Huffman codes provide a method of _____ data efficiently. Select correct option

- Reading
- Encoding
- Decoding
- Printing

37. In greedy algorithm, at each phase, you take the_____ you can get right now, without regard for future consequences.

- Worst
- Minimum
- Good
- Best

38. In Activity selection (using Greedy approach), intuitively _____.

- Short activities are not preferable
- There are always short activities as input

- We do not like long activities
- It does not matter about the length of activities

39. Breadth-first search begins at a root node and inspects all the nodes except neighboring ones.
Select correct option

- True
- False

40. The ancient Roman politicians understood an important principle of good algorithm design that is plan-sweep algorithm.

- True
- False **(p27) [Divide and Conquer]**

41. In 2d-space a point is said to be _____if it is not dominated by any other point in that space.

- Member
- Minimal
- Maximal
- Join

42. How much time merge sort takes for an array of numbers?

- T(n^2)
- T(n)
- T(log n)
- T(n log n) Or O(n log n)

43. An algorithm is a mathematical entity that is dependent on a specific programming language.

- True
- False

44. In the analysis of Selection algorithm, we make a number of passes, in fact it could be as many as

- T(n)
- T(n/2)
- Log n
- n/2+n/4

45. The running time of an algorithm would not depend upon the optimization by the compiler but that of an implementation of the algorithm would depend on it.

- True
- False

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46. f(n) and g(n) are asymptotically equivalent. This means that they have essentially the same _____ for large n.

- Results
- Variables
- Size
- Growth Rates

47. 8n2 + 2n - 3 will eventually exceed c2*(n) no matter how large we make c2.

- True
- False

- If we associate (x, y) integers pair to cars where x is the speed of the car and y is the negation of the price. High y value for a car means a _____ car.Fast
- Slow
- Expensive
- Cheap

48. For the sieve technique we solve the problem

- Recursively
- Mathematically
- Precisely
- Accurately

49. Analysis of Selection algorithm ends with,

- T(n)
- T(n/2)
- Log n
- n/2+n/4

50. The function f(n)= n(logn+1)/2 is asymptotically equivalent to n log n. Hre Upper Bound means the function f(n) grows asymptotically _____ faster than n log n.

- More
- Quiet
- Not
- At Least

51. The running time of an algorithm would not depend upon the optimization by the compiler but that of an implementation of the algorithm would depend on it.

- True
- False

52. After sorting in merge sort algorithm, merging process is invoked.

- True

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- False

53. Asymptotic growth rate of the function is taken over_____ case running time.

- Best
- Average
- Worst
- Normal

54. In analysis of f(n)=n(n/5)+n-10 log n, f(n) is asymptotically equivalent to _____.

- N
- 2n
- N+1
- N2

55. Algorithm is concerned with.....issues

- Macro
- Micro
- Both Macro & Micro
- Normal

56. We cannot make any significant improvement in the running time which is better than that of brute-force algorithm.

- True
- False

57. In addition to passing in the array itself to Merge Sort algorithm, we will pass in _____other arguments which are indices.

- TWO
- THREE
- FOUR
- FIVE

58. Brute-force algorithm uses no intelligence in pruning out decisions.

- True
- False

59. In analysis, the Brute-force algorithm uses no intelligence in pruning out decisions at least as fast as its largest term.

- TRUE
- FALSE

60. Efficient algorithm requires less computational

- Memory
- Running Time
- Memory and Running TIME
- Energy

61. The O-notation is used to state only the asymptotic _____bounds.

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- Two
- Lower
- Upper
- Both Lower & upper

62. Heaps can be stored in arrays without using any pointers; this is due to the _____ nature of the binary tree.

- Left-complete
- Right-complete
- Tree Nodes
- Tree Levees

63. .....of reference is an important fact of current processor technology

- Defining
- Assigning
- Formality
- Locality

64. In the analysis of Selection algorithm, we get the convergent _____ series.

- Harmonic
- Linear
- Arithmetic
- Geometric

65. Brute-force algorithm for 2D-Maxima is operated by comparing _____ pairs of points.

- TWO
- SOME
- MOST
- ALL

66. In _____ we have to find rank of an element from given input.

- Merge Sort Algorithm
- Selection Problem
- Brute Force Technique
- Plane Sweep algorithm

67. The sequence of merge sort algorithm is:

a. Divide Combine-Conquer

b. Conquer-Divide-Combine

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c. Divide-Conquer-Combine Page 27

d. Combine-Divide-Conquer

68. In Selection algorithm, we assume pivot selection takes theta _____ running time.

a. n Page - 36

b. n2

c. n3

d. log (n)

69. Result of asymptotical analysis of n(n -3) and 4n*n is that _____

a. n(n-1) is asymptotically Less

b. n(n-1) is asymptotically Greater

c. Both are asymptotically Not equivalent

d. Both are asymptotically Equivalent page 23 ($4n*n= 4n^2$)

70. Floor and ceiling are _____ to calculate while analyzing algorithms

a. Very easy

b. Usually considered difficult P-31

c. 3rd Option is missing

d. 4th Option is missing

71. _____ of reference is an important fact of current processor technology.

a. Defining

b. Assigning

c. Formality

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d. Locality P-8

72. Which of the following is calculated with Big O notation?

a. Medium bounds
b. Upper bounds Page - 25
c. Lower bounds
d. Both upper and lower bounds

73. Al-Khwarizmi was a/an _____

a. Artist
b. Mathematician P-7
c. Astronomer
d. Khalifah

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74. In average-case time analysis of Quick sort algorithm, the most balanced case for partition is when we divide the list of elements into _.

a. Equal no. of pieces as of input elements

b. Single piece exactly

c. Two nearly equal pieces

d. Three nearly equal pieces

75. Which of the following is calculated with Big O notation?

a. Medium bounds

b. Upper bounds Page - 25

c. Lower bounds

b. Both upper and lower bounds

76. Pseudo code of algorithms are to be read by _____.

a. People Page -12

b. RAM

c. Computer

d. Compiler

77. The definition of theta-notation relies on proving _____ asymptotic bound.

a. One

b. Lower

c. Upper

d. Both lower & upper Page - 25

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78. In merge sort algorithm, to merge two lists of size n/2 to a list of size n, takes

_____ time.

a. Theta (n) Page - 32

b. Theta log(n)

c. Theta log2(n)

d. Theta n log(n)

79. We can make _____ recursive calls in Fibonacci Sequence.

a. Infinite

b. Finite google

c. Only one

d. Zero

80. The sieve technique is a special case, where the number of sub-problems is Just

_____

a. 1 P-34

b. 2

c. 3

d. 4

81. When a recursive algorithm revisits the same problem over and over again, we say that the optimization problem has _____ sub-problems.

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a. Overlapping – Google Search

b. Over costing

c. Optimized

d. Three

82. Sieve technique is very important special case of Divide-and-Conquer strategy.

a. True P-34

b. False

83. In order to say anything meaningful about our algorithms, it will be important for us to settle on a _____.

a. Java Program

b. C++ Program

c. Pseudo program

d. Mathematical model of computation P-10

84. Merge sort is based on _____.

a. Brute-force

b. Plan-sweep

c. Axis-sweep

d. Divide and Conquer P-27

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85. What time does Merge Sort algorithm take in order to sort an array of 'n' numbers?
   a. (n)
   b. (log n)
   c. (n^2)
   d. (n log n) Google Search 31. In Heap Sort

86. In plane sweep approach, a vertical line is swept across the 2d-plane and structure is used for holding the maximal points lying to the left of the sweep line.
   a. Array
   b. Queue
   c. Stack Page - 18
   d. Tree

87. _____ time is the maximum running time over all legal inputs.
   a. Worst-case Page - 13
   b. Average-case
   c. Best-case
   d. Good-case

88. Efficient algorithm requires less computational...
   a. Memory
   b. Running Time
   c. Memory and Running Time Page - 9
   d. Energy

89. The Iteration method is used for _____
   a. Comparing sorting algorithms only
   b. Solving Recurrence relations Page 31
   c. Merging elements in Merge sort
   d. Dividing elements in Merge sort

90. Median is not useful measure of central tendency of given input set especially when the distribution of values is highly skewed.

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a. True
b. False Page – 34

91. The Omega-notation allows us to state only the asymptotic _____ bounds.
a. Middle
b. Lower Page 25
c. Upper
d. Both lower & upper

92. In the statement "output P[1].x, P[1].y", the number of times elements of P are accessed is _____.
a. 1
b.2 page 14
b. 3
c. 4

93. The main purpose of mathematical analysis is measuring the _____ required by the algorithm.
a. Space
b. Execution time P-13
c. Inputs & outputs
d. Execution time and memory

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94. _____ provides us more accurate result when input values are not closer with each other

a. Average

b. Median P-34

c. Mode

d. Mean

95. The process of _____ ends when you are left with such tiny pieces remaining that it is trivial to solve them.

a. Brute-force

b. Plan-sweep

c. Divide and Conquer P-27

d. Axis-sweep

96. Approach of solving geometric problems by sweeping a line across the plane is called _____ sweep.

a. Line

b. Plane Page 18

c. Cube

d. Box

97. In the analysis of Selection algorithm, we get the convergent _____

a. Harmonic

b. Linear

c. Arithmetic

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d. Geometric Pg:37

98. A Random Access Machine (RAM)is an idealized machine withrandom access memory.

a. Infinite large Pg:10

b. 512 MB

c. 256 MB

d. 2 GBs

99. While analyzing Selection algorithm, we make a number of passes, in fact it could be as many as

a. n(n+1)

b. log(n) Pg:37

c. n/3

d. n/4

100. In Random Access Machine (RAM), instructions are executed in

a. Parallel

b. Batch

c. One by One Pg:10

d. Multiple times

101. In selection problem, the rank of an element will be its _____ position

a. First

b. final Pg:34

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c. Second last

d. Last

102. The worst-case running time of Merge sort is _____ in order to sort an array of n elements.

a. O(log n)

b. O(n)

c. O(n log n) page 40 and google

d. O(n)

103. f(n) and g(n) are asymptotically equivalent. This means that they have essentially the same _____.

a. Results

b. Variables

c. Size

d. Growth rates P:23

104. An algorithm is a mathematical entity. Which is independent of _____.

a. Programming language

b. Machine and Programming language

c. Compiler and Programming language

d. Programming language Compiler and Machine P:07

105. In Selection problem, the Sieve technique works in _____.

a. Non-recursive manner

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b. Constant time

c. Phases page 34

d. One complete go

106. Algorithm is a sequence of computational steps that ---- the input into output.

a. Merge

b. Assign

c. Transform page 7

d. Integrate

107. If pj dominates pi and pi dominates ph then pj also dominates ph, it means dominance relation is

a. Transitive page 18

b. Non Transitive

c. Equation

d. Symbolic

108. To find maximal points in brute-force algorithm each point of the space is compared against _____ of that space.

a. One other point

b. All other points page 11

c. Few other points

d. Most of the other points

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109. In the following code the statement "cout<<j;"executes --------- times. for (j=1; j<=5; j = j+2)

```
cout<<j;
```

a. 5 times

b. 2 times

c. 3 times

d. 0 times

110. In merge sort algorithm, we split the array around the _____ index q. a. Entring

b. Mid page 17

c. Exiting

d. Summing

111. In Selection problem, the Sieve technique _____.

a. Add some more input items each time

b. Do not work recursively

c. Do not uses Divide and Conquer approach

d. Eliminates undesired data items each time

112. While Sorting, the order domain means for any two input elements x and y

_____ satisfies only.

a. x < y page 39

b. x > y

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c. x = y

d. All of the above

113. For solving Selection problem, we introduced Sieve technique due to

_____

a. Using Decrease and Conquer strategy page 34

b. Avoiding to sort all input data

c. Eliminating Rank of an element

d. Using Brute-force approach

114. _____ is one of the few problems, where provable lower bounds exist on how fast we can sort.

a. Searching

b. Sorting page 38

c. Both Searching & sorting

d. Growing

115. In plane sweep approach, a vertical line is swept across the 2d-plane from_____.

a. Right to Left

b. Left to Right page 18

c. Top to Bottom

d. Bottom to top

116. In generating Fibonacci sequence, we can avoid unnecessary repetitions by

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_____ process.

a. Tokenization

b. Memorization page 43

c. Randomization

d. Memorization

117. For _____ values of n, any algorithm is fast enough.

a. Small page 14

b. Medium

c. Large

d. Infinity

118. Single item from a larger set of _____.

a. Constant

b. Pointers

c. Phases

d. n items page 34

119. Brute-force algorithm for 2D-Maxima is operated by comparing _____

pairs of points.

a. Two

b. Some

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c. Most

d. All page 18

120. For the Sieve Technique we take time.

a. T(nk) page 34

b. IT(n / 3)

c. n^2

d. n/3

121. Asymptotic growth rate of the function is taken over _____ case running time. .

a. Best

b. Worst page 14

c. Average

d. Normal

122. The sieve technique is a special case, where the number of sub problems is just.

a. 5

b. Many

c. 1 page 34

d. Few

123. In Quick sort, we don't have the control over the sizes of recursive calls.

a. True page 49

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b. False

c. Less information to decide

d. Ether true or false

124. Before sweeping a vertical line in plane sweep approach, in start sorting of the points is done in increasing order of their _____ coordinates. .

a. X page 18

b. Y

c. Z

d. X , Y

125. Random access machine or RAM is a/an.

a. Machine build by Al-Khwarizmi

b. Mechanical machine

c. Mathematical model page 10

d. Electronics machine

126. Cubic function will _____ a quadratic function.

a. Prove

b. be equal to

c. overtake Page 25

c. find

127. Asymptotic growth of 8n^2 + 2n – 3 is:

a. Θ(n^2 + n)

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b. Θ (n^2) page 14

c. Θ(8n^2)

d. Θ(8n^2 + 2n)

128. In average-case time the probability of seeing input is denoted by _____.

a. p{I}

b. p[I]

c. p<i>

d. p(i) page 13

129. While applying the Sieve technique to selection sort, how to choose a pivot element.

a. Through mean

b. Linear

c. Randomly page 35

d. Sequentially

130. Number of _____ of the pseudo code are counted to measure the running time.

a. Inputs

b. Outputs

c. Steps page 13

d. Pages

131. 8n^2+2n+3 will exceed c28(n), no matter how large we make _____.

a. n

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b. 2

c. c2 page 2

d. this quadratic equation

132. In 2-d maxima problem a point p is said to be dominated by point q if _____.

a. p.x <= q.x

b. p.x <= q.x and p.y <= q.y page 17

c. p.y <= q.y

d. p.x >= q.x and p.y >=q.y

133. Sorting can be in _____.

a. Increasing order only

b. Decreasing order only

c. Both increasing and decreasing order

d. Random order

134. Recurrence can be described in terms of _____.

a. Array

b. Linear

c. Tree page 31

d. Graph

135. The brute-force algorithm for 2D-Maxima runs in order O(__) time.

a. n

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b. n(log n)

c. n*n page 18

d. n3

136. In plane sweep approach of solving geometric problems, a _____ is swept across the plane.

a. Line page 18

b. Plane

c. Cube

d. Box

137. Which of the following is calculated with Big Omega notation?

a. Medium bounds

b. Upper bounds

c. Lower bounds Page - 25

d. Both upper and lower bounds

138. _____ is always based on divide and conquer strategy.

a. Bubble sort

b. Selection sort

c. Pigeon sort

d. Quick sort page 46

139. If a matrix has three rows and two columns, then dimensions of matrix will be:

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a. 3x2

b. 2x3

c. 3x3

d. 2x2

140. Asymptotic notations are used to describe _____ of an algorithm.

a. Length

b. running time google

c. size

d. compile time

141. The time assumed for each basic operation to execute on RAM model of computation is _____.

a. Infinite

b. Continuous

c. Constant page 10

d. Variable

142. Boolean operation is a _____ operation on an idealized RAM model of computation.

a. Starting

b. Basic page 10

c. Advance

d. Normal

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143. If we have an equation 8n2+7f*n + 5f + 6 then is large, _____ term will be muchxxxxxxxthe n term and will dominate the running time.

a. f g (n)

b. g (n) * 2

c. n * 2 page 23

d. f (n)

144. In quick sort algorithm, we choose pivot _____.

a. Always the smallest element

b. Greater than 5

c. Randomly page 35

d. Less than 5

145. The problem with the brute-force algorithm is that is uses _____ in pruning out de

a. Worst-case time

b. No intelligence page 18

c. Outside looping

d. Artificial intelligence

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CS502 Grand Quiz

1. The sequence of merge sort algorithm is:
   a. Divide Combine-Conquer
   b. Conquer-Divide-Combine
   c. Divide-Conquer-Combine Page 27
   d. Combine-Divide-Conquer
2. In _____ Knapsack Problem, limitation is that an item can either be put in the bag or not. Fractional items are not allowed.
   a. 0
   b. 1
   c. 0/1 Page 91
   d. Fractional
3. In Selection algorithm, we assume pivot selection takes theta _____ running time.
   a. n Page - 36
   b. n2
   c. n3
   d. log (n)
4. In Heap Sort algorithm (using max heap), when every time maximum elements removed from top _____.
   a. We call merge Sort Algorithm
   b. it becomes Order n2 Algorithm
   c. Divide and Conquer strategy helps us
   d. We are left with a hole Page – 41
5. If matrix A of dimension p x q is multiply with matrix B of dimension q x r, then each entry in resultant matrix takes _____ time.
   a. O (q) Page - 84
   b. O (1)

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c. O (p x q)
d. O (q x r)

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6. _____ is a method of solving a problem in which we check all possible solutions to the problem to find the solution we need.
   a. Plane-Sweep Algorithm
   b. Sorting Algorithm
   c. Brute-Force Algorithm google
   d. Greedy approach
7. The worst case running time of Quick sort algorithm _____.
   a. Cannot be quadratic
   b. Is quadratic google QuickSort and its Analysis (codesdope.com) & http://personal.kent.edu/~rmuhamma/Algorithms/MyAlgorithms/Sorting/quickSort.htm
   c. Is always Exponential
   d. Is linear
8. In max heap (for Heap Sort algorithm), when every time maximum element is removed from top we replace it with _____ leaf in the tree.
   a. second last
   b. Last Page -41
   c. First
   d. Any
9. Quick sort algorithm was developed by -
   a. AlferdAho
   b. Sedgewick
   c. John Vincent Atanasoff
   d. Tony Hoare – Google wikipedia
10. If Matrix-A has dimensions "3x2" and Matrix-B has dimensions "2x3", then multiplication of Matrix-A and Matrix-B will result a new Matrix-C having dimensions.
    a. 3x2
    b. 2x3
    c. 2x2
    d. 3x3 http://www.calcul.com/show/calculator/matrix-multiplication

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11. For comparison-based sorting algorithms, it is possible to sort more efficiently than Omega n log(n) time.
    a. Always
    b. Not P-54
    c. Sometimes

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d. Sometimes not

12. Dynamic Programming approach is usually useful in solving optimization problems.
    a. True
    b. False

13. In Sorting the key value or attribute_____ from an ordered domain.
    a. Must be page 39
    b. Not always
    c. May be
    d. Occasionally

14. Result of asymptotical analysis of n(n -3) and 4n*n is that _____
    a. n(n-1) is asymptotically Less
    b. n(n-1) is asymptotically Greater
    c. Both are asymptotically Not equivalent
    d. Both are asymptotically Equivalent page 23 ($4n*n= 4n^2$)

15. Floor and ceiling are _____ to calculate while analyzing algorithms a. Very easy
    b. Usually considered difficult P-31
    c. 3rd Option is missing
    d. 4th Option is missing

16. _____ of reference is an important fact of current processor technology.
    a. Defining
    b. Assigning
    c. Formality
    d. Locality P-8

17. In max-heap, largest element is stored at root node. Where is the smallest element stored?
    a. Right Node
    b. Leaf Node
    c. Middle Node

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d. Left Node

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18. In average-case time analysis of Quick sort algorithm, the most balanced case for partition is when we divide the list of elements into _.
    a. Equal no. of pieces as of input elements
    b. Single piece exactly
    c. Two nearly equal pieces
    d. Three nearly equal pieces
19. Which of the following is calculated with Big O notation?
    a. Medium bounds
    b. Upper bounds Page - 25
    c. Lower bounds
    d. Both upper and lower bounds
20. Edit distance algorithm based on _____ strategy
    a. Greedy
    b. Dynamic Programming Page - 81
    c. Divide and Conquer
    d. Searching
21. In Heapsort Algorithm, total time taken by heapify procedure is _____
    a. O (log n) Page-43
    b. O (log2 n)
    c. O (n log n)
    d. O (n2 log n)
22. Al-Khwarizmi was a/an _____
    a. Artist
    b. Mathematician P-7
    c. Astronomer
    d. Khalifah
23. When matrix A of 5x3is multiply with metric B of 3x4 then the number of multiplication required is: Not found exactly
    a. 15
    b. 12

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c. 36

d. 60 **Not Found exactly but as per formula at page 84,**

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24. Pseudo code of algorithms are to be read by _____.
    a. People Page -12
    b. RAM
    c. Computer
    d. Compiler
25. The sieve technique is a special case, where the number of sub-problems is Just _____
    a. 1 P-34
    b. 2
    c. 3
    d. 4
26. When a recursive algorithm revisits the same problem over and over again, we say that the optimization problem has _____ sub-problems.
    a. Overlapping – Google Search
    b. Over costing
    c. Optimized
    d. Three
27. Sieve technique is very important special case of Divide-and-Conquer strategy.
    a. True P-34
    b. False
28. In order to say anything meaningful about our algorithms, it will be important for us to settle on a _____.
    a. Java Program
    b. C++ Program
    c. Pseudo program
    d. Mathematical model of computation P-10
29. Merge sort is based on _____.
    a. Brute-force
    b. Plan-sweep
    c. Axis-sweep

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d. Divide and Conquer **P-27**

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30. What time does Merge Sort algorithm take in order to sort an array of 'n' numbers?
    a. (n)
    b. (log n)
    c. (n^2)

    d. (n log n) Google Search 31. In Heap Sort
    31. algorithm, the first step is to _____.
    a. Call Build-Heap procedure Page - 46
    b. Sort the array in descending order
    c. Call Heapify procedure
    d. Find the number of input elements
32. The definition of theta-notation relies on proving _____ asymptotic bound.
    a. One
    b. Lower
    c. Upper
    d. Both lower & upper Page - 25
33. In merge sort algorithm, to merge two lists of size n/2 to a list of size n, takes _____ time.
    a. Theta (n) Page - 32
    b. Theta log(n)
    c. Theta log2(n)
    d. Theta n log(n)
34. We can make _____ recursive calls in Fibonacci Sequence.
    a. Infinite
    b. Finite google
    c. Only one
    d. Zero
35. Following is NOT the application of Edit Distance problem.
    a. Speech recognition

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b. Spelling Correction

c. Ascending Sort

d. Computational Molecular Biology

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36. In plane sweep approach, a vertical line is swept across the 2d-plane and structure is used for holding the maximal points lying to the left of the sweep line.
    a. Array
    b. Queue
    c. Stack Page - 18
    d. Tree
37. When a heapify procedure is applied to the root node to restore the heap, then at each level, the comparison performed takes time:
    a. It will take (log n).
    b. It can not be predicted
    c. It will take O (1). Page - 43
    d. Time will vary according to the nature of input data.
38. _____ time is the maximum running time over all legal inputs.
    a. Worst-case Page - 13
    b. Average-case
    c. Best-case
    d. Good-case
39. Efficient algorithm requires less computational...
    a. Memory
    b. Running Time
    c. Memory and Running Time Page - 9
    d. Energy
40. For average-case time analysis of Quick sort algorithm, Pivot selection is on average basis from _____
    a. half of the input values
    b. all possible random values Page - 50
    c. Pivot is input separately
    d. values greater than 5
41. Selection algorithm takes theta _____

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a. (n2)

b. (n)

c. log(n)

d. n log(n)

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42. Recurrence can be described in terms of a tree.
    a. Yes Page - 31
    b. No
43. Time complexity of Dynamic Programming based algorithm for computing the minimum cost of Chain Matrix Multiplication is _____
    a. Log n
    b. n
    c. n^2 (n square)
    d. n^3 (n cube) Page -90
44. The Iteration method is used for _____
    a. Comparing sorting algorithms only
    b. Solving Recurrence relations Page 31
    c. Merging elements in Merge sort
    d. Dividing elements in Merge sort
45. In 3-Dimensional space, a point P has _____ coordinate(s).
    a. (X, Y)
    b. (X, 0)
    c. (0, Y)
    d. (X,Y, Z)
46. Chain matrix multiplication problem can be solved through _____ strategy.
    a. Dynamic programming Page - 85
    b. Greedy
    c. Divide and conquer
    d. Sorting
47. Merge sort have running time....running time of Heap sort. Not found exactly
    a. Greater than
    b. Less than Google
    c. Equal to
    d. Different than

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48. Median is not useful measure of central tendency of given input set especially when the distribution of values is highly skewed.
    a. True
    b. False Page – 34

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49. We do not need to mathematically prove that for comparison-based sorting algorithms always takes Omega nlog (n) time.
    a. True Google & VU Tech (pg 46 not very clear)
    b. False
50. The Omega-notation allows us to state only the asymptotic _____ bounds.
    a. Middle
    b. Lower Page 25
    c. Upper
    d. Both lower & upper
51. Both lower & upperSorting can be in _____
    a. Increasing order only
    b. Decreasing order only
    c. Both Increasing and Decreasing order GOOGLR Search
    d. Random order
52. Radix sort performs sorting the numbers _____ digit (s) at a time.
    a. One Page - 71
    b. Two
    c. Three
    d. All
53. Quicksort is a/an _____ and _____ sorting algorithm.
    a. Not in place, not stable one
    b. In place , not stable one Page - 54
    c. In place , stable one
    d. Not in place , stable one
54. Consider three matrices X,Y,Z of dimensions 1x2, 2x3,3x4 respectively. The number of multiplications of (XY) Z is:
    a. 18 As per lecture slides
    b. 32
    c. 24
    d. 30

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55. In Fibonacci Sequence, unnecessary repetitions do not exist at all.

a. True

b. False **Page – 74**

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56. It is not a Fibonacci sequence . 1,1,1,2,3,5,8,13,21,34,55,…..
    a. True Page - 73
    b. False
57. Heap sort is a/ an _____ and _____ sorting algorithem.
    a. Not in place, not stable one
    b. In place , not stable one Page - 54
    c. In place , stable one
    d. Not in place , stable one
58. Identify the True Statement
    a. The knapsack problem does not belong to the domain of optimization problems.
    b. The knapsack problem belongs to the domain of optimization problems. Page - 91
    c. The Knapsack problem cannot be solved by using dynamic programming
    d. The knapsack problem is optimally solved by using brute force algorithm.
59. In Dynamic Programming, our approach is to _____
    a. Develop the solution in a top-down fashion
    b. Express the problem non-recursively
    c. Build the solution in a bottom-up fashion Page - 75
    d. Input several sub-problems simultaneously
60. Counting sort is suitable to sort the elements in range 1 to K;
    a. K is large
    b. K is small Page - 57
    c. K may be large or small
    d. None
61. We can multiply two matrices A and B only when they are compatible which means
    a. Number of columns in A must be equal to number of rows in B. it seems Correct as per page 84

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b. Number of rows and columns do not matter

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c. Number of columns in A must be equal to number of columns in B
d. Number of rows in A must be equal to number of rows in B

62. Matrix multiplication is a (n) _____ operation.
a. Commutative
b. Associative Page 85
c. Neither commutative nor associative
d. Commutative but not associative

63. In Dynamic Programming approach, solution is modified / changed
a. Always once
b. At each stage google and wikipedia
c. Only for specific problems
d. At 4th stage only

64. In Knapsack problem, the goal is to put items in the Knapsack such that the value of the items is _____ subject to weight limit of knapsack.
a. Minimized
b. Decreased
c. Maximized Page - 91
d. None of the given options

65. An in-place sorting algorithm is one that _____ uses additional array for storage.
a. Always
b. Permanently
c. Does not Page - 54
d. Sometime

66. Memoization is a part of Dynamic Programming Strategy.
a. True Page - 74
b. False

67. If matrix A of dimension 2x4 is multiply with matrix B of dimension 4x3, then the dimension of resultant matrix is Not found exactly
a. 2x4

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b. 4x3

c. 3x4

d. 2x3 It seems correct as per second last Para of page 84

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68. In Dynamic Programming approach, we do not store the solution to each sub-problem in case if it reappears.
    a. True
    b. False Page - 75
69. Dynamic Programming is a problem-solving approach in which___
    a. Problem is solved in Zero time
    b. Solution is developed only at final stage
    c. Both are correct
    d. Both are incorrect google
70. In Fibonacci sequence, each term is calculated by____ previous__ terms.
    a. Subtracting, Two
    b. Adding, Three
    c. Adding, Two Page - 73
    d. Multiplying, Two
71. Selection sort is not an in-place sorting algorithm.
    a. True Page - 54
    b. False
72. If there are θ (n^2) entries in edit distance matrix then the total running time is:
    a. θ (n)
    b. θ (1)
    c. θ (n^2) Page – 84
    d. θ (n logn)
73. The only way to convert a string of i characters into the empty string is with i deletions, represented as
    a. E(0.j) =j
    b. E(i.j) = 1
    c. E(0.i) = j
    d. E (i.0)=I Page - 78
74. Dynamic programming formulation of the matrix chain multiplication problem will store the solutions of each sub problem in an

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a. Array
b. Table Page - 86
c. Variable
d. class

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75. We can use the optimal substructure property to devise a formulation of the edit distance problem.
    a. Selective
    b. Optimum
    c. Iterative
    d. Recursive Page - 78
76. Sorting is performed on the basis of _____.
    a. Computational resources
    b. Asymptotic notation
    c. Summation
    d. Some key value of attribute page- 39
77. In Heap Sort algorithm, we call Build-heap procedure _____.
    a. Only once page 46
    b. Twice
    c. Thrice
    d. As many times as we need
78. Radix sort is not a non-comparative integer sorting algorithm.
    a. True Google Search
    b. False
79. In the statement "output P[1].x, P[1].y", the number of times elements of P are accessed is _____.
    a. 1
    b. 2 page 14
    c. 3
    d. 4
80. The main purpose of mathematical analysis is measuring the _____ required by the algorithm.
    a. Space
    b. Execution time P-13
    c. Inputs & outputs

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d. Execution time and memory

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81. _____ provides us more accurate result when input values are not closer with each other
    a. Average
    b. Median P-34
    c. Mode
    d. Mean
82. The process of _____ ends when you are left with such tiny pieces remaining that it is trivial to solve them.
    a. Brute-force
    b. Plan-sweep
    c. Divide and Conquer P-27
    d. Axis-sweep
83. _____ overcomes the limitations of _____ by working as per positional notations of numbers.
    a. Counting sort, Radix sort
    b. Radix sort, Counting sort P-71
84. Memorization is a part of Dynamic Programming strategy.
    a. True P-74
    b. False
85. Rank of an element can be defined as _____.
    a. One minus the number of elements that are smaller
    b. Two plus the number of elements that are greater
    c. One plus the number of elements that are smaller P-34
    d. Two minus the number of elements that are smaller
86. If the time complexity of an algorithm is given by O (1), then its time complexity would be
    a. Polynomial
    b. Exponential
    c. Constant - Wikipedia
    d. Average

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87. Quick sort is a recursive algorithm.

a. True **Wikipedia ; Google**
b. False

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88. The asymptotic growth of n(n+1)/2 is:
    a. $O(n^2)$ As the n^2 term has the largest contribution, the Big-O complexity is O(n^2)
    b. O(n)
    c. O(n+2)
    d. O(n log n)
89. Approach of solving geometric problems by sweeping a line across the plane is called _____ sweep.
    a. Line
    b. Plane Page 18
    c. Cube
    d. Box
90. As per algorithm of Dynamic Programing, we need to store
    a. First sub-problem only
    b. Best solution only
    c. Intermediate sub-problems Pg:75
    d. Final solution only
91. In Sieve technique, we solve the problem
    a. In recursive manner Pg:34
    b. Non recursively
    c. Using Merge Sort algorithm
    d. Using Brute force technique
92. One of the limitation in 0/1 knapsack is that an item can either be _____ in the bag or not.
    a. Use
    b. Put Pg:91
    c. Move
    d. Store
93. Which one is not passed as parameter in Quick sort algorithm?
    a. End of the array
    b. Middle of the array

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c. Array (containing input elements) **Google**

d. Start of the array

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94. In the analysis of Selection algorithm, we get the convergent _____
    a. Harmonic
    b. Linear
    c. Arithmetic
    d. Geometric Pg:37
95. A Random Access Machine (RAM)is an idealized machine withrandom access memory.
    a. Infinite large Pg:10
    b. 512 MB
    c. 256 MB
    d. 2 GBs
96. While analyzing Selection algorithm, we make a number of passes, in fact it could be as many as
    a. n(n+1)
    b. log(n) Pg:37
    c. n/3
    d. n/4
97. In Random Access Machine (RAM), instructions are executed in
    a. Parallel
    b. Batch
    c. One by One Pg:10
    d. Multiple times
98. In selection problem, the rank of an element will be its _____ position
    a. First
    b. final Pg:34
    c. Second last
    d. Last
99. The worst-case running time of Merge sort is _____ in order to sort an array of n elements.
    a. O(log n)

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b. O(n)

c. O(n log n) page 40 and google

d. O(n)

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100. f(n) and g(n) are asymptotically equivalent. This means that they have essentially the same _____.
   a. Results
   b. Variables
   c. Size
   d. Growth rates P:23

101. An algorithm is a mathematical entity. Which is independent of _____.
   a. Programming language
   b. Machine and Programming language
   c. Compiler and Programming language
   d. Programming language Compiler and Machine P:07

102. In Quick sort algorithm, Pivots form ___
   a. Stack
   b. Queue
   c. Binary Search Tree P:49
   d. Graph

103. Counting sort is suitable for sorting the elements within range 1 to P. where
   a. P is large
   b. P is small P-57
   c. P is very large
   d. P is undetermined

104. In asymptotical analysis of n'(5 2)-3, as n becomes large, the dominant (fastest growing) term is some constant times
   a. n_1
   b. n
   c. n+1
   d. n*n P-23

105. ___ Items are not allowed in the 0/1 knapsack. a. Lighter
   b. Fractional P-91

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c. Whole
d. Weighty

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106. Fibonacci Sequence was named on _____, a famous mathematician in 12th Century.
    a. Fred Brooks
    b. Grady Booch
    c. Leonardo Pisano P-73
    d. Edgar F. Codd
107. In Heap Sort algorithm, we build _____ for ascending sort.
    a. Max heap P-41
    b. Min heap
108. Bubble sort is not an in-place sorting algorithm.
    a. True
    b. False P-54
109. In partition algorithm, the subarray _____ has elements which are greater than pivot element x.
    a. A[p...r]
    b. A[p...q-1]
    c. A[q]
    d. A[q+1..r] P-46
110. In Heap Sort algorithm, if heap property is violated
    a. We call Build heap procedure P-43
    b. We call Heapify procedure
    c. We ignore
    d. Heap property can never be violated
111. _____ is not a characteristic of Random Access Machine.
    a. Single-Processor P-10
    b. Assigning a value to a variable
    c. Locality of reference
    d. Executing an arithmetic instruction
112. The only way to convert an empty string into a sting of j characters is by doing j insertions, represented as _____

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a. E(i,j) = 1
b. E(I,0) = I
c. E(0,j) = j page 78
d. E(1,j)= j

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113. In Selection problem, the Sieve technique works in _____.
    a. Non-recursive manner
    b. Constant time
    c. Phases page 34
    d. One complete go
114. Algorithm is a sequence of computational steps that ---- the input into output.
    a. Merge
    b. Assign
    c. Transform page 7
    d. Integrate
115. If pj dominates pi and pi dominates ph then pj also dominates ph, it means dominance relation is
    a. Transitive page 18
    b. Non Transitive
    c. Equation
    d. Symbolic
116. To find maximal points in brute-force algorithm each point of the space is compared against _____ of that space.
    a. One other point
    b. All other points page 11
    c. Few other points
    d. Most of the other points
117. In the following code the statement "cout<<j;"executes --------- times. for (j=1; j<=5; j = j+2)
        cout<<j;
    a. 5 times
    b. 2 times
    c. 3 times

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d. 0 times

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118. In merge sort algorithm, we split the array around the _____ index q. a. Entring
    b. Mid page 17
    c. Exiting
    d. Summing
119. In Selection problem, the Sieve technique _____.
    a. Add some more input items each time
    b. Do not work recursively
    c. Do not uses Divide and Conquer approach
    d. Eliminates undesired data items each time
120. Consider three matrices X, Y, Z of dimensions 1 x 2, 2 x 3, 3 x 4 respectively. The number of multiplications of X(YZ) is .
    a. 16
    b. 32
    c. 26
    d. 32 page 84
121. In Heap Sort algorithm, the total running time for Heapify procedure is _____
    a. Theta (log n)
    b. Order (log n)
    c. Omega (log n)
    d. O(1) i.e. Constant time
122. The sieve technique works where we have to find_____ items(s) from a large input.
    a. Single page 34
    b. Two
    c. Three
    d. Similar
123. In Dynamic Programming based solution of Knapsack Problem, if we decide to take an object i , then we gain_____

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a. W(Total Weight of Knapsack)
b. V (Total Value of all items)
c. vi (Value of object i) page 93
d. Nome of the given option

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124. While Sorting, the order domain means for any two input elements x and y _____ satisfies only.
   a. x < y page 39
   b. x > y
   c. x = y
   d. All of the above

125. For solving Selection problem, we introduced Sieve technique due to _____
   a. Using Decrease and Conquer strategy page 34
   b. Avoiding to sort all input data
   c. Eliminating Rank of an element
   d. Using Brute-force approach

126. _____ is one of the few problems, where provable lower bounds exist on how fast we can sort.
   a. Searching
   b. Sorting page 38
   c. Both Searching & sorting
   d. Growing

127. In plane sweep approach, a vertical line is swept across the 2d-plane from_____.
   a. Right to Left
   b. Left to Right page 18
   c. Top to Bottom
   d. Bottom to top

128. In generating Fibonacci sequence, we can avoid unnecessary repetitions by _____ process.
   a. Tokenization
   b. Memorization page 43
   c. Randomization
   d. Memorization

129. For _____ values of n, any algorithm is fast enough.

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a. Small page 14
b. Medium
c. Large
d. Infinity

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130. Dynamic programming comprises of _____.
    a. Recursion only
    b. Repetition only
    c. Recursion with Repetition
    d. No Repetition but Recursion page 75

131. The function f(n)=n(logn+1)/2 is asymptotically equalient t nlog n :Here Lower Bound means function f(n) grows asymptotically at __ as fast as nlog n.
    a. Least page 23
    b. Normal
    c. Most
    d. At

132. Counting sort has time complexity.
    a. O(n+k)
    b. O(n) page 58
    c. O(k)
    d. O(nlogn)

133. Due to left complete nature of binary tree, the heap can be stored in
    a. Array page 40
    b. Structures
    c. Link List
    d. Stack

134. Single item from a larger set of _____.
    a. Constant
    b. Pointers
    c. Phases
    d. n items page 34

135. In the clique cover problem, for two vertices to be in the same group, they must be _____ each other.
    a. Apart from
    b. Far from

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c. Near to

d. Adjacent to page 76

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136. How much time merge sort takes for an array of numbers?
    a. T(n^2)
    b. T(n)
    c. T(log n)
    d. T(n log n) page 40

137. In in-place sorting algorithm is one that uses arrays for storage.
    a. No additional array page 54
    b. An additional array
    c. Both of above may be true according to algorithm
    d. More than 3 arrays of one dimension

138. Brute-force algorithm for 2D-Maxima is operated by comparing _____ pairs of points.
    a. Two
    b. Some
    c. Most
    d. All page 18

139. While Sorting, the ordered domain means for any two input elements x and y ____ satisfies only.
    a. x > y
    b. x < y
    c. x = y
    d. All of the above page 38

140. Quick sort is.
    a. Stable & in place
    b. Not stable but in place page 54
    c. Stable but not in place
    d. Some time stable & some times in place

141. Which may be a stable sort?
    a. Merger
    b. Insertion

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c. Both above **page 54**

d. None of the above

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142. For the Sieve Technique we take time.
   a. T(nk) page 34
   b. IT(n / 3)
   c. n^2
   d. n/3

143. Continuation sort is suitable to sort the elements in range 1 to k.
   a. K is Large
   b. K is not known
   c. K may be small or large
   d. K is small page 54

144. Asymptotic growth rate of the function is taken over _____ case running time. .
   a. Best
   b. Worst page 14
   c. Average
   d. Normal

145. The sieve technique is a special case, where the number of sub problems is just.
   a. 5
   b. Many
   c. 1 page 34
   d. Few

146. In Quick sort, we don't have the control over the sizes of recursive calls.
   a. True page 49
   b. False
   c. Less information to decide
   d. Ether true or false

147. Before sweeping a vertical line in plane sweep approach, in start sorting of the points is done in increasing order of their _____ coordinates. .
   a. X page 18

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b. Y
c. Z
d. X , Y

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148. Random access machine or RAM is a/an.
    a. Machine build by Al-Khwarizmi
    b. Mechanical machine
    c. Mathematical model page 10
    d. Electronics machine
149. The Huffman codes provide a method of encoding data inefficiently when coded using ASCII standard.
    a. True
    b. False page 99
150. A heap is a left-complete binary tree that confirms to the _____.
    a. increasing order only
    b. decreasing order only
    c. heap order page 40
    d. log n order
151. If we associate (x, y) integers pair to cars where x is the speed of the car and y is the negation of the price. High y value for a car means a _____ car.
    a. Fast
    b. Slow
    c. Expensive
    d. Cheap
152. Which one of the following sorting algorithms is the fastest?
    a. Merge sort
    b. Quick sort
    c. Insertion sort
    d. Heap sort
153. Quick sort algorithm divide the entire array into _____ sub arrays.
    a. 2
    b. 3
    c. 4
    d. 5
154. In brute force algorithm, we measure running time T(n) based on _____.

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a. Average-case time and best-case time
b. Worst-case time and average-case time page 46
c. Worst-case time and best-case time
d. Best-case time and staring-case time

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155. For 2D Maxima problem. Plane Sweep algorithm first of all _____.
   a. Sorts all points
   b. Delete some points
   c. Output the elements
   d. Pushes all points on stack

156. There are _____ entries in the Edit Distance Matrix
   a. ɵ (n)
   b. ɵ (n_2) page 84
   c. ɵ (n+2)
   d. ɵ (n + 100)

157. Which symbol is used for Omega notation?
   a. (O)
   b. (ɵ)
   c. (Ω)
   d. (@)

158. Selection sort is a _____ sorting algorithm
   a. In-place page 54
   b. Not In-Place
   c. Stable
   d. in-partition

159. In Dynamic Programming based solution of knapsack problem, to compute entries of 'V', we will imply a(n) _____ approach.
   a. Subjective
   b. Inductive
   c. Brute Force
   d. Combination

160. We do not need to prove comparison-based sorting algorithms by mathematically. It always takes _____ time.
   a. Big Oh nlog(n)
   b. Omega nlog(n) NOT SURE

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c. Omega n(n^2)
d. Theta nlog(n)

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161. Merge sort is a/an _____ and _____ sorting algorithm
    a. Not in-place, not stable one
    b. In-place, not stable one
    c. In-place, stable one
    d. Not in-place, stable one page 54
162. Cubic function will _____ a quadratic function.
    a. Prove
    b. be equal to
    c. overtake Page 25
    d. find
163. Insertion sort is a _____ sorting algorithm
    a. Unstable
    b. In-place Page 54
    c. Not In-Place
    d. in-partition
164. To check whether a function grows faster or slower than the other function, we use some asymptotic notations, which is _____.
    a. Big-oh notation
    b. Theta notation
    c. Omega notation
    d. All of the given
165. Asymptotic growth of 8n^2 + 2n – 3 is:
    a. Θ(n^2 + n)
    b. Θ (n^2) page 14
    c. Θ(8n^2)
    d. Θ(8n^2 + 2n)
166. In the analysis of algorithms, _____ plays an important role.
    a. text analysis
    b. time
    c. growth rate

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d. money

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167. In inductive approach of knapsack problem, we consider 2 cases, _____ Or _____.
   a. Median, Mode
   b. Recursive, Iterative
   c. Leave object, Take object page 93
   d. Sequentially. Parallel

168. Random Access Machine (RAM) can execute _____ instructions
   a. only logical
   b. parallel
   c. only arithmetic
   d. logical and arithmetic

169. Using _____ algorithm, efficiency is not given much importance
   a. Greedy
   b. Merge sort
   c. Processing as there is no algorithm by this name
   d. Brute Force

170. Bubble sort takes theta _____ in the worst case
   a. (n2) page 39
   b. (n)
   c. log(n)
   d. nlog(n)

171. If matrix A of dimension p × q is multiply with matrix B of dimension q × r, then dimension of resultant matrix is:
   a. p × q
   b. p × r page 84
   c. q × r
   d. r × p

172. Dynamic Programing algorithms often use some kind of _____ to store the results of intermediate sub-problems
   a. table (Page 75)

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b. variable
c. stack
d. loop

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173. _____ is in-place sorting algorithm.
   a. Bubble sort (Page 54)
   b. Merge sort
   c. Linear search
   d. Binary Search

174. Which one of the following problems can be solved using dynamic problem?
   a. Bubble sort problem
   b. Matrix chain multiplication problem page 85
   c. Greedy search problem
   d. Fractional knapsack problem

175. In chain matrix multiplication, solutions of the sub-problems are stored in a _____.
   a. Array
   b. Table page 86
   c. Tree
   d. Link list

176. What is the average running time of a quick sort algorithm?
   a. O(n^2)
   b. O(n)
   c. O(n log n) (Page 49)
   d. O(log n)

177. Sorting Algorithms having O _____ running time are considered to be slow ones.
   a. (n)
   b. (n^2) (Page 39)
   c. (nlog(n))
   d. (log(n))

178. While solving Selection problem, in Sieve technique we partition input data _____
   a. In increasing order

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b. In decreasing order
c. According to Pivot
d. Randomly

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179. _____ is the process of avoiding unnecessary repetitions by writing down the results of recursive calls and looking them up again if we need them later.
    a. Loop
    b. Memoization page 74
    c. Recursion
    d. Function

180. In average-case time the probability of seeing input is denoted by _____.
    a. p{I}
    b. p[I]
    c. p<i>
    d. p(i) page 13

181. While applying the Sieve technique to selection sort, how to choose a pivot element.
    a. Through mean
    b. Linear
    c. Randomly page 35
    d. Sequentially

182. Number of _____ of the pseudo code are counted to measure the running time.
    a. Inputs
    b. Outputs
    c. Steps page 13
    d. Pages

183. Developing a dynamic programming algorithm generally involves _____ separate steps.
    a. One
    b. Two page 75
    c. Three
    d. Four

184. 8n^2+2n+3 will exceed c28(n), no matter how large we make _____.
    a. n

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b. 2n

c. c2 page 25

d. this quadratic equation

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185. The running time of quick sort algorithm_____.
   a. Is impossible to compute
   b. Has nothing to do with pivot selection
   c. Is Random upon each execution
   d. Greatly influenced by the selection of pivot page 49

186. _____ involves breaking up the problem into sub problems whose solutions can be combined to solve the global problem.
   a. Complexity Theory
   b. Dynamic programming solution
   c. Divide and Conquer Strategy page 34
   d. Greedy Algorithms

187. In _____ we have to find rank of an element from given input.
   a. Merge sort algorithm
   b. Selection problem page 34
   c. Brute force technique
   d. Plane Sweep algorithm

188. How many steps are involved to design the dynamic programming strategy?
   a. 2
   b. 3
   c. 1
   d. 4 page 92

189. In Bucket sort, if there are duplicates then each bin can be replaced by a a. Stack
   b. Linked list page 69
   c. Hash table
   d. Heap

190. In merge sort algorithm, we split the array _____ to find index q.
   a. from start
   b. midway page 28
   c. from end

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d. both from start or end

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191. Find the maximum value of the items which can carry using knapsack Knapsack weight capacity = 50.

Item Weight Value
11070
22020
33080
470 200

a. 280
b. 100
c. 90
d. 200

192. In 2-d maxima problem a point p is said to be dominated by point q if _____.

a. p.x <= q.x
b. p.x <= q.x and p.y <= q.y page 17
c. p.y <= q.y
d. p.x >= q.x and p.y >=q.y

193. Sorting can be in _____.

a. Increasing order only
b. Decreasing order only
c. Both increasing and decreasing order
d. Random order

194. Recurrence can be described in terms of _____.

a. Array
b. Linear
c. Tree page 31
d. Graph

195. The brute-force algorithm for 2D-Maxima runs in order O(__) time.

a. n
b. n(log n)

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c. n*n page 18

d. n3

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196. In plane sweep approach of solving geometric problems, a _____ is swept across the plane.
   a. Line page 18
   b. Plane
   c. Cube
   d. Box

197. Which of the following is calculated with Big Omega notation?
   a. Medium bounds
   b. Upper bounds
   c. Lower bounds Page - 25
   d. Both upper and lower bounds

198. _____ is always based on divide and conquer strategy.
   a. Bubble sort
   b. Selection sort
   c. Pigeon sort
   d. Quick sort page 46

199. If a matrix has three rows and two columns, then dimensions of matrix will be:
   a. 3x2
   b. 2x3
   c. 3x3
   d. 2x2

200. Asymptotic notations are used to describe _____ of an algorithm.
   a. Length
   b. running time google
   c. size
   d. compile time

201. Catalan numbers are related the number of different _____ on 'n' nodes.
   a. Arrays
   b. linked lists

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c. binary trees **page 85**

d. functions

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202. Applying the sieve technique to selection problem, _____ element is picked from array.
   a. Output
   b. Total
   c. Input
   d. Pivot page 35

203. Dynamic Programming approach is usually useful in solving _____ problems.
   a. Normal
   b. Optimization google
   c. Array
   d. Loop

204. In recursive formulation of knapsack Problem: V [0, j] = _____ for j>=0
   a. -1
   b. 0 page 93
   c. 1
   d. 2

205. _____ is a linear time sorting algorithm.
   a. Merge sort
   b. Radix sort page 71
   c. Quick sort
   d. Bubble sort

206. Quick sort is one of the _____ sorting algorithm.
   a. Fastest page 19
   b. Slowest
   c. Major
   d. Average

207. The time assumed for each basic operation to execute on RAM model of computation is _____.

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a. Infinite
b. Continuous
c. Constant page 10
d. Variable

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208. In Sieve Technique, we know the item of interest. a. True
b. False page 34

209. While analyzing algorithms, _____ and _____ usually considered difficult to calculate.
a. Finite, Infinite
b. Floor, ceiling google
c. Row, Column
d. Graph, Tree

210. While analysis of the brute-force maxima algorithm, an array sorted in the reverse order is the type of _____ case input.
a. Best
b. Worst page 14
c. Somewhat bad
d. Average

211. _____ is not useful measure of central tendency of given input set especially when the distribution of values is highly skewed.
a. Mean
b. Mode
c. Average
d. Median page 34

212. In asymptotical analysis of n(n-3) and 4n*n, as n becomes large, the dominant (fastest growing) term is some constant times _____.
a. n+1
b. n-1
c. n
d. n*n page 23

213. In addition to passing in the array itself to Merge Sort algorithm, we will pass in other arguments which are indices.

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a. Two P-38
b. Three
c. Four
d. Five

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214. In 2d-maximal problem, a point is said to be if it is not dominated by any other point in that space.
    a. Member
    b. Minimal
    c. Maximal P-11
    d. Joint

215. Counting sort assumes that the numbers to be sorted are in the range _____.
    a. K to n where n is large
    b. 1 to k where k is small (P-57)
    c. K to n where k is small
    d. k to n where n is small

216. Insertion sort is an efficient algorithm for sorting a _____ number of elements
    a. Large P-39
    b. Small
    c. Extra large
    d. Medium

217. If the indices passed to merge sort algorithm are _____ then this means that there is only one element to sort.
    a. Small page 28
    b. Large
    c. Equal
    d. Not Equal

218. In Knapsack Problem, each item must be entirely accepted or rejected, is called _____ problem.
    a. Fractional
    b. 0-1 P-92
    c. Linear
    d. Optimal

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219. If the time complexity of an algorithm is O(n). then it is called _____ time complexity.
   a. Linear Wikipedia
   b. Constant
   c. Average
   d. Exponential

220. In the case of _____ analysis does not depend upon on the distribution of input.
   a. Merge sort
   b. Quick sort P-50
   c. Insertion sort
   d. Heap sort

221. We can use the _____ Property to devise a recursive formulation of the edit distance problem.
   a. Small substructure
   b. Algorithmic
   c. Real
   d. Optimal substructure page 78

222. The following sequence is called _____
   1,2,3,5,8,13,21,34,55,.....
   a. Optimize sequence
   b. Fibonacci sequence page 73
   c. Optimal sequence
   d. Overlapping sequence

223. Which one sorting algorithm is best suited to sort an array of 2 million elements?
   a. Bubble sort
   b. Insert sort
   c. Merge sort
   d. Quick sort

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e. Ridx Sort page 71

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224. We can improve the performance of quick sort if we could be able to _,_____.
    a. Skip input elements somehow
    b. Select two or more pivots page 34
    c. Skip any sub-array completely
    d. Eliminate recursive calls
225. The problem with the brute-force algorithm is that is uses _____ in pruning out de
    a. Worst-case time
    b. No intelligence page 18
    c. Outside looping
    d. Artificial intelligence
226. In chain matrix multiplication, the order of the matrices _____.
    a. Can be changed
    b. Can not be changed page 85
    c. is equal
    d. is reverse
227. In quick sort algorithm, we choose pivot_____.
    a. Always the smallest element
    b. Greater than 5
    c. Randomly page 35
    d. Less than 5
228. In Heap Sort algorithm. Heapify procedure is _____ in nature.
    a. Recursive
    b. Non-Recursive page 43
    c. Fast
    d. Slow
229. When matrix A of 5x 3 is multiplied with matrix B of 3 x 4 then the number of multiplications required will be _____.
    a. 15

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b. 12
c. 36
d. 60

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230. An algorithm is said to be correct if for every _____ instance, it halts with the correct _____.
   a. Input, Output page 13
   b. Design, Analysis
   c. Value, Key
   d. Key, Analysis

231. In chain matrix multiplication, table is filled _____ to find the multiplication of matrix.
   a. row wise
   b. column wise
   c. diagonally
   d. bottom-to-up page 86

232. If we have an equation 8n2+7f*n + 5f + 6 then is large, _____ term will be muchxxxxxxxthe n term and will dominate the running time.
   a. f g (n)
   b. g (n) * 2
   c. n * 2 page 23
   d. f (n)

233. For quick sort algorithm. Partitioning takes theta _____.
   a. (n)
   b. log(n)
   c. n log (n)
   d. n2log (n)

234. In Heap Sort algorithm, the maximum levels an element can move upward is _____
   a. Theta (log n) page 43
   b. Big-ch (log n)
   c. Omega (log n)
   d. 0 (1) i.e. Constant time

235. _____ programming is essentially recursion without repetition.

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a. Fast
b. Dynamic page 75
c. Array
d. n (log n)

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236. There are no hard formal rules to the syntax of the _____ code.
   a. Basic
   b. Programming
   c. Pseudo
   d. Assembly

237. In Heap Sort algorithm, to remove the maximum element every time.
   a. We call Build-Heap procedure
   b. Heap Sort algorithm terminates without result
   c. We call heapify procedure
   d. Nothing happens

238. Which process is used for avoiding unnecessary repetitions and looking them up again if we need them later.
   a. Greedy Approach
   b. Memoization page 74
   c. Divide and conquer
   d. Recursion

239. The worst-case running time of Quick sort is _____ in order to sort an array of n element.
   a. O(n log n) page 49
   b. O(n)
   c. $O(n^2)$
   d. O(log n)

240. Boolean operation is a _____ operation on an idealized RAM model of computation.
   a. Starting
   b. Basic page 10
   c. Advance
   d. Normal

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241. In chain matrix multiplication, if there are n items, there are _____ ways in which outer most pair of parentheses can placed.
    a. n^2
    b. 2n
    c. n+1
    d. n-1 page 85

242. The number of nodes in a complete binary tree of height h is: a. 2 * (h+1) - 1
    b. 2^(h+1) – 1 page 40
    c. 2 * (h+1)
    d. ((h+1)^2) – 1

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1. The sequence of merge sort algorithm is:
   a. Divide Combine-Conquer
   b. Conquer-Divide-Combine
   c. Divide-Conquer-Combine Page 27
   d. Combine-Divide-Conquer
2. In _____ Knapsack Problem, limitation is that an item can either be put in the bag or not. Fractional items are not allowed.
   a. 0
   b. 1
   c. 0/1 Page 91
   d. Fractional
3. In Selection algorithm, we assume pivot selection takes theta _____ running time.
   a. n Page - 36
   b. n2
   c. n3
   d. log (n)
4. In Heap Sort algorithm (using max heap), when every time maximum elements removed from top _____.
   a. We call merge Sort Algorithm
   b. it becomes Order n2 Algorithm
   c. Divide and Conquer strategy helps us
   d. We are left with a hole Page – 41
5. If matrix A of dimension p x q is multiply with matrix B of dimension q x r, then each entry in resultant matrix takes _____ time.
   a. O (q) Page - 84
   b. O (1)
   c. O (p x q)
   d. O (q x r)

6. _____ is a method of solving a problem in which we check all possible solutions to the problem to find the solution we need.
   a. Plane-Sweep Algorithm
   b. Sorting Algorithm
   c. Brute-Force Algorithm google
   d. Greedy approach
7. The worst case running time of Quick sort algorithm _____.
   a. Cannot be quadratic
   b. Is quadratic
   c. Is always Exponential
   d. Is linear
8. In max heap (for Heap Sort algorithm), when every time maximum element is removed from top we replace it with _____ leaf in the tree.
   a. second last
   b. Last Page -41
   c. First
   d. Any
9. Quick sort algorithm was developed by -
   a. AlferdAho
   b. Sedgewick
   c. John Vincent Atanasoff
   d. Tony Hoare – Google wikipedia
10. If Matrix-A has dimensions "3x2" and Matrix-B has dimensions "2x3", then multiplication of Matrix-A and Matrix-B will result a new Matrix-C having dimensions.
    a. 3x2
    b. 2x3
    c. 2x2
    d. 3x3 http://www.calcul.com/show/calculator/matrix-multiplication
11. For comparison-based sorting algorithms, it is possible to sort more efficiently than Omega n log(n) time.
    a. Always
    b. Not P-54
    c. Sometimes

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d. Sometimes not

12. Dynamic Programming approach is usually useful in solving optimization problems.
    a. True
    b. False

13. In Sorting the key value or attribute_____ from an ordered domain.
    a. Must be page 39
    b. Not always
    c. May be
    d. Occasionally

14. Result of asymptotical analysis of n(n -3) and 4n*n is that _____
    a. n(n-1) is asymptotically Less
    b. n(n-1) is asymptotically Greater
    c. Both are asymptotically Not equivalent
    d. Both are asymptotically Equivalent page 23 ($4n*n= 4n^2$)

15. Floor and ceiling are _____ to calculate while analyzing algorithms a. Very easy
    b. Usually considered difficult P-31
    c. 3rd Option is missing
    d. 4th Option is missing

16. _____ of reference is an important fact of current processor technology.
    a. Defining
    b. Assigning
    c. Formality
    d. Locality P-8

17. In max-heap, largest element is stored at root node. Where is the smallest element stored?
    a. Right Node
    b. Leaf Node
    c. Middle Node
    d. Left Node

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18. In average-case time analysis of Quick sort algorithm, the most balanced case for partition is when we divide the list of elements into _.
    a. Equal no. of pieces as of input elements
    b. Single piece exactly
    c. Two nearly equal pieces
    d. Three nearly equal pieces
19. Which of the following is calculated with Big O notation?
    a. Medium bounds
    b. Upper bounds Page - 25
    c. Lower bounds
    d. Both upper and lower bounds
20. Edit distance algorithm based on _____ strategy
    a. Greedy
    b. Dynamic Programming Page - 81
    c. Divide and Conquer
    d. Searching
21. In Heapsort Algorithm, total time taken by heapify procedure is _____
    a. O (log n) Page-43
    b. O (log2 n)
    c. O (n log n)
    d. O (n2 log n)
22. Al-Khwarizmi was a/an _____
    a. Artist
    b. Mathematician P-7
    c. Astronomer
    d. Khalifah
23. When matrix A of 5x3is multiply with metric B of 3x4 then the number of multiplication required is: Not found exactly
    a. 15
    b. 12
    c. 36
    d. 60 Not Found exactly but as per formula at page 84,

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24. Pseudo code of algorithms are to be read by _____.
    a. People Page -12
    b. RAM
    c. Computer
    d. Compiler
25. The sieve technique is a special case, where the number of sub-problems is Just _____
    a. 1 P-34
    b. 2
    c. 3
    d. 4
26. When a recursive algorithm revisits the same problem over and over again, we say that the optimization problem has _____ sub-problems.
    a. Overlapping – Google Search
    b. Over costing
    c. Optimized
    d. Three
27. Sieve technique is very important special case of Divide-and-Conquer strategy.
    a. True P-34
    b. False
28. In order to say anything meaningful about our algorithms, it will be important for us to settle on a _____.
    a. Java Program
    b. C++ Program
    c. Pseudo program
    d. Mathematical model of computation P-10
29. Merge sort is based on _____.
    a. Brute-force
    b. Plan-sweep
    c. Axis-sweep
    d. Divide and Conquer P-27

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30. What time does Merge Sort algorithm take in order to sort an array of 'n' numbers?
    a. (n)
    b. (log n)
    c. (n^2)

    d. (n log n) Google Search 31. In Heap Sort

31. algorithm, the first step is to _____.
    a. Call Build-Heap procedure Page - 46
    b. Sort the array in descending order
    c. Call Heapify procedure
    d. Find the number of input elements

32. The definition of theta-notation relies on proving _____ asymptotic bound.
    a. One
    b. Lower
    c. Upper
    d. Both lower & upper Page - 25

33. In merge sort algorithm, to merge two lists of size n/2 to a list of size n, takes _____ time.
    a. Theta (n) Page - 32
    b. Theta log(n)
    c. Theta log2(n)
    d. Theta n log(n)

34. We can make _____ recursive calls in Fibonacci Sequence.
    a. Infinite
    b. Finite google
    c. Only one
    d. Zero

35. Following is NOT the application of Edit Distance problem.
    a. Speech recognition
    b. Spelling Correction
    c. Ascending Sort Page - 76
    d. Computational Molecular Biology

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36. In plane sweep approach, a vertical line is swept across the 2d-plane and structure is used for holding the maximal points lying to the left of the sweep line.
    a. Array
    b. Queue
    c. Stack Page - 18
    d. Tree
37. When a heapify procedure is applied to the root node to restore the heap, then at each level, the comparison performed takes time:
    a. It will take (log n).
    b. It can not be predicted
    c. It will take O (1). Page - 43
    d. Time will vary according to the nature of input data.
38. _____ time is the maximum running time over all legal inputs.
    a. Worst-case Page - 13
    b. Average-case
    c. Best-case
    d. Good-case
39. Efficient algorithm requires less computational...
    a. Memory
    b. Running Time
    c. Memory and Running Time Page - 9
    d. Energy
40. For average-case time analysis of Quick sort algorithm, Pivot selection is on average basis from _____
    a. half of the input values
    b. all possible random values Page - 50
    c. Pivot is input separately
    d. values greater than 5
41. Selection algorithm takes theta _____
    a. (n2)
    b. (n) Page - 37
    c. log(n)
    d. n log(n)

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42. Recurrence can be described in terms of a tree.
    a. Yes Page - 31
    b. No
43. Time complexity of Dynamic Programming based algorithm for computing the minimum cost of Chain Matrix Multiplication is _____
    a. Log n
    b. n
    c. n^2 (n square)
    d. n^3 (n cube) Page -90
44. The Iteration method is used for _____
    a. Comparing sorting algorithms only
    b. Solving Recurrence relations Page 31
    c. Merging elements in Merge sort
    d. Dividing elements in Merge sort
45. In 3-Dimensional space, a point P has _____ coordinate(s).
    a. (X, Y)
    b. (X, 0)
    c. (0, Y)
    d. (X,Y, Z)
46. Chain matrix multiplication problem can be solved through _____ strategy.
    a. Dynamic programming Page - 85
    b. Greedy
    c. Divide and conquer
    d. Sorting
47. Merge sort have running time....running time of Heap sort. Not found exactly
    a. Greater than
    b. Less than Google
    c. Equal to
    d. Different than
48. Median is not useful measure of central tendency of given input set especially when the distribution of values is highly skewed.
    a. True
    b. False Page – 34

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49. We do not need to mathematically prove that for comparison-based sorting algorithms always takes Omega nlog (n) time.
    a. True Google & VU Tech (pg 46 not very clear)
    b. False
50. The Omega-notation allows us to state only the asymptotic _____ bounds.
    a. Middle
    b. Lower Page 25
    c. Upper
    d. Both lower & upper
51. Both lower & upperSorting can be in _____
    a. Increasing order only
    b. Decreasing order only
    c. Both Increasing and Decreasing order GOOGLR Search
    d. Random order
52. Radix sort performs sorting the numbers _____ digit (s) at a time.
    a. One Page - 71
    b. Two
    c. Three
    d. All
53. Quicksort is a/an _____ and _____ sorting algorithm.
    a. Not in place, not stable one
    b. In place , not stable one Page - 54
    c. In place , stable one
    d. Not in place , stable one
54. Consider three matrices X,Y,Z of dimensions 1x2, 2x3,3x4 respectively. The number of multiplications of (XY) Z is:
    a. 18 As per lecture slides
    b. 32
    c. 24
    d. 30
55. In Fibonacci Sequence, unnecessary repetitions do not exist at all.
    a. True
    b. False Page – 74

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56. It is not a Fibonacci sequence . 1,1,1,2,3,5,8,13,21,34,55,…..
    a. True Page - 73
    b. False
57. Heap sort is a/ an _____ and _____ sorting algorithem.
    a. Not in place, not stable one
    b. In place , not stable one Page - 54
    c. In place , stable one
    d. Not in place , stable one
58. Identify the True Statement
    a. The knapsack problem does not belong to the domain of optimization problems.
    b. The knapsack problem belongs to the domain of optimization problems. Page - 91
    c. The Knapsack problem cannot be solved by using dynamic programming
    d. The knapsack problem is optimally solved by using brute force algorithm.
59. In Dynamic Programming, our approach is to _____
    a. Develop the solution in a top-down fashion
    b. Express the problem non-recursively
    c. Build the solution in a bottom-up fashion Page - 75
    d. Input several sub-problems simultaneously
60. Counting sort is suitable to sort the elements in range 1 to K;
    a. K is large
    b. K is small Page - 57
    c. K may be large or small
    d. None
61. We can multiply two matrices A and B only when they are compatible which means
    a. Number of columns in A must be equal to number of rows in B. it seems Correct as per page 84
    b. Number of rows and columns do not matter

c. Number of columns in A must be equal to number of columns in B
d. Number of rows in A must be equal to number of rows in B

62. Matrix multiplication is a (n) _____ operation.
a. Commutative
b. Associative Page 85
c. Neither commutative nor associative
d. Commutative but not associative

63. In Dynamic Programming approach, solution is modified / changed
a. Always once
b. At each stage google and wikipedia
c. Only for specific problems
d. At 4th stage only

64. In Knapsack problem, the goal is to put items in the Knapsack such that the value of the items is _____ subject to weight limit of knapsack.
a. Minimized
b. Decreased
c. Maximized Page - 91
d. None of the given options

65. An in-place sorting algorithm is one that _____ uses additional array for storage.
a. Always
b. Permanently
c. Does not Page - 54
d. Sometime

66. Memoization is a part of Dynamic Programming Strategy.
a. True Page - 74
b. False

67. If matrix A of dimension 2x4 is multiply with matrix B of dimension 4x3, then the dimension of resultant matrix is Not found exactly
a. 2x4
b. 4x3
c. 3x4
d. 2x3 It seems correct as per second last Para of page 84

68. In Dynamic Programming approach, we do not store the solution to each sub-problem in case if it reappears.
    a. True
    b. False Page - 75
69. Dynamic Programming is a problem-solving approach in which___
    a. Problem is solved in Zero time
    b. Solution is developed only at final stage
    c. Both are correct
    d. Both are incorrect google
70. In Fibonacci sequence, each term is calculated by____ previous__ terms.
    a. Subtracting, Two
    b. Adding, Three
    c. Adding, Two Page - 73
    d. Multiplying, Two
71. Selection sort is not an in-place sorting algorithm.
    a. True Page - 54
    b. False
72. If there are $\theta(n^2)$ entries in edit distance matrix then the total running time is:
    a. $\theta(n)$
    b. $\theta(1)$
    c. $\theta(n^2)$ Page – 84
    d. $\theta(n \log n)$
73. The only way to convert a string of i characters into the empty string is with i deletions, represented as
    a. E(0.j) =j
    b. E(i.j) = 1
    c. E(0.i) = j
    d. E (i.0)=I Page - 78
74. Dynamic programming formulation of the matrix chain multiplication problem will store the solutions of each sub problem in an
    a. Array
    b. Table Page - 86
    c. Variable
    d. class

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75. We can use the optimal substructure property to devise a formulation of the edit distance problem.
    a. Selective
    b. Optimum
    c. Iterative
    d. Recursive Page - 78

76. Sorting is performed on the basis of _____.
    a. Computational resources
    b. Asymptotic notation
    c. Summation
    d. Some key value of attribute page- 39

77. In Heap Sort algorithm, we call Build-heap procedure _____.
    a. Only once page 46
    b. Twice
    c. Thrice
    d. As many times as we need

78. Radix sort is not a non-comparative integer sorting algorithm.
    a. True Google Search
    b. False

79. In the statement "output P[1].x, P[1].y", the number of times elements of P are accessed is _____.
    a. 1
    b. 2 page 14
    c. 3
    d. 4

80. The main purpose of mathematical analysis is measuring the _____ required by the algorithm.
    a. Space
    b. Execution time P-13
    c. Inputs & outputs
    d. Execution time and memory

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81. _____ provides us more accurate result when input values are not closer with each other
    a. Average
    b. Median P-34
    c. Mode
    d. Mean
82. The process of _____ ends when you are left with such tiny pieces remaining that it is trivial to solve them.
    a. Brute-force
    b. Plan-sweep
    c. Divide and Conquer P-27
    d. Axis-sweep
83. _____ overcomes the limitations of _____ by working as per positional notations of numbers.
    a. Counting sort, Radix sort
    b. Radix sort, Counting sort P-71
84. Memorization is a part of Dynamic Programming strategy.
    a. True P-74
    b. False
85. Rank of an element can be defined as _____.
    a. One minus the number of elements that are smaller
    b. Two plus the number of elements that are greater
    c. One plus the number of elements that are smaller P-34
    d. Two minus the number of elements that are smaller
86. If the time complexity of an algorithm is given by O (1), then its time complexity would be
    a. Polynomial
    b. Exponential
    c. Constant - Wikipedia
    d. Average
87. Quick sort is a recursive algorithm.

    a. True Wikipedia ; Google
    b. False

88. The asymptotic growth of n(n+1)/2 is:
    a. $O(n^2)$ As the n^2 term has the largest contribution, the Big-O complexity is O(n^2)
    b. O(n)
    c. O(n+2)
    d. O(n log n)
89. Approach of solving geometric problems by sweeping a line across the plane is called _____ sweep.
    a. Line
    b. Plane Page 18
    c. Cube
    d. Box
90. As per algorithm of Dynamic Programing, we need to store
    a. First sub-problem only
    b. Best solution only
    c. Intermediate sub-problems Pg:75
    d. Final solution only
91. In Sieve technique, we solve the problem
    a. In recursive manner Pg:34
    b. Non recursively
    c. Using Merge Sort algorithm
    d. Using Brute force technique
92. One of the limitation in 0/1 knapsack is that an item can either be _____ in the bag or not.
    a. Use
    b. Put Pg:91
    c. Move
    d. Store
93. Which one is not passed as parameter in Quick sort algorithm?
    a. End of the array
    b. Middle of the array
    c. Array (containing input elements) Google
    d. Start of the array

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94. In the analysis of Selection algorithm, we get the convergent _____
    a. Harmonic
    b. Linear
    c. Arithmetic
    d. Geometric Pg:37
95. A Random Access Machine (RAM)is an idealized machine withrandom access memory.
    a. Infinite large Pg:10
    b. 512 MB
    c. 256 MB
    d. 2 GBs
96. While analyzing Selection algorithm, we make a number of passes, in fact it could be as many as
    a. n(n+1)
    b. log(n) Pg:37
    c. n/3
    d. n/4
97. In Random Access Machine (RAM), instructions are executed in
    a. Parallel
    b. Batch
    c. One by One Pg:10
    d. Multiple times
98. In selection problem, the rank of an element will be its _____ position
    a. First
    b. final Pg:34
    c. Second last
    d. Last
99. The worst-case running time of Merge sort is _____ in order to sort an array of n elements.
    a. O(log n)
    b. O(n)
    c. O(n log n) page 40 and google
    d. O(n)

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100. f(n) and g(n) are asymptotically equivalent. This means that they have essentially the same _____.
   a. Results
   b. Variables
   c. Size
   d. Growth rates P:23

101. An algorithm is a mathematical entity. Which is independent of _____.
   a. Programming language
   b. Machine and Programming language
   c. Compiler and Programming language
   d. Programming language Compiler and Machine P:07

102. In Quick sort algorithm, Pivots form ___
   a. Stack
   b. Queue
   c. Binary Search Tree P:49
   d. Graph

103. Counting sort is suitable for sorting the elements within range 1 to P. where
   a. P is large
   b. P is small P-57
   c. P is very large
   d. P is undetermined

104. In asymptotical analysis of n'(5 2)-3, as n becomes large, the dominant (fastest growing) term is some constant times
   a. n_1
   b. n
   c. n+1
   d. n*n P-23

105. ___ Items are not allowed in the 0/1 knapsack. a. Lighter
   b. Fractional P-91
   c. Whole
   d. Weighty

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106. Fibonacci Sequence was named on _____, a famous mathematician in 12th Century.
   a. Fred Brooks
   b. Grady Booch
   c. Leonardo Pisano P-73
   d. Edgar F. Codd

107. In Heap Sort algorithm, we build _____ for ascending sort.
   a. Max heap P-41
   b. Min heap

108. Bubble sort is not an in-place sorting algorithm.
   a. True
   b. False P-54

109. In partition algorithm, the subarray _____ has elements which are greater than pivot element x.
   a. A[p...r]
   b. A[p...q-1]
   c. A[q]
   d. A[q+1..r] P-46

110. In Heap Sort algorithm, if heap property is violated
   a. We call Build heap procedure P-43
   b. We call Heapify procedure
   c. We ignore
   d. Heap property can never be violated

111. _____ is not a characteristic of Random Access Machine.
   a. Single-Processor P-10
   b. Assigning a value to a variable
   c. Locality of reference
   d. Executing an arithmetic instruction

112. The only way to convert an empty string into a sting of j characters is by doing j insertions, represented as _____
   a. E(i,j) = 1
   b. E(I,0) = I
   c. E(0,j) = j page 78
   d. E(1,j)= j

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113. In Selection problem, the Sieve technique works in _____.
   a. Non-recursive manner
   b. Constant time
   c. Phases page 34
   d. One complete go

114. Algorithm is a sequence of computational steps that ---- the input into output.
   a. Merge
   b. Assign
   c. Transform page 7
   d. Integrate

115. If pj dominates pi and pi dominates ph then pj also dominates ph, it means dominance relation is
   a. Transitive page 18
   b. Non Transitive
   c. Equation
   d. Symbolic

116. To find maximal points in brute-force algorithm each point of the space is compared against _____ of that space.
   a. One other point
   b. All other points page 11
   c. Few other points
   d. Most of the other points

117. In the following code the statement "cout<<j;"executes ---- ----- times. for (j=1; j<=5; j = j+2)
   cout<<j;
   a. 5 times
   b. 2 times
   c. 3 times
   d. 0 times

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118. In merge sort algorithm, we split the array around the _____ index q. a. Entring
b. Mid page 17
c. Exiting
d. Summing

119. In Selection problem, the Sieve technique _____.
a. Add some more input items each time
b. Do not work recursively
c. Do not uses Divide and Conquer approach
d. Eliminates undesired data items each time

120. Consider three matrices X, Y, Z of dimensions 1 x 2, 2 x 3, 3 x 4 respectively. The number of multiplications of X(YZ) is .
a. 16
b. 32
c. 26
d. 32 page 84

121. In Heap Sort algorithm, the total running time for Heapify procedure is _____
a. Theta (log n)
b. Order (log n)
c. Omega (log n)
d. O(1) i.e. Constant time

122. The sieve technique works where we have to find_____ items(s) from a large input.
a. Single page 34
b. Two
c. Three
d. Similar

123. In Dynamic Programming based solution of Knapsack Problem, if we decide to take an object i , then we gain_____
a. W(Total Weight of Knapsack)
b. V (Total Value of all items)
c. vi (Value of object i) page 93
d. Nome of the given option

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124. While Sorting, the order domain means for any two input elements x and y _____ satisfies only.
   a. x < y page 39
   b. x > y
   c. x = y
   d. All of the above

125. For solving Selection problem, we introduced Sieve technique due to _____
   a. Using Decrease and Conquer strategy page 34
   b. Avoiding to sort all input data
   c. Eliminating Rank of an element
   d. Using Brute-force approach

126. _____ is one of the few problems, where provable lower bounds exist on how fast we can sort.
   a. Searching
   b. Sorting page 38
   c. Both Searching & sorting
   d. Growing

127. In plane sweep approach, a vertical line is swept across the 2d-plane from_____.
   a. Right to Left
   b. Left to Right page 18
   c. Top to Bottom
   d. Bottom to top

128. In generating Fibonacci sequence, we can avoid unnecessary repetitions by _____ process.
   a. Tokenization
   b. Memorization page 43
   c. Randomization
   d. Memorization

129. For _____ values of n, any algorithm is fast enough.
   a. Small page 14
   b. Medium
   c. Large
   d. Infinity

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130. Dynamic programming comprises of _____.
   a. Recursion only
   b. Repetition only
   c. Recursion with Repetition
   d. No Repetition but Recursion page 75

131. The function f(n)=n(logn+1)/2 is asymptotically equalient t nlog n :Here Lower Bound means function f(n) grows asymptotically at __ as fast as nlog n.
   a. Least page 23
   b. Normal
   c. Most
   d. At

132. Counting sort has time complexity.
   a. O(n+k)
   b. O(n) page 58
   c. O(k)
   d. O(nlogn)

133. Due to left complete nature of binary tree, the heap can be stored in
   a. Array page 40
   b. Structures
   c. Link List
   d. Stack

134. Single item from a larger set of _____.
   a. Constant
   b. Pointers
   c. Phases
   d. n items page 34

135. In the clique cover problem, for two vertices to be in the same group, they must be _____ each other.
   a. Apart from
   b. Far from
   c. Near to
   d. Adjacent to page 76

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136. How much time merge sort takes for an array of numbers?
   a. T(n^2)
   b. T(n)
   c. T(log n)
   d. T(n log n) page 40

137. In in-place sorting algorithm is one that uses arrays for storage.
   a. No additional array page 54
   b. An additional array
   c. Both of above may be true according to algorithm
   d. More than 3 arrays of one dimension

138. Brute-force algorithm for 2D-Maxima is operated by comparing _____ pairs of points.
   a. Two
   b. Some
   c. Most
   d. All page 18

139. While Sorting, the ordered domain means for any two input elements x and y ____ satisfies only.
   a. x > y
   b. x < y
   c. x = y
   d. All of the above page 38

140. Quick sort is.
   a. Stable & in place
   b. Not stable but in place page 54
   c. Stable but not in place
   d. Some time stable & some times in place

141. Which may be a stable sort?
   a. Merger
   b. Insertion
   c. Both above page 54
   d. None of the above

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142. For the Sieve Technique we take time.
   a. T(nk) page 34
   b. IT(n / 3)
   c. n^2
   d. n/3

143. Continuation sort is suitable to sort the elements in range 1 to k.
   a. K is Large
   b. K is not known
   c. K may be small or large
   d. K is small page 54

144. Asymptotic growth rate of the function is taken over _____ case running time. .
   a. Best
   b. Worst page 14
   c. Average
   d. Normal

145. The sieve technique is a special case, where the number of sub problems is just.
   a. 5
   b. Many
   c. 1 page 34
   d. Few

146. In Quick sort, we don't have the control over the sizes of recursive calls.
   a. True page 49
   b. False
   c. Less information to decide
   d. Ether true or false

147. Before sweeping a vertical line in plane sweep approach, in start sorting of the points is done in increasing order of their _____ coordinates. .
   a. X page 18
   b. Y
   c. Z
   d. X , Y

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148. Random access machine or RAM is a/an.
   a. Machine build by Al-Khwarizmi
   b. Mechanical machine
   c. Mathematical model page 10
   d. Electronics machine

149. The Huffman codes provide a method of encoding data inefficiently when coded using ASCII standard.
   a. True
   b. False page 99

150. A heap is a left-complete binary tree that confirms to the _____.
   a. increasing order only
   b. decreasing order only
   c. heap order page 40
   d. log n order

151. If we associate (x, y) integers pair to cars where x is the speed of the car and y is the negation of the price. High y value for a car means a _____ car.
   a. Fast
   b. Slow
   c. Expensive
   d. Cheap

152. Which one of the following sorting algorithms is the fastest?
   a. Merge sort
   b. Quick sort
   c. Insertion sort
   d. Heap sort

153. Quick sort algorithm divide the entire array into _____ sub arrays.
   a. 2
   b. 3
   c. 4
   d. 5

154. In brute force algorithm, we measure running time T(n) based on _____.
   a. Average-case time and best-case time
   b. Worst-case time and average-case time page 46
   c. Worst-case time and best-case time
   d. Best-case time and staring-case time

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155. For 2D Maxima problem. Plane Sweep algorithm first of all _____.
   a. Sorts all points
   b. Delete some points
   c. Output the elements
   d. Pushes all points on stack

156. There are _____ entries in the Edit Distance Matrix
   a. ɵ (n)
   b. ɵ (n_2) page 84
   c. ɵ (n+2)
   d. ɵ (n + 100)

157. Which symbol is used for Omega notation?
   a. (O)
   b. (ɵ)
   c. (Ω)
   d. (@)

158. Selection sort is a _____ sorting algorithm
   a. In-place page 54
   b. Not In-Place
   c. Stable
   d. in-partition

159. In Dynamic Programming based solution of knapsack problem, to compute entries of 'V', we will imply a(n) _____ approach.
   a. Subjective
   b. Inductive
   c. Brute Force
   d. Combination

160. We do not need to prove comparison-based sorting algorithms by mathematically. It always takes _____ time.
   a. Big Oh nlog(n)
   b. Omega nlog(n) NOT SURE
   c. Omega n(n^2)
   d. Theta nlog(n)

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161. Merge sort is a/an _____ and _____ sorting algorithm
    a. Not in-place, not stable one
    b. In-place, not stable one
    c. In-place, stable one
    d. Not in-place, stable one page 54
162. Cubic function will _____ a quadratic function.
    a. Prove
    b. be equal to
    c. overtake Page 25
    d. find
163. Insertion sort is a _____ sorting algorithm
    a. Unstable
    b. In-place Page 54
    c. Not In-Place
    d. in-partition
164. To check whether a function grows faster or slower than the other function, we use some asymptotic notations, which is _____.
    a. Big-oh notation
    b. Theta notation
    c. Omega notation
    d. All of the given
165. Asymptotic growth of 8n^2 + 2n – 3 is:
    a. Θ(n^2 + n)
    b. Θ (n^2) page 14
    c. Θ(8n^2)
    d. Θ(8n^2 + 2n)
166. In the analysis of algorithms, _____ plays an important role.
    a. text analysis
    b. time
    c. growth rate
    d. money

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167. In inductive approach of knapsack problem, we consider 2 cases, _____ Or _____.
   a. Median, Mode
   b. Recursive, Iterative
   c. Leave object, Take object page 93
   d. Sequentially. Parallel

168. Random Access Machine (RAM) can execute _____ instructions
   a. only logical
   b. parallel
   c. only arithmetic
   d. logical and arithmetic

169. Using _____ algorithm, efficiency is not given much importance
   a. Greedy
   b. Merge sort
   c. Processing as there is no algorithm by this name
   d. Brute Force

170. Bubble sort takes theta _____ in the worst case
   a. (n2) page 39
   b. (n)
   c. log(n)
   d. nlog(n)

171. If matrix A of dimension p × q is multiply with matrix B of dimension q × r, then dimension of resultant matrix is:
   a. p × q
   b. p × r page 84
   c. q × r
   d. r × p

172. Dynamic Programing algorithms often use some kind of _____ to store the results of intermediate sub-problems
   a. table (Page 75)
   b. variable
   c. stack
   d. loop

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173. _____ is in-place sorting algorithm.
   a. Bubble sort (Page 54)
   b. Merge sort
   c. Linear search
   d. Binary Search

174. Which one of the following problems can be solved using dynamic problem?
   a. Bubble sort problem
   b. Matrix chain multiplication problem page 85
   c. Greedy search problem
   d. Fractional knapsack problem

175. In chain matrix multiplication, solutions of the sub-problems are stored in a _____.
   a. Array
   b. Table page 86
   c. Tree
   d. Link list

176. What is the average running time of a quick sort algorithm?
   a. O(n^2)
   b. O(n)
   c. O(n log n) (Page 49)
   d. O(log n)

177. Sorting Algorithms having O _____ running time are considered to be slow ones.
   a. (n)
   b. (n^2) (Page 39)
   c. (nlog(n))
   d. (log(n))

178. While solving Selection problem, in Sieve technique we partition input data _____
   a. In increasing order
   b. In decreasing order
   c. According to Pivot
   d. Randomly

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179. _____ is the process of avoiding unnecessary repetitions by writing down the results of recursive calls and looking them up again if we need them later.
   a. Loop
   b. Memoization page 74
   c. Recursion
   d. Function

180. In average-case time the probability of seeing input is denoted by _____.
   a. p{I}
   b. p[I]
   c. p<i>
   d. p(i) page 13

181. While applying the Sieve technique to selection sort, how to choose a pivot element.
   a. Through mean
   b. Linear
   c. Randomly page 35
   d. Sequentially

182. Number of _____ of the pseudo code are counted to measure the running time.
   a. Inputs
   b. Outputs
   c. Steps page 13
   d. Pages

183. Developing a dynamic programming algorithm generally involves _____ separate steps.
   a. One
   b. Two page 75
   c. Three
   d. Four

184. 8n^2+2n+3 will exceed c28(n), no matter how large we make _____.
   a. n
   b. 2n
   c. c2 page 25
   d. this quadratic equation

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185. The running time of quick sort algorithm_____.
   a. Is impossible to compute
   b. Has nothing to do with pivot selection
   c. Is Random upon each execution
   d. Greatly influenced by the selection of pivot page 49

186. _____ involves breaking up the problem into sub problems whose solutions can be combined to solve the global problem.
   a. Complexity Theory
   b. Dynamic programming solution
   c. Divide and Conquer Strategy page 34
   d. Greedy Algorithms

187. In _____ we have to find rank of an element from given input.
   a. Merge sort algorithm
   b. Selection problem page 34
   c. Brute force technique
   d. Plane Sweep algorithm

188. How many steps are involved to design the dynamic programming strategy?
   a. 2
   b. 3
   c. 1
   d. 4 page 92

189. In Bucket sort, if there are duplicates then each bin can be replaced by a a. Stack
   b. Linked list page 69
   c. Hash table
   d. Heap

190. In merge sort algorithm, we split the array _____ to find index q.
   a. from start
   b. midway page 28
   c. from end
   d. both from start or end

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191. Find the maximum value of the items which can carry using knapsack Knapsack weight capacity = 50.

Item Weight Value
11070
22020
33080
470 200

a. 280
b. 100
c. 90
d. 200

192. In 2-d maxima problem a point p is said to be dominated by point q if _____.

a. p.x <= q.x
b. p.x <= q.x and p.y <= q.y page 17
c. p.y <= q.y
d. p.x >= q.x and p.y >=q.y

193. Sorting can be in _____.

a. Increasing order only
b. Decreasing order only
c. Both increasing and decreasing order
d. Random order

194. Recurrence can be described in terms of _____.

a. Array
b. Linear
c. Tree page 31
d. Graph

195. The brute-force algorithm for 2D-Maxima runs in order O(__) time.

a. n
b. n(log n)
c. n*n page 18
d. n3

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196. In plane sweep approach of solving geometric problems, a _____ is swept across the plane.
   a. Line page 18
   b. Plane
   c. Cube
   d. Box

197. Which of the following is calculated with Big Omega notation?
   a. Medium bounds
   b. Upper bounds
   c. Lower bounds Page - 25
   d. Both upper and lower bounds

198. _____ is always based on divide and conquer strategy.
   a. Bubble sort
   b. Selection sort
   c. Pigeon sort
   d. Quick sort page 46

199. If a matrix has three rows and two columns, then dimensions of matrix will be:
   a. 3x2
   b. 2x3
   c. 3x3
   d. 2x2

200. Asymptotic notations are used to describe _____ of an algorithm.
   a. Length
   b. running time google
   c. size
   d. compile time

201. Catalan numbers are related the number of different _____ on 'n' nodes.
   a. Arrays
   b. linked lists
   c. binary trees page 85
   d. functions

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202. Applying the sieve technique to selection problem, _____ element is picked from array.
   a. Output
   b. Total
   c. Input
   d. Pivot page 35

203. Dynamic Programming approach is usually useful in solving _____ problems.
   a. Normal
   b. Optimization google
   c. Array
   d. Loop

204. In recursive formulation of knapsack Problem: V [0, j] = _____ for j>=0
   a. -1
   b. 0 page 93
   c. 1
   d. 2

205. _____ is a linear time sorting algorithm.
   a. Merge sort
   b. Radix sort page 71
   c. Quick sort
   d. Bubble sort

206. Quick sort is one of the _____ sorting algorithm.
   a. Fastest page 19
   b. Slowest
   c. Major
   d. Average

207. The time assumed for each basic operation to execute on RAM model of computation is _____.
   a. Infinite
   b. Continuous
   c. Constant page 10
   d. Variable

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208. In Sieve Technique, we know the item of interest. a. True
b. False page 34

209. While analyzing algorithms, _____ and _____ usually considered difficult to calculate.
a. Finite, Infinite
b. Floor, ceiling google
c. Row, Column
d. Graph, Tree

210. While analysis of the brute-force maxima algorithm, an array sorted in the reverse order is the type of _____ case input.
a. Best
b. Worst page 14
c. Somewhat bad
d. Average

211. _____ is not useful measure of central tendency of given input set especially when the distribution of values is highly skewed.
a. Mean
b. Mode
c. Average
d. Median page 34

212. In asymptotical analysis of n(n-3) and 4n*n, as n becomes large, the dominant (fastest growing) term is some constant times _____.
a. n+1
b. n-1
c. n
d. n*n page 23

213. In addition to passing in the array itself to Merge Sort algorithm, we will pass in other arguments which are indices.
a. Two P-38
b. Three
c. Four
d. Five

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214. In 2d-maximal problem, a point is said to be if it is not dominated by any other point in that space.
   a. Member
   b. Minimal
   c. Maximal P-11
   d. Joint

215. Counting sort assumes that the numbers to be sorted are in the range _____.
   a. K to n where n is large
   b. 1 to k where k is small (P-57)
   c. K to n where k is small
   d. k to n where n is small

216. Insertion sort is an efficient algorithm for sorting a _____ number of elements
   a. Large P-39
   b. Small
   c. Extra large
   d. Medium

217. If the indices passed to merge sort algorithm are _____ then this means that there is only one element to sort.
   a. Small page 28
   b. Large
   c. Equal
   d. Not Equal

218. In Knapsack Problem, each item must be entirely accepted or rejected, is called _____ problem.
   a. Fractional
   b. 0-1 P-92
   c. Linear
   d. Optimal

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219. If the time complexity of an algorithm is O(n). then it is called _____ time complexity.
   a. Linear Wikipedia
   b. Constant
   c. Average
   d. Exponential

220. In the case of _____ analysis does not depend upon on the distribution of input.
   a. Merge sort
   b. Quick sort P-50
   c. Insertion sort
   d. Heap sort

221. We can use the _____ Property to devise a recursive formulation of the edit distance problem.
   a. Small substructure
   b. Algorithmic
   c. Real
   d. Optimal substructure page 78

222. The following sequence is called _____
   1,2,3,5,8,13,21,34,55,…..
   a. Optimize sequence
   b. Fibonacci sequence page 73
   c. Optimal sequence
   d. Overlapping sequence

223. Which one sorting algorithm is best suited to sort an array of 2 million elements?
   a. Bubble sort
   b. Insert sort
   c. Merge sort
   d. Quick sort
   e. Ridx Sort page 71

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224. We can improve the performance of quick sort if we could be able to _,_____.
   a. Skip input elements somehow
   b. Select two or more pivots page 34
   c. Skip any sub-array completely
   d. Eliminate recursive calls

225. The problem with the brute-force algorithm is that is uses _____ in pruning out de
   a. Worst-case time
   b. No intelligence page 18
   c. Outside looping
   d. Artificial intelligence

226. In chain matrix multiplication, the order of the matrices _____.
   a. Can be changed
   b. Can not be changed page 85
   c. is equal
   d. is reverse

227. In quick sort algorithm, we choose pivot_____.
   a. Always the smallest element
   b. Greater than 5
   c. Randomly page 35
   d. Less than 5

228. In Heap Sort algorithm. Heapify procedure is _____ in nature.
   a. Recursive
   b. Non-Recursive page 43
   c. Fast
   d. Slow

229. When matrix A of 5x 3 is multiplied with matrix B of 3 x 4 then the number of multiplications required will be _____.
   a. 15
   b. 12
   c. 36
   d. 60

230. An algorithm is said to be correct if for every _____ instance, it halts with the correct _____.
   a. Input, Output page 13
   b. Design, Analysis
   c. Value, Key
   d. Key, Analysis

231. In chain matrix multiplication, table is filled _____ to find the multiplication of matrix.
   a. row wise
   b. column wise
   c. diagonally
   d. bottom-to-up page 86

232. If we have an equation 8n2+7f*n + 5f + 6 then is large, _____ term will be muchxxxxxxxthe n term and will dominate the running time.
   a. f g (n)
   b. g (n) * 2
   c. n * 2 page 23
   d. f (n)

233. For quick sort algorithm. Partitioning takes theta _____.
   a. (n)
   b. log(n)
   c. n log (n)
   d. n2log (n)

234. In Heap Sort algorithm, the maximum levels an element can move upward is _____
   a. Theta (log n) page 43
   b. Big-ch (log n)
   c. Omega (log n)
   d. 0 (1) i.e. Constant time

235. _____ programming is essentially recursion without repetition.
   a. Fast
   b. Dynamic page 75
   c. Array

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d. n (log n)

236. In heap sort algorithm, the total running time for heavily procedure is _.

- Big-oh(log n)
- O (1) i.e. Constant time
- Theta (log n)
- Omega (log n)

237. Quick sort algorithm is required a lot of comparison in the ____ condition.

- Worse case
- Best and average case
- Average case
- Best case

238. In heap sort algorithm (using max heap). When every time maximum element is removed from top.

- Divide and conquer strategy helps us
- We are left with a lot
- We call merge sort algorithm
- It becomes order n2 algorithm

239. In average-case time analysis of quick sort algorithm, The most balanced case for partition is where we divide the list of element into _____

- Three nearly equal pieces
- Single piece exactly
- Two nearly piece
- Equal no. of piece as of input element

240. Consider three matrices X,Y,Z dimensions 1×2.2×.3×4 respectively. The number of multiplication of (XYZ) is:

- 32

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- 30
- 24
- 18

241. Quicksort is a/an _____ and_____ sorting algorithm.

- In-place, not stable one
- Not in-place, stable one
- In-place, stable one
- Not in-place, not stable one

242. _____items are not allowed in the 0/1 knapack.

- Lighter
- Whole
- Weighty
- Fractional

243. The main purpose of mathematical analysis is measuring the _____ required by the algorithm.

- Space
- Execution time and memory
- Input & output
- Execution time

244. Execution time of an algorithm can be measured by _____.

- Divide and conquer approach
- Both brute force and divide and conquer approach
- Mathematical analysis
- Brute force approach

245. Quick sort is based on _____ strategy.

- Graph theory
- Divide-and-conquer
- Dynamic programming
- Greedy approach

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246. A sorting algorithm is called as _____ if duplicate element remain in the same relative position after sorting.

- O(n) algorithm
- Stable
- Parallel
- Complex

247. Which one sorting algorithm is best suited to sort an array of 2 million elements?

- Insert sort
- Quick sort
- Merge sort
- Bubble sort

248. We can use the _____ property to devise a recursive formulation of the edit distance problem.

- Algorithm
- Small substructure
- Optimal substructure
- Real

249. While sorting. The ordered domain means for any two input elements x and y _____ satisfies only.

- All of the above
- x > y
- x < y
- x = y

250. 8n2 + 2n -3 will eventually exceed c2*(n) no matter how large we make _____.

- 2n
- n
- this equation

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- c2

251. _____ is a method of solving a problem in which we check all possible solution to the problem to find the solution we need.

- Sorting algorithm
- Greedy approach
- Plane-sweep algorithm
- Brute-force algorithm

252. In quick sort algorithm, provost form _____.

- Graph
- Stack
- Binary search tree
- Queue

253. In asymptotical analysis of n(n -3) and 4n*n, as n becomes large, the dominant (fastest growing) term is some constant time_____

- n +1
- n*n
- n
- n-1

254. If Matrix-A has dimensions "3×2" and Matrix-B has dimensions "2×3", then multiplication of Matrix –A and Matrix-B will result a new Matrix-C having dimensions

- 2×3
- 2×2
- 3×2
- 3×3

255. Boolean operation is a _____ operation on an idealized RAM model of computation.

- Advance
- Normal

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- Basic
- Starting

256. There are _____ entries in the Edit Distance Matrix.

- $\Theta(n^2)$
- Θ (n+100)
- Θ (n)
- Θ (n+2)

257. Counting sort is suitable for sorting the elements within range 1 to P. where_____

- P is undetermined
- P is small
- P is very large
- P is large

258. Suppose we have 4 matrices A,B,C,D. what is correct expansion of m[1,2] in chain matrix multiplication?

- m[1,2] = m[1,1] + m[2,2] +p0 . p1. p3
- m[1,2] = m[1,1] + m[2,2] +p0 . p1. p3
- m[1,2] = m[1,1] + m[2,2] +p0 . p1. p3
- m[1,2] = m[1,1] + m[2,2] +p0 . p1. p3

259. Which one is not passed as parameter in Quick Sort algorithm?

- Array (containing input elements)
- Middle of the array
- Start of the array
- End of the array

260. In asymptotical analysis of n*(5+2)-3. As n becomes large, the dominant (fastest growing) term is some constant times_____

- n+1
- n*n
- n

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- n_1

261. For _____ values of n, any algorithm is fast enough.

- Medium
- Small
- Infinity
- Large

262. Dynamic programming algorithms often use some kind of _____ to store the result of intermediate sub-problems.

- Stack
- Loop
- Table
- Variable

263. In selection problem, the Sieve technique works in_____

- One complete go
- Constant time
- Non-recursive manner
- Phases

264. In heap Sort algorithm, the maximum levels an element can move upward is _____.

- Theta (log n)
- O (1) i.e. Constant time
- Omega (log n)
- Big-oh (log n)

265. While analysis of the brute-force maxima algorithm, an array storted in the reverse order is the type of _____ case input.

- Worst
- Best
- Somewhat
- Average

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266. What type of instructions Random Access Machine (RAM) can execute?

►Algebraic and logic

►Geometric and arithmetic

►Arithmetic and logic (Page 10)

►Parallel and recursive

267. For Chain Matrix Multiplication we can not use divide and conquer approach because,

►We do not know the optimum k (Page 86)

►We use divide and conquer for sorting only

►We can easily perform it in linear time

►Size of data is not given

268. What is the total time to heapify?

- ► O(log n) (Page 43)
- ► O(n log n)
- ► O(n2 log n)
- ► O(log2 n)

269. _____ is a graphical representation of an algorithm

- ► notation
- ► notation
- ► Flowchart
- ► Asymptotic notation

270. A RAM is an idealized machine with _____ random-access memory.

- ► 256MB
- ► 512MB
- ► an infinitely large (Page 10)
- ► 100GB

271. What type of instructions Random Access Machine (RAM) can execute? Choose best answer

► Algebraic and logic

► Geometric and arithmetic

► Arithmetic and logic

► Parallel and recursive

272. What is the solution to the recurrence T(n) = T(n/2)+n .

► O(logn)

► O(n) (Page 37)

► O(nlogn)

► O(n 2)

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Mid

Quiz by Attiq Kundi

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CS-502

MIDTERM EXAMINATION

Fall 2011

CS502- Fundamentals of Algorithms

Question No: 1 (Marks: 1) - Please choose one

Due to left complete nature of binary tree, the heap can be stored in

► **Arrays (Page 40)**
► Structures
► Link Lis
►Stack

Question No: 1 (Marks: 1) - Please choose one

What type of instructions Random Access Machine (RAM) can execute?

►Algebraic and logic

►Geometric and arithmetic

►**Arithmetic and logic (Page 10)**

►Parallel and recursive

Question No: 1 (Marks: 1) - Please choose one

For Chain Matrix Multiplication we can not use divide and conquer approach because,

►**We do not know the optimum k (Page 86)**

►We use divide and conquer for sorting only

►We can easily perform it in linear time

►Size of data is not given

Question No: 1 (Marks: 1) - Please choose one

What is the total time to heapify?

► **O(log n) (Page 43)**
► O(n log n)
► O(n^2 log n)
► O($\log^2$ n)

Question No: 1 (Marks: 1) - Please choose one

word Algorithm comes from the name of the muslim author _____

►**Abu Ja'far Mohammad ibn Musa al-Khowarizmi**.

Question No: 1 (Marks: 1) - Please choose one al-Khwarizmi's work was written in a book titled _____

►**al Kitab al-mukhatasar fi hisab al-jabr wa'l-muqabalah**

MIDTERM EXAMINATION

Spring 2010

CS502- Fundamentals of Algorithms

Question No: 1 (Marks: 1) - Please choose one
Random access machine or RAM is a/an

► Machine build by Al-Khwarizmi
► Mechanical machine
► Electronics machine

► **Mathematical model (Page 10)**

Question No: 2 (Marks: 1) - Please choose one

_____ is a graphical representation of an algorithm

► Σ notation
► Θ notation
► **Flowchart** **Click here for detail**
► Asymptotic notation

Question No: 3 (Marks: 1) - Please choose one

A RAM is an idealized machine with _____ random-access memory.

► 256MB
► 512MB

► **an infinitely large (Page 10)**
► 100GB

Question No: 4 (Marks: 1) - Please choose one

What type of instructions Random Access Machine (RAM) can execute? Choose best answer

► Algebraic and logic
► Geometric and arithmetic

► **Arithmetic and logic (Rep)**
► Parallel and recursive

Question No: 5 (Marks: 1) - Please choose one

What will be the total number of max comparisons if we run brute-force maxima algorithm with n elements?

► n^2

n

► n^2

► n **(Page 14)**

► n^8

Question No: 6 (Marks: 1) - Please choose one
What is the solution to the recurrence $T(n) = T(n/2)+n$.

► $O(\log n)$

► ***O*(*n*) (Page 37)**
► $O(n\log n)$
► $O(n^2)$

Question No: 7 (Marks: 1) - Please choose one

Consider the following code:

```
For(j=1; j<n;j++)
        For(k=1; k<15;k++)
                For(l=5; l<n; l++)
                {
```

Do_something_constant();

}

What is the order of execution for this code.

- ► **O(n^3)**
- ► O(n_2)
- ► O(n^2 log n)
- ► O(n^2)

Question No: 8 (Marks: 1) - Please choose one

What is the total time to heapify?

- ► **O(log n) rep**
- ► O(n log n)
- ► O(n^2 log n)
- ► O($\log^2$ n)

Question No: 9 (Marks: 1) - Please choose one

Consider the following Algorithm:

```
Factorial (n){
  if (n=1)
    return 1
  else
     return (n * Factorial(n-1))
```

Recurrence for the following algorithm is:

- ► **T(n) = T(n-1) +1**
- ► **T(n) = nT(n-1) +1**
- ► **T(n)= T(n-1) +n**
- ► **T(n)=T(n(n-1)) +1**

Question No: 10 (Marks: 1) - Please choose one

When we call heapify then at each level the comparison performed takes time

► **It will take Θ (1)** **(Page 43)**

► Time will vary according to the nature of input data
► It can not be predicted
► It will take Θ (log n)

Question No: 11 (Marks: 1) - Please choose one

In Quick sort, we don't have the control over the sizes of recursive calls

► **True(Page 40)**
► False
► Less information to decide
► Either true or false

Question No: 12 (Marks: 1) - Please choose one Is it possible to sort without making comparisons?

► **Yes(Page 57)**
► No

Question No: 13 (Marks: 1) - Please choose one

If there are Θ (n^2) entries in edit distance matrix then the total running time is ►Θ(1)

► **Θ (n^2)** **Click here for detail**

► Θ (n)

► Θ (n log n)

Question No: 14 (Marks: 1) - Please choose one

For Chain Matrix Multiplication we can not use divide and conquer approach because,

► **We do not know the optimum k** **(Page 86)**

► We use divide and conquer for sorting only
► We can easily perform it in linear time

► Size of data is not given

Question No: 15 (Marks: 1) - Please choose one

The Knapsack problem belongs to the domain of _____ problems.

► **Optimization (Page 91)**

► NP Complete
► Linear Solution
► Sorting

Question No: 16 (Marks: 1) - Please choose one

Suppose we have three items as shown in the following table, and suppose the capacity of the knapsack is 50 i.e. W = 50.

Item	Value	Weight
1	60	10
2	100	20
3	120	30

The optimal solution is to pick

► Items 1 and 2
► Items 1 and 3

► **Items 2 and 3 (correct)**
► None of these

MIDTERM EXAMINATION

Spring 2010

CS502- Fundamentals of Algorithms

Question No: 1 (Marks: 1) - Please choose one

For the Sieve Technique we take time

► **T(nk) (Page 34)**

►T(n / 3)

►n^2

►n/3

Question No: 1 (Marks: 1) - Please choose one

Sieve Technique applies to problems where we are interested in finding a single item

from a larger set of _____

Select correct option:

►**n items (Page 34)**

►phases

►pointers

►constant

Question No: 1 (Marks: 1) - Please choose one

_____ graphical representation of algorithm.

►asymptotic

►**Flowchart** **(rep)**

Question No: 1 (Marks: 1) - Please choose one

who invented the quick sort

►**C.A.R. Hoare** Click here for detail

Question No: 1 (Marks: 1) - Please choose one

main elements to a divide-and-conquer

►**Divide, conquer, combine (Page 27)**

Question No: 1 (Marks: 1) - Please choose one Mergesort is a stable algorithm but not an in-place algorithm.

►**True (Page 54)**

►false

Question No: 1 (Marks: 1) - Please choose one

Counting sort the numbers to be sorted are in the range 1 to k where k is small.

►**True (Page 57)**

►False

MIDTERM EXAMINATION

Spring 2007

CS502- Fundamentals of Algorithms

Question No: 1 (Marks: 1) - Please choose one

Total time for heapify is:

►$O(\log^2 n)$

►O (n log n)

►$O(n^2 \log n)$

►**O (log n)** **Rep**

Question No: 1 (Marks: 1) - Please choose one

If an algorithm has a complexity of $\log_2 n + n\log_2 n + n$. we could say that it has complexity

►O(n)

►$O(n \log_2 n)$

►O(3)

►$O(\log_2 (\log_2 n))$

►$O(\log_2 n)$

Question No: 1 (Marks: 1) - Please choose one In RAM model instructions are executed

►**One after another (Page 10)**

►Parallel

►Concurrent

►Random

Question No: 1 (Marks: 1) - Please choose one

In selection algorithm, because we eliminate a constant fraction of the array with each phase, we get the

►**Convergent geometric series (Page 37)**

►Divergent geometric series

►None of these

Question No: 1 (Marks: 1) - Please choose one

Due to left-complete nature of binary tree, heaps can be stored in

►Link list

►Structure

►**Array (Page 40)**

►None of above

CS609- System Programming

Midterm Quizzes (Quiz No.1 & 2)

Quiz No.1 (04 – MAY - 2013)

Question No: 1 (Marks: 1) - Please choose one

The time assumed for each basic operation to execute on RAM model of computation is-----

Infinite

Continuous

Constant (Page 10)

Variable

Question No: 1 (Marks: 1) - Please choose one

If the indices passed to merge sort algorithm are not equal, the algorithm may return immediately.

True

False (Page 28)

Question No: 1 (Marks: 1) - Please choose one Brute-force algorithm
uses no intelligence in pruning out decisions.

True (Page 18)

False

Question No: 1 (Marks: 1) - Please choose one

In analysis, the Upper Bound means the function grows asymptotically no faster than its largest term.

True (Page 24)

False

Question No: 1 (Marks: 1) - Please choose one

For small values of n, any algorithm is fast enough. Running time does become an issue when n gets large.

True (Page 14)

Fast

Question No: 1 (Marks: 1) - Please choose one

The array to be sorted is not passed as argument to the merge sort algorithm.

True

False

Question No: 1 (Marks: 1) - Please choose one

In simple brute-force algorithm, we give no thought to efficiency.

True
False **(Page 11)**

Question No: 1 (Marks: 1) - Please choose one

The ancient Roman politicians understood an important principle of good algorithm design that is plan-sweep algorithm.

True

False (Page 27) [Divide and Conquer]

Question No: 1 (Marks: 1) - Please choose one

In 2d-space a point is said to be _____if it is not dominated by any other point in that space.

Member

Minimal

Maximal (Page 11)

Joint

Question No: 1 (Marks: 1) - Please choose one

An algorithm is a mathematical entity that is dependent on a specific programming language.

True **False**

Question No: 1 (Marks: 1) - Please choose one

The running time of an algorithm would not depend upon the optimization by the compiler but that of an implementation of the algorithm would depend on it.

True (Page 13)

False

Question No: 1 (Marks: 1) - Please choose one

F (n) and g (n) are asymptotically equivalent. This means that they have essentially the same _____ for large n.

Results

Variables

Size

Growth rates (Page 23)

Question No: 1 (Marks: 1) - Please choose one

8n2 + 2n - 3 will eventually exceed c2*(n) no matter how large we make c2.

True (Page 25)

False

Question No: 1 (Marks: 1) - Please choose one

If we associate (x, y) integers pair to cars where x is the speed of the car and y is the negation of the price. High y value for a car means a _____ car.

Fast

Slow

Expensive

Cheap (Page 11)

Question No: 1 (Marks: 1) - Please choose one

The function f(n)= n(logn+1)/2 is asymptotically equivalent to n log n. Here Upper Bound means the function f(n) grows asymptotically _____ faster than n log n.

More

Quiet

Not (Page 24)

At least

Question No: 1 (Marks: 1) - Please choose one

After sorting in merge sort algorithm, merging process is invoked.

Select correct option:

True (Page 28)

False

Question No: 1 (Marks: 1) - Please choose one

Asymptotic growth rate of the function is taken over_____ case running time.

Select correct option:

Best

Average

Worst (Page 14)

Normal

Question No: 1 (Marks: 1) - Please choose one

In analysis of f (n) =n (n/5) +n-10 log n, f (n) is asymptotically equivalent to _____.

n

2n

n+1

n2 (Page 23)

Question No: 1 (Marks: 1) - Please choose one

Algorithm is concerned with.....issues.

Macro

Micro

Both Macro & Micro (Page 8)

Normal

Question No: 1 (Marks: 1) - Please choose one

We cannot make any significant improvement in the running time which is better than that of brute-force algorithm.

True

False (Page 18)

Question No: 1 (Marks: 1) - Please choose one

In addition to passing in the array itself to Merge Sort algorithm, we will pass in _____other arguments

which are indices.

Two (Page 28)

Three

Four

Five

Question No: 1 (Marks: 1) - Please choose one

Consider the following Algorithm: Fun(n){ if (n=1) return 1 else return (n * Fun(n-1)) } Recurrence for the above algorithm is:

nT(n-1)+1

2T(n-1)+1

T(n-1)+cn

T(n-1)+1

Question No: 1 (Marks: 1) - Please choose one

In analysis, the Lower Bound means the function grows asymptotically at least as fast as its largest term.

True (Page 24)

False

Question No: 1 (Marks: 1) - Please choose one

Efficient algorithm requires less computational…….

Memory

Running Time

Memory and Running Time (Page 9)

Energy

Question No: 1 (Marks: 1) - Please choose one

The O-notation is used to state only the asymptotic _____bounds.

Two

Lower

Upper (Page 25)

Both lower & upper

Question No: 1 (Marks: 1) - Please choose one

For the worst-case running time analysis, the nested loop structure containing one “for” and one “while” loop, might be expressed as a pair of _____nested summations.

1

2 (Page 16)

3

4

Question No: 1 (Marks: 1) - Please choose one

Before sweeping a vertical line in plane sweep approach, in start sorting of the points is done in increasing order of their _____coordinates.

X (Page 18)

Y

Z

X & Y

Question No: 1 (Marks: 1) - Please choose one

Brute-force algorithm for 2D-Maxima is operated by comparing _____ pairs of points.

Two

Some

Most

All (Page 18)

Question No: 1 (Marks: 1) - Please choose one

The function f(n)=n(logn+1)/2 is asymptotically equivalent to nlog n. Here Lower Bound means function f(n) grows asymptotically at _____ as fast as nlog n.

Normal

Least (Page 23)

Most

All

Question No: 1 (Marks: 1) - Please choose one

The definition of Theta-notation relies on proving _____asymptotic bound.

One

Lower

Upper

Both lower & upper (Page 25) rep

Question No: 1 (Marks: 1) - Please choose one

In plane sweep approach, a vertical line is swept across the 2d-plane and _____structure is used for holding the maximal points lying to the left of the sweep line.

Array

Queue

Stack (Page 18)

Tree

Question No: 1 (Marks: 1) - Please choose one

Algorithm analysts know for sure about efficient solutions for NP-complete problems.

Select correct option:

True

False (Page 9)

Quiz No.1 (2012)

Question No: 1 of 10 (Marks: 1) - Please choose one The number of nodes in a complete binary tree of height h is

2^(h+1) – 1 (Page 40)

2 * (h+1) – 1

2 * (h+1)

((h+1) ^ 2) – 1

Question No: 1 of 10 (Marks: 1) - Please choose one

The analysis of Selection algorithm shows the total running time is indeed _____in n,

arithmetic

geometric

linear (Page 37)

orthogonal

Question No: 1 of 10 (Marks: 1) - Please choose one

A (an) _____ is a left-complete binary tree that conforms to the heap order

heap (Page 40)

binary tree

binary search tree

array

Question No: 1 of 10 (Marks: 1) - Please choose one Analysis of Selection algorithm ends up with,

T(n) (Page 37)

T(1 / 1 + n)

T(n / 2)

T((n / 2) + n)

Question No: 1 of 10 (Marks: 1) - Please choose one For the sieve technique we solve the problem,

recursively (Page 34)

mathematically

precisely

accurately

Question No: 1 of 10 (Marks: 1) - Please choose one

A heap is a left-complete binary tree that conforms to the _____

increasing order only

decreasing order only

heap order (Page 40)

(log n) order

Question No: 1 of 10 (Marks: 1) - Please choose one In which order we can sort?

increasing order only

decreasing order only

increasing order or decreasing order (Page 39)

both at the same time

Question No: 1 of 10 (Marks: 1) - Please choose one Divide-and-conquer as breaking the problem into a small number of

pivot

Sieve

smaller sub problems (Page 34)

Selection

Question No: 1 of 10 (Marks: 1) - Please choose one For the heap sort we store the tree nodes in

level-order traversal (Page 40)

in-order traversal

pre-order traversal

post-order traversal

Question No: 1 of 10 (Marks: 1) - Please choose one

The sieve technique works in _____ as follows

Phases (Page 34)

numbers

integers

routines

CS502 - Fundamentals of Algorithms

Quiz No.1 12-11-2012

Question No: 1 of 10 (Marks: 1) - Please choose one
We do sorting to,

keep elements in random positions

keep the algorithm run in linear order

keep the algorithm run in (log n) order

keep elements in increasing or decreasing order

Question No: 1 of 10 (Marks: 1) - Please choose one

Heaps can be stored in arrays without using any pointers; this is due to the _____ nature of the binary

tree,

left-complete(Page 40)

right-complete

tree nodes

tree leaves

Question No: 1 of 10 (Marks: 1) - Please choose one

Sieve Technique can be applied to selection problem?

True (Page 35)

False

Question No: 1 of 10 (Marks: 1) - Please choose one

In Sieve Technique we do not know which item is of interest

True (Page 34)

False

Question No: 1 of 10 (Marks: 1) - Please choose one

In the analysis of Selection algorithm, we eliminate a constant fraction of the array with each phase; we get the convergent _____ series in the analysis,

linear

arithmetic

geometric (Page 37)

exponent

Question No: 1 of 10 (Marks: 1) - Please choose one

For the heap sort, access to nodes involves simple _____ operations.

arithmetic (Page 41)

binary

algebraic

logarithmic

Question No: 1 of 10 (Marks: 1) - Please choose one Slow sorting algorithms run in,

T(n^2) (Page 39)

T(n)

T(log n)

Question No: 1 of 10 (Marks: 1) - Please choose one

In the analysis of Selection algorithm, we make a number of passes, in fact it could be as many as,

T(n)

T(n / 2)

log n (Page 37)

n / 2 + n / 4

Question No: 1 of 10 (Marks: 1) - Please choose one

The sieve technique is a special case, where the number of sub problems is just

5

many

1 (Page 34)

few

Question No: 1 of 10 (Marks: 1) - Please choose one

How many elements do we eliminate in each time for the Analysis of Selection algorithm?

(n / 2)+n elements

(n / 2) elements (Page 37)

n / 4 elements

2 n elements

Question No: 1 of 10 (Marks: 1) - Please choose one

One of the clever aspects of heaps is that they can be stored in arrays without using any _____.

pointers (Page 40)

constants

variables

functions

Question No: 1 of 10 (Marks: 1) - Please choose one

How much time merge sort takes for an array of numbers?

T(n^2)

T(n)

T(log n)

T(n log n) (Page 40)

Question No: 1 of 10 (Marks: 1) - Please choose one

The reason for introducing Sieve Technique algorithm is that it illustrates a very important special case of,

divide-and-conquer (Page 34)

decrease and conquer

greedy nature

2-dimension Maxima

Question No: 1 of 10 (Marks: 1) - Please choose one In Sieve Technique we do not know which item is of interest

True (Page 34) rep

False

Question No: 1 of 10 (Marks: 1) - Please choose one

Theta asymptotic notation for T (n) :

Set of functions described by: c1g(n)Set of functions described by c1g(n)>=f(n) for c1 s

Theta for T(n)is actually upper and worst case comp (Not sure)
Set of functions described by: c1g(n)

Question No: 1 of 10 (Marks: 1) - Please choose one

Memoization is?

To store previous results for future use

To avoid this unnecessary repetitions by writing down the results of recursive calls and looking them up again if we need them later (page 74)

To make the process accurate

None of the above

Question No: 1 of 10 (Marks: 1) - Please choose one
Which sorting algorithm is faster

O (n log n) Page 26

O n^
2 O
(n+k)
O
n^3

Question No: 1 of 10 (Marks: 1) - Please choose one Quick
sort is

Stable & in place

Not stable but in place (Page 54)

Stable but not in place

Some time stable & some times in place

Question No: 1 of 10 (Marks: 1) - Please choose one One
example of in place but not stable algorithm is

Merger Sort

Quick Sort (Page 54)

Continuation Sort

Bubble Sort

Question No: 1 of 10 (Marks: 1) - Please choose one

Cont sort is suitable to sort the elements in range 1 to k

K is Large

K is not known

K may be small or large

K is small (Page 57)

Question No: 1 of 10 (Marks: 1) - Please choose one

In place stable sorting algorithm.

If duplicate elements remain in the same relative position after sorting (Page 54)

One array is used

More than one arrays are required

Duplicating elements not handled

Question No: 1 of 10 (Marks: 1) - Please choose one

Which may be a stable sort?

Merger

Insertion (Page 54)

Both above

None of the above

Question No: 1 of 10 (Marks: 1) - Please choose one

An in place sorting algorithm is one that uses ___ arrays for storage

Two dimensional arrays

More than one array

No Additional Array (Page 54)

None of the above

Question No: 1 of 10 (Marks: 1) - Please choose one

Sieve Technique applies to problems where we are interested in finding a single item from a larger set of

_____

n items (Page 34)

phases

pointers

constant

Question No: 1 of 10 (Marks: 1) - Please choose one

Sorting is one of the few problems where provable _____ bonds exits on how fast we can sort,

upper

lower (Page 39)

average

log n

Question No: 1 of 10 (Marks: 1) - Please choose one

Counting sort has time complexity:

O(n) (Page 58)

O(n+k)

O(k)

O(nlogn)

Question No: 1 of 10 (Marks: 1) - Please choose one

The running time of quick sort depends heavily on the selection of

No of inputs

Arrangement of elements in array

Size o elements

Pivot elements (Page 49)

Question No: 1 of 10 (Marks: 1) - Please choose one

Which may be stable sort:

Bubble sort

Insertion sort

Both of above (Page 54)

Question No: 1 of 10 (Marks: 1) - Please choose one

One Example of in place but not stable sort is

Quick (Page 54)

Heap

Merge

Bubble

Question No: 1 of 10 (Marks: 1) - Please choose one
In Quick Sort Constants hidden in T(n log n) are

Large

Medium

Small Click here for detail

Not Known

Question No: 1 of 10 (Marks: 1) - Please choose one

Quick sort is based on divide and conquer paradigm; we divide the problem on base of pivot element and:

There is explicit combine process as well to conquer the solution.

No work is needed to combine the sub-arrays, the array is already sorted

Merging the sub arrays

None of above. (Page 51)

Ref: - random choices for the pivot element and each choice have an equal probability of 1/n of occurring. So we can modify the above recurrence to compute an average rather than a max

CS501 - Quiz No.2 (Spring 2013)

Question No: 1 of 10 (Marks: 1) - Please choose one

A point p in 2-dimensional space is usually given by its integer coordinate(s)_____

p.x only

p.y only

p.x & p.z

p.x & p.y (Page 10)

Question No: 1 of 10 (Marks: 1) - Please choose one

In _____ we have to find rank of an element from given input.

Merge sort algorithm

Selection problem (Page 34)

Brute force technique

Plane Sweep algorithm

Question No: 1 of 10 (Marks: 1) - Please choose one

In Heap Sort algorithm, if heap property is violated _____

We call Build heap procedure

We call Heapify procedure

We ignore

Heap property can never be violated

Question No: 1 of 10 (Marks: 1) - Please choose one

Upper bound requires that there exist positive constants c2 and n0 such that f(n) ____ c2n for all n <= n0(ye

question ghalat lag raha hai mujhae

Less than

Equal to or Less than Equal or Greater than Greater than **(Page 25)**

Question No: 1 of 10 (Marks: 1) - Please choose one

A RAM is an idealized algorithm with takes an infinitely large random-access memory.

True

False (Page 10)

Question No: 1 of 10 (Marks: 1) - Please choose one

_____ is one of the few problems, where provable lower bounds exist on how fast we can sort.

Searching

Sorting (Page)

Both Searching & Sorting

Graphing

Question No: 1 of 10 (Marks: 1) - Please choose one

Floor and ceiling are _____ to calculate while analyzing algorithms.

Very easy

Usually considered difficult (Page 31)

Question No: 1 of 10 (Marks: 1) - Please choose one

In Heap Sort algorithm, the maximum levels an element can move upward is _____

Theta (log n) (Page 43)

Order (log n)

Omega (log n)

O (1) i.e. Constant time

Question No: 1 of 10 (Marks: 1) - Please choose one

A point p in 2-dimensional space is usually given by its integer coordinate(s)_____

p.x only p.y

only p.x & p.z

p.x & p.y (Page 17)

Question No: 1 of 10 (Marks: 1) - Please choose one

In Heap Sort algorithm, the total running time for Heapify procedure is _____

Theta (log n) (Page 43)

Order (log n)

Omega (log n)

O (1) i.e. Constant time

Question No: 1 of 10 (Marks: 1) - Please choose one

Algorithm is a mathematical entity, which is independent of a specific machine and operating system.

True

False (Page 7)

Question No: 1 of 10 (Marks: 1) - Please choose one

While Sorting, the ordered domain means for any two input elements x and y _____ satisfies only.

x < y

x > y

x = y

All of the above (Page 39)

Question No: 1 of 10 (Marks: 1) - Please choose one Quick sort is best from the perspective of Locality of reference.

True
False **(Page 9)**

Question No: 1 of 10 (Marks: 1) - Please choose one

Sorting can be in _____

Increasing order only

Decreasing order only

Both Increasing and Decreasing order (Page 39)

Random order

Question No: 1 of 10 (Marks: 1) - Please choose one

In Heap Sort algorithm, we build _____ for ascending sort.

Max heap
Min heap **(Page 41)**

Question No: 1 of 10 (Marks: 1) - Please choose one In Sieve Technique, we know the item of interest.

True

False (Page 34)

Question No: 1 of 10 (Marks: 1) - Please choose one

While solving Selection problem, in Sieve technique we partition input data _____

In increasing order

In decreasing order

According to Pivot (Page 3

Randomly

Question No: 1 of 10 (Marks: 1) - Please choose one

In pseudo code, the level of details depends on intended audience of the algorithm.

True (Page 12)

False

Question No: 1 of 10 (Marks: 1) - Please choose one

The sieve technique works where we have to find _____ item(s) from a large input.

Single (Page 34)

Two

Three

Similar

Question No: 1 of 10 (Marks: 1) - Please choose one

If the indices passed to merge sort algorithm are _____,then this means that there is only one element to sort.

Small

Large

Equal (Page 28)

Not Equal

1) The sieve technique works where we have to find ---------- item(s) from a large input.

- **Single page 34**
- Two
- Three
- Similar

2) The only way to convert an empty string into a string of j characters is by doing j insertions , represented as

- E(i , j)=I
- E(i , 0)=i
- **E(0 , j)=j page 78**
- E(1 , j)=j

3) In Selection problem , the Sieve technique works in ----------

- Non-recursive manner
- Constant time
- **Phases page 34**
- One complete go

4) Algorithm is a sequence of computational steps that --------- the input into output.

- Merge
- Assign
- **Transform page 7**
- Integrate

5) In max heap (for Heap Sort algorithm), when every time maximum element is removed from top we replace it with ---------- in the leaf tree.

- Second last
- **Last page 41**
- First
- Any

6) If pj dominates pi and pi dominates ph then pj also dominates ph. It means dominance relation is

- **Transitive page 18**
- Non transitive
- Equation

Symbolic

7) In the following code the statement “cout<<j;” executes ---------- times

```
For(j=1;j<=5;j=j+2)
Cout<<j;
```

- 5 times
- 2 times
- **3 times (conceptual)**
- 0 times

9) For average-case time analysis of Quick sort algorithm, Pivot selection is on average basis from ----------.

- Half of the input values
- **All possible random values page 50**
- Pivot is input separately
- Values greater than 5

10) In the statement, “output P[i].x,P[i].y”, the number of times elements of P are accessed is ----------.

- 1
- **2 page 14**
- 3
- 4

11) Identify the TRUE statement.

- The Knapsack problem does not belong to the domain of optimization problems.
- **The Knapsack problem belongs to the domain of optimization problems Page 91**
- The Knapsack problem cannot be solved by using dynamic programming.
- The Knapsack problem is optimally solved by using brute force algorithm.

12) Matrix multiplication is a(n) ---------- operation.

- Commutative
- **Associative page 85**
- Neither commutative nor associative
- Commutative but not associative

13) ---------- provides us more accurate result when input values are not closer to each other.

- Average
- **Median page 34(conceptual)**

14) In merge sort algorithm, we split the array around the ---------- index q.

- Entering
- **Mid page 28**
- Exitin

Summing

14) In the analysis of Selection algorithm, we get the convergent ---------- series.

- Harmonic
- Linear
- Arithmetic
- **Geometric page 37**

16) In Selection problem, the Sieve technique ----------.

- Adds some more input items each time.
- Do not work recursively
- Do not uses Divide and Conquer approach
- **Eliminate undesired data items each time page 35**

17) If matrix A of dimension p x q is multiply with matrix B of dimension q x r, then each entry in resultant matrix takes ---------- time.

- **O(q) page 84**

- O(1)
- O(p x q)
- O(q x r)

18) In Sorting the key value or attribute ---------- form an ordered domain.

- Must be

Not always page 39

19) In Heap Sort algorithm, the total running time for Heapify procedure is ---------- .

Theta (log n) page 43

- Order (log n)
- Omega (log n)
- O (1) i.e. Constant time

20) Pseudo code of algorithm are to be read by ----------.

- **People page 12**
- RAM
- Computer
- Compiler

21) In Dynamic Programming based solution of Knapsack Problem, if we decide to take an object 'i', then we gain ----------.

- W (Total Weight of Knapsack)
- V (Total Value of all items)
- **vi(Value of object i) page 93**
- None of the given option

22) The worst case running time of Quick sort algorithm ----------.

- Cannot be quadratic

Is quadratic (google)

- Is always exponential
- Is linear

23) The only way to convert a string of i characters into the empty string is with i deletions , represented as

- E(0 , j)=j
- E(i , j)=I
- E(0 , i)=j
- **E(i , 0)=i page 78**

24) In Selection problem, the rank of an element will be its ---------- position if we sort the input data.

- First
- **Final page 34**
- Second last
- Last

25) Which one is not passed as parameter in Quick sort algorithm?

- End of array
- Middle of array

- **Array (containing input elements) (cs301)**

- Start of the array

26) While Sorting, the ordered domain means for any two input elements x and y --------- satisfies only.

- X<y
- X>y

- X=y
- **All of the above page 39**

27) Which of the following is calculated with Big O notation?

- Medium bounds
- **Upper bounds page 25**
- Lower bounds
- Both upper and lower bounds

28) For solving Selection problem, we introduced Sieve technique due to ---------

- Using Decrease and Conquer strategy
- Avoiding to sort all input data
- **Eliminating Rank of an element page 35**

Using Brute-Force approach

29) The worst-case running time of merge sort is ---------- in order to sort an array of n elements.

- O(log n)
- O(n)

O(n log n) page 30

-

30) Quick sort is a recursive algorithm.

- False

True (page49)

31) While analyzing Selection algorithm, we make a number of passes , in fact it could be as many as ----------.

- n(n+1)
- **log(n) page 37**

- n/3
- n/4

32) Approach for solving geometric problems by sweeping a line across the plane is called ---------- sweep.

- Line
- **Plane page 18**
- Cube
- Box

33) ---------- time is the maximum running time over all legal inputs.

- **Worst-case page 13**
- Average-case
- Best-case
- Good-case

34) The asymptotic growth of n(n+1)/2 is:

- **O(2) page 15**
- O(n)
- O(n+2)
- O(n log n)

35) One of the limitation in 0/1 Knapsack is that an item can either be ---------- in the bag or not.

- Use
- **Put page 91**
- Move
- Store

36) --------- is one of the few problems, where provable lower bounds exist on how fast we can sort.

- Searching
- **Sorting page 39**
- Both searching and sorting
- Graphing

37) Selection algorithm takes theta ----------.

- (n2)
- (**n) page 37**
- log(n)
- nlog(n)

38) The Sieve technique is a special case, where the numbers of sub-problems is just -----.

- **1 page 34**
- 2
- 3
- 4

39) Following is not the application of Edit Distance problem

- Speech recognition
- Spelling Correction
- **Ascending Sort page 76**
- Computational Molecular Biology

40) We can use the optimal substructure property to devise a ---------- formulation of the edit distance problem.

- Selective
- Optimum
- Iterative

Recursive (p78)

41) In plane sweep approach, a vertical line is swept across the 2d-plane form ----------

- Right to left
- **Left to right page 18**

- Top to bottom
- Bottom to top

42) In 3-Dimensional space, a point P has ---------- coordinate(s).

- (X,Y)
- (X,0)
- (0,Y)
- **(X,Y,Z) (google)**

43) ---------- items are not allowed in the 0/1 Knapsack.

- Lighter
- **Fractional page 91**

- Whole
- Weighty

44) Radix sort is not a non-comparative integer sorting algorithm.

- True
- **False (google)**

45) Fibonacci sequence was named on ----------, a famous mathematician in 12th Century.

- Fred Brooks
- Grady Booch
- **Leonardo Pisano page 73**

- Edgar F.Codd

46) In generating Fibonacci Sequence, we can avoid unnecessary repetitions by ---------- process.

- Tokenization
- **Memoization page 74**
- Randomization
- Memorization

47) The sequence of merge sort algorithm is:

- Divide-Combine-Conquer
- Conquer-Divide-Combine
- **Divide-Conquer-Combine page 27**
- Combine-Divide-Conquer

48) Sorting can be in ----------.

- Increasing order only
- Decreasing order only
- **Both Increasing and Decreasing order page 52**
- Random order

49) The process of --------- ends when you are left with such tiny pieces remaining that it is trivial to solve them.

- Brute-force
- Plan-sweep
- **Divide and Conquer page 27**
- Axis-sweep

50) Time complexity of the “0-1” knapsack algorithm depends on ----------.

- Number of items
- Capacity of the knapsack
- Size of the table
- **Number of items and capacity of knapsack (Google)**

51) In max-heap, largest element is stored at root node. Where is the smallest element stored?

- Right Node

Leaf Node (Google)

- Middle Node
- Left Node

52) For ---------- values of n, any----------algorithm is fast enough.

Small pg#14

- Medium
- Large
- Infinity

53) To predict the cost of an algorithm in terms of resources is called ----------.

- Key

Analysis (Google)

- Design
- Output

54) Dynamic programming comprises of ---------

- **Recursion only pg#75**
  - Repetition only
  - Recursion and Repetition
  - No Repetition but Recursion

55) For Quick sort algorithm, Partitioning takes theta ----------?

- (n)
- log(n)
- **nlog(n) (Google)**
- n2log(n)

56) An in-place sorting algorithm is one that ---------- uses additional array for storage.

- Always
- Permanently
- **Does not pg#54**
- Sometime

57) In Dynamic Programming approach, we do not store the solution to each sub-problem in case if it reappears.

- True
- **False pg#75**

58) Counting sort has time complexity:

- **O(n+k) pg#58**
- O(n)
- O(k)
- O(nlogn)

59) Due to left complete nature of binary tree, the heap ca be stored in

- **Arrays pg#40**
- Structures
- Link list
- Stack

60) For the heap sort we store the tree node in

- **Level-order traversal pg#40**

- In-order traversal
- Pre-order traversal
- Post-order traversal

61) n the clique cover problem, for two vertices to be in the same group, they must be ----------each other.

- Apart from
- Far from
- Near to
- **Adjacent to pg#176**

62) In in-place sorting algorithm is one that uses arrays for storage :

No additional array pg#54

- An additional array
- Both of above may be true according to algorithm
- More than 3 arrays of one dimension

63) Brute-force algorithm for 2D-Maxima is operated by comparing ---------- pairs of points.

- Two
- Some
- Most

All pg#18

64) F(n) and g(n) are asymptotically equivalent. This means they have essentially the same --------- for large n.

- Results

Growth rates pg#23

- Variables
- Size

65) Quick sort is

- Stable & in place
- **Not stable but in place pg#54**
- Stable but not in place
- Some time stable & some times in place

66) Heaps can be sorted in arrays without using any pointers; this is due to the ---------- nature of the binary tree.

- **Left-complete pg#40**
- Right-complete
- Tree nodes
- Tree leaves

67) Which may be a stable sort?

- **Merger**
- **Insertion**
- **Both above page 54**
- None of the above

68) For the Sieve Technique we take time

- **T(nk) page 34**
- IT(n/3)
- n^2
- n/3

69) One of clever aspects of heaps is that they can be stored in arrays without using any --------.

- **Pointers page 40**
- Constants
- Variables
- Functions

70) Counting sort is suitable to sort the elements in range 1 to k.

- K is large
- K is not known
- K may be small or large
- **K is small page 57**

71) Asymptotic growth rate of the function is taken over ---------- case running time.

- Best
- **Worst page 14**
- Average
- Normal

72) The Sieve technique is a special case, where the number of a sub-problems is just .

- 5
- **1 page 34**
- Few

73) Dynamic programming algorithms need to store the results of intermediate sub-problems.

- **True page 75**
- False

74) In counting sort, once we know the ranks, we simply ---------- numbers to their final positions in an output array.

- Delete
- **Copy page 57**
- Mark
- Arrange

75) In Quick sort, we don't have control over the sizes of recursive calls.

- **True page 49**
- False
- Less information to decide
- Either true or false

76) Before sweeping a vertical line in a plane sweep approach, in start sorting of the points is done in increasing order of their ---------- coordinates.

- **X (page #18)**
- Y
- Z
- X&Y

77) Random access machine or RAM is a/an

- Machine build by Al-Khwarizmi
- Mechanical machine
- **Mathematical model (page #10)**
- Electronics machine

78) The Huffman codes provide a method of encoding data inefficiently when coded using ASCII standard .

- True
- **False(page #99)**

79) A heap is a left-complete binary tree that conforms to the ----------.

- Increasing order only
- Decreasing order only
- **Heap order (page# 40)**
- (log n) order

80) For the sieve technique we solve the problem,

- **Recursively (page #34)**
- Mathematically
- Precisely
- Accurately

81) Median is not useful measure of central tendency of given input set especially when the distribution of values is highly skewed.

- True
- **False (page # 34)**

82) The omega-notation allows us to state only the asymptotic----------bounds.

- Middle
- **Lower (page #25)**
- Upper
- Both middle and upper

83) In plane sweep approach, a vertical line is swept across the 2-D plane and----------structure is used for holding the maximal points lying to the left of the sweep line.

- Array
- Queue
- **Stack (page #18)**
- Tree

84) When a heapify procedure is applied to the root node to restore the heap, then at each level, the comparison performed takes time.

- It will take O(log n)
- It cannot be predicted
- **It will takeO (1) (page #43)**
- Time will vary according to the nature of input data

85) Efficient algorithm requires less computational----------.

- Memory
- Running time
- **Memory and running time (page #9)**
- Energy

86) Recurrence can be described in terms of a tree.

- **Yes (page#31)**
- No

87) Time complexity of dynamic programming based algorithm for computing the minimum cost of Chain Matrix multiplication is----------.

- log n
- n
- n^2(n square)
- **n^3(n cube) (page#90)**

88) In order to say anything meaningful about our algorithms, it will be important for us to settle on a----------.

- Java programming
- C++ programming
- Pseudo program
- **Mathematical model of computation (page # 10)**

89) The iteration method is used for ----------.

- Comparing sorting algorithms only
- **Solving recurrence relation (page # 31)**
- Merging elements in merging sort
- Divide elements in merge sort

90) Chain matrix multiplication problem can be solved through----------strategy.

- **Dynamic programming (page #85)**
- Greedy
- Divide and conquer
- Sorting

91) For comparison-based sorting algorithms, it is ----------possible to sort more efficiently than Omega nlog (n) time.

- Always
- **Not (page #54)**
- Sometimes not
- Sometimes

92) Dynamic Programming approaches are usually useful in solving optimization problems.

- **True (page # 97)**
- False

93) In sorting the key value or attribute----------from an ordered domain.

- **Must be (page #39)**
- Not always

94) Result of asymptotical analysis of n (n-3) and 4n*n is that----------.

- n(n-1) is asymptotically less
- n(n-1) is asymptotically Greater
- both are asymptotically Not equivalent
- **both are asymptotically Equivalent (page #23)**

95) Floor and ceiling are----------to calculate while analyzing algorithms.

- Very easy
- **Usually consider difficult pg#31**

96) ---------- of reference is an important fact of current processor technology.

- Defining
- Assigning
- Formality
- **•Locality (page # 8)**

97) As per algorithm of dynamic programming , we need to store

- First sub- problem only
- Best solution only
- **Intermediate sub-problems (page #75)**
- Final solution only

8) In Sieve technique, we solve the problem----------.

- **•In recursive manner (page # 34)**
- Non recursively
- Using Merge sort algorithm
- Using Brute force technique

99) Upper bound requires that there exist positive constant c2and n0 such that f(n)---------c2n for all n<= n0.

- Less than
- **Equal to or less than (page # 24)**
- Equal to or greater than
- Greater than

100) In 2-D matrix problem, A point p is said to be dominated by point q if--------.

- P.x=q.x and p.y=q.y
- P.x≠q.x and p.y≠q.y
- P.x≥ q.x and p.y≥q.y
- **P.x≤q.x and p.y≤q.y (page # 11)**

101) A Random Access Machine (RAM) is an idealized machine with---------- random access.

- **Infinite large (page # 10)**
- 512 MB
- 256 MB
- 2 GBs

102) There are----------entries in the Edit Distance Matrix.

- Θ (n)
- **Θ (n2) (page # 84)**
- Θ (n+2)
- Θ (n+100)

103) Counting sort is suitable for sorting the elements within range 1 to P, where----------.

- P is large
- **P is small (page # 57)**
- P is very large
- P is undetermined

104) While analyzing searching algorithm, we make a number of passes, in fact it called----

- n(n+1)

log(n) (page #37)

- n/3

- n/4

105) It is not a Fibonacci Sequence.

1,1,1,2,3,5,8,13,21,34,

55,----

- **True (page #73)**
- False

106) In Random Access Machine (RAM), instructions are executed in----------.

- Parallel
- Batch

One by one (page # 10)

- Multiple times

107) Dynamic programming formulation of the matrix chain multiplication problem will store the solutions of each sub problem in a(n):

- Array

Table (page #86)

- Variable
- Class

108) An algorithm is a mathematical entity, which is independent of----------.

- Programming language
- Machine and programming language
- Compiler and programming language

Programming language Compiler and Machine (page #7)

109) In Fibonacci sequence, unnecessary repetitions do not exist at all.

True (page# 74)

- False

110) In Quick sort algorithm, pivots form----------.

- Stack

Queue

- Graph

111) In Heap Sort Algorithm, Heapify procedure is---------- in nature.

- **Recursive (page #43)**
- Non-recursive

112) In Bucket sort, if there are duplicates then each bin can be replaced by a :

- Stack

Link list (page #69)

- Hash table
- Heap

113) Counting sort assumes that the numbers to be sorted are in the range ----------.

- k to n where n is large

1 to k where k is small (page # 57)

- k to n where k is small
- k to n where n is small

114) In Dynamic Programming, our approach is to ----------.

- Develop the solution in a top-down fashion.
- Express the problem non-recursively.

o Build the solution in a bottom- up fashion. Pg#92

- Input several sub-problems simultaneously.

115) In knapsack Problem, the goal is to put items in the knapsack such that the value of the items is ---------- ------ subject to weight limit of knapsack.

- Minimized
- Decreased

Maximized

- None of the given option

116) If there are $\Theta(n^2)$ entries in edit distance matrix then each entry E(i,j) takes ---------- time to compute.

- $\Theta(n)$
- $\Theta(n^2)$

Θ (1) (page #84)

- $\Theta(n \log n)$

117) In ---------- knapsack Problem, limitation is that an item can either be put in the bag or not.
Fractional items are not allowed.

- 0
- 1

0/1 (page #91)

- Fractional

118) Heapsort is a/an ---------- and ---------- sorting algorithms.

- Not in-place, not stable one
- In-place , stable one

In-place, not stable one (page#54)

- Not in-place , stable one

119) Selection sort is not an in-place sorting algorithm.

- True

False (page #54)

120) Memoization is a part of Dynamic Programming strategy.

True (page 74)

- False

127) Quick sort algorithms divide the entire array into----------sub arrays.

- One

Two (Google)

- Three
- Four

121) Insertion sort is an in-place algorithm.

a) True (page #54)

b) False

122) When matrix A of 5x3 is multiply with matrix B of 3x4 then the number of multiplication required is :

- 15
- 12
- 36
- **60 page 84**

123) If matrix A of dimensions 2x4 is multiply with matrix B of dimensions 4x3, then the dimensions of resultant matrix is :

- 2x4
- 4x3
- 3x4
- **2x3 page 84(conceptual)**

124) Dynamic Programming strategy is useful when sub- problems are independent.

- True

False google

125) If Matrix-A of dimension 2x4 multiply with Matrix-B of dimension 4x3, then the dimension of resultant Matrix is:

- 2x4
- 4x3
- 3x4
- **2x3 page 84(conceptual)**

126) In the following code the statement “cout<< i” executes---------

----times. **for (int i=1; i<=n; i++)**

cout<< i;

- **n times (conceptual)**
- n+5 times
- Infinite times
- Zero times

127) Edit distance algorithm based on----------strategy.

- Greedy
- **Dynamic programming page 77**
- Divide and conquer
- Searching

128) If Matrix A has dimensions “3x2” and Matrix B has dimensions “2x3” , then multiplication of Matrix-A and Matrix-B will result a new Matrix-C having dimensions.

- 3x2
- 2x3
- 2x2
- **3x3 page 84(conceptual)**

129) The function f (n) =n (logn+1)/2 is asymptotically equivalent to nlogn. Here Lower Bound means function f

(n) grows asymptotically at ---------- as fast as nlogn.

- **Least page 24**
- Normal
- Most
- All

130) Single atom from a larger set of ----------.

- Constant
- Pointers
- Phases
- **n items page 34**

131) Fibonacci sequence was named on a----------, a famous mathematician in 12th century.

- Fred brooks
- Grady booch
- **Leonardo Pisano (page # 73)**
- Edgar F.Codd

132) In dynamic programming, our approach is to----------.

- Develop the solution in a top down fashion
- Express the problem non-recursively
- **Build the solution in a bottom up fashion (page #92)**
- Input several sub-programs simultaneously

133) In heap sort algorithm, we build----------for ascending order.

- **Max heap page 42(conceptual)**
- Min heap

134) Bubble sort is not an in-place sorting algorithm.

- True

False (page #54)

135) In partition algorithm, the subarray----------has elements which are greater than pivot element X.

- A[p….r]
- A[p….q-1]
- A[q]

A[q+1…..r] (page #46)

136) In Heap sort algorithm, if heap property is violated----------.

- We call build heap procedure

We call heapify procedure (page # 41)

- We ignore
- Heap property can never be violated

137) ----------is not a characteristics of Random Access Machine.

- Single-processor
- Assigning a value to a variable

Locality of reference (page #10)

- Executing an arithmetic instruction

138) Boolean operation is a----------operation on an idealized RAM model of computation.

- Starting

Basic (page #10)

- Advance
- Normal

139) How much time merge sort takes for an array of numbers?

- T(n^2)
- T(n)
- T(log n)
- **T(n log n) page 4**

140) Sorting is one of the---------problems in Computer Sciences.

Easiest page 39

- Solution-less
- Least popular
- Tony Hoare

142) We can make----------recursive calls in Fibonacci sequence.

- **Infinite google**
- Finite

140) Insertion sort is a ---------- sorting algorithm.

- Unstable
- **In-place page 54**
- Not in-place
- In-portion

141) We do not need to prove comparison- based sorting algorithms by mathematically , it always takes----------

----------:

- Big Oh nlog(n)
- **Omega nlog(n) page 54**
- Omega n(n^2)
- Theta nlog(n)

142) A Principal operation for maintaining the heap property is called heapify, it is also called :

- **Shifting Down page 43**
- Shifting Up
- Shifting Right
- Shifting Left

143) The number of nodes in a complete binary tree of height h is:

- 2*(h+1)-1
- 2^(h+1)-1
- **2^(h+1) page 45**
- ((h+1)^2)-1

144) After partitioning array in Quick sort, pivot is placed in a position such that:

- Pivot is the first element of array

- Pivot is the last element of array
- **Values smaller than pivot are on left and larger than pivot are on right page 35**
- Values larger than pivot are on left and smaller than pivot are on right

145) ---------- programming is essentially recursion without repetition:

- Fast
- **Dynamic page 75**
- Array
- N log n

146) We can use the _____ property to devise a recursive formulation of the edit distance problem.

- small substructure
- algorithmic

real

- **optimal substructure page 78**

147) Applying the sieve technique to selection problem,_____ element is picked from array.

- Output
- Total
- Input
- **Pivot page 35**

148) If matrix A of dimension p* q is multiply with matrix B of dimension q*r, then the dimension of the resultant matrix is:

- p*q
- **p*r page 84**
- q*r
- r*p

149) In knapsack problem, each item must be entirely accepted or rejected, is called _____ problem.

- Fractional
- **“0-1” page 92**
- Linear
- Optimal

150) In the case of _____ analysis does not depend upon on the distribution of input.

- Merge sort
- **Quick sort page 50**
- Insertion sort
- Heap sort

151) In Heapsort Algorithm, total time taken by heapify procedure is:

- **O(log n) page 43**
- O(log2 n)
- O(n log n)
- O(n2 log n)

152) In merge sort algorithm, we split the array _____ to find index q.

- from start
- **midway page 28**
- from end
- both from start or end

153) Al-Khwarizmi was a/an _____

- artist
- **mathematician page 7**
- astronomer
- khalifah

154) In Sieve Technique, we know the item of interest.

- True
- **False page 34**

155) Radix sort is a _____ integer sorting algorithm.

- Comparative
- **Non-comparative google**
- In-Place
- Unstable

156) Random Access Machine (RAM) can execute _____ instruction.

- only logical
- parallel
- only arithmetic
- **logical and arithmetic page 10**

157) Which of the following is calculated with Big Omega notation?

- Medium bounds
- **Upper bounds page 25**
- Lower bounds
- Both upper and lower bounds

158) Asymptotic growth of 8n^2 + 2n – 3 is:

- O(n^2+n)
- **O(n^2) page 23**
- O(Bn^2)
- O(Bn^2+2n)

159) _____ is the process of avoiding unnecessary repetitions by writing down the results of recursive calls and looking them up again if we need them later.

- Loop
- **Memoization page 74**
- Recursion
- Function

160) In Heap Sort algorithm, the first step is to_____.

- **Call Build-Heap procedure page 43**
- Sort the array in descending order
- Call Heapify procedure
- Find the number of input elements

161) The time assumed for each basic operation to execute on RAM model of computation is_____.

- Infinite
- Continuous
- **Constant page 10**
- Variable

162) If input “n” is odd, then median will be_____.

- **(n+1)/2 page 34**

- n/2
- (n-1)/2
- n+2

163) In quick sort algorithm, _____ decides nature of Binary Search Tree formed by pivots.

- middle element from input
- smallest element from input
- **rank of the pivot page 49**
- largest element from input

164) The brute-force algorithm for 2D-Maxima runs in order O(----------) time:

- n
- n(log n)
- **n*n page 18**
- n3

165) In average-case time analysis of quick sort algorithm, the most balanced case for partition is when we divide

the list of elements into----------

- Equal no. of pieces as input elements
- Single piece exactly
- **Two nearly equal pieces google**
- Three nearly equal pieces

166) The main purpose of mathematical analysis is measuring the ---------- required by the algorithm:

- Space
- **Execution time page 13**
- Inputs & outputs
- Execution time and memory

167) Dynamic Programming algorithms often use some kind of ---------- to store the results of intermediate Sub-problems:

- **Table page 75**
- Variable
- Stack
- Loop

168) The average case running time of quick sort algorithm is theta ----------:

- Log(n)
- (n)
- **nlog(n) page 53**
- nxn log(n)

169) What is the best case time complexity of merge sort?

- O((n^2logn)
- O((nlog^2)
- **O((nlogn) page 40**
- O((n^2)

170) While solving Selection problem, in Sieve technique we partition input data ----------:

- In increasing order
- In decreasing order
- **According to Pivot page 35**
- Randomly

171) In average-case time the probability of seeing input is denoted by----------:

- P{I}
- P[I}

- **P(I) page 13**
- P<I>

172) To check whether a function grows faster or slower than the other function, we use some asymptotic notations, which Is ----------:

- Big-oh notation
- Theta notation
- Omega notation
- **All of the given google**

173) Asymptotic notations are used to describe -------- of an algorithm:

- Length
- **Running time (google)**
- Size
- Compile time

174) In recursive formulation of Knapsack Problem

V[0,j] = ---------- for j>=0

- -1

0 (page #93)

- 1
- 2

175) If a matrix has three rows and two columns, then dimensions of matrix will be:

3x2 (google0

- 2x2
- 2x3
- 3x3

176) Selection sort takes theta --- in the worst case:

(n^2)

- (n)
- Log(n)
- nlog(n)

177) Which of the following problems can be solved using dynamic problem?

- Bubble sort problem
- **Matrix chain multiplication problem page 92**
- Greedy search problem
- Fractional knapsack problem

178) ---------- algorithm based on Dynamic Programming strategy:

- Heap sort
- Binary tree
- Quick sort
- **Edit distance page 77**

179) The definition of theta-notation relies on proving ------- asymptotic bound:

- One
- Lower
- Upper
- **Both lower and upper page 25**

180) In order to say anything meaningful about our algorithms, it will be important for us to settle on a --------:

- Java program
- C++ program
- Pseudo program
- **Mathematical model of computation page 10**

181) In asymptotical analysis of n(n-3) and 4n*n , as n becomes large, the dominant (fastest growing) term is some Constant times----------,

- n+1
- n-1
- n
- **n*n page 23**

182) ---------- is in- place sorting algorithm

- **Bubble sort page 54**
- Merge sort
- Linear sort
- Binary sort

183) In chain matrix multiplication, if there are --------- ways in which outer most pair of parentheses can placed:

- n^2
- 2n
- n+1
- **n-1 page 85**

184) In chain matrix multiplication, table is filled ----- to find the multiplication of matrix:

- row wise
- column wise
- **diagonally page 86**
- bottom-to-up

185) Which one sorting algorithm is best suited to sort an array of 2 million elements?

- Bubble sort
- Insertion sort
- Merge sort
- **Quick sort google**

186) The problem with the brute-force algorithm is that it uses ----- in pruning out decisions:

- Worst-case time
- **No intelligence page 18**

- Outside looping
- Artificial intelligence

187) Quicksort is a/an --------- and ---------- sorting algorithm:

- No in-place , not-stable one

In-place , not stable one page 54

- In-place , stable one
- Not in-place , stable one

191) In Heap Sort algorithm, the maximum levels an element can move upward is ----------:

- **Theta (log n) page 43**
- Big-oh(log n)
- Omega (log n)
- O (1) i.e. Constant time

192) If the time complexity of an algorithm is O(n) , then it is called ---------- time complexity:

- **Linear page 37**
- Constant
- Average
- Exponential

193) Dynamic Programming approach is usually useful in solving ---------- problems:

- Normal
- **Optimization page 97**
- Array
- Loop

194) What time does Merge Sort algorithm take in order to sort an array of ‘n’ numbers?

- Θ(n)
- Θ(log n)
- Θ(n^2)
- **Θ(n log n) page 30**

CS-502 (Fundamentals of Algorithms)

Grand Quiz - Fall 2020

If you found any error in the file, Please correct it accordingly.

163) The sequence of merge sort algorithm is:

- **Divide Combine-Conquer**
- **Conquer-Divide-Combine**

c. Divide-Conquer-Combine Page 27

- **Combine-Divide-Conquer**

- **In _____ Knapsack Problem, limitation is that an item can either be put in the bag or not. Fractional items are not allowed.**

0

1

c. 0/1 Page 91

Fractional

165) In Selection algorithm, we assume pivot selection takes theta _____ running time.

a. n Page - 36

n2

n3

log (n)

- In Heap Sort algorithm (using max heap), when every time maximum elements removed from top _____.

We call merge Sort Algorithm

it becomes Order n2 Algorithm

Divide and Conquer strategy helps us

d. We are left with a hole Page – 41

167) If matrix A of dimension p x q is multiply with matrix B of dimension q x r, then each entry in resultant matrix takes _____ time.

a. O (q) Page - 84

168) O (1)

169) O (p x q)

170) O (q x r)

169) _____ is a method of solving a problem in which we check all possible solutions to the problem to find the solution we need.

- Plane-Sweep Algorithm
- Sorting Algorithm

c. Brute-Force Algorithm google

- Greedy approach

171) The worst case running time of Quick sort algorithm _____.

- Cannot be quadratic

b. Is quadratic google QuickSort and its Analysis (codesdope.com) &

http://personal.kent.edu/~rmuhamma/Algorithms/MyAlgorithms/Sorting/quickSort.htm

Is always Exponential

Is linear

172) In max heap (for Heap Sort algorithm), when every time maximum element is removed from top we replace it with _____ leaf in the tree.

- second last

b. Last Page -41

- First
- Any

174) Quick sort algorithm was developed by -

AlferdAho

Sedgewick

John Vincent Atanasoff

d. Tony Hoare – Google wikipedia

- **If Matrix-A has dimensions "3x2" and Matrix-B has dimensions "2x3", then multiplication of Matrix-A and Matrix-B will result a new Matrix-C having dimensions.**

  3x2

  2x3

  2x2

  d. 3x3 **http://www.calcul.com/show/calculator/matrix-multiplication**

- **For comparison-based sorting algorithms, it is possible to sort more efficiently than Omega n log(n) time.**

  a. Always

  b. Not **P-54**

  c. Sometimes

Sometimes not

176) Dynamic Programming approach is usually useful in solving optimization problems.

a. True https://en.wikipedia.org/wiki/Dynamic_programming &

http://web.mit.edu/15.053/www/AMP-Chapter-11.pdf

178) False

179) In Sorting the key value or attribute_____ from an ordered domain.

a. Must be page 39

- Not always
- May be
- Occasionally

181) Result of asymptotical analysis of n(n -3) and 4n*n is that _____

- n(n-1) is asymptotically Less
- n(n-1) is asymptotically Greater
- Both are asymptotically Not equivalent

d. Both are asymptotically Equivalent page 23 (4n*n= 4n2)

182) Floor and ceiling are _____ to calculate while analyzing algorithms a. Very easy

b. Usually considered difficult P-31

3rd Option is missing

4th Option is missing

184) _____ of reference is an important fact of current processor technology.

- Defining
- Assigning

- **Formality**

d. Locality **P-8**

185) In max-heap, largest element is stored at root node. Where is the smallest element stored?

- **Right Node**

Leaf Node

- **Middle Node**
- **Left Node**

- **In average-case time analysis of Quick sort algorithm, the most balanced case for partition is when we divide the list of elements into _.**

Equal no. of pieces as of input elements

Single piece exactly

Two nearly equal pieces

Three nearly equal pieces

- **Which of the following is calculated with Big O notation?**

Medium bounds

b. Upper bounds **Page - 25**

- **Lower bounds**
- **Both upper and lower bounds**

187) Edit distance algorithm based on _____ strategy

- **Greedy**

b. Dynamic Programming **Page - 81**

Divide and Conquer

Searching

- **In Heapsort Algorithm, total time taken by heapify procedure is _____**

a. O (log n) **Page-43**

- **O (log2 n)**
- **O (n log n)**
- **O (n2 log n)**

193) Al-Khwarizmi was a/an _____

- **Artist**

b. Mathematician **P-7**

Astronomer

Khalifah

194) When matrix A of 5x3is multiply with metric B of 3x4 then the number of multiplication required is: Not found exactly

15

12

36

60Not Found exactly but as per formula at page 84,

24. Pseudo code of algorithms are to be read by _____.

a. People Page -12

RAM

Computer

Compiler

195) The sieve technique is a special case, where the number of sub-problems is Just

_____

a. 1 P-34

b. 2

c. 3

d. 4

26. When a recursive algorithm revisits the same problem over and over again, we say that the optimization problem has _____ sub-problems.

a. Overlapping – Google Search

b. Over costing

c. Optimized

d. Three

27. Sieve technique is very important special case of Divide-and-Conquer strategy.

a. True P-34

b. False

28. In order to say anything meaningful about our algorithms, it will be important for us to settle on a _____.

a. Java Program

b. C++ Program

c. Pseudo program

d. Mathematical model of computation P-10

29. Merge sort is based on _____.

a. Brute-force

b. Plan-sweep

c. Axis-sweep

d. Divide and Conquer P-27

30. What time does Merge Sort algorithm take in order to sort an array of 'n' numbers?

a. (n)

b. (log n)

c. (n^2)

d. (n log n) Google Search

31. In Heap Sort algorithm, the first step is to _____. a. Call Build-Heap

a. procedure Page - 46

b. Sort the array in descending order

c. Call Heapify procedure

d. Find the number of input elements

32. The definition of theta-notation relies on proving _____ asymptotic bound.

a. One

b. Lower

c. Upper

b. d. Both lower & upper Page - 25

33. In merge sort algorithm, to merge two lists of size n/2 to a list of size n, takes _____ time.

a. Theta (n) Page - 32

b. Theta log(n)

c. Theta log2(n)

d. Theta n log(n)

34. We can make _____ recursive calls in Fibonacci Sequence.

a. Infinite

b. Finite google

c. Only one

d. Zero

35. Following is NOT the application of Edit Distance problem.

a. Speech recognition

b. Spelling Correction

c. Ascending Sort Page - 76

d. Computational Molecular Biology

36. **In plane sweep approach, a vertical line is swept across the 2d-plane and structure is used for holding the maximal points lying to the left of the sweep line.**

    a. **Array**

    b. **Queue**

    c. **Stack** **Page - 18**

    d. **Tree**

37. **When a heapify procedure is applied to the root node to restore the heap, then at each level, the comparison performed takes time:**

    a. **It will take (log n).**

    b. **It can not be predicted**

    c. **It will take O (1).** **Page - 43**

    d. **Time will vary according to the nature of input data.**

38. **_____ time is the maximum running time over all legal inputs.**

    a. **Worst-case** **Page - 13**

    b. **Average-case**

    c. **Best-case**

    d. **Good-case**

39. **Efficient algorithm requires less computational...**

    a. **Memory**

    b. **Running Time**

    c. **Memory and Running Time** **Page - 9**

    d. **Energy**

40. For average-case time analysis of Quick sort algorithm, Pivot selection is on average basis from _____

    a. half of the input values

    b. all possible random values Page - 50

    c. Pivot is input separately

    d. values greater than 5

41. Selection algorithm takes theta _____

    a. (n2)

    b. (n) Page - 37

    c. log(n)

    d. n log(n)

42. Recurrence can be described in terms of a tree.

a. Yes Page - 31

b. No

43. Time complexity of Dynamic Programming based algorithm for computing the minimum cost of Chain Matrix Multiplication is _____

a. Log n

b. n

c. n^2 (n square)

d. n^3 (n cube) Page -90

44. The Iteration method is used for _____

a. Comparing sorting algorithms only

b. Solving Recurrence relations Page 31

c. Merging elements in Merge sort

d. Dividing elements in Merge sort

45. In 3-Dimensional space, a point P has _____ coordinate(s).

a. (X, Y)

b. (X, 0)

c. (0, Y)

d. (X,Y, Z)

46. Chain matrix multiplication problem can be solved through _____ strategy.

a. Dynamic programming Page - 85

b. Greedy

c. Divide and conquer

d. Sorting

47. Merge sort have running time....running time of Heap sort. Not found exactly

a. Greater than

b. Less than Google

c. Equal to

d. Different than

48. Median is not useful measure of central tendency of given input set especially when the distribution of values is highly skewed.

a. True

b. False Page – 34

49. We do not need to mathematically prove that for comparison-based sorting algorithms always takes Omega nlog (n) time.

a. True Google & VU Tech (pg 46 not very clear)

b. False

50. The Omega-notation allows us to state only the asymptotic _____ bounds.

a. Middle

b. Lower Page 25

c. Upper

d. Both lower & upper

51. Both lower & upperSorting can be in _____

a. Increasing order only

b. Decreasing order only

Both Increasing and Decreasing order – (My opinion)

c. Random order

52. Radix sort performs sorting the numbers _____ digit (s) at a time.

a. One Page - 71

b. Two

c. Three

d. All

53. Quicksort is a/an _____ and _____ sorting algorithm.

a. Not in place, not stable one

b. In place , not stable one Page - 54

c. In place , stable one

d. Not in place , stable one

54. Consider three matrices X,Y,Z of dimensions 1x2, 2x3,3x4 respectively. The number of multiplications of (XY) Z is:

a. 18 As per lecture slides

b. 32

c. 24

d. 30

55. In Fibonacci Sequence, unnecessary repetitions do not exist at all.

a. True

b. False Page – 74

56. It is not a Fibonacci sequence . 1,1,1,2,3,5,8,13,21,34,55,…..

a. True Page - 73

b. False

57. Heap sort is a/ an _____ and _____ sorting algorithem.

a. Not in place, not stable one

b. In place , not stable one Page - 54

c. In place , stable one

d. Not in place , stable one

58. Identify the True Statement

a. The knapsack problem does not belong to the domain of optimization problems.

b. The knapsack problem belongs to the domain of optimization problems. Page - 91

c. The Knapsack problem cannot be solved by using dynamic programming

d. The knapsack problem is optimally solved by using brute force algorithm.

59. In Dynamic Programming, our approach is to _____

a. Develop the solution in a top-down fashion

b. Express the problem non-recursively

c. Build the solution in a bottom-up fashion Page - 75

d. Input several sub-problems simultaneously

60. Counting sort is suitable to sort the elements in range 1 to K;

a. K is large

b. K is small Page - 57

c. K may be large or small

d. None

61. We can multiply two matrices A and B only when they are compatible which means

a. Number of columns in A must be equal to number of rows in B.

it seems Correct as per page 84

(https://www.varsitytutors.com/hotmath/hotmath_help/topics/matrix-multiplication#:~:text=You%20can%20only%20multiply%20two,an%20m%C3%97p%20mat rix.)

b. Number of rows and columns do not matter

c. Number of columns in A must be equal to number of columns in B

d. Number of rows in A must be equal to number of rows in B

62. Matrix multiplication is a (n) _____ operation.

a. Commutative

b.
c. b. Associative Page 85

c. Neither commutative nor associative

d. Commutative but not associative

63. In Dynamic Programming approach, solution is modified / changed

a. Always once

b. At each stage google and wikipedia

c. Only for specific problems

d. At 4th stage only

64. In Knapsack problem, the goal is to put items in the Knapsack such that the value of the items is _____ subject to weight limit of knapsack.

a. Minimized

b. Decreased

c. Maximized Page - 91

d. None of the given options

65. An in-place sorting algorithm is one that _____ uses additional array for storage.

a. Always

b. Permanently

c. Does not Page - 54

d. Sometime

66. Memoization is a part of Dynamic Programming Strategy.

a. True Page - 74

b. False

67. If matrix A of dimension 2x4 is multiply with matrix B of dimension 4x3, then the dimension of resultant matrix is Not found exactly

a. 2x4

b. 4x3

c. 3x4

d. 2x3 It seems correct as per second last Para of page 84

68. In Dynamic Programming approach, we do not store the solution to each sub-problem in case if it reappears.

a. True

b. False Page - 75

69. Dynamic Programming is a problem-solving approach in which___

a. Problem is solved in Zero time

b. Solution is developed only at final stage

c. Both are correct

d. Both are incorrect google

70. In Fibonacci sequence, each term is calculated by____ previous__ terms.

a. Subtracting, Two

b. Adding, Three

c. Adding, Two Page - 73

d. Multiplying, Two

71. Selection sort is not an in-place sorting algorithm.

a. True Page - 54

b. False

72. If there are θ (n^2) entries in edit distance matrix then the total running time is:

a. θ (n)

b. θ (1)

c. θ (n2) Page – 84

d. θ (n logn)

73. **The only way to convert a string of i characters into the empty string is with i deletions, represented as**

    a. **E(0.j) =j**

    b. **E(i.j) = 1**

    c. **E(0.i) = j**

    d. **E (i.0)=I** **Page - 78**

74. **Dynamic programming formulation of the matrix chain multiplication problem will store the solutions of each sub problem in an**

    a. **Array**

    b. **Table** **Page - 86**

    c. **Variable**

    d. **class**

75. We can use the optimal substructure property to devise a formulation of the edit distance problem.

    a. Selective

    b. Optimum

    c. Iterative

    d. Recursive Page - 78

76. Sorting is performed on the basis of _____.

    a. Computational resources

    b. Asymptotic notation

    c. Summation

    d. Some key value of attribute page- 39

77. In Heap Sort algorithm, we call Build-heap procedure _____.

    a. Only once page 46

    b. Twice

    c. Thrice

    d. As many times as we need

78. Radix sort is not a non-comparative integer sorting algorithm.

    a. True Google Search

    b. False

79. In the statement “output P[1].x, P[1].y”, the number of times elements of P are accessed is _____.

    a. 1

    b. 2 page 14

c. 3

d. 4

80. The main purpose of mathematical analysis is measuring the _____ required by the algorithm.

a. Space

b. Execution time P-13

c. Inputs & outputs

d. Execution time and memory

81. _____ provides us more accurate result when input values are not closer with each other

a. Average

Median (p-34)

c. Mode

d. Mean

82. The process of _____ ends when you are left with such tiny pieces remaining that it is trivial to solve them.

a. Brute-force

b. Plan-sweep

c. Divide and Conquer P-27

d. Axis-sweep

83. _____ overcomes the limitations of _____ by working as per positional notations of numbers.

a. Counting sort, Radix sort

b. Radix sort, Counting sort P-71

84. Memorization is a part of Dynamic Programming strategy.

a. True P-74

b. False

85. Rank of an element can be defined as _____.

a. One minus the number of elements that are smaller

b. Two plus the number of elements that are greater

One plus the number of elements that are smaller P-34

c. Two minus the number of elements that are smaller

86. If the time complexity of an algorithm is given by O (1), then its time complexity would be

a. Polynomial

b. Exponential

c. Constant - Wikipedia

d. Average

87. Quick sort is a recursive algorithm.

a. True Wikipedia ; Google

b. False

88. The asymptotic growth of n(n+1)/2 is:

O(n2) As the n^2 term has the largest contribution, the Big-O complexity is O(n^2)

a. O(n)

b. O(n+2)

c. O(n log n)

89. Approach of solving geometric problems by sweeping a line across the plane is called _____ sweep.

a. Line

b. Plane Page 18

c. Cube

d. Box

90. As per algorithm of Dynamic Programing, we need to store

a. First sub-problem only

b. Best solution only

c. Intermediate sub-problems Pg:75

d. Final solution only

91. In Sieve technique, we solve the problem

a. In recursive manner Pg:34

b. Non recursively

c. Using Merge Sort algorithm

d. Using Brute force technique

92. One of the limitation in 0/1 knapsack is that an item can either be _____ in the bag or not.

a. Use

b. Put Pg:91

c. Move

d. Store

93. Which one is not passed as parameter in Quick sort algorithm?

a. End of the array

b. Middle of the array

c. Array (containing input elements) Google

d. Start of the array

94. In the analysis of Selection algorithm, we get the convergent _____

a. Harmonic

b. Linear

c. Arithmetic

d. Geometric Pg:37

95. A Random Access Machine (RAM)is an idealized machine withrandom access memory.

a. Infinite large Pg:10

b. 512 MB

c. 256 MB

d. 2 GBs

96. While analyzing Selection algorithm, we make a number of passes, in fact it could be as many as

a. n(n+1)

b. log(n) Pg:37

c. n/3

d. n/4

97. In Random Access Machine (RAM), instructions are executed in

a. Parallel

b. Batch

c. One by One Pg:10

d. Multiple times

98. In selection problem, the rank of an element will be its _____ position

a. First

b. final Pg:34

c. Second last

d. Last

99. The worst-case running time of Merge sort is _____ in order to sort an array of n elements.

a. O(log n)

b. O(n)

c. O(n log n) page 40 and google

d. O(n)

100. f(n) and g(n) are asymptotically equivalent. This means that they have essentially the same _____.

a. Results

b. Variables

c. Size

d. Growth rates P:23

101. An algorithm is a mathematical entity. Which is independent of _____.

a. Programming language

b. Machine and Programming language

c. Compiler and Programming language

d. Programming language Compiler and Machine P: 07

102. In Quick sort algorithm, Pivots form ___

a. Stack

b. Queue

c. Binary Search Tree P:49

d. Graph

103. Counting sort is suitable for sorting the elements within range 1 to P. where

a. P is large

b. P is small P-57

c. P is very large

d. P is undetermined

104. In asymptotical analysis of n'(5 2)-3, as n becomes large, the dominant (fastest growing) term is some constant times

a. n_1

b. n

c. n+1

d. n*n P-23

105. ___ Items are not allowed in the 0/1 knapsack. a. Lighter

b. Fractional P-91

c. Whole

d. Weighty

106. Fibonacci Sequence was named on _____, a famous mathematician in 12th Century.

a. Fred Brooks

b. Grady Booch

c. Leonardo Pisano P-73

d. Edgar F. Codd

107. In Heap Sort algorithm, we build _____ for ascending sort.

a. Max heap P-41

b. Min heap

108. Bubble sort is not an in-place sorting algorithm.

a. True

b. False P-54

109. In partition algorithm, the subarray _____ has elements which are greater than pivot element x.

a. A[p...r]

b. A[p...q-1]

c. A[q]

d. A[q+1..r] P-46

110. In Heap Sort algorithm, if heap property is violated

a. We call Build heap procedure P-43

b. We call Heapify procedure

c. We ignore

d. Heap property can never be violated

111. _____ is not a characteristic of Random Access Machine.

a. Single-Processor P-10

b. Assigning a value to a variable

c. Locality of reference

d. Executing an arithmetic instruction

112. The only way to convert an empty string into a sting of j characters is by doing j insertions, represented as _____

a. E(i,j) = 1

b. E(I,0) = I

c. E(0,j) = j page 78

d. E(1,j)= j

113. In Selection problem, the Sieve technique works in _____.

a. Non-recursive manner

b. Constant time

c. Phases page 34

d. One complete go

114. Algorithm is a sequence of computational steps that ---- the input into output.

a. Merge

b. Assign

c. Transform page 7

d. Integrate

115. If pj dominates pi and pi dominates ph then pj also dominates ph, it means dominance relation is

a. Transitive page 18

b. Non Transitive

c. Equation

d. Symbolic

116. To find maximal points in brute-force algorithm each point of the space is compared against _____ of that space.

a. One other point

b. All other points page 11

c. Few other points

d. Most of the other points

117. In the following code the statement “cout<<j;”executes ---- ----- times. for (j=1; j<=5; j = j+2)

```
cout<<j;
```

a. 5 times

b. 2 times

c. 3 times

d. 0 times

118. In merge sort algorithm, we split the array around the _____ index q. a. Entring

e. b. Mid page 17

c. Exiting

d. Summing

119. In Selection problem, the Sieve technique _____.

a. Add some more input items each time

b. Do not work recursively

c. Do not uses Divide and Conquer approach

f. Eliminates undesired data items each time

120. Consider three matrices X, Y, Z of dimensions 1 x 2, 2 x 3, 3 x 4 respectively. The number of multiplications of X(YZ) is .

a. 16

b. 32

c. 26

g. d. 32 page 84

121. In Heap Sort algorithm, the total running time for Heapify procedure is _____

a. Theta (log n)

b. Order (log n)

c. Omega (log n)

h. O(1) i.e. Constant time

122. The sieve technique works where we have to find_____ items(s) from a large input.

a. Single page 34

Two

b. Three

c. Similar

123. In Dynamic Programming based solution of Knapsack Problem, if we decide to take an object i , then we gain_____

a. W(Total Weight of Knapsack)

b. V (Total Value of all items)

c. vi (Value of object i) page 93

d. Nome of the given option

124. While Sorting, the order domain means for any two input elements x and y _____ satisfies only.

a. x < y page 39

b. x > y

c. x = y

d. All of the above

125. For solving Selection problem, we introduced Sieve technique due to _____

a. Using Decrease and Conquer strategy page 34

b. Avoiding to sort all input data

c. Eliminating Rank of an element

d. Using Brute-force approach

126. _____ is one of the few problems, where provable lower bounds exist on how fast we can sort.

a. Searching

b. Sorting page 38

c. Both Searching & sorting

d. Growing

127. In plane sweep approach, a vertical line is swept across the 2d-plane from_____.

a. Right to Left

b. Left to Right page 18

c. Top to Bottom

d. Bottom to top

128. In generating Fibonacci sequence, we can avoid unnecessary repetitions by _____ process.

a. Tokenization

b. Memorization page 43

c. Randomization

d. Memorization

129. For _____ values of n, any algorithm is fast enough.

a. Small page 14

b. Medium

c. Large

d. Infinity

130. Dynamic programming comprises of _____.

a. Recursion only

b. Repetition only

c. Recursion with Repetition

d. No Repetition but Recursion page 75

131. The function f(n)=n(logn+1)/2 is asymptotically equalient t nlog n :Here Lower Bound means function f(n) grows asymptotically at __ as fast as nlog n.

a. Least page 23

b. Normal

c. Most

d. At

132. Counting sort has time complexity.

a. O(n+k)

b. O(n) page 58

c. O(k)

d. O(nlogn)

133. Due to left complete nature of binary tree, the heap can be stored in

a. Array page 40

b. Structures

c. Link List

d. Stack

134. Single item from a larger set of _____.

a. Constant

b. Pointers

c. Phases

d. n items page 34

135. In the clique cover problem, for two vertices to be in the same group, they must be _____ each other.

a. Apart from

b. Far from

c. Near to

d. Adjacent to page 76

136. How much time merge sort takes for an array of numbers?

a. T(n^2)

b. T(n)

c. T(log n)

d. T(n log n) page 40

137. In in-place sorting algorithm is one that uses arrays for storage.

a. No additional array page 54

b. An additional array

c. Both of above may be true according to algorithm

d. More than 3 arrays of one dimension

138. Brute-force algorithm for 2D-Maxima is operated by comparing _____ pairs of points.

a. Two

b. Some

c. Most

d. All page 18

139. While Sorting, the ordered domain means for any two input elements x and y ____ satisfies only.

a. x > y

b. x < y

c. x = y

d. All of the above page 38

140. Quick sort is.

a. Stable & in place

b. Not stable but in place page 54

c. Stable but not in place

d. Some time stable & some times in place

141. Which may be a stable sort?

a. Merger

b. Insertion

c. Both above page 54

d. None of the above

142. For the Sieve Technique we take time.

a. T(nk) page 34

b. IT(n / 3)

c. n^2

d. n/3

143. Continuation sort is suitable to sort the elements in range 1 to k.

a. K is Large

b. K is not known

c. K may be small or large

d. K is small page 54

144. Asymptotic growth rate of the function is taken over _____ case running time. .

a. Best

b. Worst page 14

c. Average

d. Normal

145. The sieve technique is a special case, where the number of sub problems is just.

a. 5

b. Many

c. 1 page 34

d. Few

146. In Quick sort, we don't have the control over the sizes of recursive calls.

a. True page 49

b. False

c. Less information to decide

d. Ether true or false

147. Before sweeping a vertical line in plane sweep approach, in start sorting of the points is done in increasing order of their _____ coordinates. .

a. X page 18

b. Y

c. Z

d. X , Y

148. Random access machine or RAM is a/an.

a. Machine build by Al-Khwarizmi

b. Mechanical machine

c. Mathematical model page 10

d. Electronics machine

149. The Huffman codes provide a method of encoding data inefficiently when coded using ASCII standard.

a. True

b. False page 99

150. A heap is a left-complete binary tree that confirms to the _____.

a. increasing order only

b. decreasing order only

c. heap order page 40

d. log n order

151. If we associate (x, y) integers pair to cars where x is the speed of the car and y is the negation of the price. High y value for a car means a _____ car.

a. Fast

b. Slow

c. Expensive

Cheap

152. Which one of the following sorting algorithms is the fastest?

a. Merge sort

Quick sort

b. Insertion sort

c. Heap sort

153. Quick sort algorithm divide the entire array into _____ sub arrays.

2

a. 3

b. 4

c. 5

154. In brute force algorithm, we measure running time T(n) based on _____.

a. Average-case time and best-case time

b. Worst-case time and average-case time page 46

c. Worst-case time and best-case time

d. Best-case time and staring-case time

155. For 2D Maxima problem. Plane Sweep algorithm first of all _____.

a. Sorts all points

b. Delete some points

c. Output the elements

d. Pushes all points on stack

156. There are _____ entries in the Edit Distance Matrix

a. ɵ (n)

b. b. Θ (n2) page 84

c. ɵ (n+2)

d. ɵ (n + 100)

157. Which symbol is used for Omega notation?

a. (O)

b. (ɵ)

c. (Ω)

c. (@)

158. Selection sort is a _____ sorting algorithm

d. a. In-place page 54

b. Not In-Place

c. Stable

d. in-partition

159. In Dynamic Programming based solution of knapsack problem, to compute entries of 'V', we will imply a(n) _____ approach.

a. Subjective

b. Inductive

c. Brute Force

d. Combination

160. We do not need to prove comparison-based sorting algorithms by mathematically. It always takes _____ time.

a. Big Oh nlog(n)

b. Omega nlog(n)

c. Omega n(n^2)

d. Theta nlog(n)

161. Merge sort is a/an _____ and _____ sorting algorithm

a. Not in-place, not stable one

b. In-place, not stable one

c. In-place, stable one

d. Not in-place, stable one page 54

162. Cubic function will _____ a quadratic function.

a. Prove

b. be equal to

c. overtake Page 25

d. find

163. Insertion sort is a _____ sorting algorithm

a. Unstable

b. In-place Page 54

c. Not In-Place

d. in-partition

164. To check whether a function grows faster or slower than the other function, we use some asymptotic notations, which is _____.

a. **Big-oh notation**

b. **Theta notation**

c. **Omega notation**

d. **All of the given**

165. Asymptotic growth of 8n^2 + 2n – 3 is:

a. **Θ(n^2 + n)**

b. **Θ (n^2)** page 14

c. **Θ(8n^2)**

d. **Θ(8n^2 + 2n)**

166. In the analysis of algorithms, _____ plays an important role.

a. **text analysis**

time

b. **growth rate**

c. **money**

167. In inductive approach of knapsack problem, we consider 2 cases, _____

Or _____.

a. Median, Mode

b. Recursive, Iterative

c. Leave object, Take object page 93

d. Sequentially. Parallel

168. Random Access Machine (RAM) can execute _____ instructions

a. only logical

b. parallel

c. only arithmetic

d. logical and arithmetic

169. Using _____ algorithm, efficiency is not given much importance

a. Greedy

b. Merge sort

c. Processing as there is no algorithm by this name

d. Brute Force

170. Bubble sort takes theta _____ in the worst case

a. (n2) page 39

b. (n)

c. log(n)

d. nlog(n)

171. If matrix A of dimension p × q is multiply with matrix B of dimension q × r, then dimension of resultant matrix is:

a. p × q

b. p × r page 84

c. q × r

d. r × p

172. Dynamic Programing algorithms often use some kind of _____ to store the results of intermediate sub-problems

a. table (Page 75)

b. variable

c. stack

d. loop

173. _____ is in-place sorting algorithm.

a. Bubble sort (Page 54)

b. Merge sort

c. Linear search

d. Binary Search

174. Which one of the following problems can be solved using dynamic problem?

a. Bubble sort problem

b. Matrix chain multiplication problem page 85

c. Greedy search problem

d. Fractional knapsack problem

175. In chain matrix multiplication, solutions of the sub-problems are stored in a _____.

a. Array

b. Table page 86

c. Tree

d. Link list

176. What is the average running time of a quick sort algorithm?

a. O(n^2)

b. O(n)

c. O(n log n) (Page 49)

d. O(log n)

177. Sorting Algorithms having O _____ running time are considered to be slow ones.

a. (n)

b. (n^2) (Page 39)

c. (nlog(n))

d. (log(n))

178. While solving Selection problem, in Sieve technique we partition input data _____

a. In increasing order

b. In decreasing order

c. According to Pivot

d. Randomly

179. _____ is the process of avoiding unnecessary repetitions by writing down the results of recursive calls and looking them up again if we need them later.

a. Loop

b. Memoization page 74

c. Recursion

d. Function

180. In average-case time the probability of seeing input is denoted by _____.

a. p{I}

b. p[I]

c. p<i>

d. p(i) page 13

181. While applying the Sieve technique to selection sort, how to choose a pivot element.

a. Through mean

b. Linear

c. Randomly page 35

d. Sequentially

182. Number of _____ of the pseudo code are counted to measure the running time.

a. Inputs

b. Outputs

c. Steps page 13

d. Pages

183. Developing a dynamic programming algorithm generally involves _____ separate steps.

a. One

b. Two page 75

c. Three

d. Four

184. 8n^2+2n+3 will exceed c28(n), no matter how large we make _____.

a. n

b. 2n

c. c2 page 25

d. this quadratic equation

185. The running time of quick sort algorithm_____.

a. Is impossible to compute

b. Has nothing to do with pivot selection

c. Is Random upon each execution

d. Greatly influenced by the selection of pivot page 49

186. _____ involves breaking up the problem into sub problems whose solutions can be combined to solve the global problem.

a. Complexity Theory

b. Dynamic programming solution

c. Divide and Conquer Strategy page 34

d. Greedy Algorithms

187. In _____ we have to find rank of an element from given input.

a. Merge sort algorithm

b. Selection problem page 34

c. Brute force technique

d. Plane Sweep algorithm

188. How many steps are involved to design the dynamic programming strategy?

a. 2

b. 3

c. 1

d. 4 page 92

189. In Bucket sort, if there are duplicates then each bin can be replaced by a a. Stack

b. Linked list page 69

c. Hash table

d. Heap

190. In merge sort algorithm, we split the array _____ to find index q.

a. from start

b. midway page 28

c. from end

d. both from start or end

191. Find the maximum value of the items which can carry using knapsack Knapsack weight capacity = 50.

Item Weight Value

11070,22020,33080,470 200

a. 280

b. 100

c. 90

200

192. In 2-d maxima problem a point p is said to be dominated by point q if _____.

a. p.x <= q.x

b. p.x <= q.x and p.y <= q.y page 17

c. p.y <= q.y

d. p.x >= q.x and p.y >=q.y

193. Sorting can be in _____.

a. Increasing order only

b. Decreasing order only

c. Both increasing and decreasing order

d. Random order

194. Recurrence can be described in terms of _____.

a. Array

b. Linear

c. Tree page 31

d. Graph

195. The brute-force algorithm for 2D-Maxima runs in order O(__) time.

a. n

b. n(log n)

c. n*n page 18

d. n3

196. In plane sweep approach of solving geometric problems, a _____ is swept across the plane.

a. Line page 18

b. Plane

c. Cube

d. Box

197. Which of the following is calculated with Big Omega notation?

a. Medium bounds

b. Upper bounds

c. Lower bounds Page - 25

d. Both upper and lower bounds

198. _____ is always based on divide and conquer strategy.

a. Bubble sort

b. Selection sort

c. Pigeon sort

d. Quick sort page 46

199. If a matrix has three rows and two columns, then dimensions of matrix will be:

a. 3x2

2x3

b. 3x3

c. 2x2

200. Asymptotic notations are used to describe _____ of an algorithm.

a. Length

b. running time google

c. size

d. compile time

201. Catalan numbers are related the number of different _____ on 'n' nodes.

a. Arrays

b. linked lists

c. binary trees page 85

d. functions

202. Applying the sieve technique to selection problem, _____ element is picked from array.

a. Output

b. Total

c. Input

d. Pivot page 35

203. Dynamic Programming approach is usually useful in solving _____ problems.

a. Normal

b. Optimization google

c. Array

d. Loop

204. In recursive formulation of knapsack Problem: V [0, j] = _____ for j>=0

a. -1

b. 0 page 93

c. 1

d. 2

205. _____ is a linear time sorting algorithm.

a. Merge sort

b. Radix sort page 71

c. Quick sort

d. Bubble sort

206. Quick sort is one of the _____ sorting algorithm.

a. Fastest page 19

b. Slowest

c. Major

d. Average

207. The time assumed for each basic operation to execute on RAM model of computation is _____.

a. Infinite

b. Continuous

c. Constant page 10

d. Variable

208. In Sieve Technique, we know the item of interest.

a. True

b. False page 34

209. While analyzing algorithms, _____ and _____ usually considered difficult to calculate.

a. Finite, Infinite

b. Floor, ceiling google

c. Row, Column

d. Graph, Tree

210. While analysis of the brute-force maxima algorithm, an array sorted in the reverse order is the type of _____ case input.

a. Best

b. Worst page 14

c. Somewhat bad

d. Average

211. _____ is not useful measure of central tendency of given input set especially when the distribution of values is highly skewed.

a. Mean

b. Mode

c. Average

d. Median page 34

212. In asymptotical analysis of n(n-3) and 4n*n, as n becomes large, the dominant (fastest growing) term is some constant times _____.

a. n+1

b. n-1

c. n

d. n*n page 23

213. In addition to passing in the array itself to Merge Sort algorithm, we will pass in other arguments which are indices.

a. Two P-38

b. Three

c. Four

d. Five

214. In 2d-maximal problem, a point is said to be if it is not dominated by any other point in that space.

a. Member

b. Minimal

c. Maximal P-11

d. Joint

215. Counting sort assumes that the numbers to be sorted are in the range _____.

a. K to n where n is large

b. 1 to k where k is small (P-57)

c. K to n where k is small

d. k to n where n is small

216. Insertion sort is an efficient algorithm for sorting a _____ number of elements

a. Large P-39

b. Small

c. Extra large

d. Medium

217. If the indices passed to merge sort algorithm are _____ then this means that there is only one element to sort.

a. Small page 28

b. Large

c. Equal

d. Not Equal

218. In Knapsack Problem, each item must be entirely accepted or rejected, is called _____ problem.

a. Fractional

b. 0-1 P-92

c. Linear

d. Optimal

219. If the time complexity of an algorithm is O(n). then it is called _____ time complexity.

a. Linear Wikipedia

b. Constant

c. Average

d. Exponential

220. In the case of _____ analysis does not depend upon on the distribution of input.

a. Merge sort

b. Quick sort P-50

c. Insertion sort

d. Heap sort

221. We can use the _____ Property to devise a recursive formulation of the edit distance problem.

a. Small substructure

b. Algorithmic

c. Real

d. Optimal substructure page 78

222. The following sequence is called _____

1,2,3,5,8,13,21,34,55,…..

a. Optimize sequence

b. Fibonacci sequence page 73

c. Optimal sequence

d. Overlapping sequence

223. Which one sorting algorithm is best suited to sort an array of 2 million elements?

a. Bubble sort

b. Insert sort

c. Merge sort

d. Quick sort

e. Ridx Sort page 71

224. We can improve the performance of quick sort if we could be able to _,_____.

a. Skip input elements somehow

b. Select two or more pivots page 34

c. Skip any sub-array completely

d. Eliminate recursive calls

225. The problem with the brute-force algorithm is that is uses _____ in pruning out de

a. Worst-case time

b. No intelligence page 18

c. Outside looping

d. Artificial intelligence

226. In chain matrix multiplication, the order of the matrices _____.

a. Can be changed

b. Can not be changed page 85

c. is equal

d. is reverse

227. In quick sort algorithm, we choose pivot_____.

a. Always the smallest element

b. Greater than 5

c. Randomly page 35

d. Less than 5

228. In Heap Sort algorithm. Heapify procedure is _____ in nature.

a. Recursive

b. Non-Recursive page 43

c. Fast

d. Slow

229. When matrix A of 5x 3 is multiplied with matrix B of 3 x 4 then the number of multiplications required will be _____.

a. 15

b. 12

c. 36

d.

60

230. An algorithm is said to be correct if for every _____ instance, it halts with the correct _____.

a. Input, Output page 13

b. Design, Analysis

c. Value, Key

d. Key, Analysis

231. In chain matrix multiplication, table is filled _____ to find the multiplication of matrix.

a. row wise

b. column wise

c. diagonally

d. bottom-to-up page 86

232. If we have an equation 8n2+7f*n + 5f + 6 then is large, _____ term will be muchxxxxxxxthe n term and will dominate the running time.

a. f g (n)

b. g (n) * 2

c. n * 2 page 23

d. f (n)

233. For quick sort algorithm. Partitioning takes theta _____.

a. (n)

b. log(n)

c. n log (n)

n2log (n)

234. In Heap Sort algorithm, the maximum levels an element can move upward is

_____

a. Theta (log n) page 43

b. Big-ch (log n)

c. Omega (log n)

d. 0 (1) i.e. Constant time

235. _____ programming is essentially recursion without repetition.

a. Fast

b. Dynamic page 75

c. Array

d. n (log n)

236. There are no hard formal rules to the syntax of the _____ code.

a. Basic

b. Programming

c. Pseudo

d. Assembly

237. In Heap Sort algorithm, to remove the maximum element every time.

a. We call Build-Heap procedure

b. Heap Sort algorithm terminates without result

c. We call heapify procedure

d. Nothing happens

238. Which process is used for avoiding unnecessary repetitions and looking them up again if we need them later.

a. Greedy Approach

b. Memoization page 74

c. Divide and conquer

d. Recursion

239. The worst-case running time of Quick sort is _____ in order to sort an array of n element.

a. O(n log n) page 49

b. O(n)

c. $O(n^2)$

d. O(log n)

240. Boolean operation is a _____ operation on an idealized RAM model of computation.

a. Starting

b. Basic page 10

c. Advance

d. Normal

241. In chain matrix multiplication, if there are n items, there are _____ ways in which outer most pair of parentheses can placed.

a. n^2

b. 2n

c. n+1

d. n-1 page 85

242. The number of nodes in a complete binary tree of height h is: a. 2 * (h+1) - 1

b. 2^(h+1) – 1 page 40

c. 2 * (h+1)

d. ((h+1)^2) – 1

Cs502 Solved MCQS for Final Term

July 2013

Including Current Quizzes

Question #

Bellman Ford Algorithm applies relaxations to every _____

Vertex of …

Edge of Graph and repeats this V-1 times (p159)

Question #

Floyd-Warshall Algorithm is _____

Based on dynamic programming (p161)

Question #

Running time of Floyd-Warshall Algorithm is _____

Θn^2

Θn^4

Θn^3 (p164)

3^n

Question #

Space used by Floyd-Warshall Algorithm is _____

Θn^2 (p164)

Θn^4

Θn^3

3^n

Question #

Huffman encoding Algorithm is a ____

Greedy Algorithm (p99)

Dynamic and Greedy

Question #

Graphs cannot be traversed by brute-force technique.

True (p117) False

Question #

Brute-force technique is always optimal for Graph traversal.

True **False (p117)**

Question #

For graph traversal, Breadth-first search strategy _____

is always recursive

Cannot be recursive

Cannot be non-recursive

Can be both recursive and non-recursive

Question #

Greedy algorithm can do very poorly for some problems.

True (p97) False

Question #

A greedy algorithm sometimes works well for optimization problems.

True (p97) False

Question #

A greedy algorithm does not work in phases.

True **False (p97)**

Question #

Greedy algorithm approach solves both 0/1 Knapsack and Fractional Knapsack problems.

True **False**

Question #

Dynamic Programming approach solves both 0/1 Knapsack and Fractional Knapsack problems.

True **False**

Question #

In _____ based solution of knapsack problem, we consider 2 cases. Leave object Or Take object.

Brute force

Dynamic programming (p93)

Question #

In brute force based solution of knapsack problem, we consider 2 cases. Leave object Or Take object.

True **False (p93)**

Question #

In DP based solution of knapsack problem, V[0, j] = _____ if j>=0

-1

0 (p93)

1
2

Question #

In DP based solution of knapsack problem, V[0, j] = 1 if
j>=0 True

False (p93)

Question #

In DP based solution of knapsack problem, to compute entries of V we will imply a/an _____ approach.

Subjective

Inductive (p93)

Brute force

Combination

Question #

In DP based solution of knapsack problem, if we decide to take an object i, then we gain a value of _____.

w i-1

vi-1
w i

vi (p93)

Question #

In Fractional Knapsack problem, one is allowed to take fraction of an item for _____

for fraction of the weight

for fraction of the value

Both, fraction of the weight and value (p109)

None

Question #

_____ approach is optimal for the fractional knapsack problem.

Divide and Conquer

Dynamic Programming

Greedy algorithm (p110)

Brute force

Question #

In ____ Knapsack Problem, limitation is that an item can either be put in the bag or not-fractional items are not allowed.

0 1 **0/1** Fractional

Question #

In Fractional Knapsack problem, the goal is to _____

minimize the value of items without exceeding the total weight limit of W

maximize the value of items without exceeding the total weight limit of W (p109)

minimize the value of items even if exceeding the total weight limit of W

maximize the value of items even if exceeding the total weight limit of W

Question #

In solving Fractional Knapsack problem, we do not need sorting.

True

False (p109)

Question #

Suppose that a graph G = (V,E) is implemented using adjacency lists. What is the complexity of a breadth-first traversal of G? Select correct option:

O(|V |^2)

O(|V | |E|)

O(|V |^2|E|)

O(|V | + |E|) (p116)

Question #

For traversing graphs, Breadth-first search can be visualized as a wave front propagating inwards towards root (or source) node.

True

False (p117)

Question #

Breadth-first search is not a traversal strategy for graphs.

True

False (p119)

Question #

Breadth-first search is not a popular algorithm technique used for traversing graphs.

True

False (p119)

Question #

Breadth-first search begins at a root node and inspects all the nodes except neighboring ones.
True

False (117)

Question #

Each time we traverse graph by Breath-first search algorithm, we count the distance from _____

Starting node (p117)

neighbors of the starting node

right most node

left most node

Question #

Suppose we are given a set S = {a1, a2, ... an} of n activities that are to be scheduled to use some resource. Each activity ai must be started at a given start time si and ends at a given finish time fi. We want to schedule the largest number of activities on the resource. It is an example of _____. Searching

Dynamic Programming

Activity selection (p105)

Divide and Conquer

Question #

In general, the Activity selection problem is to select a _____

minimum-size set of interfering activities

maximum-size set of mutually non-interfering activities (p105)

maximum-size set of interfering activities

minimum-size set of mutually non-interfering activities

Question #

In Activity selection (using Greedy approach), intuitively _____

Short activities are not preferable

There are always short activities as input

We do not like long activities (p105)

It does not matter about the length of activities

Question #

The activity schedule is a simple scheduling problem for which the ____ approach provides an optimal solution.

Dynamic Programming

Graph

Divide and Conquer

Greedy Algorithm (p105)

Question #

The activity scheduling is a simple scheduling problem for which the greedy algorithm approach provides a/an

_____solution.

Simple

Sub optimal

Optimal (p105)

Non optimal

Question #

A number of lectures are to be given in a single lecture hall. Optimum scheduling for this is an example of Activity selection.

True (p105)

False

Question #

In Activity scheduling algorithm, each activity is represented by a _____

Circle

Square

Triangle

Rectangle (p106)

Question #

In Activity scheduling algorithm, the width of a rectangle _____

Is always ignored

Directs towards recursion

Should be maximized

Indicates the duration of an activity (p106)

Question #

In Activity scheduling algorithm, the time is dominated by sorting of the activities by _____

start times

finish times (p106)

Question #

The complexity of Activity scheduling algorithm is _____

O (n)

O log n

O (n) log n (p106)

O n^2

Question #

In Activity scheduling algorithm, as base case if there are no activities then Greedy algorithm _____

cannot be optimized

is solved using Recursion

is transformed into Dynamic Programming

is trivially optimal (p109)

Question #

Bag is a _____

type of algorithm

data structure (p119)

program

compiler

Question #

For _____ problems, greedy approach always gets optimum.

Select correct option:

Two

Some (p97)

All

None of the

Question #

Fixed-length codes may not be efficient from the perspective of _____the total quantity of data.

Minimizing (p99)

Averaging

Maximizing

Summing

Question #

There are _____ nested loops in DP based algorithm for computing the minimum cost of chain matrix multiplication.

2

3 (p90)

4

5

Question #

Time complexity of DP based algorithm for computing the minimum cost of chain matrix Multiplication is ----.

log n n n^2 **n3 (p90)**

Question #

In greedy algorithm, at each phase, you take the_____ you can get right now, without regard for future consequences.

Worst Minimum Good **Best (p97)**

Question #

An optimization problem is one in which you want to find, not just a solution, but the _____ solution.

Simple Good **Best (p97)** Worst

Question #

In _____ algorithm, you hope that by choosing a local optimum at each step, you will end up at a global optimum.

Simple Non-Greedy. **Greedy (p97)** Brute force

Question #

The string |xyz|, if coded with ASCII code, the message length would be 24 bits.

True False

Question #

If the string |lmncde| is coded with ASCII code, the message length would be --- bits.

24 36 **48** 60

Question #

If there are n items, there are _____ possible combinations of the items.

---

2 n **2^n (p92)** 3^n

Question #

In Huffman encoding, for a given message string, the frequency of occurrence (relative probability) of each character in the message is determined last.

True **False (p100)**

Question #

In DP based solution of knapsack problem, to compute entries of V we will imply a/an _____ approach.

Subjective **Inductive (p93)** Brute force Combination

Question #

Inductive approach to compute entries of V is implied in _____ based solution of knapsack problem.

Brute force **Dynamic programming (p93)**

Question #

In brute force based solution of knapsack problem, we consider 2 cases, Leave object Or Take object.

True (p93) False

Question #

There are _____ nested loops in DP based algorithm for computing the minimum cost of chain matrix multiplication.

2

3

4

5 (not confirm page 90)

Question #

Those problems in which Greedy finds good, but not always best is called a greedy_____.

Algorithm

Solution

Heuristic (p97)

Result

Question #

The Huffman codes provide a method of _____ data efficiently.

Reading

Encoding (p99)

Decoding

Printing

Question #

Fixed-length codes are known for easy break up of a string into its individual characters.

True (p99) False

Question #

The Huffman codes provide a method of _____ data efficiently.

Reading **Encoding (p99)** Decoding Printing

Question #

In Huffman encoding, the _____is the number of occurrence of a character divided by the total characters in the message.

Counting

Parsing

Probability (p100)

Weight

Question #

In Huffman encoding, for a given message string, the frequency of occurrence (relative probability) of each _____in the message is done first.

Word Space Digit **Character (p100)**

Question #

The Huffman algorithm finds a (n) _____ solution.

Optimal (Reference) Non-optimal Exponential Polynomial

Question #

The Huffman algorithm finds an **exponential** solution True **False (Reference)**

Question #

The Huffman algorithm finds a **polynomial** solution True **False (Reference)**

Prefix Property:

The codewords assigned to characters by the Huffman algorithm have the property that no codeword is a prefix of any other:

Question #

Non-optimal or greedy algorithm for money change takes_____

O(k) (p99) O(kN) O(2k) O(N)

Question #

The Huffman codes provide a method of encoding data **inefficiently** when coded using ASCII standard.

True **False (p99)**

- The Huffman codes provide a method of encoding data **efficiently**.

Question #

The codeword assigned to characters by the Huffman algorithm have the property that no codeword is the **postfix** of any other. True **False**

Question #

The codeword assigned to characters by the Huffman algorithm have the property that no codeword is the **prefix** of any other. **True (p101)** False

Question #

Using ASCII standard the string “abacdaacacwe” will be encoded with _____ bits

Select correct option:

64

128

96 (p101 12x8=96)

120

Question #

Using ASCII standard the string abacdaacac will be encoded with _____ bits.

80 (p99) 160 320 100

- Consider the string " abacdaacac". if the string is coded with ASCII codes, the message length would **be**
- **10 × 8 = 80 bits.**

Question #

Using ASCII standard the string abacdaacac will be encoded with 160 bits. True **False (p99)**

Question #

Using ASCII standard the string abacdaacac will be encoded with 320 bits. True **False (p99)**

Question #

Using ASCII standard the string abacdaacac will be encoded with 100 bits. True **False (p99)**

Question #

Using ASCII standard the string abacdaacac will be encoded with 32 bytes. True **False (p99)**

Question #

The greedy part of the Huffman encoding algorithm is to first find two nodes with **smallest** frequency.

True (p100) False

Question #

The greedy part of the Huffman encoding algorithm is to first find two nodes with **character** frequency.

True **False (p100)**

Question #

The greedy part of the Huffman encoding algorithm is to first find two nodes with **larger** frequency.

True **False**

Question #

Huffman algorithm uses a greedy approach to generate an antefix code T that minimizes the expected length B (T) of the encoded string. True **False (p102)**

Question #

Huffman algorithm uses a greedy approach to generate a postfix code T that minimizes the expected length B (T) of the encoded string. True **False (p102)**

Question #

Huffman algorithm uses a greedy approach to generate a prefix code T that minimizes the expected length B (T) of the encoded string. **True (p102)** False

Question #

Huffman algorithm generates an optimum prefix code.

True (p102) False

Question #

A graph is said to acyclic if it contains _____

at least one cycle

exactly one cycle

---

always more than one cycle

no cycle (p116)

Question #

Graph is not a good way to model some sort of "connection" or "relationship" or "interaction" between pairs of objects taken from a set of objects.

True **False (p113)**

Question #

A graph is not connected if every vertex can reach every other vertex.

True **False (p116)**

Question #

Traversing a graph means visiting _____ in the graph.

no node

at least one

more than one node

every node (p119)

Question #

Graphs are important _____ model for many application problems.

Unsystematic

Mathematical (p113)

Haphazard

Unpredictable

Question #

In a digraph, the number of edges coming in of a vertex is **not** called the in-degree of that vertex.

True **False (p114)**

Question #

In a digraph, the number of edges coming out of a vertex is **not** called the out-degree of that vertex.

True **False** **(p114)**

Question #

For a digraph G = (V, E), Sum of in-degree(v) _____

Not equal to Sum of out-degree(v)

= Sum of out-degree(v) (p115)

< Sum of out-degree(v)
> Sum of out-degree(v)

Question #

Graphs cannot be represented by an adjacency matrix.

True **False** **(p116)**

Question #

A vertex a is not adjacent to vertex b if there is an edge from a to b.

True **False** **(p113)**

Question #

A **dense undirected** graph is:

Select correct option:

A graph in which E = O (V^2) (Reference)

A graph in which E = O (V)

A graph in which E = O (log V)

All items above may be used to characterize a dense undirected graph

Question #

What algorithm technique is used in the implementation of **Kruskal solution** for the MST?

Greedy Technique (p142)

Divide-and-Conquer Technique

Dynamic Programming Technique

The algorithm combines more than one of the above techniques

Question #

A digraph is strongly connected under what condition?

A digraph is strongly connected if for every pair of vertices u, v e V, u can reach v .

A digraph is strongly connected if for every pair of vertices u, v e V, u can reach v and vice versa.
(p135) A digraph is strongly connected if for at least one pair of vertex u, v e V, u can reach v and vice versa.

A digraph is strongly connected if at least one third pair of vertices u, v e V, u can reach v and vice versa.

Question #

The relationship between number of back edges and number of cycles in DFS is,

Both are equal

Back edges are half of cycles

Back edges are one quarter of cycles

There is no relationship between no. of edges and cycles (p131)

Question #

Back edge is?

(u, v) where v is an ancestor of u in the tree. (p128)

(u,v) where u is an ancestor of v in the tree.

(u, v) where v is an predecessor of u in the tree.

None of above

Question #

Forward edge is?

(u, v) where u is a proper descendent of v in the tree.

(u, v) where v is a proper descendent of u in the tree. (p129)

(u, v) where v is a proper ancestor of u in the tree.

(u, v) where u is a proper ancestor of v in the tree.

Question #

Edge (u, v) is a forward edge if

u is a proper descendant of v in the tree

v is a proper descendant of u in the tree

None of these

Question #

Cross edge is?

(u, v) where u and v are not ancestor of one another

(u, v) where u is ancestor of v and v is not descendent of u.

(u, v) where u and v are not ancestor or descendent of one another (p129)

(u, v) where u and v are either ancestor or descendent of one another.

Question #

If you find yourself in maze the better traversel approach will be?

Select correct option:

BFS

BFS and DFS both are valid (p119)

Level order

DFS

Question #

In digraph G=(V,E) ;G **has cycle** if and only if

Select correct option:

The DFS forest has forward edge.

The DFS forest has back edge (p131)

The DFS forest has both back and forward edge

BFS forest has forward edge

Question #

What is generally true of Adjacency List and Adjacency Matrix representations of graphs?

Select correct option:

Lists require less space than matrices but take longer to find the weight of an edge (v1,v2)

Lists require less space than matrices and they are faster to find the weight of an edge (v1, v2)

Lists require more space than matrices and they take longer to find the weight of an edge (v1, v2)

Lists require more space than matrices but are faster to find the weight of an edge (v1, v2)

Question #

There is relationship between number of **back edges** and **number of cycles** in DFS

Select correct option:

Both are equal.

Cycles are half of back edges.

Cycles are one fourth of back edges.

There is no relationship between back edges and number of cycle **(p131)**

Question #

You have an adjacency list for G, what is the time complexity to compute Graph transpose G^T.?

Select correct option:

(V+E) **(Reference)**

V.E

V

E

Question #

Although it requires more complicated data structures, **Prim's algorithm** for a minimum spanning tree **is better** than **Kruskal's** when the graph has a **large number** of vertices.

True False

Question #

Maximum number of vertices in a Directed Graph may be $|V^2|$

True False

Question #

Dijkstra's algorithm:

Has greedy approach to find all shortest paths

Has both greedy and dynamic approach to find all shortest paths

Has greedy approach to compute single source shortest paths to all other vertices (p154) Has both greedy and dynamic approach to compute single source shortest paths to all other vertices.

Question #

Dijkestra's single source shortest path algorithm works if all edges weights are non-negative and there are negative cost cycles. True **False**

Question #

Dijkestra s single source shortest path algorithm works if all edges weights are non-negative and there are no

negative cost cycles. **True (p159)** False

Question #

Dijkestra s single source shortest path algorithm works if all edges weights are negative and there are no negative cost cycles.

True **False**

Question #

Dijkstra's algorithm is operates by maintaining a subset of vertices.

True False

Question #

Floyd-Warshall algorithm is a dynamic programming algorithm; the genius of the algorithm is in the clever

recursive formulation of the shortest path problem. **True (p162)** False

Question #

Floyd-Warshall algorithm, as in the case with DP algorithms, we avoid recursive evaluation by generating a table for $d_{ij}^{(k)}$

True (p164) False

Question #

The term coloring came from the original application which was in **map drawing**. **True (p173)** False

Question #

The term "coloring" came from the original application which was in **architectural design**. True **False**

Question #

In the **clique cover** problem, for two vertices to be in the same group, they must be _____ each other.

Apart from Far from Near to **Adjacent to (p176)**

Question #

In the clique cover problem, for two vertices to be in the same group, they **must be adjacent** to each other.

True (p176) False

Question #

In the **clique cover** problem, for two vertices to be in the same group, they **must be apart** from each other.

True **False (p176)**

Question #

The difference between Prims algorithm and Dijkstra s algorithm is that Dijkstra s algorithm uses a different key.

True (p156) False

Question #

The difference between Prim s algorithm and Dijkstra s algorithm is that Dijkstra s algorithm uses a same key.

True **False (p156)**

Question #

You have an adjective list for G, what is the time complexity to computer graph transpose G^T.?

(V+E) (p138)

Question #

Given an adjacency list for G, it is possible to compute G^T in $\Theta(V + E)$ time.

It takes O(log V) to extract a vertex from the priority queue.

Question #

Overall time for Kruskal is $\Theta(E \log E) = \Theta(E \log V)$ if the graph is sparse. **True (p149)**

Question #

An optimization problem is one in which you want to find,

Not a solution

An algorithm

Good solution

The best solution (p97)

Question #

Although it requires more complicated data structures, Prim's algorithm for a minimum spanning tree is better than Kruskal's when the graph has a large number of vertices.

True False

Question #

If a graph has v vertices and e edges then to obtain a spanning tree we have to delete v edges.

v – e + 5 edges

v + e edges.

None of these

Question #

Bellman-Ford allows negative weights edges and negative cost cycles.

True **False (p159)**

- Bellman-Ford allows negative weights edges and no negative cost cycles.

Question #

We do sorting to,

keep elements in random positions

keep the algorithm run in linear order

keep the algorithm run in (log n) order

keep elements in increasing or decreasing order (p39)

Question #

After partitioning array in Quick sort, pivot is placed in a position such that

Values smaller than pivot are on left and larger than pivot are on right (Ref)

Values larger than pivot are on left and smaller than pivot are on right

Pivot is the first element of array

Pivot is the last element of array

Question #

Dynamic programming algorithms need to store the results of intermediate sub-problems.

True False

Question #

Which statement is true?

If a dynamic-programming problem satisfies the optimal-substructure property, then a locally optimal solution is globally optimal.

If a greedy choice property satisfies the optimal-substructure property, then a locally optimal solution is globally optimal.

Both of above None of above

Question #

What **general property** of the list indicates that the graph has an **isolated vertex**?

There is Null pointer at the end of list.

The Isolated vertex is not handled in list.

Only one value is entered in the list.

There is at least one null list.

Question #

Depth first search is **shortest path** algorithm that works on **un-weighted** graphs.

True **False (p153)**

- The **breadth-first-search** algorithm is a shortest-path algorithm that works on **un-weighted graphs.**

Question #

Which is true statement?

Select correct option:

Breadth first search is shortest path algorithm that works on un-weighted graphs (p153)
Depth first search is shortest path algorithm that works on un-weighted graphs.

Both of above are true.

None of above are true.

Question #

Shortest path problems can be solved efficiently by modeling the road map as a graph.

True False

Question #

Which is true statement in the following?

Kruskal algorithm is multiple source technique for finding MST.

Kruskal's algorithm is used to find minimum spanning tree of a graph, time complexity of this algorithm is O(EV) Both of above

Kruskal's algorithm (choose best non-cycle edge) is better than Prim's (choose best tree edge) when the graph has relatively few edges.

Question #

Kruskal's algorithm (choose best non-cycle edge) is better than Prim's (choose best tree edge) when the graph has relatively few edges.

True False

Question #

What is the time complexity to **extract a vertex** from the **priority queue** in **Prim's algorithm**?

Select correct option:

O (log E)

(V)

(V+E)

O (log V) (p152)

Question #

In Prim's algorithm, the additional information maintained by the algorithm is the **length of the shortest edge** from vertex v to points already in the tree.

a) True **b) False** c) unknown

Question #

Although it requires more complicated data structures, Prim's algorithm for a minimum spanning tree is better than Kruskal's when the graph has a **large number of vertices**.

a) True b) False c) unknown

Question #

Which statement is true?

► The running time of Bellman-Ford algorithm is T (VE)
► Both Dijkstra's algorithm and Bellman-Ford are based on performing **repeated relaxations**
► The 0-1 knapsack problem is hard to solve

Only i Only iii Both i and iii **All of these**

Question #

Which of the following arrays represent **descending (max) heaps**?

i. [10,7,7,2,4,6] ii. [10,7,6,2,4,7]

iii. [10,6,7,2,4,6] iv. [6,6,7,2,4,10]

Only ii Only iv Both ii and iv Both i and iii

Question #

Which of the following statement(s) is/are correct?

(a) O(n log n + n2) = O(n2). (b) O(n log n + n2) = O(n2 log 2n)

(c) O(c n2) = O(n2) where c is a constant. (d) O(c n2) = O(c) where c is a constant.

(e) O(c) = O(1) where c is a constant.

Only (a) & (e) Both (c) and (e)

Question #

Which of the shortest path algorithms would be most appropriate for finding paths in the graph with **negative edge weights** and **cycles**?

- Dijkstra's Algorithm
- Bellman-Ford Algorithm.
- Floyd Warshall Algorithm

Only ii Only iii

Both ii & iii

Question #

If, in a DFS forest of digraph G = (V, E), f[u] = f[v] for an edge (u, v) ? E then the edge is called

Back edge Forward edge Cross Edge Tree Edge None of these

Question #

Best and worst case times of an algorithm may be same. **True** False

Question #

Can an adjacency matrix for a directed graph ever not be square in shape? **Yes** No

Question #

Consider the following Huffman Tree. The binary code for the string **TEA** is:

10 00 010

011 00 010

10 00 110

11 10 110

String	Encoding
TEA	10 00 010
SEA	011 00 010
TEN	10 00 110

Question #

Can an adjacency matrix for a directed graph ever not be square in shape? Yes **No**

- No. since we want to describe the relationship between each node and each other node, we need precisely n^2 matrix entries.

Question #

Suppose we have two problems A and B .Problem A is polynomial-time reducible and problem B is NP-complete. If we reduce problem A into B then problem A becomes NP-complete **Yes** No

Question #

If a problem is NP-complete, it must also be in NP.

a) True b) False c) unknown

Question #

If a problem is in NP, it must also be in P.

True **False (p178)** unknown

Question #

If a problem is in NP-complete, it must also be in NP.

True False

Cs502 Solved MCQS for MidTerm

Including Current Quizes

26 May 2013

Question # 1

Word Algorithm comes from the name of the Muslim author:

Abu Ja'far Mohammad ibn Musaal-Khowarizmi. (p7)

Question # 2

Al-Khwarizmi's work was written in a book titled

al Kitab al-mukhatasar fi hisab al-jabr wa'l-muqabalah (p7)

Question # 3

Analysis of Selection algorithm ends up with,

Select correct option:

T(n) (p37) T(1 / 1 + n) T(n / 2) T((n / 2) + n)

Question # 4

The function f(n)= n(logn+1)/2 is asymptotically equivalent to n log n. Hre Upper Bound means the function f(n) grows asymptotically _____ faster than n log n.

Select correct option:

More Quiet **Not (p24)** At least

Question # 5

The running time of an algorithm would not depend upon the optimization by the compiler but that of an implementation of the algorithm would depend on it. Select correct option:

True **False (p13)**

Question # 6

After sorting in merge sort algorithm, merging process is invoked.

Select correct option:

True (p27) False

Question # 7

Asymptotic growth rate of the function is taken over_____ case running time.

Select correct option:

Best

Average

Worst (p14)

Normal

Question # 8

In analysis of f(n)=n(n/5)+n-10 log n, f(n) is asymptotically equivalent to _____.

n

2n

n+1

n^2 (p23)

Question # 9

f(n) and g(n) are asymptotically equivalent. This means that they have essentially the same _____ for large n.

Results Variables Size **Growth rates (p23)**

Question # 10

Algorithm is concerned with _____ issues.

Macro Micro 1 **Both Macro & Micro (p8)** Normal

Question # 11

We cannot make any significant improvement in the running time which is better than that of brute-force algorithm.

True **False (p18, p92)**

Question # 12

In addition to passing in the array itself to Merge Sort algorithm, we will pass in _____other arguments which are indices.

Two (p28) Three Four Five

Question # 13

For the worst-case running time analysis, the nested loop structure containing one “for” and one “while” loop, might be expressed as a pair of _____nested summations.

1 **2 (p14)** 3 4

Question # 14

The O-notation is used to state only the asymptotic _____bounds.

Two Lower **Upper (p25)** Both lower & upper

Question # 15

Which of the following is calculated with **big O notation**?

Lower bounds **Upper bounds (p25)**

Both upper and lower bound Medium bounds

Question # 16

The Ω-notation allows us to state only the asymptotic _____ bounds.

Lower (p25) Upper

Both upper and lower bound Medium bounds

Question # 17

The time assumed for each basic operation to execute on RAM model of computation is _____

Infinite Continuous **Constant (p10)** Variable

Question # 18A

If the indices passed to merge sort algorithm are not equal, the algorithm may return immediately.

True **False (p28)**

Question #18B

If the indices passed to merge sort algorithm are _____,then this means that there is only one element to sort.

Small Large **Equal (p28)** Not Equal

Question # 19

Brute-force algorithm uses no intelligence in pruning out decisions.

True (p18) False

Question # 20

In analysis, the **Lower Bound** means the function grows asymptotically **at least** as fast as its largest term.

True (p24) False

Question # 21

In analysis, the **Upper Bound** means the function grows asymptotically **no faster** than its largest term.

True (p24) False

Question # 22

For small values of n, any algorithm is fast enough. Running time does become an issue when n gets large.

True (p14) Fast

Question # 23

The array to be sorted is not passed as argument to the merge sort algorithm.

True **False (p28)**

Question # 24

In simple brute-force algorithm, we give no thought to efficiency.

True (p11) False

Question # 25

The ancient Roman politicians understood an important principle of good algorithm design that is plan-sweep algorithm.

True **False (p27) [Divide and Conquer]**

Question # 26

In 2d-space a point is said to be _____if it is not dominated by any other point in that space.

Select correct option:

Member Minimal **Maximal (p11)** Joint

Question # 27

An algorithm is a mathematical entity that is dependent on a specific programming language.

Select correct option:

True **False (p7)**

Question # 28

Fibonacci sequence was posted by _____

Leonardo Pisano (p73)

Question # 29

$8n^2 + 2n - 3$ will eventually exceed $c_2(n)$ no matter how large we make c2.

True (p25) False

Question # 30

If we associate (x, y) integers pair to cars where x is the speed of the car and y is the negation of the price. High y value for a car means a _____ car.

Fast Slow Expensive **Cheap (p11)**

Question # 31

The running time of quick sort depends heavily on the selection of:

No of inputs Arrangement of elements in array

Size of elements **Pivot element (p49)**

Question # 32

Which sorting algorithm is faster?

O(n^2) O(nlogn) **O(n+k) (p58)** O(n^3)

Question # 33

Q knapsack problem is called a "0-1" problem, because

Each item must be entirely accepted or rejected (p92)

Question # 34

Which statement is true?

Select correct option:

If a **dynamic-programming problem** satisfies the optimal-substructure property, then a locally optimal solution is globally optimal.

If a **greedy choice property** satisfies the optimal-substructure property, then a locally optimal solution is globally optimal.

Both of above (p77, p98) None of above

Question # 35

Merge sort is stable sort, but not an in-place algorithm

True (p54) False

Question # 36

In counting sort, once we know the ranks, we simply _____ numbers to their final positions in an output array.

Delete **copy (p57)** Mark arrange

Question # 37

Dynamic programming algorithms need to store the results of intermediate sub-problems. **True (p75)** False

Question # 38

A p × q matrix A can be multiplied with a q × r matrix B. The result will be a p × r matrix C. There are (p . r) total entries in C and each takes _____ to compute.

O (q) **O (1) (p84)** O (n^2) O (n^3)

Question # 39

Due to left-complete nature of binary tree, heaps can be stored in

Link list Structure **Array (p40)** None of above

Question # 40

Efficient algorithm requires less computational _____

Memory Running Time **Memory and Running Time (p9)** Energy

Question # 41

Merge sort makes two recursive calls. Which statement is true after these recursive calls finish, but before the merge step?

The array elements form a heap

Elements in each half of the array are sorted amongst themselves

Elements in the first half of the array are less than or equal to elements in the second half of the array

None of the above

Question # 42

Who invented Quick sort procedure?

Hoare Sedgewick Mellroy Coreman

Question # 43

Merge sort requires extra array storage. **True (p54)** False

- Mergesort is a stable algorithm but **not** an in-place algorithm. It requires extra array storage.

Question # 44

In which order we can sort?

Increasing order only decreasing order only

Increasing order or decreasing order both at the same time

Question # 45

In the analysis of **Selection algorithm**, we make a number of passes, in fact it could be as many as,

T(n) T(n / 2) **log n (p37)** n / 2 + n / 4

Question # 46

How much time **merge sort** takes for an array of numbers?

T(n^2) T(n) T(log n) **T(n log n) (p40)**

Question # 47A

Sieve Technique applies to problems where we are interested in finding a single item from a larger set of ___

n items (p34) phases pointers constant

Question # 47B

The sieve technique works in phases as follows. It applies to problems where we are interested in finding

a **single** item from a larger set of n items. **(p34)**

Question # 47C

The sieve technique works where we have to find _____ item(s) from a large input.

Single (p34) Two Three Similar

Question # 48

The sieve technique works in _____ as follows

Phases (p34) numbers integers routines

Question # 49

For the heap sort, access to nodes involves simple _____ operations.

Arithmetic (p41) binary algebraic logarithmic

Question # 50

The analysis of Selection algorithm shows the total running time is indeed _____in n,

Arithmetic geometric **linear (p39)** orthogonal

Question # 51

Slow sorting algorithms run in,

T(n^2) (p39) T(n) T(log n) T(n log n)

Question # 52

A heap is a left-complete binary tree that conforms to the

Increasing order only decreasing order only **heap order** (log n) order

Question # 53

For the heap sort we store the tree nodes in

Level-order traversal (p40) in-order traversal pre-order traversal post-order traversal

Question # 54A

In Sieve Technique we do not know which item is of interest **True (p34)** False

Question # 54B

In Sieve Technique, we know the item of interest.

True **False (p34)**

Question # 55

Divide-and-conquer as breaking the problem into a small number of

Pivot Sieve **smaller sub problems (p34)** Selection

Question # 56

For the sieve technique we solve the problem,

Recursively (p34) mathematically precisely accurately

Question # 57

The reason for introducing **Sieve Technique** algorithm is that it illustrates a very important special case of, **Divide-and-conquer (p34)** decrease and conquer greedy nature 2-dimension Maxima

Question # 58

Sorting is one of the few problems where provable _____ bonds exits on how fast we can sort,

Upper **Lower (p39)** Average log n

Question # 59

Sieve Technique can be applied to selection problem? **True (p35)** False

Question # 60

One of the clever aspects of heaps is that they can be stored in arrays without using any_____

Pointers (p40) constants variables functions

Question # 61

Heaps can be stored in arrays without using any pointers; this is due to the ___ nature of the binary tree,

Left-complete (p40) right-complete tree nodes tree leaves

Question # 62

How many elements do we eliminate in each time for the Analysis of Selection algorithm?

n / 2 elements (p36) (n / 2) + n elements n / 4 elements 2 n elements

Question # 63

The sieve technique is a special case, where the number of sub problems is just

5 Many **1 (p34)** few

Question # 64

For the Sieve Technique we take time

T(nk) (p34) T(n / 3) n^2 n/3

Question # 65

The number of nodes in a complete binary tree of height h is

Select correct option:

2^(h+1) – 1

2 * (h+1) – 1

2 * (h+1)

((h+1) ^ 2) – 1

Question # 66

Random access machine or RAM is a/an _____

Machine build by Al-Khwarizmi Mechanical machine

Electronics machine **Mathematical model (p10)**

Question # 67

Analysis of Selection algorithm ends up with,

T(n)

T(1 / 1 + n)

T(n / 2)

T((n / 2) + n)

Question # 68

Continuation sort is suitable to sort the elements in range 1 to k

1. K is Large 2. K is not known 3. K may be small or large **4. K is small (p57)**

Question # 69

Counting sort is suitable to sort the elements in range 1 to k:

K is large **K is small (p57)** K may be large or small None

Question # 70

Continuing sort has time complexity of?

1. O (n) (p58) 2. O(n+k) 3. O(nlogn)

Question # 71

Counting sort has time complexity:

O (n) (p58) O(n+k) O(k) O(nlogn)

Question # 72

Memoization is _____:

-To store previous results for further use.

-To avoid unnecessary repetitions by writing down the results of recursive calls and looking them again if needed later. (p74)

-To make the process accurate -None of the above

Question # 73

In Quick sort algorithm, constants hidden in T(n lg n) are:

Large Medium Not known **Small (Ref)**

Question # 74

Quick sort is based on divide and conquer paradigm; we divide the problem on base of pivot element and:

There is explicit combine process as well to conquer the solution.

No work is needed to combine the sub-arrays, the array is already sorted

Merging the subarrays None of above.

Question # 75

In RAM model instructions are executed _____

In Random Access Machine, instructions are executed_____

One after another (p10) Parallel Concurrent Random

Question # 76

In the analysis of Selection algorithm, we eliminate a constant fraction of the array with each phase; we get the

convergent _____ series in the analysis,

linear arithmetic **geometric (p37)** exponent

Question # 77

_____ is a graphical representation of an algorithm

Σ notation Θnotation **Flowchart (Ref)** Asymptotic notation

Question # 78

A RAM is an idealized machine with _____ random-access memory.

256MB 512MB **An infinitely large (p10)** 100GB

Question # 79

What type of instructions Random Access Machine (RAM) can execute? Choose best answer

Algebraic and logic Geometric and arithmetic

Arithmetic and logic (p10) Parallel and recursive

Question # 80

What will be the total number of **max comparisons** if we run **brute-force maxima** algorithm with n elements?

n^2 (p14) 2n/n n 8n

Question # 81

What is the solution to the recurrence T(n) = T(n/2)+n .

O(logn) **O(n)** O(nlogn) O(n2)

Question # 82

Consider the following code:

```
For(j=1; j<n;j++)
        For(k=1; k<15;k++)
                For(l=5; l<n; l++)
                {
                        Do_something_constant();
                }
```

What is the order of execution for this code.

O(n)

O(n3)

O(n2 log n)

O(n2)

Question # 83

Consider the following Algorithm: Fun(n){ if (n=1) return 1 else return (n * Fun(n-1)) } Recurrence for the above algorithm is:

nT(n-1)+1

2T(n-1)+1

T(n-1)+cn

T(n-1)+1

Question # 84

Consider the following Algorithm:

```
Factorial (n){
  if (n=1)
    return 1
  else
    return (n * Factorial(n-1))
{
```

Recurrence for the following algorithm is:

T(n) = T(n-1) +1

T(n) = nT(n-1) +1

T(n)= T(n-1) +n

T(n)=T(n(n-1)) +1

Question # 85

What is the total time to heapify?

O(log n) **(p43)** O(n log n) O(n2 log n) O(log2 n)

Question # 86

When we call heapify then at each level the comparison performed takes time

It will take Θ (1) (p43)

Time will vary according to the nature of input data

It cannot be predicted

It will take Θ (log n)

Question # 87

In Quick sort, we don't have the control over the sizes of recursive calls

-True (p49) -False -Less information to decide -Either true or false

Question # 88

Is it possible to sort without making comparisons? **Yes (p57)** No

Question # 89

If there are **Θ (n2) entries** in **edit distance matrix** then the total running time is

Θ (1) **Θ (n^2) (p84)** Θ (n) Θ (n log n)

Question # 90

For **Chain Matrix Multiplication** we cannot use divide and conquer approach because,

We do not know the optimum k (p86)

We use divide and conquer for sorting only

We can easily perform it in linear time

Size of data is not given

Question # 91

The Knapsack problem belongs to the domain of _____ problems.

Optimization (p91) NP Complete Linear Solution

Sorting

Question # 92

Suppose we have three items as shown in the following table, and suppose the capacity of the knapsack is 50 i.e.

W=50.

Item	Value	Weight
1	60	10
2	100	20
3	120	30

The optimal solution is to pick

Items 1 and 2

Items 1 and 3

Items 2 and 3

None of these

Question # 93

Theta asymptotic notation for T (n):

Select correct option:

Set of functions described by: c1g(n)<=f(n) for c1 some constant and n=n0

Set of functions described by c1g(n)>=f(n) for c1 some constant and n=n0

Theta for T(n)is actually upper and worst case complexity of the code

Set of functions described by: c1g(n)<=f(n)<=c2g(n) for c1 and c2 some constants and n=n0

Question # 94

A (an) _____ is a left-complete binary tree that conforms to the heap order.

Heap (page 40) Binary tree Binary search tree Array

Question # 95

A heap is a left-complete binary tree that conforms to the _____

increasing order only decreasing order only **heap order (p40)** (log n) order

Question # 96

Consider the following Algorithm: Fun(n){ if (n=1) return 1 else return (n * Fun(n-1)) } Recurrence for the above algorithm is: Select correct option:

nT(n-1)+1

2T(n-1)+1

T(n-1)+cn

T(n-1)+1

Question # 97

The recurrence relation of Tower of Hanoi is given below T(n)={1 if n=1 and 2T(n-1) if n >1 In order to move a tower of 5 rings from one peg to another, how many ring moves are required?

Select correct option: 16 10 **32 (p30)** 31

Question # 98

In Quick Sort Constants hidden in T(n log n) are

1. Large 2. Medium **3. Small (Reference)** 4. Not Known

Question # 99

In stable sorting algorithm:

One array is used

In which duplicating elements are not handled.

More than one arrays are required.

Duplicating elements remain in same relative position after sorting. (p54)

Question # 100

In in-place sorting algorithm is one that uses arrays for storage:

An additional array

No additional array (p54)

Both of above may be true according to algorithm

More than 3 arrays of one dimension

Question # 101

Which may be a stable sort?

1. Merger 2. Insertion **3. Both above (p54)** 4. None of the above

Question # 102

An in place sorting algorithm is one that uses _____ arrays for storage

1. Two dimensional arrays 2. More than one array

3. No Additional Array (p54) 4. None of the above

Question # 103

Quick sort is _____

Stable and In place **Not stable but in place (p54)**

Stable and not in place Sometime in place and sometime stable

Question # 104

One Example of **in place** but **not stable** sort is

Quick (p54) Heap Merge Bubble

Question # 105

Which may be stable sort?

Bubble sort Insertion sort **both of above (p54)**

Question # 106

Merge sort is stable sort, but not an in-place algorithm **True (p54)** False

A sorting algorithm is **stable** if duplicate elements remain in the same relative position after sorting. **(p54)**

Inplace (SBHIQ)	**Stable (MCBI)**
Selection Sort	Merge Sort
Bubble Sort	Counting Sort
Heap Sort	Bubble Sort
Insertion Sort	
Quick Sort	

Question # 107

$8n^2 + 2n - 3$ grows asymptotically at least as fast as $\mathbf{n^2}$.

Question # 108

Given a set of points P = {p1, p2, . . . , pn} in 2-space, output the set of maximal points of P, i.e., those points pi such that pi is not dominated by any other point of P.

True (p11) False

Question # 109

While Sorting, the ordered domain means for any two input elements x and y _____ satisfies only.

x < y

x > y

x = y

All of the above (p34)

Question # 110

Brute-force algorithm for 2D-Maxima is operated by comparing _____ pairs of points.

Two Some Most **All (p18)**

Question # 111

Merge sort is based on ____

Brute-force Plan-sweep Axis-sweep **Divide and Conqer (p27)**

Question # 112

In the statement "if (P[i].x < P[j].x) & (P[i].y < P[j].y)", the number of times elements of P are accessed is ____

1 2 3 **4 (p14)**

Question # 113

Al-Khawarizmi was a/an....

Artist **Mathematician** Astronomer Khalifah

Question # 114

In Sorting the key value or attribute ____ from an ordered domain.

Must be Not always

The key value need not be a number. It can be any object from a totally ordered domain. (p39)

Question # 115

In Selection problem, the Sieve technique works in ____

Non-recursive manner Constant time

Phases (p34) One complete go

Question # 116

In max heap (for Heap Sort algorithm), when every time maximum element is removed from top we replace it with ____ leaf in the tree.

Second last **Last (p41)** First Any

Question # 117

If input "n" is odd, the median will be ____

(n+1)/2 (p34) n/2

Question # 118

The function f(n)= n(logn+1)/2 is asymptotically equivalent to n log n. Hre Upper Bound means the function f(n) grows asymptotically _____ faster than n log n.

More Quiet **Not (p23)** At least

Question # 119

In asymptotical analysis of n(n - 3) and 4n*n, as n becomes large, the dominant (fastest growing) term is some constant times ____.

n+1

n-1

n

n*n (p23)

Question # 120

In asymptotical analysis of n*(5 + 2)-3 , as n becomes large, the dominant (fastest growing) term is some constant times ____.

n+1

n-1

n

n*n

Question # 121

A point p in 2-dimensional space is usually given by its integer coordinate(s)_____

Select correct option:

p.x only

p.y only

p.x & p.z

p.x & p.y (p10)

Question # 122

Algorithm analysts know for sure about efficient solutions for NP-complete problems.

True **False (p9)**

Question # 123

In _____ we have to find rank of an element from given input.

Select correct option:

Merge sort algorithm **Selection problem (p34)**

Brute force technique Plane Sweep algorithm

Question # 124

In Heap Sort algorithm, if heap property is violated _____

Select correct option:

We call Build heap procedure **We call Heapify procedure (p41)**

We ignore Heap property can never be violated

Question # 125

Upper bound requires that there exist positive constants c2 and n0 such that f(n) ____ c2n for all n <= n0

Select correct option:

Less than

Equal to or Less than (p25 => for all n >= n0)

Equal or Greater than

Greater than

Question # 126

A RAM is an idealized algorithm with takes an infinitely large random-access memory.

True **False (p10 => A RAM is an idealized machine)**

Question # 127

In simple brute-force algorithm, we give no thought to efficiency.

True (p11) False

Question # 128

_____ is one of the few problems, where provable lower bounds exist on how fast we can sort.

Select correct option:

Searching **Sorting (p39)**

Both Searching & Sorting Graphing

Question # 129

The total no of arguments passed to merge sort algorithm is _____

2

3 (p28 => Array itself + two indices)

4

5

Question # 130

While solving Selection problem, in Sieve technique we partition input data _____

In increasing order In decreasing order **According to Pivot (p35)** Randomly

Question # 131

The number of accesses to any element of space is not counted for the running time calculation of algorithm.

True **False (p14)**

Question # 132

In merge sort algorithm, to merge two lists of size n/2 to a list of size n, takes _____ time.

Theta (n) (p30) Theta log(n) Theta log2(n) Theta nlog(n)

Question # 133

In Heap Sort algorithm, Heapify procedure is _____ in nature.

Recursive (p43) Non-Recursive

Question # 134

In Heap Sort algorithm, we call Build-heap procedure _____

Only once Twice Thrice **As many times as we need**

Question # 135

In merge sort algorithm, we split the array around the _____ index q.

Entring **Mid (p28)** Exiting Summing

Question # 136

The Iteration method is used for _____

Comparing sorting algorithms only **Solving Recurrence relations (p31)**

Merging elements in Merge sort Dividing elements in Merge sort

Question # 137

Rank of an element can be defined as _____

One minus the number of elements that are smaller

Two plus the number of elements that are greater

One plus the number of elements that are smaller (p34)

Two minus the number of elements that are smaller

Question # 138

f(n) is a set of functions such that there exist positive constants c1, c2 and n0 such that 0 <= c1g(n) <= f(n) <= c2g(n) for all n >= n0 Then f(n) is asymptotically _____ g(n).

Select correct option:

Less than Greater than

Equivalent to (p23) Not equivalent to

Question # 139

Floor and ceiling are _____ to calculate while analyzing algorithms.

Select correct option:

Very easy **Usually considered difficult (p31)**

Question # 140 In Heap Sort algorithm, the maximum levels an element can move upward is _____

Select correct option:

Theta (log n) (p43) Order (log n) Omega (log n) O (1) i.e. Constant time

Question # 141

In Heap Sort algorithm, the total running time for Heapify procedure is _____

Select correct option:

Theta (log n) **Order (log n) (p43)** Omega (log n) O (1) i.e. Constant time

Question # 142

In pseudo code, the level of details depends on intended audience of the algorithm.

True (p12) False

Question # 143

Counting sort assumes that the numbers to be sorted are in the range ____

1 to k where k is small (p57)

CS502 Solved Grand Quiz 2020

Question No 1

In the analysis of algorithms, _____ plays an important role.

Select Correct Option

- Text analysis
- **Time**
- Growth rate
- Money

Question No 2

_____ overcomes the limitations of _____ by working as per positional notations of numbers.

Select Correct Option

- **Bubble sort, Radix sort**……(**For More Visit VUStudents.pk)**
- Counting sort, Radix sort
- Radix sort, Bubble sort
- Radix sort, Counting sort

Question No 3

In generating Fibonacci Sequence, we can avoid unnecessary repetitions by _____ process.

Select Correct Option

- Tokenization
- **Memoization**……(**For More Visit VUStudents.pk)**
- Randomization
- Memorization

Question No 4

In Dynamic Programming, our approach is to _____.

Select Correct Option

- **Develop the solution in a top-down fashion**
- Express the problem non-recursively
- Build the solution in a bottom-up fashion
- Input several sub-problems simultaneously

For More Visit VUStudents.pk

Question No 5

We can user the optimal substructure property to devise a _____ formulation of the edit distance problem.

Select Correct Option

- Selective
- Optimum
- Iterative
- **Recursive……(For More Visit VUStudents.pk)**

Question No 6

Chain matrix multiplication problem can be solved through _____ strategy.

Select Correct Option

- **Dynamic programming**……(**For More Visit VUStudents.pk)**
- Greedy
- Divide and conquer
- Implementing arrays

Question No 7

For 2D Maxima problem, Plane Sweep algorithm first of all _____-.

Select Correct Option

- **Sorts all points……(For More Visit VUStudents.pk)**
- Delete some points
- Output the elements
- Pushes all points on stack

Question No 8

_____ algorithm based on Dynamic Programming strategy.

Select Correct Option

- **Heap Sort……(For More Visit VUStudents.pk)**
- Binary Tree
- Quick Sort
- Edit distance

For More Visit VUStudents.pk

Question No 9

The only way to convert a string of i characters into the empty string is with i deletions, represented as _____--.

Select Correct Option

- E (0, j) = j
- E (i, j) = l
- E (0, i) = l
- **E (i, 0) = i……(For More Visit VUStudents.pk)**

Question No 10

Edit Distance algorithm based on _____ strategy.

Select Correct Option

- Greedy
- **Dynamic Programming……(For More Visit VUStudents.pk)**
- Divide and Conquer
- Brute force

Question No 11

In _____ we have to find rank of an element from given input.

Select Correct Option

5. Merge sort algorithm
6. **Selection problem**……(For More Visit VUStudents.pk)
7. Brute force technique
8. Plane Sweep algorithm

Question No 12

In the statement “output P[i].x, P[i].y”, the number of time elements of P are accessed is _____

Select Correct Option

P 1

Q 2……(For More Visit VUStudents.pk)

R 3

S 4

For More Visit VUStudents.pk

Question No 13

Rank of an element can be defined as _____-.

Select Correct Option

- ✓ One minus the number of elements that are smaller
- ✓ Two plus the number of elements that are greater
- ✓ **One plus the number of elements that are smaller**
- ✓ Two minus the number of elements that are smaller

Question No 14

8n2 + 3n – 3 will eventually exceed c2*(n) no matter how large we make _____-.

Select Correct Option

- ✓ n
- ✓ c2
- ✓ **2n……(For More Visit VUStudents.pk)**
- ✓ this equation

Question No 15

Matrix Multiplication is a(n) _____ operation.

Select Correct Option

- ✓ Commutative
- ✓ **Associative……(For More Visit VUStudents.pk)**
- ✓ Neither commutative nor associative
- ✓ Transitive

Question No 16

The number of nodes in a complete binary tree of height h is:

Select Correct Option

- ✓ 2 * (h+1) -1
- ✓ **2^(h+10 – 1**……(**For More Visit VUStudents.pk)**
- ✓ 2*(h+1)
- ✓ ((h+1) ^ 2) – 1

Question No 17

Applying the sieve technique to selection problem, _____ element is picked from array.

Select Correct Option

- ✓ Output
- ✓ Total
- ✓ Input
- ✓ **Pivot……(For More Visit VUStudents.pk)**

Question No 18

In Quick sort algorithm, we choose pivot _____--.

Select Correct Option

- ✓ **Always the smallest element** ……(**For More Visit VUStudents.pk)**
- ✓ Greater than 5
- ✓ Randomly
- ✓ Less than 5

Question No 19

Approach of solving geometric problems by sweeping a line across the plane is called

_____ sweep.

Select Correct Option

- ✓ Line
- ✓ **Plane……(For More Visit VUStudents.pk)**
- ✓ Cube
- ✓ Box

Question No 20

In Knapsack Problem, the goal is to put items in the knapsack such that the value of the items is _____ subject to weight limit of knapsack.

Select Correct Option

- ✓ Minimized
- ✓ Decrease
- ✓ **Maximized……(For More Visit VUStudents.pk)**
- ✓ None of the given options

For More Visit VUStudents.pk

For More Visit VUStudents.pk

Question No 21

Sorting is performed on the basis of _____.

Select Correct Option

- ✓ Computational resources
- ✓ Asymptotic notation
- ✓ Summation
- ✓ **Some key value or attribute**……(For More Visit VUStudents.pk)

Question No 22

In Asymptotical analysis of n(n-3) and 4n*n, as n becomes large, the dominant (fastest growing) term is some constant times_____.

Select Correct Option

- ✓ n+1
- ✓ n-1
- ✓ n
- ✓ **n*n……(**For More Visit VUStudents.pk)

Question No 23

In Heap Sort algorithm, we build _____ for ascending sort.

Select Correct Option

- ✓ Max heap
- ✓ **Min heap**……(For More Visit VUStudents.pk)
- ✓ Full heap
- ✓ Order heap

Question No 24

If we have an equation 8n2+7f*n+5f+6 then n is large, _____ term will be much larger than the n term and will dominate the running time.

Select Correct Option

- ✓ f(g(n)
- ✓ g(n)^2
- ✓ **n^2……(****For More Visit VUStudents.pk)**
- ✓ f(n)

Question No 25

_____ is the process of avoiding unnecessary repetitions by writing down the results of recursive calls and looking them up again if we need them later.

Select Correct Option

- ✓ Loop
- ✓ **Memoization……(For More Visit VUStudents.pk)**
- ✓ Recursion
- ✓ Function

Question No 26

The worst-case running time of merge sort is _____ in order to sort an array of n elements.

Select Correct Option

- ✓ O(log n)
- ✓ O(n)
- ✓ **O(nlogn)** ……(For More Visit VUStudents.pk)
- ✓ O(n^2)

Question No 27

If matrix A of dimension 2 x 4 is multiplied with matrix b of dimension 4 x 3, then the dimension of resultant matrix will be _____.

Select Correct Option

- ✓ **2 x 4**……(For More Visit VUStudents.pk)

- ✓ 4 x 3
- ✓ 3 x 4
- ✓ 2 x 3

Question No 28

Fibonacci Sequence was named on _____ , a famous mathematician in 12th Century.

Select Correct Option

- ✓ Fred Brooks
- ✓ Grady Booch
- ✓ **Leonardo Pisano**……(**For More Visit VUStudents.pk)**
- ✓ Edger F.Codd

For More Visit VUStudents.pk

For More Visit VUStudents.pk

Question No 29

Algorithm is a sequence of computational steps that _____ the input into output.

Select Correct Option

- ✓ Merge
- ✓ Assign
- ✓ **Transform**……(For More Visit VUStudents.pk)
- ✓ Integrate

Question No 30

If the indices passed to marge sort algorithm are _____, then this means that there is only one element to sort.

Select Correct Option

- ✓ Small
- ✓ Large
- ✓ **Equal**……
- ✓ Not Equal

CS502 Quiz 2 Fall 2020

Question No 1

The tricky part of _____ algorithm(s) is/are, how to detect whether the addition of an edge will create a cycle in viable set A.

Kruskal's (Page 147)

Question No 2

Overall Running time of Prim's algorithm is _____.

Θ(ElogV) (Page 153)

Question No 3

Finding the faster result of the shortest path from u to v for every pair of vertices u and v we use_____.

All-pairs shortest-paths problem (Page 154)

Question No 4

What is the time complexity to extract a vertex from the priority queue in Prim's algorithm?

O(log V) (Page 152)

Question No 5

Edge weights can be interpreted as distance _____.

in the shortest-paths (Page 153)

Question No 6

Dijkstra's Algorithm is used to solve_____problems.

Single-source shortest path (Page 159)

Question No 7

Problems such as the shortest route between cities can be solved efficiently by modeling the road map as a_____.

Graph (Page 153)

Question No 8

For _____ graphs, there is no distinction between forward and back edges.

Undirected (Page 130)

Question No 9

Bellman-Ford Algorithm does not allow G(graph) to have _____.

negative cost cycles (Page 159)

Question No 10

A topological sort of a DAG is a _____ ordering of the vertices of the DAG such that for each edge (u, v), u appears before v in the ordering.

Linear (Page 133)

Quiz File 2

Question No 1

Networks are _____ in the sense that it is possible from any location in the network to reach any other location in the digraph.

Complete (Page 135)

Question No 2

According to parenthesis lemma vertex u is a descendent of v vertex if and only if:

f [d[u], f[u]] ⊆ [d[v], f[v]] (Page 129)

Question No 3

For _____ graphs, there is no distinction between forward and back edges.

Undirected (Page 130)

Question No 4

There exist a unique path between any _____ vertices of a free tree.

Two (Page 142)

Question No 5

Keeping in mind the shortest-path, if given scenarios occur in computer networks like the internet where data packets have to be routed. The vertices are_____and Edges are

_____which may be wired or wireless.

Routers, communication links (Page 153)

Question No 6

In undirected graph, by convention all the edges are called _____ edges.

back (Page 130)

Question No 7

Bellman-Ford algorithm is slower than_____.

Dijkstra's (Page 159)

Question No 8

From the given option's which one is correct regarding the time complexity of Dijkstra's algorithm:

O(N^2)

Question No 9

In the shortest-paths problem, we are given a weighted of_____G = (V, E).

Un-directed graph

Question No 10

You have an adjacency list for G, what is the time complexity to compute Graph transpose G^T

?

(V+E)

CS502 – Midterm Spring 2013 (26 May-05 June)

How would you modify QUICKSORT to sort into non-increasing order?
Answer: - Click here for detail
To make QUICKSORT sort in non-increasing order we must modify PARTITION.

Q- We can avoid unnecessary repetitions for recursive calls?
Answer: - (Page 74)
We can avoid these unnecessary repetitions by writing down the results of recursive calls and looking them up again if we need them later. This process is called *memoization*.

Q-Write a pseudo code Fibonacci With memorization? -- (3)
Answer: - (Page 74)
MEMOFIB(n)
1 **if** (n < 2)
2 **then return** n
3 **if** (F[n] is undefined)
4 **then** F[n] MEMOFIB(n − 1) + MEMOFIB(n − 2)
5 **return** F[n]

Spelling correction in edit distance? 3 marks
Answer: - (Page 76)
If a text contains a word that is not in the dictionary, a 'close' word, i.e. one with a small edit distance, may be suggested as a correction. Most word processing applications, such as Microsoft Word, have spelling checking and correction facility. When Word, for example, finds an incorrectly spelled word, it makes suggestions of possible replacements.

Bubble sort?
Answer: - (Page 39)
Scan the array. Whenever two consecutive items are found that are out of order, swap them. Repeat until all consecutive items are in order.

What is the worst case running time for the Quick sort? What simple change is required in the algorithm to preserve its linear expected running time and makes it worst case time Θ(n log n)
Answer: - (Page 49)
Worst case running time is O (n).
The simple change which can change the running time of the edit distance algorithm is the number of entries n2.

CS502 – Midterm Spring 2013 (26 May-05 June)

QUICK, HEAP, COUNTING, MERG KA TABLE KA JIS ME STABLE OR IN-PLACE BTANEY THE 5 MARKS
Answer: Page 54

IN Place	Stable
Selection Sort	
Quick Sort	
Heap sort	

Bubble sort	Bubble sort
Insertion Sort	Insertion Sort
	Merge sort
	Counting sort

ADIT DISTANCE KI 3 APPLICATION K NAME BTANEY THE 3 MARKS
Answer: - (Page 76)
Spelling Correction
Plagiarism Detection
Speech Recognition

MEMOIZATION KI DIFINATION THI 2 MARKS
Answer: - Click here for detail
Memoization is an optimization technique used primarily to speed up computer programs by having function calls avoid repeating the calculation of results for previously processed inputs

WORS CASE OR AVERAGE CASE DEFINE KRNE THE 2 MARKS
Answer: - (Page 13)
Worst-case time is the maximum running time over all (legal) inputs of size n
Average-case time is the average running time over all inputs of size n.

CS502 – Midterm Spring 2013 (26 May-05 June)

Write Essential constraint of Counting sort.
Answer:
Essential constraint for the counting sort is that numbers to be sorted must be small integers i.e., 1…..k where k is small.

What are total numbers of entries in matrix for edit distance?
Answer: - (Page 84)
There are O(n2) entries in the matrix. Each entry E(i, j) takes O(1) time to compute. The total running time is O(n2).

WHAT IS the necessary assumption for average case analysis quick sort?
Answer: - (Page 50)
Average case depends on some assumption about the distribution of inputs. However, in the case of quicksort, the analysis does not depend on the distribution of input at all. It only depends upon the random choices of pivots that the algorithm makes

Suggest & describe modifications of the implementation of "Quick sort" that will improve its performance?
Answer: - (Page 76)

CS502 – Midterm Spring 2013 (26 May-05 June)

Q1 : How we Heapify?
Answer:
If an element in the Heap is not at its proper place means it is violating the Heap Order, the Heapify procedure is used to fix it and place it at its proper position. In Heapify, we recursively swap the element with its larger one child and stop at a stage when this element is larger than both of its children or it becomes the leaf node.

Q2 : Describe Heap sort algorithm?
Answer: - (Page 43)

```
HEAPIFY( array A, int i, int m)
1 l   LEFT(i)
2 r   RIGHT(i)
3 max   i
4 if (l _ m)and(A[l] > A[max])
5 then max   l
6 if (r _ m)and(A[r] > A[max])
7 then max   r
8 if (max 6= i)
9 then SWAP(A[i],A[max])
10 HEAPIFY(A,max,m)
```

Q3 : How many elements are in matrix of edit distance?
Answer: Rep

Q6 : suggest and describe one modification of implementing quick sort? 5marks

CS502 – Midterm Spring 2013 (26 May-05 June)

Edit Distance in Speech Recognition
Answer: - (Page 77)
Speech Recognition
Algorithms similar to those for the edit-distance problem are used in some speech recognition systems. Find a close match between a new utterance and one in a library of classified utterances.

What is sorting? Describe slow running sorting algorithms. 5 marks
Answer: - (Page 39)

Sorting is any process of arranging items in some sequence and/or in different sets, and accordingly,
There are a number of well-known slow O(n2) sorting algorithms. These include the following:
Bubble sort: Scan the array. Whenever two consecutive items are found that are out of order, swap them. Repeat until all consecutive items are in order.
Insertion sort: Assume that A[1..i − 1] have already been sorted. Insert A[i] into its proper position in this sub array. Create this position by shifting all larger elements to the right.
Selection sort: Assume that A[1..i − 1] contain the i − 1 smallest elements in sorted order. Find the smallest element in A[i..n] Swap it with A[i]. These algorithms are easy to implement. But they run in _(n2) time in the worst case

Catalan Numbers and their formula 3 marks
Answer: - (Page 85)
Catalan numbers are related the number of different binary trees on n nodes. Catalan number is given by the formula:

$$C(n) = \frac{1}{n+1}\binom{2n}{n}$$

CS502 – Midterm Spring 2013 (26 May-05 June)

Q1.Heap Sort , Counting Sort , Quick Sort ,Merge Sort , in ,table ,And required to tell , which is inplace Algorithm and stable ? Marks 5
Answer: rep

Q2 . Suggest that how can make better improvement in Quick Sort algorithm .Marks 5
Answer: - (Page 54)
The best we have seen so far is O(n log n) algorithms for sorting. Is it possible to do better than O(n log n)? If a sorting algorithm is solely based on comparison of keys in the array then it is *impossible* to sort more efficiently than (n log n) time.

Q3. Consider three numbers with comparison based sortig algorithim and write possible combination in a1,a2,a3 .Marks 3
Answer: - (Page 85)
Consider sorting three numbers a1, a2, a3. There are 3! = 6 possible combinations:
(a1, a2, a3), (a1, a3, a2), (a3, a2, a1)
(a3, a1, a2), (a2, a1, a3), (a2, a3, a1)

Q4. What is better aproach of multiplication rather than straight form of Multiplication . Named that . Marks 3
Answer: - (Page 54)

Chain Matrix Multiplication-Dynamic Programming Formulation

Q6.worst case and average case algorithm of Quick Sort .Marks 2
Answer: - (Page 50)
Answer: Worst case = O(n2)
Average Case = O(n log n)

CS502 – Midterm Spring 2013 (26 May-05 June)

1.worst case of bucket algorithm
Answer: - O(n)

Catalan Numbers and their formula 3 marks
Answer: rep

speech reorganization.2
Answer: rep

Worst and average case of quick sort algorithm.2 marks
Answer: rep

CS502 – Midterm Spring 2013 (26 May-05 June)

Radix sort table ? 5 marks
Answer:
Radix sort:
In linear time sorting, counting sort is used for the numbers in the range 1 to k where k is small and it is based on determining the rank of each number in final sorted array. But it is useful only for small integers i.e., 1 ….k where k is small. But if k were very large, then the size of the rank array formed would also be very large which is not efficient. So solution for such cases is the Radix sort which works by sorting one digit at a time.

Example:
841 84[1] 8[4]1 [1]85

185 è 37[3] è 3[7]3 è [3]73

373 18[5] 1[8]5 [8]41

CS502 – Midterm Spring 2013 (26 May-05 June)

1. **What are the worst and average run times of quick sort? (2 marks)**
   Answer: rep

2. **What are total numbers of entries in matrix for edit distance? (2 marks)**
   Answer: rep

Q.3 *What does dynamic programming have in common with divide and conquer ?*

Ans. : Both the divide and conquer and dynamic programming solve the problem by breaking it into number of subproblems. In both these methods solutions from subproblems are collected together to form a solution to given problem.

Q.4 *What is the principle difference between the two techniques ?*

Ans. : Both divide and conquer as well as dynamic programming solve the given problem by breaking it into subproblems. But in divide and conquer the subproblems are solved independently whereas in dynamic programming the subproblems share the solutions among themselves. The dynamic programming works on principle of optimality to solve the problem.

5. **Show that the running time of quick sort when the array A contains distinct elements and sorted in decreasing order ? (5 marks)**
   Answer:

3. (CLRS 7.2-3) Show that the running time of QUICKSORT is $\Theta(n^2)$ when the array A contains distinct elements and is sorted in decreasing order.

   Solution: On the first iteration of PARTITION the pivot element is chosen as the first element of A. Index i is incremented once and j is decremented until it reaches the pivot, i.e. the entire length of A. PARTITION returns to QUICKSORT the first element of A, which recursively sorts one subarray of size 1 and one of size $n-1$. This process is repeated for the subarray of size $n-1$. The running time of the entire computation is then given by the recurrence:

$$\begin{aligned} T(n) &= \begin{cases} \Theta(1) & n \leq 2 \\ T(n-1) + \Theta(n) & \text{otherwise} \end{cases} \\ &= \Theta(n^2). \end{aligned}$$

6. **Illustrate how Radix Sort works on the following words: Show the result of first 2 passes only.**
   COW, DOW, SEA, RUG, ROW, MOB, BOX, TAB, BAR, EAR, TAR, DIG, BIG, TEA, NOW, FOX
   Answer:
COW DOG SEA RUG ROW MOB BOX TAB BAR EAR TAR DIG BIG TEA NOW FOX
SEA TAB TEA BAR MOB EAR TAB TAR DOG SEA RUG TEA DIG DIG BIG BIG BAR
MOB EAR DOG TAR COW COW ROW ROW NOW NOW BOX BOX FOX FOX RUG BAR
BIG...

CS502 – Midterm Spring 2013 (26 May-05 June)

Q: define heap and heap oerder
A *heap* is a left-complete binary tree that conforms to the *heap order*.

Q:Average-case Analysis of Quicksort
Average case analysis of Quick sort: The running time of Quick sort depends on the selection of pivot q which is done randomly. For average case analysis of quick sort, average is computed over all possible random choices of the pivot index q. The average case running time for quick sort is Θ (n log n).

Q:Worst case Analysis of Quick sort
Worst case analysis of Quick sort: For worst case we maximize over all possible values of q, means the selection of pivot q which gives the maximum (worst) time for sorting. The chances are that the pivot values q=1 (start) or n (end) or n/2 (middle) happen to give maximum time values. The worst case running time is O (n2).

CS502 – Midterm Spring 2013 (26 May-05 June)

In Random Access Machine, instructions are executed _____ Instructions are executed
Answer:- one-by-one (there is no parallelism). (Page 10)

Dynamic Programming algorithm:
Answer: - The genius of the algorithm is in the clever recursive formulation of the shortest path problem. (Page 162)

For the heap sort, access to nodes involves simple __ **arithmetic** ___ operations. (Page 40)

Comparison based sorting algorithm cannot run faster than __ **Ω(n log n)** __ (Page 46)

Fibonacci sequence was posted by __ **Leonardo Pisano**, ___ (Page 73)

Mid

Quiz by Attiq Kundi

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CS-502

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Cs-502 Important Macq's For Final Term
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1. Dijkstra's Algorithm is used to solve _____ problems.
   a) All pair shortest path
   b) Single source shortest path
   c) Network flow
   d) Sorting

2. Which of the following is the most commonly used data structure for implementing Dijkstra's Algorithm?
   a) Max priority queue
   b) Stack
   c) Circular queue
   d) Min priority queue

3. What is the time complexity of Dijikstra's algorithm?
   a) O(N)
   b) $O(N^3)$
   c) $O(N^2)$
   d) O(logN)

- Please choose one

---

4. Dijkstra's Algorithm cannot be applied on _____
   a) Directed and weighted graphs
   b) Graphs having negative weight function
   c) Unweighted graphs
   d) Undirected and unweighted graphs

5. How many priority queue operations are involved in Dijkstra's Algorithm?
   a) 1
   b) 3
   c) 2
   d) 4

6. How many times the insert and extract min operations are invoked per vertex?
   a) 1
   b) 2
   c) 3
   d) 0

7. The maximum number of times the decrease key operation performed in Dijkstra's algorithm will be equal to _____
   a) Total number of vertices
   b) Total number of edges
   c) Number of vertices – 1
   d) Number of edges – 1

8. What is running time of Dijkstra's algorithm using Binary min- heap method?
   a) O(V)
   b) O(VlogV)
   c) O(E)
   d) O(ElogV)

True False

- Please choose one

---

9. The running time of Bellmann Ford algorithm is lower than that of Dijkstra's Algorithm.

a) True

b) False

10. Dijkstra's Algorithm run on a weighted, directed graph G={V,E} with non-negative weight function w and source s, terminates with d[u]=delta(s,u) for all vertices u in V.

a) True

b) False

11. Dijkstra's Algorithm is the prime example for _____

a) Greedy algorithm

b) Branch and bound

c) Back tracking

d) Dynamic programming

1. Computing the strongly connected components of a graph is a generalization of the problem to determine whether a digraph is strongly connected or not.

   True

2. If u finds yourself in maze the better traversal approach will be:

   BFS and DFS are valid

3. In ---------- algorithm, at any time, the subset of edges A forms a single tree.

   Prim's

True False

- Please choose one

---

4. In computing the strongly connected components of a digraph, vertices of the digraph are not partitioned into subsets.

   False

5. Which is true statement:

   The breadth-first-search is a shortest-path algorithm that works on un-weighted graphs.

6. There are no ---------- edges in undirected graph.

   Cross

7. In undirected graph, by convention all the edges are called ---------- edges.

   Back

8. A digraph is strongly connected under what condition?

   A digraph is strongly connected if for every pair of vertices u, v ∈ V, u can reach v and vice versa.

9. The tricky part of the ---------- algorithm is how to detect whether the addition of an edge will create a cycle in viable set A.

   Kruskal's

10. Digraphs are not used in communication and transportation networks.

    True

11. If u and v are mutually reachable in G, then in the graph formed by reversing all the edges, these vertices are not reachable.

True False

- Please choose one

False

12. A topological sort of a DAG is a---------- ordering of the vertices of the DAG such that for each (u, v), u appears before v in the ordering.

Linear

13. Kruskal's algorithm works by adding vertices in increasing order of weight (lightest edge first).

False

14. Adding an edge to a free tree creates:

Cycle

15. Forward edge is:

(u, v) where v is a proper descendent of u in the tree

16. In strong components algorithm, vertices are sorted in ---------- order of finish times.

Decreasing

17. In Kruskal's algorithm the next edge is not added to viable set A, if it's adding Induce a cycle.

True

18. What is the time complexity to extract a vertex from priority queue in Prim's algorithm?

(log V)

19. A graph may contain----------.

True False

- Please choose one

More than one MST

20. Prim's algorithm builds MST by starting with any vertex and at any time the subset of edges form:

A single tree

21. In Prim's algorithm, we start with the---------- vertex r; it can be any vertex.

Root

22. The ---------- given by DFS allow us to determine whether the graph contains any cycles.

Time Stamps

23. You have an adjacency list for G, what is time complexity to compute Graph transpose G^T?

Θ (V + E)

24. Back edge is:

Ans. (u, v) where v is an ancestor of u in the tree

25. What algorithm technique is used in the implementation of Kruskal solution for the MST?

Ans. Greedy Technique (Pg. # 142)

26. If a subset of edges A is viable for building MST. It cannot contain a/an----------.

Ans. Cycle (Pg. # 143)

True False

- Please choose one

---

27. Adding any edge to a free tree creates a unique----------

Ans. Cycle (Pg#142)

28. Prim's algorithm is based on -------- strategy.

Ans. Greedy (Pg. # 142)

29. In strong components algorithm, First of all DFS is run for getting ---------- times of vertices.

Ans. Finish

30. There exists a unique path between any two vertices of a free tree.

Ans. True (pg. # 142)

31. In strong components algorithm, first of all DFS is run for computing finish times of vertices.

Ans. true

32. There is relationship between number of back edges and number of cycle in DFS

Ans. there is no relationship between number of back edges and number of cycles

33. In Prim's algorithm at any time, the subset of edges A forms a single forest.

Ans. False (pg#149)

34. In Kruskal's algorithm at any time, the subset of edges A forms a single tree.

True False

- Please choose one

Ans. False (pg#149)

35. Kruskal's algorithm works by adding ---------- in increasing order of weight (lightest edge first).

Ans. edges (Pg. # 147)

36. In Prims algorithm we will make use of priority----------.

Ans. queue (pg. # 150)

37. Cross edge is :

Ans. (u, v) where u and v are not ancestor or descendent of one another

38. The ancestor and descendent relation cannot be inferred by the parenthesis Lemma.

Ans. False (Pg. # 129)

39. In Kruskal's algorithm, the next ---------- is not added to viable set A, if it's adding induce a/an cycle.

Ans. edge (Pg. # 147)

40. Equivalence relation partitions the vertices into ---------- classes of mutually reachable vertices and these are the strong components.

Ans. Equivalence (Pg#136)

41. For undirected graph, there is no distinction between forward and back edges.

Ans. True (Pg#130)

True False

- Please choose one

---

42. In strong components algorithm, the form of graph is used in which all the ---------- of original graph G have been reversed in direction.

Ans. edges (Pg. # 138)

43. Networks are complete in the sense that it is not possible from any location in the network to reach any other location in the digraph.

Ans. True (Pg#135)

44. In digraph G = (V, E) ; G has cycle if and only if:

Ans. The DFS forest has a back edge (Pg. # 131)

45. As the Kruskal's algorithm runs the edges in viable set A induce a ---------- on the vertices.

Ans. Forest (Pg#147)

46. Timestamp structure of ---------- is used in determining the strong components of a digraph.

Ans. Both DFS and BFS

47. In Prim's algorithm, we start with the _____ vertex r; it can be any vertex.

Root

48. In Prim's algorithm, at any time, the subset of edges A forms a single _____.

Tree

True False

- Please choose one

---

49. Kruskal's algorithm (choose best non-cycle edge) is better than Prim's (choose best tree edge) when the graph has relatively few edges.

    True

50. There is relationship between number of back edges and number of cycles in DFS

    There is no relationship between back edges and number of cycles.

51. In Prim's algorithm, we will make use of priority _____.

    Queue

52. We say that two vertices u and v are mutually not reachable if u can reach v and vice versa.

    False

53. In strong components algorithm, the form of graph is used in which all the vertices of original graph G have been reversed in direction.

    False

54. In Timestamped DFS-cycles lemma, if edge (u, v) is a back edge, then _____

    $f[u] \leqslant f[v]$

55. There exist a unique path between any _____ vertices of a free tree.

True False

- Please choose one

Two

56. In strong components algorithm, vertices are sorted in_____ order of finish times.

Decreasing

57. According to parenthesis lemma, vertex u is unrelated to v vertex if and only if d[u],f[u]] and [d[v],f[v]] are disjoint.

True

58. A digraph is strongly connected under what condition?

A digraph is strongly connected if for every pair of vertices u, v e V, u can reach v and vice versa.

59. The tricky part of the _____ algorithm is how to detect whether the addition of an edge will create a cycle in viable set A.

Kruskal's

60. If a subset of edges A is viable for building MST, it can not contain a/an _____.

Cycle

61. Digraphs are not used in communication and transportation networks.

True

62. A strongly connected component only apply to:

True False

- Please choose one

Directed Graph

63. According to parenthesis lemma, vertex u is a descendent of v vertex if and only if;

[d[u], f[u]] ⊆ [d[v], f[v]

64. What is the time complexity to extract a vertex from the priority queue in Prim's algorithm?

(log V)

65. Back edge is:

(u, v) where v is an ancestor of u in the tree.

66. In computing the strongly connected components of a digraph, vertices of the digraph are not partitioned into subsets.

False

67. In Timestamped DFS-cycles lemma, if edge (u, v) is a tree, forward or cross edge, then _____

f[u]⩾f[v]

68. In Kruskal's algorithm, the next edge is added to viable set A, if its adding does not induce a/an _____ .

Cycle

69. In Generic approach determining of Greedy MST, we maintain a subset A of _____ .

Edges

True False

- Please choose one

---

70. If u and v are mutually reachable in G, then in the graph formed by reversing all the edges, these vertices are not reachable.

    False

71. Kruskal's algorithm works by adding _____ in increasing order of weight (lightest edge first).

    Edges

72. In Kruskal's algorithm, the next _____ is not added to viable set A, if its adding induce a/an cycle.

    Edge

73. Networks are complete in the sense that it is possible from any location in the network to reach any other location in the digraph.

    True

74. In Generic approach determining of Greedy MST, we maintain a subset A of _____ .

    Edges

75. There exist a unique path between any _____ vertices of a free tree.

    Two

76. What is the time complexity to extract a vertex from the priority queue in Prim's algorithm?

    log (V)

True False

- Please choose one

---

77. The time stamps given by DFS do not allow us to determine whether the graph contains any cycles.

False

78. Timestamp structure of _____ is used in determining the strong components of a digraph.

Both DFS & BFS

79. In Kruskal's algorithm, the next edge is not added to viable set A, if its adding induce a cycle.

False

80. A topological sort of a DAG is a _____ ordering of the vertices of the DAG such that for each edge (u, v), u appears before v in the ordering.

Linear

81. In Kruskal's algorithm, at any time, the subset of edges A forms a single tree.

False

82. Adding any edge to a free tree creates a unique cycle.

True

83. In digraph G=(V,E) ;G has cycle if and only if The DFS forest has back edge

Cycle

84. In undirected graph, by convention all the edges are called _____ edges.

True False

- Please choose one

Back

85. Prim’s algorithm is based on ---------- strategy.

Greedy

86. There are no _____ edges in undirected graph.

Cross

87. You have an adjacency list for G, what is the time complexity to compute Graph transpose G^T ?

(V+E)

Which is true statement in the following.

Kruskal algorithm is multiple source technique for finding MST.

Kruskal's algorithm is used to find minimum spanning tree of a graph, time complexity of this algorithm is O(EV)

Both of above

Kruskal's algorithm (choose best non-cycle edge) is better than Prim's (choose best Tree edge) when the graph has relatively few edges) (Right Answer)

The relationship between number of back edges and number of cycles in DFS is,

Both are equal

Back edges are half of cycles

Back edges are one quarter of cycles

There is no relationship between no. of edges and cycles (Right Answer)

Kruskal's algorithm (choose best non-cycle edge) is better than Prim's (choose best tree

edge) when the graph has relatively few edges.

True (Right Answer)
False

What is the time complexity to extract a vertex from the priority queue in Prim's

algorithm?

True False

- Please choose one

Select correct option:

log (V)
V.V

E.E

log (E)

Suppose that a graph G = (V,E) is implemented using adjacency lists. What is the complexity of a breadth-first traversal of G?

Select correct option:

O(|V |^2)

O(|V | |E|) (Right Answer)
O(|V |^2|E|)

O(|V | + |E|)

What is generally true of Adjacency List and Adjacency Matrix representations of graphs?

Select correct option:

Lists require less space than matrices but take longer to find the weight of an edge (v1,v2)

Lists require less space than matrices and they are faster to find the weight of an edge (v1, v2) Right Answer)
Lists require more space than matrices and they take longer to find the weight of an edge (v1, v2)

Lists require more space than matrices but are faster to find the weight of an edge (v1, v2)

What general property of the list indicates that the graph has an isolated vertex?

Select correct option:

There is Null pointer at the end of list.

The Isolated vertex is not handled in list. (not Sure)
Only one value is entered in the list.

There is at least one null list.

A dense undirected graph is:

Select correct option:

A graph in which E = O(V^2) (Right Answer)
A graph in which E = O(V)

A graph in which E = O(log V)

True False

- Please choose one

All items above may be used to characterize a dense undirected graph

In digraph G=(V,E) ;G has cycle if and only if

Select correct option:

The DFS forest has forward edge.

The DFS forest has back edge (Right Answer)
The DFS forest has both back and forward edge

BFS forest has forward edge

Back edge is:

Select correct option:

(u, v) where v is an ancestor of u in the tree. (Right Answer)
(u,v) where u is an ancesstor of v in the tree.

(u, v) where v is an predcessor of u in the tree.

None of above

There is no relationship between back edges and number of cycle(Right Answer)

You have an adjacency list for G, what is the time complexity to compute Graph transpose G^T ?

Select correct option:

(V+E) (Right Answer)
V.E

V

E

Question # 3 of 10 (Start time: 06:54:27 PM) Total Marks: 1

You have an adjacency list for G, what is the time complexity to compute Graph transpose G^T.?

?(V + E) Right Answer)
?(V E)

?(V)

?(V^2)

True False

- Please choose one

What is the time complexity to extract a vertex from the priority queue in Prim's algorithm?

Select correct option:

log (V) (Right Answer)
V.V

E.E

log (E)

Dijkstra's algorithm :

Select correct option:

Has greedy approach to find all shortest paths

Has both greedy and Dynamic approach to find all shortest paths

Has greedy approach to compute single source shortest paths to all other vertices (Right Answer)
Has both greedy and dynamic approach to compute single source shortest paths to all other vertices.

What algorithm technique is used in the implementation of Kruskal solution for the MST?

Greedy Technique (Right Answer)
Divide-and-Conquer Technique

Dynamic Programming Technique

The algorithm combines more than one of the above techniques

What is the time complexity to extract a vertex from the priority queue in Prim's algorithm?

Select correct option:

O (log E)

? (V)

? (V+E)

O (log V) (Right Answer)

1. What algorithm technique is used in the implementation of Kruskal solution for MST?

a) Greedy Technique page 142

True False

- Please choose one

b) Divide-and-Conquer Technique

c) Dynamic Programming Technique

d) The algorithm combines more than one of the above techniques i.e. Divide-and-Conquer and dynamic programing

1. There exists a unique path between any _____ vertices of a free tree.

a) One

b) Two page 142

c) Three

d) All

1. If a subset of edges A is viable for building MST, it cannot contain a/an _____

a) Vertex

b) Edge

c) Cycle page 143

d) Graph

1. As the Kruskal's runs, the edges in viable set A induce a _____ on the vertices.

a) Set

b) Graph

c) Tree

d) Forest

1. The _____ given by DFS allow us to determine whether the graph contains any cycles.

a) Order

b) Time stamps page 130

c) BFS traversing

d) Topological sort

1. The time stamps given by DFS do not allow determining whether the graph contains any cycles.

a) True

b) False page130

1. By breaking any edge on a cycle created in free tree, the free _____ is restored.

a) Edge

b) Tree

c) Cycle page 143

d) Vertex

1. Forward edge is

a) (u, v) where u is a proper descendent of v in the tree

b) (u, v) where v is a proper descendent of u in the tree page 129

True False

- Please choose one

c) (u, v) where v is a proper ancestor of u in the tree

d) (u, v) where u is a proper ancestor of v in the tree

1. In Kruskal's algorithm, the next _____ is not added to viable set A, if its adding induces a/an cycle.

a) Vertex

b) Edge page147

c) Cycle

d) Tree

1. In _____ algorithm, at any time, the subset of edges A forms a single tree

a) Kruskal's

b) Prim's page 149

c) Both

d) None

Question # 1 of 10 (Start time: 10:14:37 PM) Total Marks: 1
There exist a unique path between any _____ vertices of a free tree.
Select correct option:
One
Two Ans
Three
All

Question # 2 of 10 (Start time: 10:15:15 PM) Total Marks: 1
In _____ algorithm, at any time, the subset of edges A forms a single tree.
Select correct option:
Kruskal's
Prim's Ans
Both
None

Question # 3 of 10 (Start time: 10:15:56 PM) Total Marks: 1
A digraph is strongly connected under what condition?
Select correct option:
A digraph is strongly connected if for every pair of vertices u, v e V, u can reach v .
A digraph is strongly connected if for every pair of vertices u, v e V, u can reach v and vice versa. Ans
A digraph is strongly connected if for at least one pair of vertex u, v e V, u can reach v and vice versa.
A digraph is strongly connected if at least one third pair of vertices u, v e V, u can reach v and vice versa.

Question # 4 of 10 (Start time: 10:16:38 PM) Total Marks: 1
Digraphs are not used in communication and transportation networks.
Select correct option:

True False

- Please choose one

True

False Ans

Question # 5 of 10 (Start time: 10:17:31 PM) Total Marks: 1
There are no _____ edges in undirected graph.
Select correct option:
Forward
Back
Cross
Both forward and back Ans

Question # 6 of 10 (Start time: 10:17:53 PM) Total Marks: 1
In Kruskal's algorithm, at any time, the subset of edges A forms a single tree.
Select correct option:
True
False Ans

Question # 7 of 10 (Start time: 10:18:12 PM) Total Marks: 1
Kruskal's algorithm (choose best non-cycle edge) is better than Prim's (choose best tree edge) when the graph has relatively few edges.
Select correct option:
True
False Ans

Question # 8 of 10 (Start time: 10:18:54 PM) Total Marks: 1
The _____ given by DFS allow us to determine whether the graph contains any cycles.
Select correct option:
Order
Time stamps Ans
BFS traversing
Topological sort

Question # 9 of 10 (Start time: 10:19:41 PM) Total Marks: 1
Kruskal’s algorithm works by adding _____ in increasing order of weight (lightest edge first).
Select correct option:
Vertices
Edges Ans
Trees
Weights

Question # 10 of 10 (Start time: 10:20:15 PM) Total Marks: 1
Runtime complexity of Prim's algorithm is _____.
Select correct option:
V log V
E log V Ans
log V
None of the above

-

Question # 1 of 10 (Start time: 10:23:08 PM) Total Marks: 1
Adding any edge to a free tree creates a unique cycle.

True False

- Please choose one

Select correct option:

True
False

Question # 2 of 10 (Start time: 10:23:52 PM) Total Marks: 1
Forward edge is:
Select correct option:
(u, v) where u is a proper descendent of v in the tree.
(u, v) where v is a proper descendent of u in the tree. Ans
(u, v) where v is a proper ancesstor of u in the tree.
(u, v) where u is a proper ancesstor of v in the tree.

Question # 3 of 10 (Start time: 10:25:10 PM) Total Marks: 1
There is relationship between number of back edges and number of cycles in DFS
Select correct option:
Both are equal.
Cycles are half of back edges.
Cycles are one fourth of back edges.
There is no relationship between back edges and number of cycles. Ans

Question # 4 of 10 (Start time: 10:25:31 PM) Total Marks: 1
In undirected graph, by convention all the edges are called _____ edges.
Select correct option:
Forward
Back Ans
Cross
Both forward and back

Question # 5 of 10 (Start time: 10:26:38 PM) Total Marks: 1
For undirected graph, there is no distinction between forward and back edges.
Select correct option:
True
False

Question # 6 of 10 (Start time: 10:26:55 PM) Total Marks: 1
Adding any edge to a free tree creates a unique _____ .
Select correct option:
Vertex
Edge
Cycle
Strong component

Question # 7 of 10 (Start time: 10:27:09 PM) Total Marks: 1
In _____ algorithm, at any time, the subset of edges A forms a single tree.
Select correct option:
Kruskal's
Prim's Ans
Both
None

Question # 8 of 10 (Start time: 10:27:23 PM) Total Marks: 1
In strong components algorithm, the form of graph is used in which all the _____ of original graph G have been reversed in direction.
Select correct option:
Vertices
Edges Ans (not sure)
Both edges & vertices
None of the above

True False

- Please choose one

Question # 9 of 10 (Start time: 10:28:31 PM) Total Marks: 1

According to parenthesis lemma, vertex u is a descendent of v vertex if and only if;
Select correct option:

[d[u], f[u]] ⊆ [d[v], f[v]] Ans

[d[u], f[u]] ⊇ [d[v], f[v]]

Unrelated

Disjoint

Question # 10 of 10 (Start time: 10:29:23 PM) Total Marks: 1
Strongly connected components are not affected by reversal of all edges in terms of vertices reachability.
Select correct option:
True Ans
False

-

Question # 1 of 10 (Start time: 10:23:08 PM) Total Marks: 1
Adding any edge to a free tree creates a unique cycle.
Select correct option:
True Ans
False

Question # 2 of 10 (Start time: 10:23:52 PM) Total Marks: 1
Forward edge is:
Select correct option:
(u, v) where u is a proper descendent of v in the tree.
(u, v) where v is a proper descendent of u in the tree. Ans
(u, v) where v is a proper ancesstor of u in the tree.
(u, v) where u is a proper ancesstor of v in the tree.

Question # 3 of 10 (Start time: 10:25:10 PM) Total Marks: 1
There is relationship between number of back edges and number of cycles in DFS
Select correct option:
Both are equal.
Cycles are half of back edges.
Cycles are one fourth of back edges.
There is no relationship between back edges and number of cycles. Ans

Question # 4 of 10 (Start time: 10:25:31 PM) Total Marks: 1
In undirected graph, by convention all the edges are called _____ edges.
Select correct option:
Forward
Back Ans
Cross
Both forward and back

Question # 5 of 10 (Start time: 10:26:38 PM) Total Marks: 1
For undirected graph, there is no distinction between forward and back edges.
Select correct option:
True Ans
False

Question # 6 of 10 (Start time: 10:26:55 PM) Total Marks: 1
Adding any edge to a free tree creates a unique _____ .
Select correct option:

True False

- Please choose one

Vertex

Edge
Cycle Ans
Strong component

Question # 7 of 10 (Start time: 10:27:09 PM) Total Marks: 1
In _____ algorithm, at any time, the subset of edges A forms a single tree.
Select correct option:
Kruskal's
Prim's Ans
Both
None

Question # 8 of 10 (Start time: 10:27:23 PM) Total Marks: 1
In strong components algorithm, the form of graph is used in which all the _____ of original graph G have been reversed in direction.
Select correct option:
Vertices
Edges Ans
Both edges & vertices
None of the above

Question # 9 of 10 (Start time: 10:28:31 PM) Total Marks: 1
According to parenthesis lemma, vertex u is a descendent of v vertex if and only if;
Select correct option:

[d[u], f[u]] ⊆ [d[v], f[v]] Ans

[d[u], f[u]] ⊇ [d[v], f[v]]

Unrelated

Disjoint

Question # 10 of 10 (Start time: 10:29:23 PM) Total Marks: 1
Strongly connected components are not affected by reversal of all edges in terms of vertices reachability.
Select correct option:
True Ans
False
Question # 1 of 10 (Start time: 08:34:15 PM) Total Marks: 1
According to parenthesis lemma, vertex u is unrelated to v vertex if and only if d[u],f[u]] and [d[v],f[v]] are disjoint.
Select correct option:
True Ans
False

Question # 2 of 10 (Start time: 08:35:47 PM) Total Marks: 1
Kruskal's algorithm (choose best non-cycle edge) is better than Prim's (choose best tree edge) when the graph has relatively few edges.
Select correct option:
True
False Ans

Question # 3 of 10 (Start time: 08:37:05 PM) Total Marks: 1
You have an adjacency list for G, what is the time complexity to compute Graph transpose G^T.?
Select correct option:
? (V + E) Ans

True False

- Please choose one

? (V E)

? (V)
? (V^2)

Question # 4 of 10 (Start time: 08:38:13 PM) Total Marks: 1
Kruskal's algorithm works by adding vertices in increasing order of weight (lightest edge first).
Select correct option:
True Ans
False

Question # 5 of 10 (Start time: 08:38:38 PM) Total Marks: 1
In Prim's algorithm, we will make use of priority _____.
Select correct option:
Stack
Queue Ans
Array
Graph

Question # 6 of 10 (Start time: 08:39:03 PM) Total Marks: 1
If a vertex v is a descendent of vertex u, then v's start-finish interval is contained within u's start-finish interval.
Select correct option:
True Ans
False

Question # 7 of 10 (Start time: 08:39:53 PM) Total Marks: 1
Computing the strongly connected components of a digraph is a generalization of the problem to determine whether a digraph is strongly connected or not.
Select correct option:
True Ans
False

Question # 8 of 10 (Start time: 08:41:01 PM) Total Marks: 1
Adding any edge to a free tree creates a unique _____ .
Select correct option:
Vertex
Edge
Cycle Ans
Strong component

Question # 9 of 10 (Start time: 08:41:41 PM) Total Marks: 1
Networks are complete in the sense that it is possible from any location in the network to reach any other location in the digraph.
Select correct option:
True Ans
False

Question # 10 of 10 (Start time: 08:42:10 PM) Total Marks: 1
By breaking any edge on a cycle created in free tree, the free _____ is restored.
Select correct option:
Edge
Tree Ans
Cycle
Vertex

Timestamp structure of _____ is used in determining the strong components of a digraph.
Select correct option

True False

- Please choose one

DFS

BFS

Both DFS & BFS

None

For a digraph G = (V, E), Sum of in-degree(v) _____
Select correct option

Not equal to Sum of out-degree(v)

= Sum of out-degree(v) ok

< Sum of out-degree(v)

> Sum of out-degree(v)
Question # 2 of 10 (Start time: 09:32:33 PM) Total Marks: 1
We say that two vertices u and v are mutually not reachable if u can reach v and vice versa.
Select correct option

True ok

False

Question # 6 of 10 (Start time: 09:36:53 PM) Total Marks: 1
The Huffman codes provide a method of _____ data efficiently.
Select correct option

Reading

Encoding ok

Decoding

Printing
Question # 7 of 10 (Start time: 09:37:46 PM) Total Marks: 1
In greedy algorithm, at each phase, you take the_____ you can get right now, without regard for future consequences.
Select correct option

Worst

Minimum

Good

Best ok
Question # 8 of 10 (Start time: 09:38:38 PM) Total Marks: 1
In digraph G=(V,E) ;G has cycle if and only if
Select correct option

The DFS forest has forward edge.

The DFS forest has back edge ok

The DFS forest has both back and forward edge

BFS forest has forward edge
Question # 9 of 10 (Start time: 09:39:38 PM) Total Marks: 1
In Activity selection (using Greedy approach), intuitively _____.
Select correct option

Short activities are not preferable

True False

- Please choose one

There are always short activities as input

We do not like long activities ok

It does not matter about the length of activities
Question # 10 of 10 (Start time: 09:40:50 PM) Total Marks: 1
Breadth-first search begins at a root node and inspects all the nodes except neighboring ones.
Select correct option

True

False ok

-

In in-place sorting algorithm is one that uses arrays for storage :
An additional array
No additional array
Both of above may be true according to algorithm
More than 3 arrays of one dimension.

The running time of quick sort depends heavily on the selection of
No of inputs
Arrangement of elements in array
Size o elements
Pivot element

In stable sorting algorithm
One array is used
In which duplicating elements are not handled.
More then one arrays are required.
Duplicating elements remain in same relative position after sorting.

Which sorting algorithn is faster :
O(n^2)
O(nlogn)
O(n+k)
O(n^3)

In Quick sort algorithm,constants hidden in T(n lg n) are
Large
Medium
Not known
Small

Quick sort is based on divide and conquer paradigm; we divide the problem on base of pivot element and:
There is explicit combine process as well to conquer the solution.
No work is needed to combine the sub-arrays, the array is already sorted
Merging the subarrays
None of above.

There is relationship between number of back edges and number of cycles in DFS
Both are equal.
Cycles are half of back edges.
Cycles are one fourth of back edges.
There is no relationship between back edges and number of cycle

True False

- Please choose one

You have an adjacency list for G, what is the time complexity to compute Graph transpose G^T ?

(V+E)
V.E
V
E

You have an adjacency list for G, what is the time complexity to compute Graph transpose G^T.?

?(V + E)
?(V E)
?(V)
?(V^2)

What is the time complexity to extract a vertex from the priority queue in Prim's algorithm?

log (V)
V.V
E.E
log (E)

Dijkstra's algorithm :

Has greedy approach to find all shortest paths
Has both greedy and Dynamic approach to find all shortest paths
Has greedy approach to compute single source shortest paths to all other vertices
Has both greedy and dynamic approach to compute single source shortest paths to all other vertices.

What algorithm technique is used in the implementation of Kruskal solution for the MST?

Greedy Technique
Divide-and-Conquer Technique
Dynamic Programming Technique
The algorithm combines more than one of the above techniques

Which is true statement in the following.

Kruskal algorithm is multiple source technique for finding MST.
Kruskal's algorithm is used to find minimum spanning tree of a graph, time complexity of this algorithm is O(EV)
Both of above
Kruskal's algorithm (choose best non-cycle edge) is better than Prim's (choose best Tree edge) when the graph has relatively few edges)

The relationship between number of back edges and number of cycles in DFS is,

Both are equal
Back edges are half of cycles
Back edges are one quarter of cycles
There is no relationship between no. of edges and cycles

Kruskal's algorithm (choose best non-cycle edge) is better than Prim's (choose best tree edge) when the graph has relatively few edges

True
False

Suppose that a graph G = (V,E) is implemented using adjacency lists. What is the complexity of a breadth-first traversal of G?

O(|V |^2)
O(|V | |E|)

True False

- Please choose one

O(|V |^2|E|)

O(|V | + |E|)

What is generally true of Adjacency List and Adjacency Matrix representations of graphs?
Lists require less space than matrices but take longer to find the weight of an edge (v1,v2)
Lists require less space than matrices and they are faster to find the weight of an edge (v1, v2)
Lists require more space than matrices and they take longer to find the weight of an edge (v1, v2)
Lists require more space than matrices but are faster to find the weight of an edge (v1, v2)

What general property of the list indicates that the graph has an isolated vertex?
There is Null pointer at the end of list.
The Isolated vertex is not handled in list.
Only one value is entered in the list.
There is at least one null list.

A dense undirected graph is:
A graph in which E = O(V^2)
A graph in which E = O(V)
A graph in which E = O(log V)
All items above may be used to characterize a dense undirected graph

In digraph G=(V,E) ;G has cycle if and only if
The DFS forest has forward edge.
The DFS forest has back edge
The DFS forest has both back and forward edge
BFS forest has forward edge

Back edge is:
(u, v) where v is an ancestor of u in the tree.
(u,v) where u is an ancesstor of v in the tree.
(u, v) where v is an predcessor of u in the tree.
None of above

Using ASCII standard the string “abacdaacacwe” will be encoded with _____ bits
64
128
96
120

Cross edge is :
(u, v) where u and v are not ancestor of one another
(u, v) where u is ancesstor of v and v is not descendent of u.
(u, v) where u and v are not ancestor or descendent of one another
(u, v) where u and v are either ancestor or descendent of one another.

Which statement is true?
If a dynamic-programming problem satisfies the optimal-substructure property, then a locally optimal solution is globally optimal.
If a greedy choice property satisfies the optimal-substructure property, then a locally optimal solution is globally optimal.
Both of above
None of above

If you find yourself in maze the better traversel approach will bEA dense undirected graph is:

True False

- Please choose one

A graph in which E = O(V^2)

A graph in which E = O(V)
A graph in which E = O(log V)
All items above may be used to characterize a dense undirected graph

Which is true statement.
Breadth first search is shortest path algorithm that works on un-weighted graphs
Depth first search is shortest path algorithm that works on un-weighted graphs.
Both of above are true.
None of above are true.

Forward edge is:
(u, v) where u is a proper descendent of v in the tree.
(u, v) where v is a proper descendent of u in the tree.
(u, v) where v is a proper ancesstor of u in the tree.
(u, v) where u is a proper ancesstor of v in the tree.

In digraph G=(V,E) ;G has cycle if and only if
The DFS forest has forward edge.
The DFS forest has back edge
The DFS forest has both back and forward edge
BFS forest has forward edge

If you find yourself in maze the better traversel approach will be :
BFS
BFS and DFS both are valid
Level order
DFS

What algorithm technique is used in the implementation of Kruskal solution for the MST?
Greedy Technique
Divide-and-Conquer Technique
Dynamic Programming Technique
The algorithm combines more than one of the above techniques

Kruskal's algorithm (choose best non-cycle edge) is better than Prim's (choose best tree edge) when the graph has relatively few
True
False

A digraph is strongly connected under what condition?
A digraph is strongly connected if for every pair of vertices u, v e V, u can reach v .
A digraph is strongly connected if for every pair of vertices u, v e V, u can reach v and vice versa.
A digraph is strongly connected if for at least one pair of vertex u, v e V, u can reach v and vice versa.
A digraph is strongly connected if at least one third pair of vertices u, v e V, u can reach v and vice versa.

The relationship between number of back edges and number of cycles in DFS is,
Both are equal
Back edges are half of cycles
Back edges are one quarter of cycles
There is no relationship between no. of edges and cycles

What algorithm technique is used in the implementation of Kruskal solution for the M ST?
Greedy Technique

True False

- Please choose one

Divide-and-Conquer Technique

Dynamic Programming Technique
The algorithm combines more than one of the above techniques

The _____ given by DFS allow us to determine whether the graph contains any cycles. CS502

- Order
- **Time stamps** Page 130
- BFS traversing
- Topological sort

A graph is not connected if every vertex can reach every other vertex. CS502

- True
- **False** Page No. 116

In strong components algorithm, first of all DFS is run for computing finish times of vertices. CS502

- **True** Page No.138
- False

In strong components algorithm, first of all DFS is run for computing finish times of vertices. CS502

- [d[u], f[u]]⊆[d[v], f[v]]
- [d[u], f[u]]⊇[d[v], f[v]]
- **unrelated** Page No.129
- Disjoint

For traversing graphs, Breadth-first search can be visualized as a wave front propagating inwards towards root (or source) node. CS502

- True
- **False** Page No.117

Digraphs are not used in communication and transportation networks. CS502

- True
- **False** Page No.135

You have an adjacency list for G, what is the time complexity to compute Graph transpose G^T.? CS502 ?

True False

- Please choose one

- **(V + E)** Page No.138
- V E
- V
- E

In computing the strongly connected components of a digraph, vertices of the digraph are not partitioned into subsets. CS502

- True
- **False** Page No.135

There are no _____ edges in undirected graph. CS502

- Forward
- Back
- Cross
- **Both Forward and Back** Page No. 130

We say that two vertices u and v are mutually not reachable if u can reach v and vice versa. CS502

- **True** Page No. 135
- False

- There is relationship between number of back edges and number of cycles in DFS Select correct option:
- Both are equal.
- Cycles are half of back edges.
- Cycles are one fourth of back edges.
- **There is no relationship between back edges and number of cycles**. **(Correct)**
-
-
- In strong components algorithm, the form of graph is used in which all the vertices of original graph G have been reversed in direction. Select correct option:
- **True** **(Correct)**
- False
-

True False

- Please choose one

- In Kruskal's algorithm, the next edge is added to viable set A, if its adding does not induce a/an _____ . Select correct option:
-
- Vertex
- Edge
- **Cycle (Correct)**
- Tree
-
-
- For undirected graph, there is no distinction between forward and back edges. Select correct option:
- **True (Correct)**
- False
-
- You have an adjacency list for G, what is the time complexity to compute Graph transpose G^T ? Select correct option:
- **(V+E) (Correct)**
- V.E
- V
- E
-
- There exist a unique path between any _____ vertices of a free tree. Select correct option:
- One
- **Two (Correct)**
- Three
- All
-

True False

- Please choose one

- A digraph is strongly connected under what condition? Select correct option:
-
- A digraph is strongly connected if for every pair of vertices u, v e V, u can reach v .
- **A digraph is strongly connected if for every pair of vertices u, v e V, u can reach v and vice versa. (Correct)**
- A digraph is strongly connected if for at least one pair of vertex u, v e V, u can reach v and vice versa.
- A digraph is strongly connected if at least one third pair of vertices u, v e V, u can reach v and vice versa.
-
-
- If u and v are mutually reachable in G, then in the graph formed by reversing all the edges, these vertices are not reachable. Select correct option:
- True
- **False (Correct)**
-
-
- In Prim's algorithm, at any time, the subset of edges A forms a single forest. Select correct option:
- True
- **False (Correct)**
-
- In digraph G=(V,E) ;G has cycle if and only if
- The DFS forest has forward edge.
- **The DFS forest has back edge**
- The DFS forest has both back and forward edge
- BFS forest has forward edge

True False

- Please choose one

Question No: 1 (Marks: 1) - Please choose one

In_____ algorithm, at anytime, the subset of edges A forms a single tree.

Kruskal's
Prim's
Both
None

Question No: 2 (Marks: 1) - Please choose one

According to parenthesis lemma, vertex u is unrelated to v vertex if and only if [d[u], f[u]] and [d[v], f[v]] are disjoint.

True
False

Question No: 3 (Marks: 1) - Please choose one

A free tree with n vertices have exactly n+1 edges.

True
Fale

True False

- Please choose one

Question No: 4 (Marks: 1) - Please choose one

For undirected graph, there is no distinction between forward and back edges.

True
Fale

Question No: 5

In computing the strongly connected components of a digraph, vertices of the digraph are not partitioned into subsets.

True
False

Question No: 6 (Marks: 1) - Please choose one

What is the time complexity to extract a vertex from the priority queue in Prim's algorithm?

log (V)
V.V
E.E
log (E)

True False

- Please choose one

Question No: 7 (Marks: 1) - Please choose one

What is true statement?

Breadth first search is shortest path algorithm that works on un-weighted graphs
Depth first search is shortest path algorithm that works on un-weighted graphs
Both of above are true
None of above are true

Question No: 8 (Marks: 1) - Please choose one

Networks are_____ in the sense that it is possible from any location in the network to reach any other location in the digraph.

Complete
Incomplete
Not graphs
Transportation

Question No: 9 (Marks: 1) - Please choose one

In Generic approach determining of Greedy MST, we maintain a subset A of are_____

Edges
Vertices
Cycles
Paths

True False

- Please choose one

Question No: 10 (Marks: 1)

There are no_____ edges in undirected graph.

Forward
Back
Cross
Both forward and back

Question No: 11 (Marks: 1) - Please choose one

If you find yourself in maze the better traversal approach will be:

BFS
BFS and DFS both are valid
Level order
DFS

Question No: 12 (Marks: 1) - Please choose one

In strong components algorithm, the form of graph is used in which all the vertices of original graph G have been reversed in direction.

True
False

True False

- Please choose one

Question No: 13 (Marks: 1) - Please choose one

What algorithm technique is used in the implementation of Kruskal solution for the MST?

Greedy Technique
Divide-and-Conquer Technique
Dynamic Programming Technique
The algorithm combines more than one of the above techniques

Question No: 14 (Marks: 1) - Please choose one

In Kruskal's algorithm, the next edge is added to viable set A, if its adding does not induce a cycle.

True False

Question No: (Marks: 1) - Please choose one

15

There exists a unique path between any_____ vertices of a free tree.

One
Two
Three
All

Question No: 16 (Marks: 1) - Please choose one

Computing the strongly connected components of a digraph is a generalization of the problem to determine whether a digraph is strongly connected or not.

True
False

Question No: 17 (Marks: 1) - Please choose one

If a vertex is a descendent of vertex u, then v's start-finish interval is contained within u's start-finish interval.

True
False

True False

- Please choose one

Question No: 18 (Marks: 1) - Please choose one

The_____ given by DFS allow us to determine whether the graph contains any cycles.

Order
Time stamps
BFS traversing
Topological sort

Question No: 19 (Marks: 1) - Please choose one

Networks are complete in the sense that it is possible from any location in the network to reach any other location in the digraph.

Question No: 20 (Marks: 1)

There are no_____ edges in undirected graph.

Forward
Back
Cross
Both forward and back

Question No: 21 (Marks: 1) - Please choose one

True False

- Please choose one

You have an adjacency list for G, what is the time complexity to compute Graph transpose G^T.?

? (V + E)
? (V E)
? (V)
? (V^2)

Question No: 22 (Marks: 1) - Please choose one

There are no_____ edges in undirected graph.

Forward
Back
Cross
Both forward and back

Question No: 23 (Marks: 1) - Please choose one

In Prim's algorithm, at anytime, the subset of edges A forms a single_____.

Vertex
Forest
Tree
Graph

True False

- Please choose one

Question No: 24 (Marks: 1) - Please choose one

Kruskal's algorithm works by adding vertices in increasing order of weight (lightest edge first).

25

Runtime complexity of Prim's algorithm is _____.

V log V
E log V
log V
None of the above

Question No: 26 (Marks: 1) - Please choose one

According to parenthesis lemma, vertex u is an ancestor of v vertex if and only if;

[d[u], f[u]] ⊆ [d[v], f[v]]
[d[u], f[u]] ⊇ [d[v], f[v]]
Unrelated
Disjoint

Question No: 27 (Marks: 1) - Please choose one

In Kruskal's algorithm, the next _____ is not added to viable set A, if its adding induce a/an

True False

- Please choose one

cycle.

Vertex
Edge
Cycle
Tree

Question No: 28 (Marks: 1) - Please choose one

In Prim's algorithm, we start with the _____ vertex r; it can be any vertex.

First
Leaf
Mid
Root

Question No: 29 (Marks: 1) - Please choose one

A free tree with n vertices has exactly n+1 edges.

True
False

True False

(Marks: 1) - Please choose one

Question No: 30

There exists a unique path between any _____ vertices of a free tree.

One
Two
Three
All

Question No: 31 (Marks: 1) - Please choose one

The tricky part of the _____ algorithm is how to detect whether the addition of an edge will create a cycle in viable set A.

Kruskal's
Prim's
Both
None

Question No: 32 (Marks: 1) - Please choose one

In Prim's algorithm, we start with the root vertex r; it can be any vertex.

True
False

Question No: -

Question No: 33 (Marks: 1) - Please choose one

The relationship between number of back edges and number of cycles in DFS is,

Both are equal
Back edges are half of cycles
Back edges are one quarter of cycles
There is no relationship between no. of edges and cycles

Question No: 34 (Marks: 1) - Please choose one

If you find yourself in maze the better traversal approach will be:

BFS
BFS and DFS both are valid
Level order
DFS

35 Please choose one

According to parenthesis lemma, vertex u is an ancestor of v vertex if and only if;

[d[u], f[u]] ⊆ [d[v], f[v]]
[d[u], f[u]] ⊇ [d[v], f[v]]
Unrelated
Disjoint

Question No: -

Question No: 36 (Marks: 1) - Please choose one

We say that two vertices u and v are mutually not reachable if u can reach v and vice versa.

True
False

Question No: 37 (Marks: 1) - Please choose one

In Generic approach determining of Greedy MST, we maintain a subset A of are_____

Edges
Vertices
Cycles
Paths

Question No: 38 (Marks: 1) - Please choose one

Kruskal's algorithm (choose best non-cycle edge) is better than Prim's (choose best tree edge) when the graph has relatively few edges.

True
False

Question No: -

Question No: 39 (Marks: 1) - Please choose one

Digraphs are not used in communication and transportation networks.

True
False

40 (Marks: 1) Please choose one

We say that two vertices u and v are mutually not reachable if u can reach v and vice versa.

True
False

Question No: 41 (Marks: 1) - Please choose one

Runtime complexity of Prim's algorithm is _____.

V log V **E**
log V log
V
None of the above

Question No: -

Question No: 42 (Marks: 1) - Please choose one

In Prim's algorithm, we start with the root vertex r; it can be any vertex.

True
False

Question No: 43 (Marks: 1) - Please choose one

Networks are _____ in the sense that it is possible from any location in the network to reach any other location in the digraph.

Complete
Incomplete
Not graphs
Transportation

Question No: 44 (Marks: 1) - Please choose one

Kruskal's algorithm (choose best non-cycle edge) is better than Prim's (choose best tree edge) when the graph has relatively few edges.

True False

Question No: -

45

In strong components algorithm, vertices are sorted in_____ order of finish times.

Any
Increasing
Decreasing
None

Question No: 46 (Marks: 1) - Please choose one

In Kruskal's algorithm, at anytime, the subset of edges A forms a single tree.

True
False

Question No: 47 (Marks: 1) - Please choose one

If a subset of edges A is viable for building MST, it can not contain a/an_____.

Vertex
Edge
Cycle
Graph

Question No: -

Question No: 48 (Marks: 1) - Please choose one

In Timestamped DFS-cycles lemma, if edge (u,v) is a tree, forward or cross edge, then_____.

{\text{f[u]}} < {\text{f[v]}} {\text{f[u]}} >
{\text{f[v]}}
f[u] ≤ f[v]
f[u] ≥ f[v]

Question No: 49 (Marks: 1) - Please choose one

Strongly connected components are not affected by reversal of all edges in terms of vertices reachability.

True False **50**
If u and v are mutually reachable in G, then in the graph formed by reversing all the edges, these vertices are not reachable.

True
False

Question No: 51 (Marks: 1) - Please choose one

Forward edge is:

(u, v) where u is a proper descendent of v in the tree.
(u, v) where v is a proper descendent of u in the tree.
(u, v) where v is a proper ancestor of u in the tree.
(u, v) where u is a proper ancestor of v in the tree.

Question No: -

Question No: 52 (Marks: 1) - Please choose one

According to parenthesis lemma, vertex u is an ancestor of v vertex if and only if;

[d[u], f[u]] ⊆ [d[v], f[v]]
[d[u], f[u]] ⊇ [d[v], f[v]]
Unrelated
Disjoint

Question No: 53 (Marks: 1) - Please choose one

In Timestamped DFS-cycles lemma, if edge (u,v) is a tree, forward or cross edge, then_____.

{\text{f[u]}} < {\text{f[v]}} {\text{f[u]}} >
{\text{f[v]}}
f[u] ≤ f[v]
f[u] ≥ f[v]

Question No: 54 (Marks: 1) - Please choose one

As the Kruskal's algorithm runs, the edges in viable set A induce a_____ on the vertices.

Set
Graph
Tree
Forest

55

Kruskal's algorithm (choose best non-cycle edge) is better than Prim's (choose best tree edge) when the

Question No: -

graph has relatively few edges.

True
False

Question No: 56 (Marks: 1) - Please choose one

Runtime complexity of Prim's algorithm is _____.

V log V E log V
log V
None of the above

Question No: 57 (Marks: 1) - Please choose one

Kruskal's algorithm works by adding _____vertices in increasing order of weight (lightest edge first).

Vertices
Edges
Trees
Weights

Question No: 58 (Marks: 1) - Please choose one

In strong components algorithm, the form of graph is used in which all the vertices of original graph G have been reversed in direction.

Question No: -

True

False

CS610-Computer Network

SOLVED MCQS QUIZ 1

LEACTURE (1 to 10)

Solved by JUNAID MALIK

AL-Junaid Institude

Contact no: 03041659294

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1. In heap sort algorithm, the total running time for heavily procedure is _.

- Big-oh(log n)
- O (1) i.e. Constant time
- Theta (log n)
- Omega (log n)

2. Quick sort algorithm is required a lot of comparison in the _____ condition.

- Worse case
- Best and average case
- Average case
- Best case

3. In heap sort algorithm (using max heap). When every time maximum element is removed from top.

- Divide and conquer strategy helps us
- We are left with a lot
- We call merge sort algorithm
- It becomes order n2 algorithm

4. In average-case time analysis of quick sort algorithm,
The most balanced case for partition is where we divide the list of element into _____

- Three nearly equal pieces
- Single piece exactly
- Two nearly piece
- Equal no. of piece as of input element

5. Consider three matrices X,Y,Z dimensions 1×2.2×.3×4 respectively. The number of multiplication of (XYZ) is:

- 32
- 30
- 24
- 18

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6. Quicksort is a/an _____ and_____ sorting algorithm.

- In-place, not stable one
- Not in-place, stable one
- In-place, stable one
- Not in-place, not stable one

7. _____items are not allowed in the 0/1 knapack.

- Lighter
- Whole
- Weighty
- Fractional

8. The main purpose of mathematical analysis is measuring the _____ required by the algorithm.

- Space
- Execution time and memory
- Input & output
- Execution time

9. Execution time of an algorithm can be measured by _____.

- Divide and conquer approach
- Both brute force and divide and conquer approach
- Mathematical analysis
- Brute force approach

10. Quick sort is based on _____ strategy.

- Graph theory
- Divide-and-conquer
- Dynamic programming
- Greedy approach

11. A sorting algorithm is called as _____ if duplicate element remain in the same relative position after sorting.

- O(n) algorithm
- Stable
- Parallel

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- Complex

12. Which one sorting algorithm is best suited to sort an array of 2 million elements?

- Insert sort
- Quick sort
- Merge sort
- Bubble sort

13. We can use the _____ property to devise a recursive formulation of the edit distance problem.

- Algorithm
- Small substructure
- Optimal substructure
- Real

14. While sorting. The ordered domain means for any two input elements x and y _____ satisfies only.

- All of the above
- x > y
- x < y
- x = y

15. 8n2 + 2n -3 will eventually exceed c2*(n) no matter how large we make _____.

- 2n
- n
- this equation
- c2

16. _____ is a method of solving a problem in which we check all possible solution to the problem to find the solution we need.

- Sorting algorithm
- Greedy approach
- Plane-sweep algorithm
- Brute-force algorithm

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17. In quick sort algorithm, provost form _____.

- Graph
- Stack
- Binary search tree
- Queue

18. In asymptotical analysis of n(n -3) and 4n*n, as n becomes large, the dominant (fastest growing) term is some constant time_____

- n +1
- n*n
- n
- n-1

19. If Matrix-A has dimensions "3×2" and Matrix-B has dimensions "2×3", then multiplication of Matrix –A and Matrix-B will result a new Matrix-C having dimensions

- 2×3
- 2×2
- 3×2
- 3×3

20. Boolean operation is a _____ operation on an idealized RAM model of computation.

- Advance
- Normal
- Basic
- Starting

21. There are _____ entries in the Edit Distance Matrix.

- Θ (n^2)
- Θ (n+100)
- Θ (n)
- Θ (n+2)

22. Counting sort is suitable for sorting the elements within range 1 to P. where_____

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- P is undetermined
- P is small
- P is very large
- P is large

23. Suppose we have 4 matrices A,B,C,D. what is correct expansion of m[1,2] in chain matrix multiplication?

- m[1,2] = m[1,1] + m[2,2] +p0 . p1. p3
- m[1,2] = m[1,1] + m[2,2] +p0 . p1. p3
- m[1,2] = m[1,1] + m[2,2] +p0 . p1. p3
- m[1,2] = m[1,1] + m[2,2] +p0 . p1. p3

24. Which one is not passed as parameter in Quick Sort algorithm?

- Array (containing input elements)
- Middle of the array
- Start of the array
- End of the array

25. In asymptotical analysis of n*(5+2)-3. As n becomes large, the dominant (fastest growing) term is some constant times_____

- n+1
- n*n
- n
- n_1

26. For _____ values of n, any algorithm is fast enough.

- Medium
- Small
- Infinity
- Large

27. Dynamic programming algorithms often use some kind of _____ to store the result of intermediate sub-problems.

- Stack
- Loop
- Table

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- Variable

28. In selection problem, the Sieve technique works in_____

- One complete go
- Constant time
- Non-recursive manner
- Phases

29. In heap Sort algorithm, the maximum levels an element can move upward is _____.

- Theta (log n)
- O (1) i.e. Constant time
- Omega (log n)
- Big-oh (log n)

30. While analysis of the brute-force maxima algorithm, an array storted in the reverse order is the type of _____ case input.

- Worst
- Best
- Somewhat
- Average

31.1Divide-and-Conquer is as breaking the problem into small number of

- **Smaller Sub Problems**
- Pivot
- Sieve

32. Analysis of Selection Sort ends up with

- **T(n)**
- T(1/1+n)
- T(n/2)
- T((n/2) +n)

33. For a digraph G = (V, E), Sum of in-degree(v) _____

- Not equal to Sum of out-degree(v)
- = Sum of out-degree(v)
- < Sum of out-degree(v)
- Sum of out-degree(v)

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34. In _____ problem, we want to find the best solution.

- Minimization
- Averaging
- Optimization
- Maximization

35. Timestamp structure of _____ is used in determining the strong components of a digraph.

- DFS
- BFS
- Both DFS & BFS
- None

36. The Huffman codes provide a method of _____ data efficiently. Select correct option

- Reading
- Encoding
- Decoding
- Printing

37. In greedy algorithm, at each phase, you take the_____ you can get right now, without regard for future consequences.

- Worst
- Minimum
- Good
- Best

38. In Activity selection (using Greedy approach), intuitively _____.

- Short activities are not preferable
- There are always short activities as input

- We do not like long activities
- It does not matter about the length of activities

39. Breadth-first search begins at a root node and inspects all the nodes except neighboring ones.
Select correct option

- True
- False

40. The ancient Roman politicians understood an important principle of good algorithm design that is plan-sweep algorithm.

- True
- False **(p27) [Divide and Conquer]**

41. In 2d-space a point is said to be _____if it is not dominated by any other point in that space.

- Member
- Minimal
- Maximal
- Join

42. How much time merge sort takes for an array of numbers?

- T(n^2)
- T(n)
- T(log n)
- T(n log n) Or O(n log n)

43. An algorithm is a mathematical entity that is dependent on a specific programming language.

- True
- False

44. In the analysis of Selection algorithm, we make a number of passes, in fact it could be as many as

- T(n)
- T(n/2)
- Log n
- n/2+n/4

45. The running time of an algorithm would not depend upon the optimization by the compiler but that of an implementation of the algorithm would depend on it.

- True
- False

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46. f(n) and g(n) are asymptotically equivalent. This means that they have essentially the same _____ for large n.

- Results
- Variables
- Size
- Growth Rates

47. 8n2 + 2n - 3 will eventually exceed c2*(n) no matter how large we make c2.

- True
- False

- If we associate (x, y) integers pair to cars where x is the speed of the car and y is the negation of the price. High y value for a car means a _____ car.Fast
- Slow
- Expensive
- Cheap

48. For the sieve technique we solve the problem

- Recursively
- Mathematically
- Precisely
- Accurately

49. Analysis of Selection algorithm ends with,

- T(n)
- T(n/2)
- Log n
- n/2+n/4

50. The function f(n)= n(logn+1)/2 is asymptotically equivalent to n log n. Hre Upper Bound means the function f(n) grows asymptotically _____ faster than n log n.

- More
- Quiet
- Not
- At Least

51. The running time of an algorithm would not depend upon the optimization by the compiler but that of an implementation of the algorithm would depend on it.

- True
- False

52. After sorting in merge sort algorithm, merging process is invoked.

- True

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- False

53. Asymptotic growth rate of the function is taken over_____ case running time.

- Best
- Average
- Worst
- Normal

54. In analysis of f(n)=n(n/5)+n-10 log n, f(n) is asymptotically equivalent to _____.

- N
- 2n
- N+1
- N2

55. Algorithm is concerned with.....issues

- Macro
- Micro
- Both Macro & Micro
- Normal

56. We cannot make any significant improvement in the running time which is better than that of brute-force algorithm.

- True
- False

57. In addition to passing in the array itself to Merge Sort algorithm, we will pass in _____other arguments which are indices.

- TWO
- THREE
- FOUR
- FIVE

58. Brute-force algorithm uses no intelligence in pruning out decisions.

- True
- False

59. In analysis, the Brute-force algorithm uses no intelligence in pruning out decisions at least as fast as its largest term.

- TRUE
- FALSE

60. Efficient algorithm requires less computational

- Memory
- Running Time
- Memory and Running TIME
- Energy

61. The O-notation is used to state only the asymptotic _____bounds.

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- Two
- Lower
- Upper
- Both Lower & upper

62. Heaps can be stored in arrays without using any pointers; this is due to the _____ nature of the binary tree.

- Left-complete
- Right-complete
- Tree Nodes
- Tree Levees

63. .....of reference is an important fact of current processor technology

- Defining
- Assigning
- Formality
- Locality

64. In the analysis of Selection algorithm, we get the convergent _____ series.

- Harmonic
- Linear
- Arithmetic
- Geometric

65. Brute-force algorithm for 2D-Maxima is operated by comparing _____ pairs of points.

- TWO
- SOME
- MOST
- ALL

66. In _____ we have to find rank of an element from given input.

- Merge Sort Algorithm
- Selection Problem
- Brute Force Technique
- Plane Sweep algorithm

67. The sequence of merge sort algorithm is:

a. Divide Combine-Conquer

b. Conquer-Divide-Combine

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c. Divide-Conquer-Combine Page 27

d. Combine-Divide-Conquer

68. In Selection algorithm, we assume pivot selection takes theta _____ running time.

a. n Page - 36

b. n2

c. n3

d. log (n)

69. Result of asymptotical analysis of n(n -3) and 4n*n is that _____

a. n(n-1) is asymptotically Less

b. n(n-1) is asymptotically Greater

c. Both are asymptotically Not equivalent

d. Both are asymptotically Equivalent page 23 ($4n*n= 4n^2$)

70. Floor and ceiling are _____ to calculate while analyzing algorithms

a. Very easy

b. Usually considered difficult P-31

c. 3rd Option is missing

d. 4th Option is missing

71. _____ of reference is an important fact of current processor technology.

a. Defining

b. Assigning

c. Formality

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d. Locality P-8

72. Which of the following is calculated with Big O notation?

a. Medium bounds
b. Upper bounds Page - 25
c. Lower bounds
d. Both upper and lower bounds

73. Al-Khwarizmi was a/an _____

a. Artist
b. Mathematician P-7
c. Astronomer
d. Khalifah

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74. In average-case time analysis of Quick sort algorithm, the most balanced case for partition is when we divide the list of elements into _.

a. Equal no. of pieces as of input elements

b. Single piece exactly

c. Two nearly equal pieces

d. Three nearly equal pieces

75. Which of the following is calculated with Big O notation?

a. Medium bounds

b. Upper bounds Page - 25

c. Lower bounds

b. Both upper and lower bounds

76. Pseudo code of algorithms are to be read by _____.

a. People Page -12

b. RAM

c. Computer

d. Compiler

77. The definition of theta-notation relies on proving _____ asymptotic bound.

a. One

b. Lower

c. Upper

d. Both lower & upper Page - 25

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78. In merge sort algorithm, to merge two lists of size n/2 to a list of size n, takes

_____ time.

a. Theta (n) Page - 32

b. Theta log(n)

c. Theta log2(n)

d. Theta n log(n)

79. We can make _____ recursive calls in Fibonacci Sequence.

a. Infinite

b. Finite google

c. Only one

d. Zero

80. The sieve technique is a special case, where the number of sub-problems is Just

_____

a. 1 P-34

b. 2

c. 3

d. 4

81. When a recursive algorithm revisits the same problem over and over again, we say that the optimization problem has _____ sub-problems.

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a. Overlapping – Google Search

b. Over costing

c. Optimized

d. Three

82. Sieve technique is very important special case of Divide-and-Conquer strategy.

a. True P-34

b. False

83. In order to say anything meaningful about our algorithms, it will be important for us to settle on a _____.

a. Java Program

b. C++ Program

c. Pseudo program

d. Mathematical model of computation P-10

84. Merge sort is based on _____.

a. Brute-force

b. Plan-sweep

c. Axis-sweep

d. Divide and Conquer P-27

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85. What time does Merge Sort algorithm take in order to sort an array of 'n' numbers?
   a. (n)
   b. (log n)
   c. (n^2)
   d. (n log n) Google Search 31. In Heap Sort

86. In plane sweep approach, a vertical line is swept across the 2d-plane and structure is used for holding the maximal points lying to the left of the sweep line.
   a. Array
   b. Queue
   c. Stack Page - 18
   d. Tree

87. _____ time is the maximum running time over all legal inputs.
   a. Worst-case Page - 13
   b. Average-case
   c. Best-case
   d. Good-case

88. Efficient algorithm requires less computational...
   a. Memory
   b. Running Time
   c. Memory and Running Time Page - 9
   d. Energy

89. The Iteration method is used for _____
   a. Comparing sorting algorithms only
   b. Solving Recurrence relations Page 31
   c. Merging elements in Merge sort
   d. Dividing elements in Merge sort

90. Median is not useful measure of central tendency of given input set especially when the distribution of values is highly skewed.

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a. True
b. False Page – 34

91. The Omega-notation allows us to state only the asymptotic _____ bounds.
a. Middle
b. Lower Page 25
c. Upper
d. Both lower & upper

92. In the statement "output P[1].x, P[1].y", the number of times elements of P are accessed is _____.
a. 1
b.2 page 14
b. 3
c. 4

93. The main purpose of mathematical analysis is measuring the _____ required by the algorithm.
a. Space
b. Execution time P-13
c. Inputs & outputs
d. Execution time and memory

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94. _____ provides us more accurate result when input values are not closer with each other

a. Average

b. Median P-34

c. Mode

d. Mean

95. The process of _____ ends when you are left with such tiny pieces remaining that it is trivial to solve them.

a. Brute-force

b. Plan-sweep

c. Divide and Conquer P-27

d. Axis-sweep

96. Approach of solving geometric problems by sweeping a line across the plane is called _____ sweep.

a. Line

b. Plane Page 18

c. Cube

d. Box

97. In the analysis of Selection algorithm, we get the convergent _____

a. Harmonic

b. Linear

c. Arithmetic

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d. Geometric Pg:37

98. A Random Access Machine (RAM)is an idealized machine withrandom access memory.

a. Infinite large Pg:10

b. 512 MB

c. 256 MB

d. 2 GBs

99. While analyzing Selection algorithm, we make a number of passes, in fact it could be as many as

a. n(n+1)

b. log(n) Pg:37

c. n/3

d. n/4

100. In Random Access Machine (RAM), instructions are executed in

a. Parallel

b. Batch

c. One by One Pg:10

d. Multiple times

101. In selection problem, the rank of an element will be its _____ position

a. First

b. final Pg:34

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c. Second last

d. Last

102. The worst-case running time of Merge sort is _____ in order to sort an array of n elements.

a. O(log n)

b. O(n)

c. O(n log n) page 40 and google

d. O(n)

103. f(n) and g(n) are asymptotically equivalent. This means that they have essentially the same _____.

a. Results

b. Variables

c. Size

d. Growth rates P:23

104. An algorithm is a mathematical entity. Which is independent of _____.

a. Programming language

b. Machine and Programming language

c. Compiler and Programming language

d. Programming language Compiler and Machine P:07

105. In Selection problem, the Sieve technique works in _____.

a. Non-recursive manner

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b. Constant time

c. Phases page 34

d. One complete go

106. Algorithm is a sequence of computational steps that ---- the input into output.

a. Merge

b. Assign

c. Transform page 7

d. Integrate

107. If pj dominates pi and pi dominates ph then pj also dominates ph, it means dominance relation is

a. Transitive page 18

b. Non Transitive

c. Equation

d. Symbolic

108. To find maximal points in brute-force algorithm each point of the space is compared against _____ of that space.

a. One other point

b. All other points page 11

c. Few other points

d. Most of the other points

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109. In the following code the statement "cout<<j;"executes --------- times. for (j=1; j<=5; j = j+2)

```
cout<<j;
```

a. 5 times

b. 2 times

c. 3 times

d. 0 times

110. In merge sort algorithm, we split the array around the _____ index q. a. Entring

b. Mid page 17

c. Exiting

d. Summing

111. In Selection problem, the Sieve technique _____.

a. Add some more input items each time

b. Do not work recursively

c. Do not uses Divide and Conquer approach

d. Eliminates undesired data items each time

112. While Sorting, the order domain means for any two input elements x and y

_____ satisfies only.

a. x < y page 39

b. x > y

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c. x = y

d. All of the above

113. For solving Selection problem, we introduced Sieve technique due to

_____

a. Using Decrease and Conquer strategy page 34

b. Avoiding to sort all input data

c. Eliminating Rank of an element

d. Using Brute-force approach

114. _____ is one of the few problems, where provable lower bounds exist on how fast we can sort.

a. Searching

b. Sorting page 38

c. Both Searching & sorting

d. Growing

115. In plane sweep approach, a vertical line is swept across the 2d-plane from_____.

a. Right to Left

b. Left to Right page 18

c. Top to Bottom

d. Bottom to top

116. In generating Fibonacci sequence, we can avoid unnecessary repetitions by

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_____ process.

a. Tokenization

b. Memorization page 43

c. Randomization

d. Memorization

117. For _____ values of n, any algorithm is fast enough.

a. Small page 14

b. Medium

c. Large

d. Infinity

118. Single item from a larger set of _____.

a. Constant

b. Pointers

c. Phases

d. n items page 34

119. Brute-force algorithm for 2D-Maxima is operated by comparing _____

pairs of points.

a. Two

b. Some

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c. Most

d. All page 18

120. For the Sieve Technique we take time.

a. T(nk) page 34

b. IT(n / 3)

c. n^2

d. n/3

121. Asymptotic growth rate of the function is taken over _____ case running time. .

a. Best

b. Worst page 14

c. Average

d. Normal

122. The sieve technique is a special case, where the number of sub problems is just.

a. 5

b. Many

c. 1 page 34

d. Few

123. In Quick sort, we don't have the control over the sizes of recursive calls.

a. True page 49

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b. False

c. Less information to decide

d. Ether true or false

124. Before sweeping a vertical line in plane sweep approach, in start sorting of the points is done in increasing order of their _____ coordinates. .

a. X page 18

b. Y

c. Z

d. X , Y

125. Random access machine or RAM is a/an.

a. Machine build by Al-Khwarizmi

b. Mechanical machine

c. Mathematical model page 10

d. Electronics machine

126. Cubic function will _____ a quadratic function.

a. Prove

b. be equal to

c. overtake Page 25

c. find

127. Asymptotic growth of 8n^2 + 2n – 3 is:

a. Θ(n^2 + n)

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b. Θ (n^2) page 14

c. Θ(8n^2)

d. Θ(8n^2 + 2n)

128. In average-case time the probability of seeing input is denoted by _____.

a. p{I}

b. p[I]

c. p<i>

d. p(i) page 13

129. While applying the Sieve technique to selection sort, how to choose a pivot element.

a. Through mean

b. Linear

c. Randomly page 35

d. Sequentially

130. Number of _____ of the pseudo code are counted to measure the running time.

a. Inputs

b. Outputs

c. Steps page 13

d. Pages

131. 8n^2+2n+3 will exceed c28(n), no matter how large we make _____.

a. n

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b. 2

c. c2 page 2

d. this quadratic equation

132. In 2-d maxima problem a point p is said to be dominated by point q if _____.

a. p.x <= q.x

b. p.x <= q.x and p.y <= q.y page 17

c. p.y <= q.y

d. p.x >= q.x and p.y >=q.y

133. Sorting can be in _____.

a. Increasing order only

b. Decreasing order only

c. Both increasing and decreasing order

d. Random order

134. Recurrence can be described in terms of _____.

a. Array

b. Linear

c. Tree page 31

d. Graph

135. The brute-force algorithm for 2D-Maxima runs in order O(__) time.

a. n

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b. n(log n)

c. n*n page 18

d. n3

136. In plane sweep approach of solving geometric problems, a _____ is swept across the plane.

a. Line page 18

b. Plane

c. Cube

d. Box

137. Which of the following is calculated with Big Omega notation?

a. Medium bounds

b. Upper bounds

c. Lower bounds Page - 25

d. Both upper and lower bounds

138. _____ is always based on divide and conquer strategy.

a. Bubble sort

b. Selection sort

c. Pigeon sort

d. Quick sort page 46

139. If a matrix has three rows and two columns, then dimensions of matrix will be:

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a. 3x2

b. 2x3

c. 3x3

d. 2x2

140. Asymptotic notations are used to describe _____ of an algorithm.

a. Length

b. running time google

c. size

d. compile time

141. The time assumed for each basic operation to execute on RAM model of computation is _____.

a. Infinite

b. Continuous

c. Constant page 10

d. Variable

142. Boolean operation is a _____ operation on an idealized RAM model of computation.

a. Starting

b. Basic page 10

c. Advance

d. Normal

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143. If we have an equation 8n2+7f*n + 5f + 6 then is large, _____ term will be muchxxxxxxxthe n term and will dominate the running time.

a. f g (n)

b. g (n) * 2

c. n * 2 page 23

d. f (n)

144. In quick sort algorithm, we choose pivot _____.

a. Always the smallest element

b. Greater than 5

c. Randomly page 35

d. Less than 5

145. The problem with the brute-force algorithm is that is uses _____ in pruning out de

a. Worst-case time

b. No intelligence page 18

c. Outside looping

d. Artificial intelligence

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Design and Analysis of Algorithms

Sohail Aslam

January 2004

Contents

1 Introduction **7**

1.1 Origin of word: *Algorithm* 7

1.2 Algorithm: Informal Definition 7

1.3 Algorithms, Programming 7

1.4 Implementation Issues 8

1.5 Course in Review 9

1.6 Analyzing Algorithms 9

1.7 Model of Computation 10

1.8 Example: 2-dimension maxima 10

1.9 Brute-Force Algorithm 11

1.10 Running Time Analysis 13

1.10.1 Analysis of the brute-force maxima algorithm. 14

1.11 Analysis: A Harder Example 16

1.11.1 2-dimension Maxima Revisited 17

1.11.2 Plane-sweep Algorithm 18

1.11.3 Analysis of Plane-sweep Algorithm 21

1.11.4 Comparison of Brute-force and Plane sweep algorithms 21

2 Asymptotic Notation **23**

3 Divide and Conquer Strategy **27**

3.1 Merge Sort 27

3.1.1 Analysis of Merge Sort 29

3.1.2 The Iteration Method for Solving Recurrence Relations 31

3.1.3 Visualizing Recurrences Using the Recursion Tree 31

3.1.4 A Messier Example . . . 32
3.2 Selection Problem . . . 34
3.2.1 Sieve Technique . . . 34
3.2.2 Applying the Sieve to Selection . . . 35
3.2.3 Selection Algorithm . . . 35
3.2.4 Analysis of Selection . . . 36

4 Sorting **39**
4.1 Slow Sorting Algorithms . . . 39
4.2 Sorting in $O(n \log n)$ time . . . 40
4.2.1 Heaps . . . 40
4.2.2 Heapsort Algorithm . . . 41
4.2.3 Heapify Procedure . . . 43
4.2.4 Analysis of Heapify . . . 43
4.2.5 BuildHeap . . . 43
4.2.6 Analysis of BuildHeap . . . 44
4.2.7 Analysis of Heapsort . . . 46
4.3 Quicksort . . . 46
4.3.1 Partition Algorithm . . . 46
4.3.2 Quick Sort Example . . . 47
4.3.3 Analysis of Quicksort . . . 49
4.3.4 Worst Case Analysis of Quick Sort . . . 49
4.3.5 Average-case Analysis of Quicksort . . . 50
4.4 In-place, Stable Sorting . . . 54
4.5 Lower Bounds for Sorting . . . 54

5 Linear Time Sorting **57**
5.1 Counting Sort . . . 57
5.2 Bucket or Bin Sort . . . 69
5.3 Radix Sort . . . 71

6 Dynamic Programming **73**
6.1 Fibonacci Sequence . . . 73

6.2 Dynamic Programming . . . 75
6.3 Edit Distance . . . 75
6.3.1 Edit Distance: Applications . . . 76
6.3.2 Edit Distance Algorithm . . . 77
6.3.3 Edit Distance: Dynamic Programming Algorithm . . . 77
6.3.4 Analysis of DP Edit Distance . . . 84
6.4 Chain Matrix Multiply . . . 84
6.4.1 Chain Matrix Multiplication-Dynamic Programming Formulation . . . 85
6.5 0/1 Knapsack Problem . . . 91
6.5.1 0/1 Knapsack Problem: Dynamic Programming Approach . . . 93

7 Greedy Algorithms **97**
7.1 Example: Counting Money . . . 97
7.1.1 Making Change: Dynamic Programming Solution . . . 98
7.1.2 Complexity of Coin Change Algorithm . . . 99
7.2 Greedy Algorithm: Huffman Encoding . . . 99
7.2.1 Huffman Encoding Algorithm . . . 100
7.2.2 Huffman Encoding: Correctness . . . 102
7.3 Activity Selection . . . 105
7.3.1 Correctness of Greedy Activity Selection . . . 107
7.4 Fractional Knapsack Problem . . . 109

8 Graphs **113**
8.1 Graph Traversal . . . 116
8.1.1 Breadth-first Search . . . 117
8.1.2 Depth-first Search . . . 119
8.1.3 Generic Graph Traversal Algorithm . . . 119
8.1.4 DFS - Timestamp Structure . . . 125
8.1.5 DFS - Cycles . . . 130
8.2 Precedence Constraint Graph . . . 131
8.3 Topological Sort . . . 133
8.4 Strong Components . . . 135
8.4.1 Strong Components and DFS . . . 137

8.5 Minimum Spanning Trees . . . 142
8.5.1 Computing MST: Generic Approach . . . 143
8.5.2 Greedy MST . . . 144
8.5.3 Kruskal's Algorithm . . . 147
8.5.4 Prim's Algorithm . . . 149
8.6 Shortest Paths . . . 153
8.6.1 Dijkstra's Algorithm . . . 154
8.6.2 Correctness of Dijkstra's Algorithm . . . 158
8.6.3 Bellman-Ford Algorithm . . . 159
8.6.4 Correctness of Bellman-Ford . . . 160
8.6.5 Floyd-Warshall Algorithm . . . 161

9 Complexity Theory **169**
9.1 Decision Problems . . . 170
9.2 Complexity Classes . . . 170
9.3 Polynomial Time Verification . . . 171
9.4 The Class NP . . . 172
9.5 Reductions . . . 173
9.6 Polynomial Time Reduction . . . 177
9.7 NP-Completeness . . . 177
9.8 Boolean Satisfiability Problem: Cook's Theorem . . . 178
9.9 Coping with NP-Completeness . . . 184

Chapter 1

Introduction

1.1 Origin of word: *Algorithm*

The word *Algorithm* comes from the name of the muslim author *Abu Ja'far Mohammad ibn Musa al-Khowarizmi*. He was born in the eighth century at Khwarizm (Kheva), a town south of river Oxus in present Uzbekistan. Uzbekistan, a Muslim country for over a thousand years, was taken over by the Russians in 1873.

His year of birth is not known exactly. Al-Khwarizmi parents migrated to a place south of Baghdad when he was a child. It has been established from his contributions that he flourished under Khalifah Al-Mamun at Baghdad during 813 to 833 C.E. Al-Khwarizmi died around 840 C.E.

Much of al-Khwarizmi's work was written in a book titled *al Kitab al-mukhatasar fi hisab al-jabr wa'l-muqabalah* (The Compendious Book on Calculation by Completion and Balancing). It is from the titles of these writings and his name that the words *algebra* and *algorithm* are derived. As a result of his work, al-Khwarizmi is regarded as the most outstanding mathematician of his time

1.2 Algorithm: Informal Definition

An algorithm is any well-defined computational procedure that takes some values, or set of values, as input and produces some value, or set of values, as output. An algorithm is thus a sequence of computational steps that transform the input into output.

1.3 Algorithms, Programming

A good understanding of algorithms is essential for a good understanding of the most basic element of computer science: programming. Unlike a program, an algorithm is a mathematical entity, which is independent of a specific programming language, machine, or compiler. Thus, in some sense, algorithm

design is all about the mathematical theory behind the design of good programs.

Why study algorithm design? There are many facets to good program design. Good algorithm design is one of them (and an important one). To be really complete algorithm designer, it is important to be aware of programming and machine issues as well. In any important programming project there are two major types of issues, macro issues and micro issues.

Macro issues involve elements such as how does one coordinate the efforts of many programmers working on a single piece of software, and how does one establish that a complex programming system satisfies its various requirements. These macro issues are the primary subject of courses on software engineering.

A great deal of the programming effort on most complex software systems consists of elements whose programming is fairly mundane (input and output, data conversion, error checking, report generation). However, there is often a small critical portion of the software, which may involve only tens to hundreds of lines of code, but where the great majority of computational time is spent. (Or as the old adage goes: 80% of the execution time takes place in 20% of the code.) The micro issues in programming involve how best to deal with these small critical sections.

It may be very important for the success of the overall project that these sections of code be written in the most efficient manner possible. An unfortunately common approach to this problem is to first design an inefficient algorithm and data structure to solve the problem, and then take this poor design and attempt to fine-tune its performance by applying clever coding tricks or by implementing it on the most expensive and fastest machines around to boost performance as much as possible. The problem is that if the underlying design is bad, then often no amount of fine-tuning is going to make a substantial difference.

Before you implement, first be sure you have a good design. This course is all about how to design good algorithms. Because the lesson cannot be taught in just one course, there are a number of companion courses that are important as well. CS301 deals with how to design good data structures. This is not really an independent issue, because most of the fastest algorithms are fast because they use fast data structures, and vice versa. In fact, many of the courses in the computer science program deal with efficient algorithms and data structures, but just as they apply to various applications: compilers, operating systems, databases, artificial intelligence, computer graphics and vision, etc. Thus, a good understanding of algorithm design is a central element to a good understanding of computer science and good programming.

1.4 Implementation Issues

One of the elements that we will focus on in this course is to try to study algorithms as pure mathematical objects, and so ignore issues such as programming language, machine, and operating system. This has the advantage of clearing away the messy details that affect implementation. But these details may be very important.

For example, an important fact of current processor technology is that of locality of reference. Frequently accessed data can be stored in registers or cache memory. Our mathematical analysis will usually ignore these issues. But a good algorithm designer can work within the realm of mathematics, but still keep an

open eye to implementation issues down the line that will be important for final implementation. For example, we will study three fast sorting algorithms this semester, heap-sort, merge-sort, and quick-sort. From our mathematical analysis, all have equal running times. However, among the three (barring any extra considerations) quick sort is the fastest on virtually all modern machines. Why? It is the best from the perspective of locality of reference. However, the difference is typically small (perhaps 10-20% difference in running time).

Thus this course is not the last word in good program design, and in fact it is perhaps more accurately just the first word in good program design. The overall strategy that I would suggest to any programming would be to first come up with a few good designs from a mathematical and algorithmic perspective. Next prune this selection by consideration of practical matters (like locality of reference). Finally prototype (that is, do test implementations) a few of the best designs and run them on data sets that will arise in your application for the final fine-tuning. Also, be sure to use whatever development tools that you have, such as profilers (programs which pin-point the sections of the code that are responsible for most of the running time).

1.5 Course in Review

This course will consist of four major sections. The first is on the mathematical tools necessary for the analysis of algorithms. This will focus on asymptotics, summations, recurrences. The second element will deal with one particularly important algorithmic problem: sorting a list of numbers. We will show a number of different strategies for sorting, and use this problem as a case-study in different techniques for designing and analyzing algorithms.

The final third of the course will deal with a collection of various algorithmic problems and solution techniques. Finally we will close this last third with a very brief introduction to the theory of NP-completeness. NP-complete problem are those for which no efficient algorithms are known, but no one knows for sure whether efficient solutions might exist.

1.6 Analyzing Algorithms

In order to design good algorithms, we must first agree the criteria for measuring algorithms. The emphasis in this course will be on the design of efficient algorithm, and hence we will measure algorithms in terms of the amount of computational resources that the algorithm requires. These resources include mostly running time and memory. Depending on the application, there may be other elements that are taken into account, such as the number disk accesses in a database program or the communication bandwidth in a networking application.

In practice there are many issues that need to be considered in the design algorithms. These include issues such as the ease of debugging and maintaining the final software through its life-cycle. Also, one of the luxuries we will have in this course is to be able to assume that we are given a clean, fully-specified mathematical description of the computational problem. In practice, this is often not the case, and the algorithm must be designed subject to only partial knowledge of the final specifications. Thus, in practice

it is often necessary to design algorithms that are simple, and easily modified if problem parameters and specifications are slightly modified. Fortunately, most of the algorithms that we will discuss in this class are quite simple, and are easy to modify subject to small problem variations.

1.7 Model of Computation

Another goal that we will have in this course is that our analysis be as independent as possible of the variations in machine, operating system, compiler, or programming language. Unlike programs, algorithms to be understood primarily by people (i.e. programmers) and not machines. Thus gives us quite a bit of flexibility in how we present our algorithms, and many low-level details may be omitted (since it will be the job of the programmer who implements the algorithm to fill them in).

But, in order to say anything meaningful about our algorithms, it will be important for us to settle on a mathematical model of computation. Ideally this model should be a reasonable abstraction of a standard generic single-processor machine. We call this model a *random access machine* or RAM.

A RAM is an idealized machine with an infinitely large random-access memory. Instructions are executed one-by-one (there is no parallelism). Each instruction involves performing some basic operation on two values in the machines memory (which might be characters or integers; let's avoid floating point for now). Basic operations include things like assigning a value to a variable, computing any basic arithmetic operation (+, - , $\times$, integer division) on integer values of any size, performing any comparison (e.g. $x \leq 5$) or boolean operations, accessing an element of an array (e.g. $A[10]$). We assume that each basic operation takes the same constant time to execute.

This model seems to go a good job of describing the computational power of most modern (nonparallel) machines. It does not model some elements, such as efficiency due to locality of reference, as described in the previous lecture. There are some "loop-holes" (or hidden ways of subverting the rules) to beware of. For example, the model would allow you to add two numbers that contain a billion digits in constant time. Thus, it is theoretically possible to derive nonsensical results in the form of efficient RAM programs that cannot be implemented efficiently on any machine. Nonetheless, the RAM model seems to be fairly sound, and has done a good job of modelling typical machine technology since the early 60's.

1.8 Example: 2-dimension maxima

Let us do an example that illustrates how we analyze algorithms. Suppose you want to buy a car. You want the pick the fastest car. But fast cars are expensive; you want the cheapest. You cannot decide which is more important: speed or price. Definitely do not want a car if there is another that is both faster and cheaper. We say that the fast, cheap car *dominates* the slow, expensive car relative to your selection criteria. So, given a collection of cars, we want to list those cars that are not dominated by any other.

Here is how we might model this as a formal problem.

- Let a point p in 2-dimensional space be given by its integer coordinates, $p = (p.x, p.y)$.

- A point p is said to be dominated by point q if $p.x \leq q.x$ and $p.y \leq q.y$.
- Given a set of n points, $P = \{p_1, p_2, \ldots, p_n\}$ in 2-space a point is said to be maximal if it is not dominated by any other point in P.

The car selection problem can be modelled this way: For each car we associate (x, y) pair where x is the speed of the car and y is the negation of the price. High y value means a cheap car and low y means expensive car. Think of y as the money left in your pocket after you have paid for the car. Maximal points correspond to the fastest and cheapest cars.

The *2-dimensional Maxima* is thus defined as

- Given a set of points $P = \{p_1, p_2, \ldots, p_n\}$ in 2-space, output the set of maximal points of P, i.e., those points p_i such that p_i is not dominated by any other point of P.

Here is set of maximal points for a given set of points in 2-d.

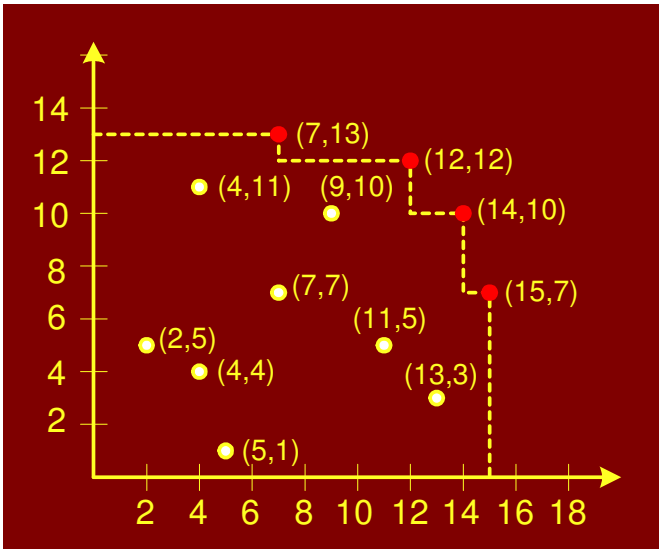


Figure 1.1: Maximal points in 2-d

Our description of the problem is at a fairly mathematical level. We have intentionally not discussed how the points are represented. We have not discussed any input or output formats. We have avoided programming and other software issues.

1.9 Brute-Force Algorithm

To get the ball rolling, let's just consider a simple brute-force algorithm, with no thought to efficiency. Let $P = \{p_1, p_2, \ldots, p_n\}$ be the initial set of points. For each point p_i, test it against all other points p_j. If p_i is not dominated by any other point, then output it.

This English description is clear enough that any (competent) programmer should be able to implement it. However, if you want to be a bit more formal, it could be written in pseudocode as follows:

```
MAXIMA(int n, Point P[1...n])
1  for i ← 1 to n
2  do maximal ← true
3     for j ← 1 to n
4     do
5        if (i ≠ j) and (P[i].x ≤ P[j].x) and (P[i].y ≤ P[j].y)
6           then maximal ← false;  break
7     if (maximal = true)
8        then output P[i]
```

There are no formal rules to the syntax of this pseudo code. In particular, do not assume that more detail is better. For example, I omitted type specifications for the procedure Maxima and the variable maximal, and I never defined what a Point data type is, since I felt that these are pretty clear from context or just unimportant details. Of course, the appropriate level of detail is a judgement call. Remember, algorithms are to be read by people, and so the level of detail depends on your intended audience. When writing pseudo code, you should omit details that detract from the main ideas of the algorithm, and just go with the essentials.

You might also notice that I did not insert any checking for consistency. For example, I assumed that the points in P are all distinct. If there is a duplicate point then the algorithm may fail to output even a single point. (Can you see why?) Again, these are important considerations for implementation, but we will often omit error checking because we want to see the algorithm in its simplest form.

Here are a series of figures that illustrate point domination.

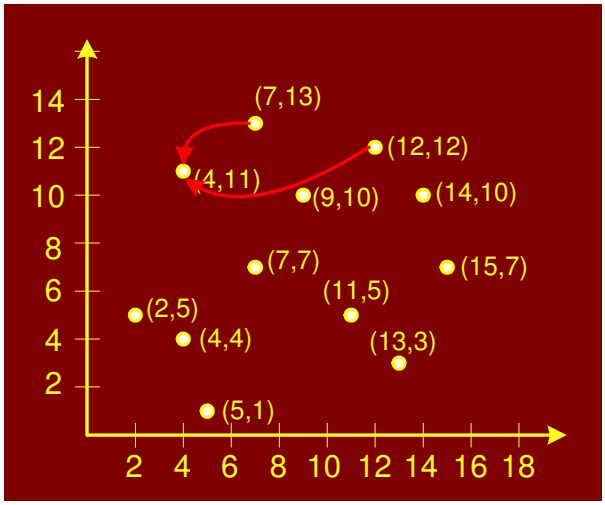


Figure 1.2: Points that dominate $(4, 11)$

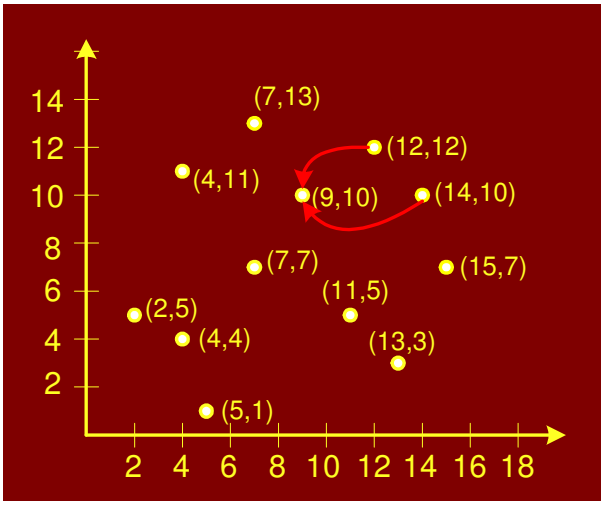


Figure 1.3: Points that dominate $(9, 10)$

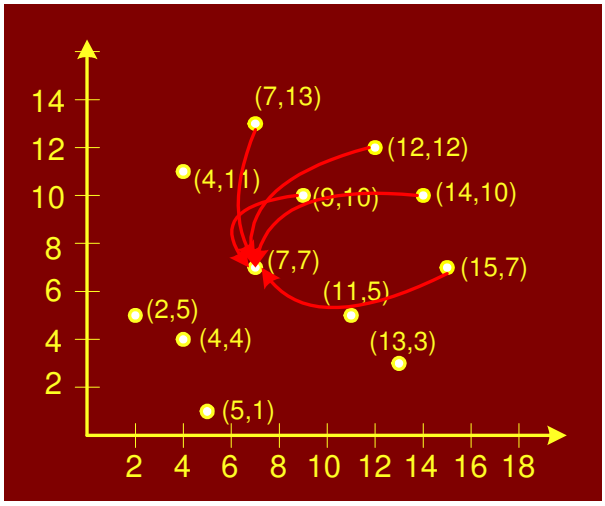


Figure 1.4: Points that dominate $(7, 7)$

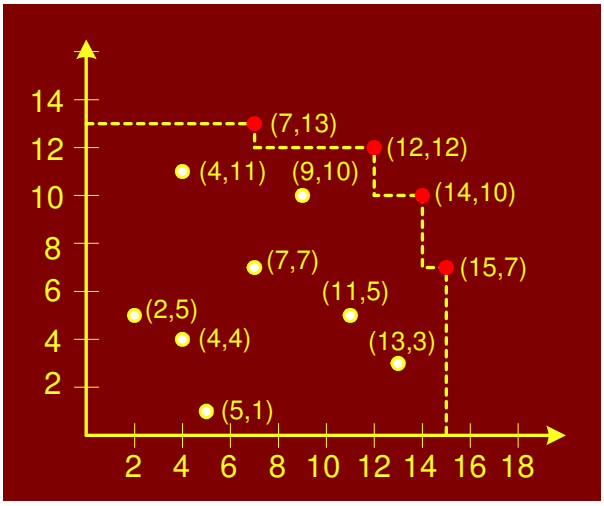


Figure 1.5: The maximal points

1.10 Running Time Analysis

The main purpose of our mathematical analysis will be measuring the execution time. We will also be concerned about the space (memory) required by the algorithm.

The running time of an implementation of the algorithm would depend upon the speed of the computer, programming language, optimization by the compiler etc. Although important, we will ignore these technological issues in our analysis.

To measure the running time of the brute-force 2-d maxima algorithm, we could count the number of steps of the pseudo code that are executed or, count the number of times an element of P is accessed or, the number of comparisons that are performed.

The running time depends upon the input size, e.g. n Different inputs of the same size may result in different running time. For example, breaking out of the inner loop in the brute-force algorithm depends not only on the input size of P but also the structure of the input.

Two criteria for measuring running time are *worst-case time* and *average-case time*.

Worst-case time is the maximum running time over all (legal) inputs of size n. Let I denote an input instance, let $|I|$ denote its length, and let $T(I)$ denote the running time of the algorithm on input I. Then

$$T_{worst}(n) = \max_{|I|=n} T(I)$$

Average-case time is the average running time over all inputs of size n. Let $p(I)$ denote the probability of seeing this input. The average-case time is the weighted sum of running times with weights

being the probabilities:

$$T_{avg}(n) = \sum_{|I|=n} p(I)T(I)$$

We will almost always work with worst-case time. Average-case time is more difficult to compute; it is difficult to specify probability distribution on inputs. Worst-case time will specify an upper limit on the running time.

1.10.1 Analysis of the brute-force maxima algorithm.

Assume that the input size is n, and for the running time we will count the number of time that any element of P is accessed. Clearly we go through the outer loop n times, and for each time through this loop, we go through the inner loop n times as well. The condition in the if-statement makes four accesses to P. The output statement makes two accesses for each point that is output. In the worst case every point is maximal (can you see how to generate such an example?) so these two access are made for each time through the outer loop.

```
MAXIMA(int n, Point P[1...n])
1  for i ← 1 to n                    n times
2  do maximal ← true
3     for j ← 1 to n                 n times
4     do
5        if (i ≠ j)&(P[i].x ≤ P[j].x)&(P[i].y ≤ P[j].y)   4 accesses
6           then maximal ← false break
7     if maximal
8       then output P[i].x, P[i].y    2 accesses
```

Thus we might express the worst-case running time as a pair of nested summations, one for the i-loop and the other for the j-loop:

$$\begin{aligned} T(n) &= \sum_{i=1}^{n} \left(2 + \sum_{j=1}^{n} 4\right) \\ &= \sum_{i=1}^{n} (4n + 2) \\ &= (4n + 2)n = 4n^2 + 2n \end{aligned}$$

For small values of n, any algorithm is fast enough. What happens when n gets large? Running time does become an issue. When n is large, n^2 term will be much larger than the n term and will *dominate* the running time.

We will say that the worst-case running time is $\Theta(n^2)$. This is called the asymptotic growth rate of the function. We will discuss this Θ-notation more formally later.

The analysis involved computing a summation. Summation should be familiar but let us review a bit here. Given a finite sequence of values $a_1, a_2, \ldots, a_n$, their sum $a_1 + a_2 + \ldots + a_n$ is expressed in summation notation as

$$\sum_{i=1}^{n} a_i$$

If $n = 0$, then the sum is additive identity, 0.

Some facts about summation: If c is a constant

$$\sum_{i=1}^{n} ca_i = c \sum_{i=1}^{n} a_i$$

and

$$\sum_{i=1}^{n} (a_i + b_i) = \sum_{i=1}^{n} a_i + \sum_{i=1}^{n} b_i$$

Some important summations that should be committed to memory.

Arithmetic series

$$\begin{aligned} \sum_{i=1}^{n} i &= 1 + 2 + \ldots + n \\ &= \frac{n(n+1)}{2} = \Theta(n^2) \end{aligned}$$

Quadratic series

$$\begin{aligned} \sum_{i=1}^{n} i^2 &= 1 + 4 + 9 + \ldots + n^2 \\ &= \frac{2n^3 + 3n^2 + n}{6} = \Theta(n^3) \end{aligned}$$

Geometric series

$$\begin{aligned} \sum_{i=1}^{n} x^i &= 1 + x + x^2 + \ldots + x^n \\ &= \frac{x^{(n+1)} - 1}{x - 1} = \Theta(n^2) \end{aligned}$$

If $0 < x < 1$ then this is $\Theta(1)$, and if $x > 1$, then this is $\Theta(x^n)$.

Harmonic series For $n \geq 0$

$$\begin{aligned} H_n &= \sum_{i=1}^{n} \frac{1}{i} \\ &= 1 + \frac{1}{2} + \frac{1}{3} + \ldots + \frac{1}{n} \approx \ln n \\ &= \Theta(\ln n) \end{aligned}$$

1.11 Analysis: A Harder Example

Let us consider a harder example.

```
NESTED-LOOPS()
1  for i ← 1 to n
2  do
3     for j ← 1 to 2i
4     do k = j ...
5        while (k ≥ 0)
6        do k = k − 1 ...
```

How do we analyze the running time of an algorithm that has complex nested loop? The answer is we write out the loops as summations and then solve the summations. To convert loops into summations, we work from inside-out.

Consider the *inner most while* loop.

```
NESTED-LOOPS()
1  for i ← 1 to n
2  do for j ← 1 to 2i
3     do k = j
4        while (k ≥ 0)
5        do k = k − 1
```

It is executed for $k = j, j-1, j-2, \ldots, 0$. Time spent inside the while loop is constant. Let $I()$ be the time spent in the while loop. Thus

$$I(j) = \sum_{k=0}^{j} 1 = j + 1$$

Consider the *middle for* loop.

```
NESTED-LOOPS()
1  for i ← 1 to n
2  do for j ← 1 to 2i
3     do k = j
4        while (k ≥ 0)
5        do k = k − 1
```

Its running time is determined by i. Let $M()$ be the time spent in the for loop:

$$\begin{aligned} M(i) &= \sum_{j=1}^{2i} I(j) \\ &= \sum_{j=1}^{2i} (j+1) \\ &= \sum_{j=1}^{2i} j + \sum_{j=1}^{2i} 1 \\ &= \frac{2i(2i+1)}{2} + 2i \\ &= 2i^2 + 3i \end{aligned}$$

Finally the *outer-most for* loop.

```
NESTED-LOOPS()
1  for i ← 1 to n
2  do for j ← 1 to 2i
3       do k = j
4          while (k ≥ 0)
5          do k = k − 1
```

Let $T()$ be running time of the entire algorithm:

$$\begin{aligned} T(n) &= \sum_{i=1}^{n} M(i) \\ &= \sum_{i=1}^{n} (2i^2 + 3i) \\ &= \sum_{i=1}^{n} 2i^2 + \sum_{i=1}^{n} 3i \\ &= 2\frac{2n^3 + 3n^2 + n}{6} + 3\frac{n(n+1)}{2} \\ &= \frac{4n^3 + 15n^2 + 11n}{6} \\ &= \Theta(n^3) \end{aligned}$$

1.11.1 2-dimension Maxima Revisited

Recall the 2-d maxima problem: Let a point p in 2-dimensional space be given by its integer coordinates, $p = (p.x, p.y)$. A point p is said to *dominated* by point q if $p.x \leq q.x$ and $p.y \leq q.y$. Given a set of n

points, $P = \{p_1, p_2, \ldots, p_n\}$ in 2-space a point is said to be *maximal* if it is not dominated by any other point in P. The problem is to output all the maximal points of P. We introduced a brute-force algorithm that ran in $\Theta(n^2)$ time. It operated by comparing *all pairs* of points. Is there an approach that is significantly better?

The problem with the brute-force algorithm is that it uses no intelligence in *pruning* out decisions. For example, once we know that a point p_i is dominated by another point p_j, we do not need to use p_i for eliminating other points. This follows from the fact that dominance relation is *transitive*. If p_j dominates p_i and p_i dominates p_h then p_j also dominates p_h; p_i is not needed.

1.11.2 Plane-sweep Algorithm

The question is whether we can make an significant improvement in the running time? Here is an idea for how we might do it. We will sweep a vertical line across the plane from left to right. As we sweep this line, we will build a structure holding the maximal points lying to the left of the sweep line. When the sweep line reaches the rightmost point of P , then we will have constructed the complete set of maxima. This approach of solving geometric problems by sweeping a line across the plane is called *plane sweep*.

Although we would like to think of this as a continuous process, we need some way to perform the plane sweep in discrete steps. To do this, we will begin by sorting the points in increasing order of their x-coordinates. For simplicity, let us assume that no two points have the same y-coordinate. (This limiting assumption is actually easy to overcome, but it is good to work with the simpler version, and save the messy details for the actual implementation.) Then we will advance the sweep-line from point to point in n discrete steps. As we encounter each new point, we will update the current list of maximal points.

We will sweep a vertical line across the 2-d plane from left to right. As we sweep, we will build a structure holding the maximal points lying to the left of the sweep line. When the sweep line reaches the rightmost point of P, we will have the complete set of maximal points. We will store the existing maximal points in a list The points that p_i dominates will appear at the end of the list because points are sorted by x-coordinate. We will scan the list left to right. Every maximal point with y-coordinate less than p_i will be eliminated from computation. We will add maximal points onto the end of a list and delete from the end of the list. We can thus use a stack to store the maximal points. The point at the top of the stack will have the highest x-coordinate.

Here are a series of figures that illustrate the plane sweep. The figure also show the content of the stack.

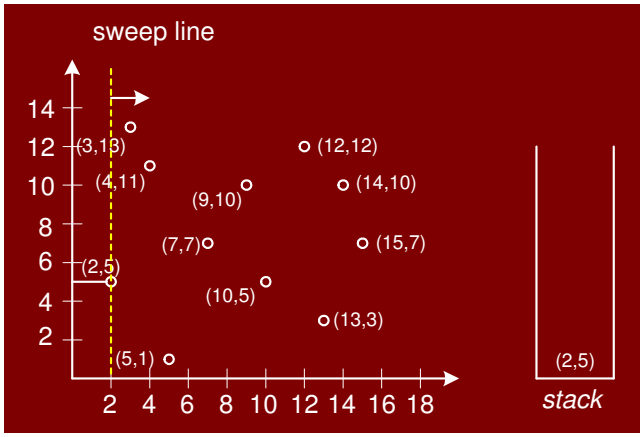


Figure 1.6: Sweep line at $(2, 5)$

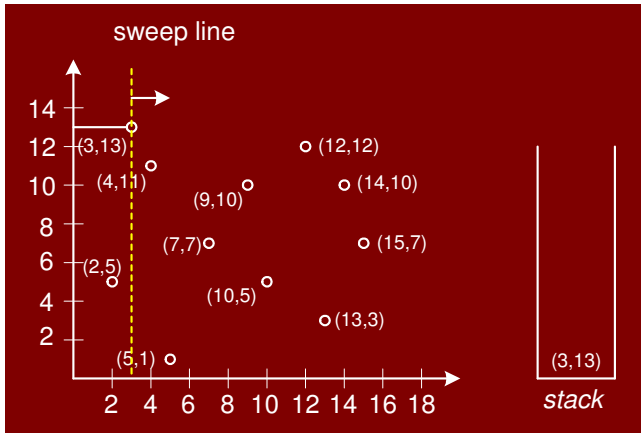


Figure 1.7: Sweep line at $(3, 13)$

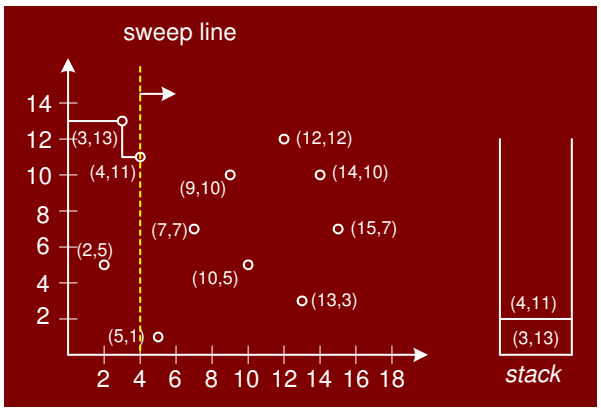


Figure 1.8: Sweep line at $(4, 11)$

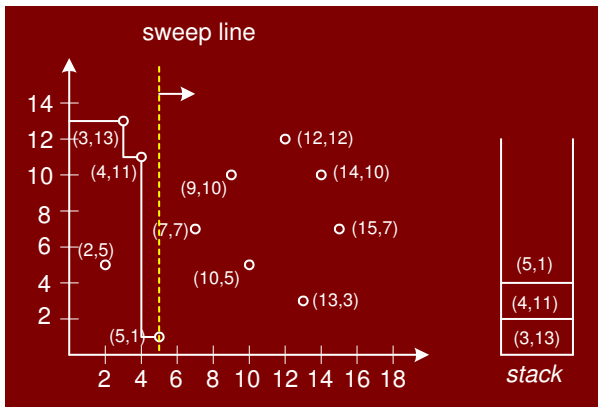


Figure 1.9: Sweep line at $(5, 1)$

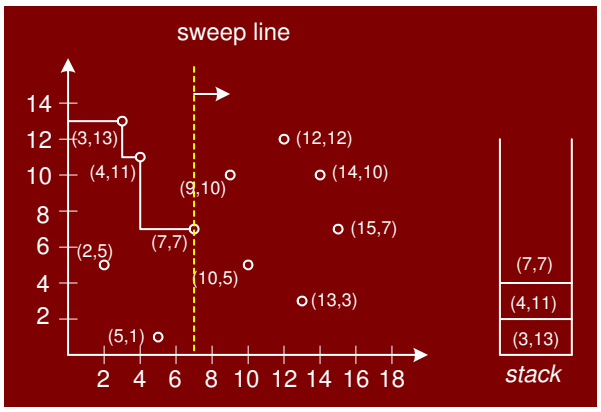


Figure 1.10: Sweep line at $(7, 7)$

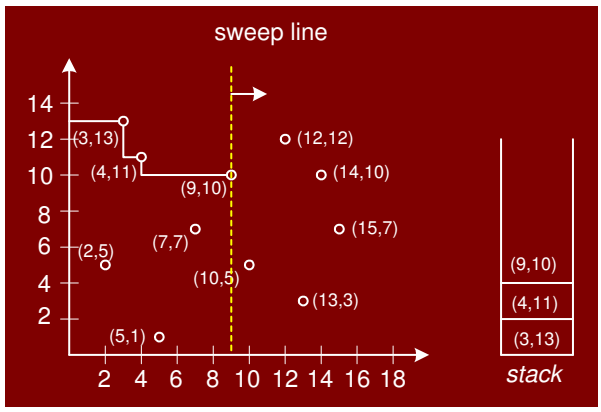


Figure 1.11: Sweep line at $(9, 10)$

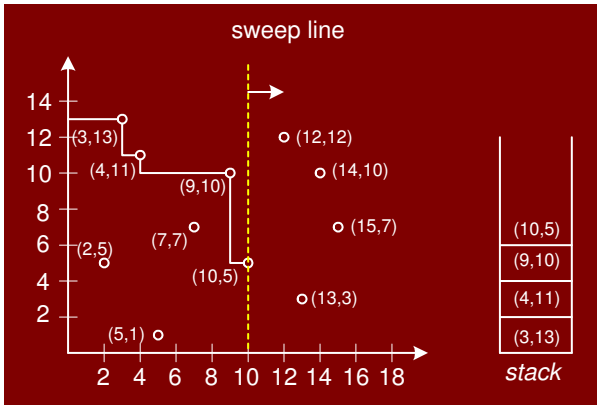


Figure 1.12: Sweep line at $(10, 5)$

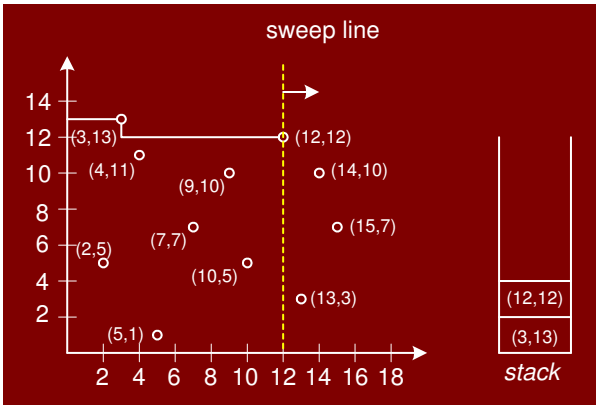


Figure 1.13: Sweep line at $(12, 12)$

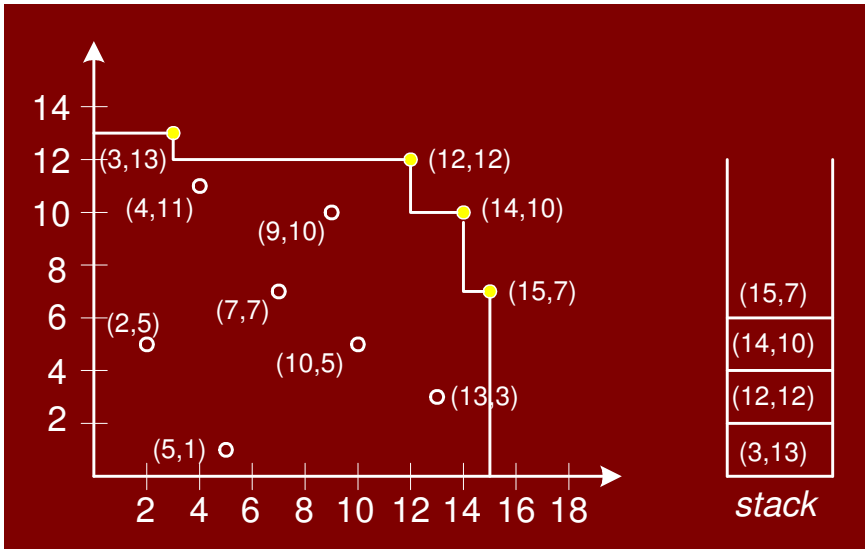


Figure 1.14: The final maximal set

Here is the algorithm.

PLANE-SWEEP-MAXIMA$(n,\ P[1..n])$

```
1  sort P in increasing order by x;
2  stack s;
3  for i ← 1 to n
4  do
5      while (s.notEmpty() & s.top().y ≤ P[i].y)
6      do s.pop();
7      s.push(P[i]);
8  output the contents of stack s;
```

1.11.3 Analysis of Plane-sweep Algorithm

Sorting takes $\Theta(n \log n)$; we will show this later when we discuss sorting. The for loop executes n times. The inner loop (seemingly) could be iterated $(n-1)$ times. It seems we still have an $n(n-1)$ or $\Theta(n^2)$ algorithm. Got fooled by simple minded loop-counting. The while loop will not execute more n times over the entire course of the algorithm. Why is this? Observe that the total number of elements that can be pushed on the stack is n since we execute exactly one push each time during the outer for-loop.

We pop an element off the stack each time we go through the inner while-loop. It is impossible to pop more elements than are ever pushed on the stack. Therefore, the inner while-loop cannot execute more than n times over the entire course of the algorithm. (*Make sure that you understand this*).

The for-loop iterates n times and the inner while-loop also iterates n time for a total of $\Theta(n)$. Combined with the sorting, the runtime of entire plane-sweep algorithm is $\Theta(n \log n)$.

1.11.4 Comparison of Brute-force and Plane sweep algorithms

How much of an improvement is plane-sweep over brute-force? Consider the ratio of running times:

$$\frac{n^2}{n \log n} = \frac{n}{\log n}$$

n	$\log n$	$\frac{n}{\log n}$
100	7	15
1000	10	100
10000	13	752
100000	17	6021
1000000	20	50171

For $n = 1,000,000$, if plane-sweep takes 1 second, the brute-force will take about 14 hours!. From this we get an idea about the importance of asymptotic analysis. It tells us which algorithm is better for large values of n. As we mentioned before, if n is not very large, then almost any algorithm will be fast. But efficient algorithm design is most important for large inputs, and the general rule of computing is that input sizes continue to grow until people can no longer tolerate the running times. Thus, by designing algorithms efficiently, you make it possible for the user to run large inputs in a reasonable amount of time.

Chapter 2

Asymptotic Notation

You may be asking that we continue to use the notation $\Theta()$ but have never defined it. Let's remedy this now. Given any function $g(n)$, we define $\Theta(g(n))$ to be a set of functions that *asymptotically equivalent* to $g(n)$. Formally:

$$\begin{aligned}\Theta(g(n)) = \{f(n) \mid &\text{there exist positive}\\ &\text{constants } c_1, c_2 \text{ and } n_0 \text{ such that}\\ &0 \leq c_1 g(n) \leq f(n) \leq c_2 g(n)\\ &\text{for all } n \geq n_0\}\end{aligned}$$

This is written as "$f(n) \in \Theta(g(n))$" That is, $f(n)$ and $g(n)$ are *asymptotically equivalent.* This means that they have essentially the same growth rates for large n. For example, functions like

- $4n^2$,
- $(8n^2 + 2n - 3)$,
- $(n^2/5 + \sqrt{n} - 10\log n)$
- $n(n-3)$

are all asymptotically equivalent. As n becomes large, the *dominant* (fastest growing) term is some constant times n^2.

Consider the function

$$f(n) = 8n^2 + 2n - 3$$

Our informal rule of keeping the largest term and ignoring the constant suggests that $f(n) \in \Theta(n^2)$. Let's see why this bears out formally. We need to show two things for

$$f(n) = 8n^2 + 2n - 3$$

Lower bound $f(n) = 8n^2 + 2n - 3$ grows asymptotically at least as fast as n^2,

Upper bound $f(n)$ grows no faster asymptotically than n^2,

Lower bound: $f(n)$ grows asymptotically at least as fast as n^2. For this, need to show that there exist positive constants c_1 and n_0, such that $f(n) \geq c_1 n^2$ for all $n \geq n_0$. Consider the reasoning

$$f(n) = 8n^2 + 2n - 3 \geq 8n^2 - 3 = 7n^2 + (n^2 - 3) \geq 7n^2$$

Thus $c_1 = 7$. We implicitly assumed that $2n \geq 0$ and $n^2 - 3 \geq 0$ These are not true for all n but if $n \geq \sqrt{3}$, then both are true. So select $n_0 \geq \sqrt{3}$. We then have $f(n) \geq c_1 n^2$ for all $n \geq n_0$.

Upper bound: $f(n)$ grows asymptotically no faster than n^2. For this, we need to show that there exist positive constants c_2 and n_0, such that $f(n) \leq c_2 n^2$ for all $n \geq n_0$. Consider the reasoning

$$f(n) = 8n^2 + 2n - 3 \leq 8n^2 + 2n \leq 8n^2 + 2n^2 = 10n^2$$

Thus $c_2 = 10$. We implicitly made the assumption that $2n \leq 2n^2$. This is not true for all n but it is true for all $n \geq 1$ So select $n_0 \geq 1$. We thus have $f(n) \leq c_2 n^2$ for all $n \geq n_0$.

From lower bound we have $n_0 \geq \sqrt{3}$ From upper bound we have $n_0 \geq 1$. Combining the two, we let n_0 be the larger of the two: $n_0 \geq \sqrt{3}$. In conclusion, if we let $c_1 = 7$, $c_2 = 10$ and $n_0 \geq \sqrt{3}$, we have

$$7n^2 \leq 8n^2 + 2n - 3 \leq 10n^2 \quad \text{for all } n \geq \sqrt{3}$$

We have thus established

$$0 \leq c_1 g(n) \leq f(n) \leq c_2 g(n) \qquad \text{for all } n \geq n_0$$

Here are plots of the three functions. Notice the bounds.

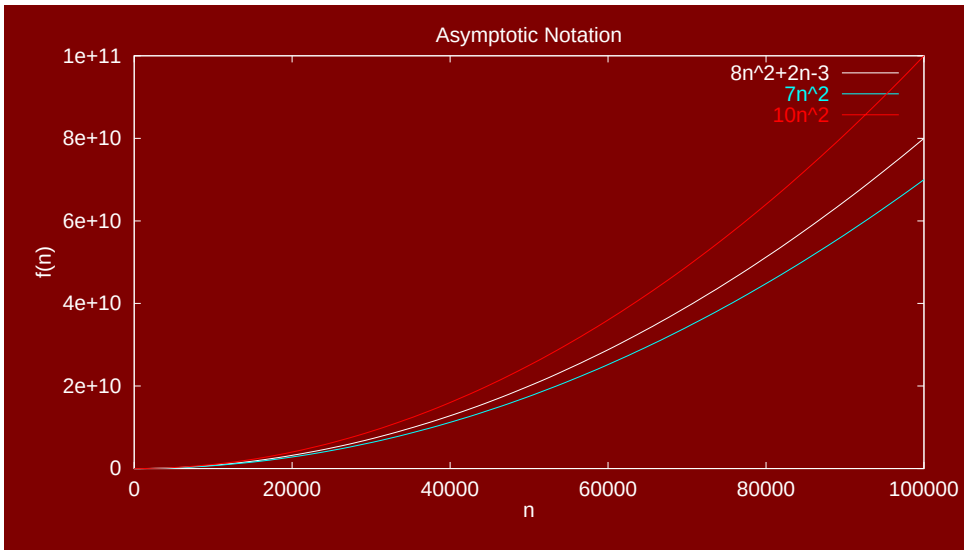


Figure 2.1: Asymptotic Notation Example

We have established that $f(n) \in n^2$. Let's show why $f(n)$ is not in some other asymptotic class. First, let's show that $f(n) \notin \Theta(n)$. Show that $f(n) \notin \Theta(n)$. If this were true, we would have had to satisfy

both the upper and lower bounds. The lower bound is satisfied because $f(n) = 8n^2 + 2n - 3$ does grow at least as fast asymptotically as n. But the upper bound is false. Upper bounds requires that there exist positive constants c_2 and n_0 such that $f(n) \leq c_2 n$ for all $n \geq n_0$.

Informally we know that $f(n) = 8n^2 + 2n - 3$ will eventually exceed $c_2 n$ no matter how large we make c_2. To see this, suppose we assume that constants c_2 and n_0 did exist such that $8n^2 + 2n - 3 \leq c_2 n$ for all $n \geq n_0$ Since this is true for all sufficiently large n then it must be true in the limit as n tends to infinity. If we divide both sides by n, we have

$$\lim_{n \to \infty} \left(8n + 2 - \frac{3}{n} \right) \leq c_2.$$

It is easy to see that in the limit, the left side tends to ∞. So, no matter how large c_2 is, the statement is violated. Thus $f(n) \notin \Theta(n)$.

Let's show that $f(n) \notin \Theta(n^3)$. The idea would be to show that the lower bound $f(n) \geq c_1 n^3$ for all $n \geq n_0$ is violated. (c_1 and n_0 are positive constants). Informally we know this to be true because any cubic function will overtake a quadratic.

If we divide both sides by n^3:

$$\lim_{n \to \infty} \left(\frac{8}{n} + \frac{2}{n^2} - \frac{3}{n^3} \right) \geq c_1$$

The left side tends to 0. The only way to satisfy this is to set $c_1 = 0$. But by hypothesis, c_1 is positive. This means that $f(n) \notin \Theta(n^3)$.

The definition of Θ-notation relies on proving both a lower and upper asymptotic bound. Sometimes we only interested in proving one bound or the other. The O-notation is used to state only the asymptotic *upper* bounds.

$$\begin{aligned} O(g(n)) = \{ f(n) \mid & \text{there exist positive} \\ & \text{constants } c \text{ and } n_0 \text{ such that} \\ & 0 \leq f(n) \leq cg(n) \\ & \text{for all } n \geq n_0 \} \end{aligned}$$

The Ω-notation allows us to state only the asymptotic *lower* bounds.

$$\begin{aligned} \Omega(g(n)) = \{ f(n) \mid & \text{there exist positive} \\ & \text{constants } c \text{ and } n_0 \text{ such that} \\ & 0 \leq cg(n) \leq f(n) \\ & \text{for all } n \geq n_0 \} \end{aligned}$$

The three notations:

$$\begin{aligned}\Theta(g(n)):&\ 0 \le c_1 g(n) \le f(n) \le c_2 g(n)\\ O(g(n)):&\ 0 \le f(n) \le cg(n)\\ \Omega(g(n)):&\ 0 \le cg(n) \le f(n)\\ &\text{for all } n \ge n_0\end{aligned}$$

These definitions suggest an alternative way of showing the asymptotic behavior. We can use limits for define the asymptotic behavior. Limit rule for Θ-notation:

$$\lim_{n\to\infty} \frac{f(n)}{g(n)} = c,$$

for some constant $c > 0$ (strictly positive but not infinity) then $f(n) \in \Theta(g(n))$. Similarly, the limit rule for O-notation is

$$\lim_{n\to\infty} \frac{f(n)}{g(n)} = c,$$

for some constant $c \ge 0$ (nonnegative but not infinite) then $f(n) \in O(g(n))$ and limit rule for Ω-notation:

$$\lim_{n\to\infty} \frac{f(n)}{g(n)} \neq 0,$$

(either a strictly positive constant or infinity) then $f(n) \in \Omega(g(n))$.

Here is a list of common asymptotic running times:

- $\Theta(1)$: Constant time; can't beat it!
- $\Theta(\log n)$: Inserting into a balanced binary tree; time to find an item in a sorted array of length n using binary search.
- $\Theta(n)$: About the fastest that an algorithm can run.
- $\Theta(n \log n)$: Best sorting algorithms.
- $\Theta(n^2)$, $\Theta(n^3)$: Polynomial time. These running times are acceptable when the exponent of n is small or n is not to large, e.g., $n \le 1000$.
- $\Theta(2^n)$, $\Theta(3^n)$: Exponential time. Acceptable only if n is small, e.g., $n \le 50$.
- $\Theta(n!)$, $\Theta(n^n)$: Acceptable only for really small n, e.g. $n \le 20$.

Chapter 3

Divide and Conquer Strategy

The ancient Roman politicians understood an important principle of good algorithm design (although they were probably not thinking about algorithms at the time). You divide your enemies (by getting them to distrust each other) and then conquer them piece by piece. This is called *divide-and-conquer*. In algorithm design, the idea is to take a problem on a large input, break the input into smaller pieces, solve the problem on each of the small pieces, and then combine the piecewise solutions into a global solution. But once you have broken the problem into pieces, how do you solve these pieces? The answer is to apply divide-and-conquer to them, thus further breaking them down. The process ends when you are left with such tiny pieces remaining (e.g. one or two items) that it is trivial to solve them. Summarizing, the main elements to a divide-and-conquer solution are

Divide: the problem into a small number of pieces

Conquer: solve each piece by applying divide and conquer to it *recursively*

Combine: the pieces together into a global solution.

3.1 Merge Sort

Divide and conquer strategy is applicable in a huge number of computational problems. The first example of divide and conquer algorithm we will discuss is a simple and efficient sorting procedure called We are given a sequence of n numbers A, which we will assume are stored in an array $A[1..n]$. The objective is to output a permutation of this sequence sorted in increasing order. This is normally done by permuting the elements within the array A. Here is how the merge sort algorithm works:

- (Divide:) split A down the middle into two subsequences, each of size roughly $n/2$
- (Conquer:) sort each subsequence by calling merge sort recursively on each.
- (Combine:) merge the two sorted subsequences into a single sorted list.

Let's design the algorithm top-down. We'll assume that the procedure that merges two sorted list is available to us. We'll implement it later. Because the algorithm is called recursively on sublists, in addition to passing in the array itself, we will pass in two indices, which indicate the first and last indices of the sub-array that we are to sort. The call $MergeSort(A, p, r)$ will sort the sub-array $A[p : r]$ and return the sorted result in the same sub-array.Here is the overview. If $r = p$, then this means that there is only one element to sort, and we may return immediately. Otherwise if $(p \neq r)$ there are at least two elements, and we will invoke the divide-and-conquer. We find the index q, midway between p and r, namely $q = (p + r)/2$ (rounded down to the nearest integer). Then we split the array into sub-arrays $A[p : q]$ and $A[q + 1 : r]$. Call MergeSort recursively to sort each sub-array. Finally, we invoke a procedure (which we have yet to write) which merges these two sub-arrays into a single sorted array.

Here is the MergeSort algorithm.

```
MERGE-SORT( array A, int p, int r)
1  if (p < r)
2    then
3          q ← (p + r)/2
4          MERGE-SORT(A, p, q)       // sort A[p..q]
5          MERGE-SORT(A, q + 1, r) // sort A[q + 1..r]
6          MERGE(A, p, q, r)    // merge the two pieces
```

The following figure illustrates the dividing (splitting) procedure.

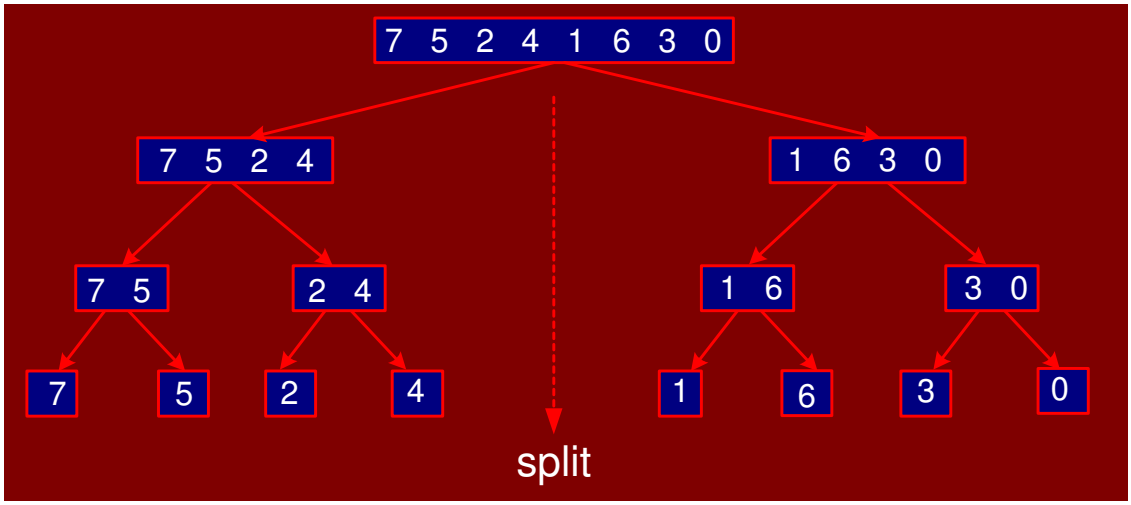


Figure 3.1: Merge sort divide phase

Merging: All that is left is to describe the procedure that merges two sorted lists. $Merge(A, p, q, r)$ assumes that the left sub-array, $A[p : q]$, and the right sub-array, $A[q + 1 : r]$, have already been sorted. We merge these two sub-arrays by copying the elements to a temporary working array called B. For convenience, we will assume that the array B has the same index range A, that is, $B[p : r]$. (One nice thing about pseudo-code, is that we can make these assumptions, and leave them up to the programmer to figure out how to implement it.) We have to indices i and j, that point to the current elements of each

sub-array. We move the smaller element into the next position of B (indicated by index k) and then increment the corresponding index (either i or j). When we run out of elements in one array, then we just copy the rest of the other array into B. Finally, we copy the entire contents of B back into A. (The use of the temporary array is a bit unpleasant, but this is impossible to overcome entirely. It is one of the shortcomings of MergeSort, compared to some of the other efficient sorting algorithms.)

Here is the merge algorithm:

```
MERGE( array A, int p, int q int r)
 1  int B[p..r]; int i ← k ← p; int j ← q + 1
 2  while (i ≤ q) and (j ≤ r)
 3  do if (A[i] ≤ A[j])
 4        then B[k++] ← A[i++]
 5        else B[k++] ← A[j++]
 6  while (i ≤ q)
 7  do B[k++] ← A[i++]
 8  while (j ≤ r)
 9  do B[k++] ← A[j++]
10  for i ← p to r
11  do A[i] ← B[i]
```

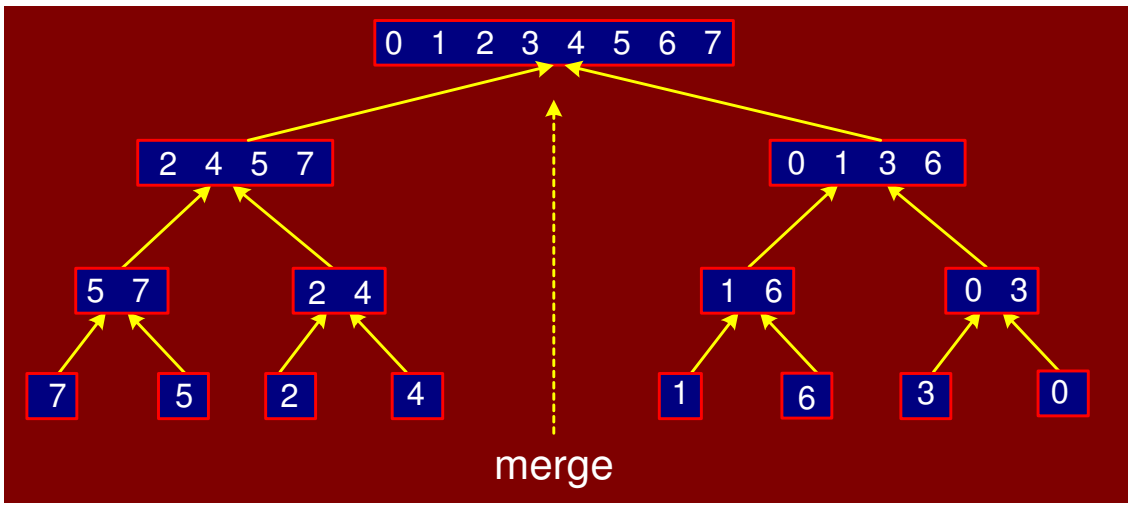


Figure 3.2: Merge sort: combine phase

3.1.1 Analysis of Merge Sort

First let us consider the running time of the procedure $Merge(A, p, q, r)$. Let $n = r - p + 1$ denote the total length of both the left and right sub-arrays. What is the running time of Merge as a function of n? The algorithm contains four loops (none nested in the other). It is easy to see that each loop can be executed at most n times. (If you are a bit more careful you can actually see that all the while-loops

together can only be executed n times in total, because each execution copies one new element to the array B, and B only has space for n elements.) Thus the running time to Merge n items is $\Theta(n)$. Let us write this without the asymptotic notation, simply as n. (We'll see later why we do this.)

Now, how do we describe the running time of the entire MergeSort algorithm? We will do this through the use of a recurrence, that is, a function that is defined recursively in terms of itself. To avoid circularity, the recurrence for a given value of n is defined in terms of values that are strictly smaller than n. Finally, a recurrence has some basis values (e.g. for $n = 1$), which are defined explicitly.

Let $T(n)$ denote the worst case running time of MergeSort on an array of length n. If we call MergeSort with an array containing a single item ($n = 1$) then the running time is constant. We can just write $T(n) = 1$, ignoring all constants. For $n > 1$, MergeSort splits into two halves, sorts the two and then merges them together. The left half is of size $\lceil n/2 \rceil$ and the right half is $\lfloor n/2 \rfloor$. How long does it take to sort elements in sub array of size $\lceil n/2 \rceil$? We do not know this but because $\lceil n/2 \rceil < n$ for $n > 1$, we can express this as $T(\lceil n/2 \rceil)$. Similarly the time taken to sort right sub array is expressed as $T(\lfloor n/2 \rfloor)$. In conclusion we have

$$T(n) = \begin{cases} 1 & \text{if } n = 1, \\ T(\lceil n/2 \rceil) + T(\lfloor n/2 \rfloor) + n & \text{otherwise} \end{cases}$$

This is called *recurrence relation*, i.e., a recursively defined function. Divide-and-conquer is an important design technique, and it naturally gives rise to recursive algorithms. It is thus important to develop mathematical techniques for solving recurrences, either exactly or asymptotically.

Let's expand the terms.

$$\begin{aligned} T(1) &= 1 \\ T(2) &= T(1) + T(1) + 2 = 1 + 1 + 2 = 4 \\ T(3) &= T(2) + T(1) + 3 = 4 + 1 + 3 = 8 \\ T(4) &= T(2) + T(2) + 4 = 4 + 4 + 4 = 12 \\ T(5) &= T(3) + T(2) + 5 = 8 + 4 + 5 = 17 \\ &\cdots \\ T(8) &= T(4) + T(4) + 8 = 12 + 12 + 8 = 32 \\ &\cdots \\ T(16) &= T(8) + T(8) + 16 = 32 + 32 + 16 = 80 \\ &\cdots \\ T(32) &= T(16) + T(16) + 32 = 80 + 80 + 32 = 192 \end{aligned}$$

What is the pattern here? Let's consider the ratios $T(n)/n$ for powers of 2:

$$\begin{aligned} T(1)/1 &= 1 & \qquad T(8)/8 &= 4 \\ T(2)/2 &= 2 & \qquad T(16)/16 &= 5 \\ T(4)/4 &= 3 & \qquad T(32)/32 &= 6 \end{aligned}$$

This suggests $T(n)/n = \log n + 1$ Or, $T(n) = n \log n + n$ which is $\Theta(n \log n)$ (using the limit rule).

3.1.2 The Iteration Method for Solving Recurrence Relations

Floor and ceilings are a pain to deal with. If n is assumed to be a power of 2 ($2^k = n$), this will simplify the recurrence to

$$T(n) = \begin{cases} 1 & \text{if } n = 1, \\ 2T(n/2) + n & \text{otherwise} \end{cases}$$

The iteration method turns the recurrence into a summation. Let's see how it works. Let's expand the recurrence:

$$\begin{aligned} T(n) &= 2T(n/2) + n \\ &= 2(2T(n/4) + n/2) + n \\ &= 4T(n/4) + n + n \\ &= 4(2T(n/8) + n/4) + n + n \\ &= 8T(n/8) + n + n + n \\ &= 8(2T(n/16) + n/8) + n + n + n \\ &= 16T(n/16) + n + n + n + n \end{aligned}$$

If n is a power of 2 then let $n = 2^k$ or $k = \log n$.

$$\begin{aligned} T(n) &= 2^k T(n/(2^k)) + \underbrace{(n + n + n + \cdots + n)}_{k \text{ times}} \\ &= 2^k T(n/(2^k)) + kn \\ &= 2^{(\log n)} T(n/(2^{(\log n)})) + (\log n)n \\ &= 2^{(\log n)} T(n/n) + (\log n)n \\ &= nT(1) + n \log n = n + n \log n \end{aligned}$$

3.1.3 Visualizing Recurrences Using the Recursion Tree

Iteration is a very powerful technique for solving recurrences. But, it is easy to get lost in all the symbolic manipulations and lose sight of what is going on. Here is a nice way to visualize what is going on in iteration. We can describe any recurrence in terms of a tree, where each expansion of the recurrence takes us one level deeper in the tree.

Recall that the recurrence for MergeSort (which we simplified by assuming that n is a power of 2, and hence could drop the floors and ceilings)

$$T(n) = \begin{cases} 1 & \text{if } n = 1, \\ 2T(n/2) + n & \text{otherwise} \end{cases}$$

Suppose that we draw the recursion tree for MergeSort, but each time we merge two lists, we label that node of the tree with the time it takes to perform the associated (nonrecursive) merge. Recall that to

merge two lists of size $m/2$ to a list of size m takes $\Theta(m)$ time, which we will just write as m. Below is an illustration of the resulting recursion tree.

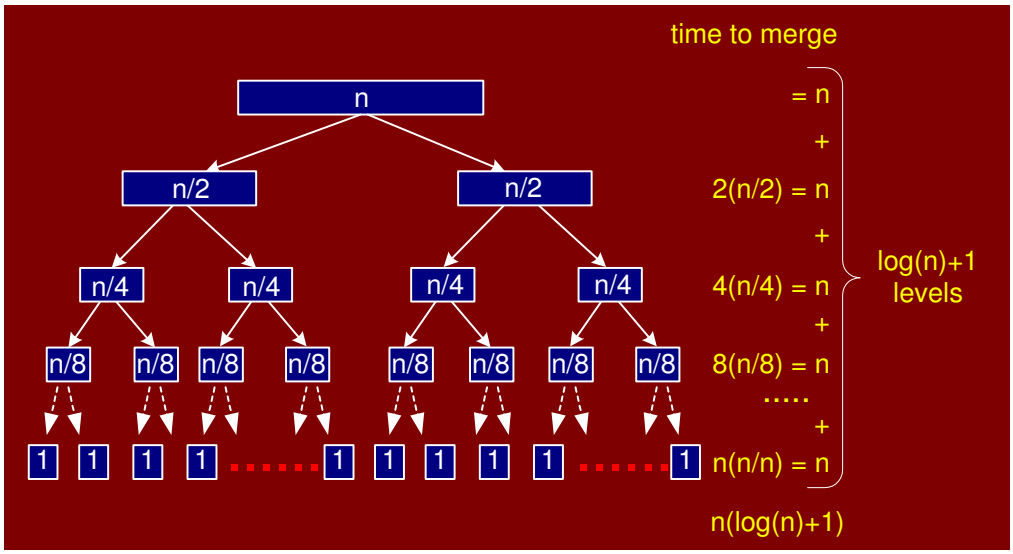


Figure 3.3: Merge sort Recurrence Tree

3.1.4 A Messier Example

The merge sort recurrence was too easy. Let's try a messier recurrence.

$$T(n) = \begin{cases} 1 & \text{if } n = 1, \\ 3T(n/4) + n & \text{otherwise} \end{cases}$$

Assume n to be a power of 4, i.e., $n = 4^k$ and $k = \log_4 n$

$$\begin{aligned} T(n) &= 3T(n/4) + n \\ &= 3(3T(n/16) + (n/4) + n \\ &= 9T(n/16) + 3(n/4) + n \\ &= 27T(n/64) + 9(n/16) + 3(n/4) + n \\ &= \dots \\ &= 3^k T(\frac{n}{4^k}) + 3^{k-1}(n/4^{k-1}) \\ &\qquad + \cdots + 9(n/16) + 3(n/4) + n \\ &= 3^k T(\frac{n}{4^k}) + \sum_{i=0}^{k-1} \frac{3^i}{4^i} n \end{aligned}$$

With $n = 4^k$ and $T(1) = 1$

$$\begin{aligned} T(n) &= 3^k T(\frac{n}{4^k}) + \sum_{i=0}^{k-1} \frac{3^i}{4^i} n \\ &= 3^{\log_4 n} T(1) + \sum_{i=0}^{(\log_4 n)-1} \frac{3^i}{4^i} n \\ &= n^{\log_4 3} + \sum_{i=0}^{(\log_4 n)-1} \frac{3^i}{4^i} n \end{aligned}$$

We used the formula $a^{\log_b n} = n^{\log_b a}$. n remains constant throughout the sum and $3^i/4^i = (3/4)^i$; we thus have

$$T(n) = n^{\log_4 3} + n \sum_{i=0}^{(\log_4 n)-1} \left(\frac{3}{4}\right)^i$$

The sum is a geometric series; recall that for $x \neq 1$

$$\sum_{i=0}^{m} x_i = \frac{x^{m+1} - 1}{x - 1}$$

In this case $x = 3/4$ and $m = \log_4 n - 1$. We get

$$T(n) = n^{\log_4 3} + n \frac{(3/4)^{\log_4 n + 1} - 1}{(3/4) - 1}$$

Applying the log identity once more

$$\begin{aligned} (3/4)^{\log_4 n} &= n^{\log_4 (3/4)} = n^{\log_4 3 - \log_4 4} \\ &= n^{\log_4 3 - 1} = \frac{n^{\log_4 3}}{n} \end{aligned}$$

If we plug this back, we get

$$\begin{aligned} T(n) &= n^{\log_4 3} + n \frac{\frac{n^{\log_4 3}}{n} - 1}{(3/4) - 1} \\ &= n^{\log_4 3} + \frac{n^{\log_4 3} - n}{-1/4} \\ &= n^{\log_4 3} + 4(n - n^{\log_4 3}) \\ &= 4n - 3n^{\log_4 3} \end{aligned}$$

With $\log_4 3 \approx 0.79$, we finally have the result!

$$T(n) = 4n - 3n^{\log_4 3} \approx 4n - 3n^{0.79} \in \Theta(n)$$

3.2 Selection Problem

Suppose we are given a set of n numbers. Define the *rank* of an element to be one plus the number of elements that are smaller. Thus, the rank of an element is its final position if the set is sorted. The minimum is of rank 1 and the maximum is of rank n.

Consider the set: $\{5, 7, 2, 10, 8, 15, 21, 37, 41\}$. The rank of each number is its position in the sorted order.

position	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*
Number	2	5	7	8	10	15	21	37	41

For example, rank of 8 is 4, one plus the number of elements smaller than 8 which is 3.

The selection problem is stated as follows:

> Given a set A of n distinct numbers and an integer k, $1 \leq k \leq n$, output the element of A of rank k.

Of particular interest in statistics is the *median*. If n is odd then the median is defined to be element of rank $(n+1)/2$. When n is even, there are two choices: $n/2$ and $(n+1)/2$. In statistics, it is common to return the average of the two elements.

Medians are useful as a measures of the *central tendency* of a set especially when the distribution of values is highly skewed. For example, the median income in a community is a more meaningful measure than average. Suppose 7 households have monthly incomes 5000, 7000, 2000, 10000, 8000, 15000 and 16000. In sorted order, the incomes are 2000, 5000, 7000, 8000, 10000, 15000, 16000. The median income is 8000; median is element with rank 4: $(7+1)/2 = 4$. The average income is 9000. Suppose the income 16000 goes up to 450,000. The median is still 8000 but the average goes up to 71,000. Clearly, the average is not a good representative of the majority income levels.

The selection problem can be easily solved by simply sorting the numbers of A and returning $A[k]$. Sorting, however, requires $\Theta(n \log n)$ time. The question is: can we do better than that? In particular, is it possible to solve the selections problem in $\Theta(n)$ time? The answer is yes. However, the solution is far from obvious.

3.2.1 Sieve Technique

The reason for introducing this algorithm is that it illustrates a very important special case of divide-and-conquer, which I call the *sieve technique*. We think of divide-and-conquer as breaking the problem into a small number of smaller subproblems, which are then solved recursively. The sieve technique is a special case, where the number of subproblems is just 1.

The sieve technique works in phases as follows. It applies to problems where we are interested in finding a single item from a larger set of n items. We do not know which item is of interest, however after doing some amount of analysis of the data, taking say $\Theta(nk)$ time, for some constant k, we find that we do not

know what the desired the item is, but we can identify a large enough number of elements that cannot be the desired value, and can be eliminated from further consideration. In particular "large enough" means that the number of items is at least some fixed constant fraction of n (e.g. $n/2$, $n/3$). Then we solve the problem recursively on whatever items remain. Each of the resulting recursive solutions then do the same thing, eliminating a constant fraction of the remaining set.

3.2.2 Applying the Sieve to Selection

To see more concretely how the sieve technique works, let us apply it to the selection problem. We will begin with the given array $A[1..n]$. We will pick an item from the array, called the *pivot element* which we will denote by x. We will talk about how an item is chosen as the pivot later; for now just think of it as a random element of A.

We then *partition* A into three parts.

1. $A[q]$ contains the pivot element x,
2. $A[1..q-1]$ will contain all the elements that are less than x and
3. $A[q+1..n]$ will contains all elements that are greater than x.

Within each sub array, the items may appear in any order. The following figure shows a partitioned array:

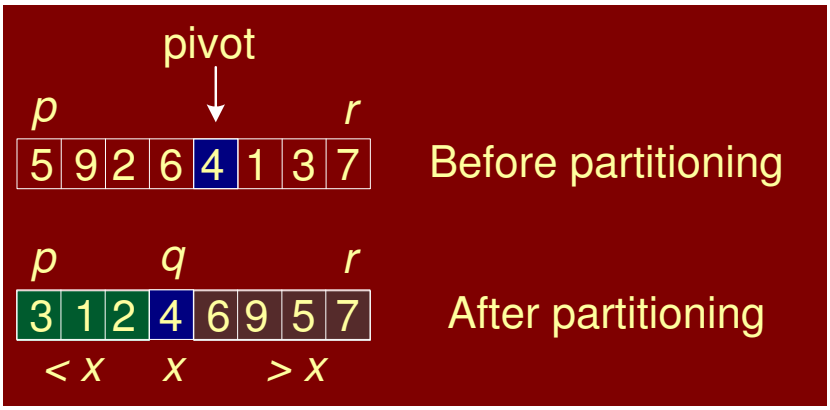


Figure 3.4: $A[p..r]$ partitioned about the pivot x

3.2.3 Selection Algorithm

It is easy to see that the rank of the The rank of the pivot x is $q-p+1$ in $A[p..r]$. Let $rank_x = q-p+1$. If $k = rank_x$ then the pivot is k^{th} smallest. If $k < rank_x$ then search $A[p..q-1]$ recursively. If $k > rank_x$ then search $A[q+1..r]$ recursively. Find element of rank $(k-q)$ because we eliminated q smaller elements in A.

SELECT(array A, int p, int r, int k)

```
1  if (p = r)
2    then return A[p]
3    else  x ← CHOOSE_PIVOT(A, p, r)
4          q ← PARTITION(A, p, r, x)
5          rank_x ← q − p + 1
6          if k = rank_x
7            then return x
8          if k < rank_x
9            then return SELECT(A, p, q − 1, k)
10           else  return SELECT(A, q + 1, r, k − q)
```

Example: select the 6^{th} smallest element of the set $\{5, 9, 2, 6, 4, 1, 3, 7\}$

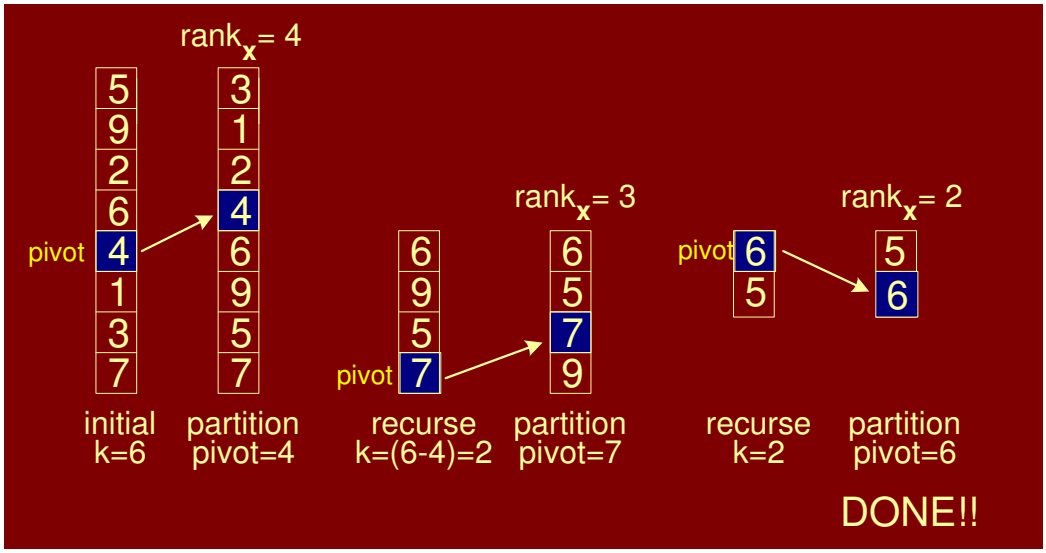


Figure 3.5: Sieve example: select 6^{th} smallest element

3.2.4 Analysis of Selection

We will discuss how to choose a pivot and the partitioning later. For the moment, we will assume that they both take $\Theta(n)$ time. How many elements do we eliminate in each time? If x is the largest or the smallest then we may only succeed in eliminating one element.

$$5, 9, 2, 6, 4, \boxed{1}, 3, 7 \qquad \text{pivot is } 1$$
$$\boxed{1}, 5, 9, 2, 6, 4, 3, 7 \qquad \text{after partition}$$

Ideally, x should have a rank that is neither too large or too small.

Suppose we are able to choose a pivot that causes exactly half of the array to be eliminated in each phase.

This means that we recurse on the remaining $n/2$ elements. This leads to the following recurrence:

$$T(n) = \begin{cases} 1 & \text{if } n = 1, \\ T(n/2) + n & \text{otherwise} \end{cases}$$

If we expand this recurrence, we get

$$\begin{aligned} T(n) &= n + \frac{n}{2} + \frac{n}{4} + \dots \\ &\leq \sum_{i=0}^{\infty} \frac{n}{2^i} \\ &= n \sum_{i=0}^{\infty} \frac{1}{2^i} \end{aligned}$$

Recall the formula for infinite geometric series; for any $|c| < 1$,

$$\sum_{i=0}^{\infty} c^i = \frac{1}{1-c}$$

Using this we have

$$T(n) \leq 2n \in \Theta(n)$$

Let's think about how we ended up with a $\Theta(n)$ algorithm for selection. Normally, a $\Theta(n)$ time algorithm would make a single or perhaps a constant number of passes of the data set. In this algorithm, we make a number of passes. In fact it could be as many as $\log n$.

However, because we eliminate a constant fraction of the array with each phase, we get the convergent geometric series in the analysis. This shows that the total running time is indeed *linear* in n. This lesson is well worth remembering. It is often possible to achieve linear running times in ways that you would not expect.

Chapter 4

Sorting

For the next series of lectures, we will focus on sorting. There a number of reasons for sorting. Here are a few important ones. Procedures for sorting are parts of many large software systems. Design of efficient sorting algorithms is necessary to achieve overall efficiency of these systems.

Sorting is well studied problem from the analysis point of view. Sorting is one of the few problems where provable lower bounds exist on how fast we can sort. In sorting, we are given an array $A[1..n]$ of n numbers We are to reorder these elements into increasing (or decreasing) order.

More generally, A is an array of objects and we sort them based on one of the attributes - the *key value*. The key value need not be a number. It can be any object from a *totally ordered domain*. Totally ordered domain means that for any two elements of the domain, x and y, either $x < y$, $x = y$ or $x > y$.

4.1 Slow Sorting Algorithms

There are a number of well-known slow $O(n^2)$ sorting algorithms. These include the following:

Bubble sort: Scan the array. Whenever two consecutive items are found that are out of order, swap them. Repeat until all consecutive items are in order.

Insertion sort: Assume that $A[1..i-1]$ have already been sorted. Insert $A[i]$ into its proper position in this sub array. Create this position by shifting all larger elements to the right.

Selection sort: Assume that $A[1..i-1]$ contain the $i-1$ smallest elements in sorted order. Find the smallest element in $A[i..n]$ Swap it with $A[i]$.

These algorithms are easy to implement. But they run in $\Theta(n^2)$ time in the worst case.

4.2 Sorting in $O(n \log n)$ time

We have already seen that Mergesort sorts an array of numbers in $\Theta(n \log n)$ time. We will study two others: *Heapsort* and *Quicksort*.

4.2.1 Heaps

A *heap* is a left-complete binary tree that conforms to the *heap order*. The heap order property: in a (min) heap, for every node X, the key in the parent is smaller than or equal to the key in X. In other words, the parent node has key smaller than or equal to both of its children nodes. Similarly, in a max heap, the parent has a key larger than or equal both of its children Thus the smallest key is in the root in a min heap; in the max heap, the largest is in the root.

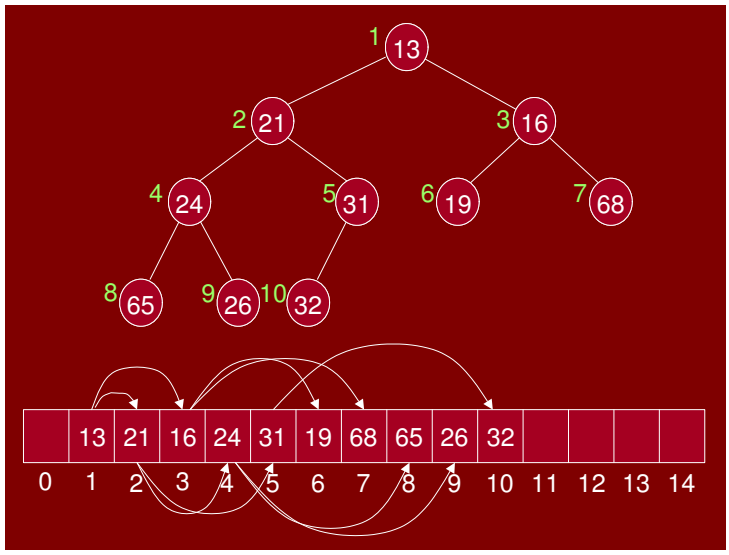


Figure 4.1: A min-heap

The number of nodes in a complete binary tree of height h is

$$n = 2^0 + 2^1 + 2^2 + \cdots + 2^h = \sum_{i=0}^{h} 2^i = 2^{h+1} - 1$$

h in terms of n is

$$h = (\log(n+1)) - 1 \approx \log n \in \Theta(\log n)$$

One of the clever aspects of heaps is that they can be stored in arrays without using any pointers. This is due to the left-complete nature of the binary tree. We store the tree nodes in level-order traversal. Access

to nodes involves simple arithmetic operations:

$$\begin{aligned} \texttt{left}(i) &: \text{returns } 2i, \text{ index of left child of node } i. \\ \texttt{right}(i) &: \text{returns } 2i+1, \text{ the right child.} \\ \texttt{parent}(i) &: \text{returns } \lfloor i/2 \rfloor, \text{ the parent of } i. \end{aligned}$$

The root is at position 1 of the array.

4.2.2 Heapsort Algorithm

We build a max heap out of the given array of numbers $A[1..n]$. We repeatedly extract the the maximum item from the heap. Once the max item is removed, we are left with a hole at the root. To fix this, we will replace it with the last leaf in tree. But now the heap order will very likely be destroyed. We will apply a heapify procedure to the root to restore the heap. Figure 4.2 shows an array being sorted.

```
HEAPSORT( array A, int n)
1  BUILD-HEAP(A, n)
2  m ← n
3  while (m ≥ 2)
4  do SWAP(A[1], A[m])
5     m ← m − 1
6     HEAPIFY(A, 1, m)
```

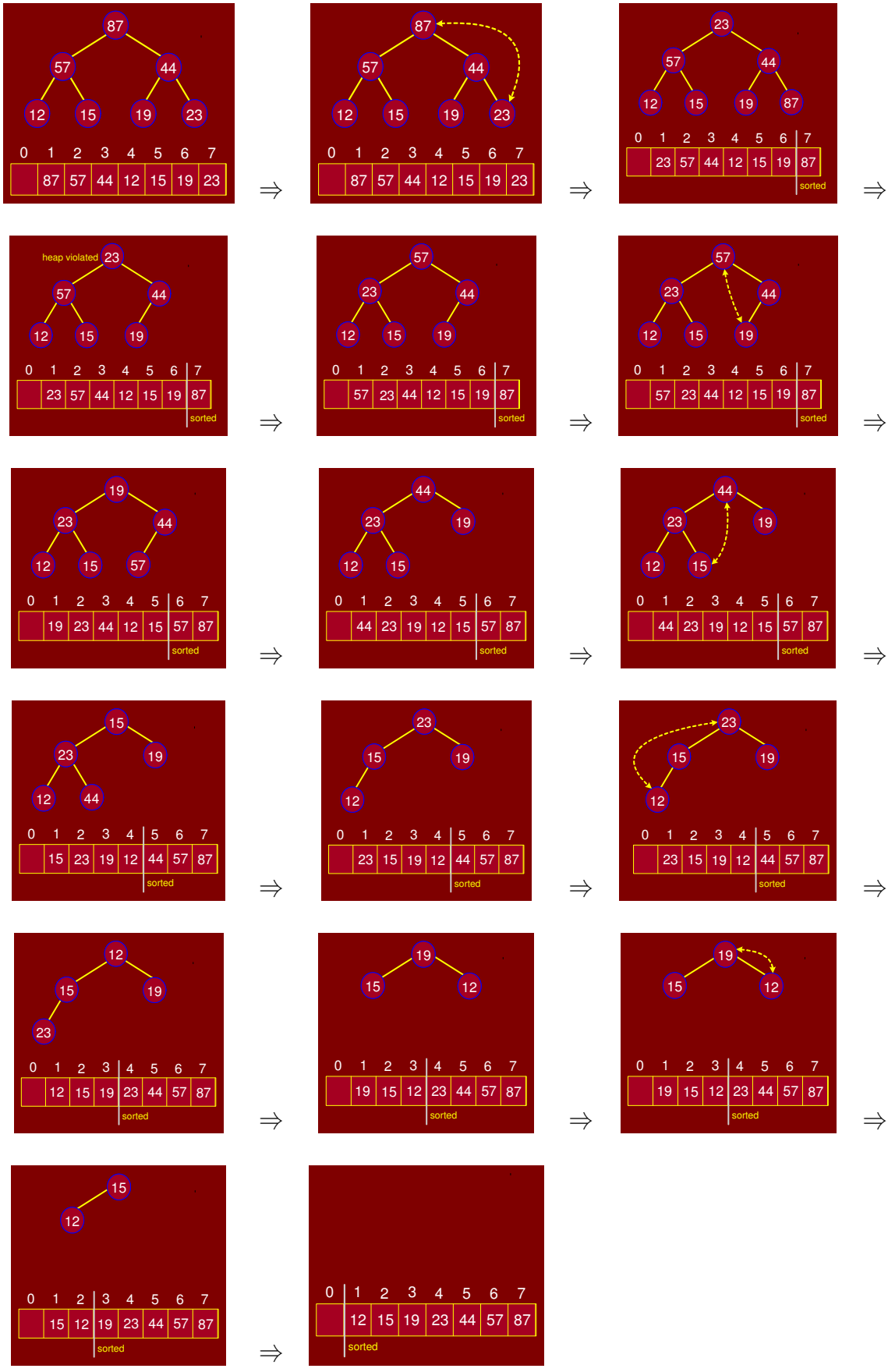


Figure 4.2: Example of heap sort

4.2.3 Heapify Procedure

There is one principal operation for maintaining the heap property. It is called Heapify. (In other books it is sometimes called sifting down.) The idea is that we are given an element of the heap which we suspect may not be in valid heap order, but we assume that all of other the elements in the subtree rooted at this element are in heap order. In particular this root element may be too small. To fix this we "sift" it down the tree by swapping it with one of its children. Which child? We should take the larger of the two children to satisfy the heap ordering property. This continues recursively until the element is either larger than both its children or until its falls all the way to the leaf level. Here is the algorithm. It is given the heap in the array A, and the index i of the suspected element, and m the current active size of the heap. The element $A[max]$ is set to the maximum of $A[i]$ and it two children. If $max \neq i$ then we swap $A[i]$ and $A[max]$ and then recurse on $A[max]$.

```
HEAPIFY( array A, int i, int m)
 1  l ← LEFT(i)
 2  r ← RIGHT(i)
 3  max ← i
 4  if (l ≤ m)and(A[l] > A[max])
 5     then max ← l
 6  if (r ≤ m)and(A[r] > A[max])
 7     then max ← r
 8  if (max ≠ i)
 9     then SWAP(A[i], A[max])
10          HEAPIFY(A, max, m)
```

4.2.4 Analysis of Heapify

We call heapify on the root of the tree. The maximum levels an element could move up is $\Theta(\log n)$ levels. At each level, we do simple comparison which $O(1)$. The total time for heapify is thus $O(\log n)$. Notice that it is not $\Theta(\log n)$ since, for example, if we call heapify on a leaf, it will terminate in $\Theta(1)$ time.

4.2.5 BuildHeap

We can use Heapify to build a heap as follows. First we start with a heap in which the elements are not in heap order. They are just in the same order that they were given to us in the array A. We build the heap by starting at the leaf level and then invoke Heapify on each node. (Note: We cannot start at the top of the tree. Why not? Because the precondition which Heapify assumes is that the entire tree rooted at node i is already in heap order, except for i.) Actually, we can be a bit more efficient. Since we know that each leaf is already in heap order, we may as well skip the leaves and start with the first non-leaf node. This will be in position $n/2$. (Can you see why?)

Here is the code. Since we will work with the entire array, the parameter m for Heapify, which indicates the current heap size will be equal to n, the size of array A, in all the calls.

```
BUILDHEAP( array A, int n)
1  for i ← n/2 downto 1
2  do
3      HEAPIFY(A, i, n)
```

4.2.6 Analysis of BuildHeap

For convenience, we will assume $n = 2^{h+1} - 1$ where h is the height of tree. The heap is a left-complete binary tree. Thus at each level j, $j < h$, there are 2^j nodes in the tree. At level h, there will be 2^h or less nodes. How much work does buildHeap carry out? Consider the heap in Figure 4.3:

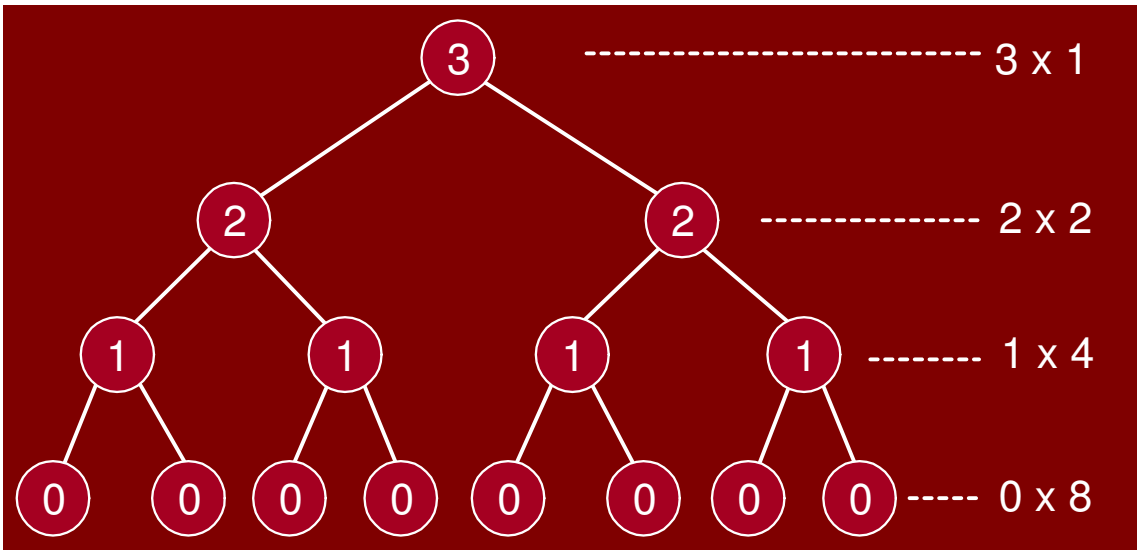


Figure 4.3: Total work performed in buildheap

At the bottom most level, there are 2^h nodes but we do not heapify these. At the next level up, there are 2^{h-1} nodes and each might shift down 1. In general, at level j, there are 2^{h-j} nodes and each may shift down j levels.

So, if count from bottom to top, level-by-level, the total time is

$$T(n) = \sum_{j=0}^{h} j2^{h-j} = \sum_{j=0}^{h} j\frac{2^h}{2^j}$$

We can factor out the 2^h term:

$$T(n) = 2^h \sum_{j=0}^{h} \frac{j}{2^j}$$

How do we solve this sum? Recall the geometric series, for any constant $x < 1$

$$\sum_{j=0}^{\infty} x^j = \frac{1}{1-x}$$

Take the derivative with respect to x and multiply by x

$$\sum_{j=0}^{\infty} jx^{j-1} = \frac{1}{(1-x)^2} \qquad \sum_{j=0}^{\infty} jx^j = \frac{x}{(1-x)^2}$$

We plug $x = 1/2$ and we have the desired formula:

$$\sum_{j=0}^{\infty} \frac{j}{2^j} = \frac{1/2}{(1-(1/2))^2} = \frac{1/2}{1/4} = 2$$

In our case, we have a bounded sum, but since we the infinite series is bounded, we can use it as an easy approximation:

$$\begin{aligned} T(n) &= 2^h \sum_{j=0}^{h} \frac{j}{2^j} \\ &\leq 2^h \sum_{j=0}^{\infty} \frac{j}{2^j} \\ &\leq 2^h \cdot 2 = 2^{h+1} \end{aligned}$$

Recall that $n = 2^{h+1} - 1$. Therefore

$$T(n) \leq n + 1 \in O(n)$$

The algorithm takes at least $\Omega(n)$ time since it must access every element at once. So the total time for BuildHeap is $\Theta(n)$.

BuildHeap is a relatively complex algorithm. Yet, the analysis yield that it takes $\Theta(n)$ time. An intuitive way to describe why it is so is to observe an important fact about binary trees The fact is that the vast majority of the nodes are at the lowest level of the tree. For example, in a complete binary tree of height h, there is a total of $n \approx 2^{h+1}$ nodes.

The number of nodes at the bottom three levels alone is

$$2^h + 2^{h-1} + 2^{h-2} = \frac{n}{2} + \frac{n}{4} + \frac{n}{8} = \frac{7n}{8} = 0.875n$$

Almost 90% of the nodes of a complete binary tree reside in the 3 lowest levels. Thus, algorithms that operate on trees should be efficient (as BuildHeap is) on the bottom-most levels since that is where most of the weight of the tree resides.

4.2.7 Analysis of Heapsort

Heapsort calls BuildHeap once. This takes $\Theta(n)$. Heapsort then extracts roughly n maximum elements from the heap. Each extract requires a constant amount of work (swap) and $O(\log n)$ heapify. Heapsort is thus $O(n \log n)$.

Is HeapSort $\Theta(n \log n)$? The answer is yes. In fact, later we will show that comparison based sorting algorithms can not run faster than $\Omega(n \log n)$. Heapsort is such an algorithm and so is Mergesort that we saw ealier.

4.3 Quicksort

Our next sorting algorithm is Quicksort. It is one of the fastest sorting algorithms known and is the method of choice in most sorting libraries. Quicksort is based on the divide and conquer strategy. Here is the algorithm:

```
QUICKSORT( array A, int p, int r)
1  if (r > p)
2    then
3         i ← a random index from [p..r]
4          swap A[i] with A[p]
5         q ← PARTITION(A, p, r)
6         QUICKSORT(A, p, q − 1)
7         QUICKSORT(A, q + 1, r)
```

4.3.1 Partition Algorithm

Recall that the partition algorithm partitions the array $A[p..r]$ into three sub arrays about a pivot element x. $A[p..q-1]$ whose elements are less than or equal to x, $A[q] = x$, $A[q+1..r]$ whose elements are greater than x

We will choose the first element of the array as the pivot, i.e. $x = A[p]$. If a different rule is used for selecting the pivot, we can swap the chosen element with the first element. We will choose the pivot randomly.

The algorithm works by maintaining the following *invariant condition*. $A[p] = x$ is the pivot value. $A[p..q-1]$ contains elements that are less than x, $A[q+1..s-1]$ contains elements that are greater than

or equal to x $A[s..r]$ contains elements whose values are currently unknown.

```
PARTITION( array A, int p, int r)
1  x ← A[p]
2  q ← p
3  for s ← p + 1 to r
4  do if (A[s] < x)
5        then q ← q + 1
6             swap A[q] with A[s]
7   swap A[p] with A[q]
8  return q
```

Figure 4.4 shows the execution trace partition algorithm.

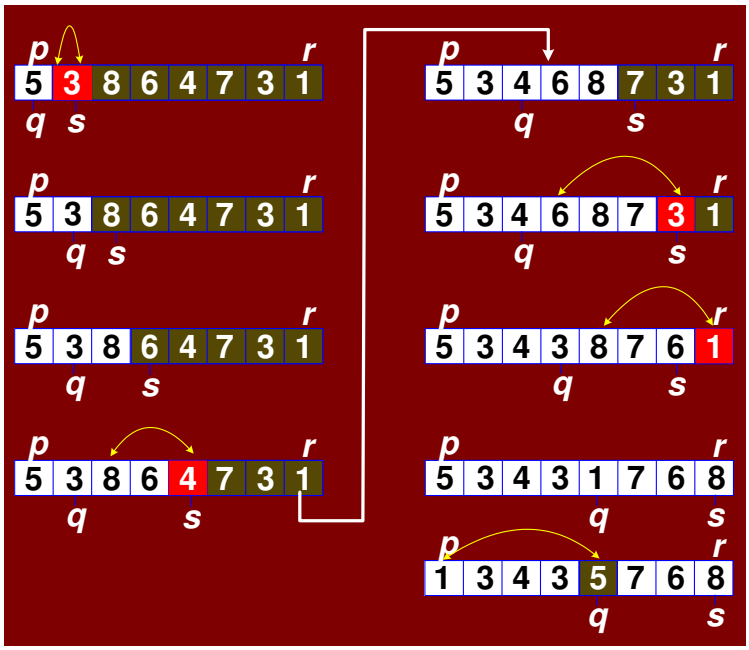


Figure 4.4: Trace of partitioning algorithm

4.3.2 Quick Sort Example

The Figure 4.5 trace out the quick sort algorithm. The first partition is done using the last element, 10, of the array. The left portion are then partitioned about 5 while the right portion is partitioned about 13. Notice that 10 is now at its final position in the eventual sorted order. The process repeats as the algorithm recursively partitions the array eventually sorting it.

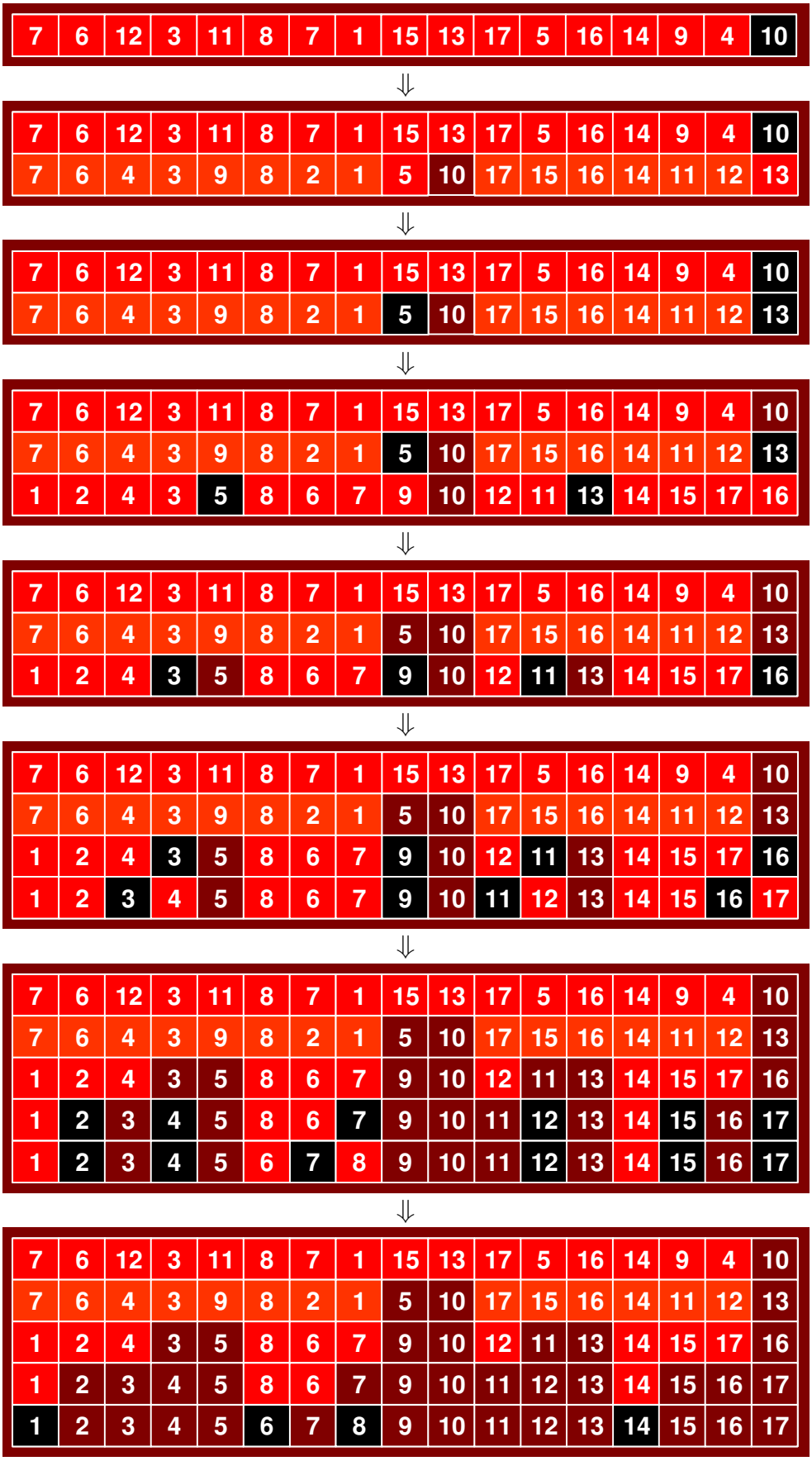


Figure 4.5: Example of quick sort

It is interesting to note (but not surprising) that the pivots form a binary search tree. This is illustrated in Figure 4.6.

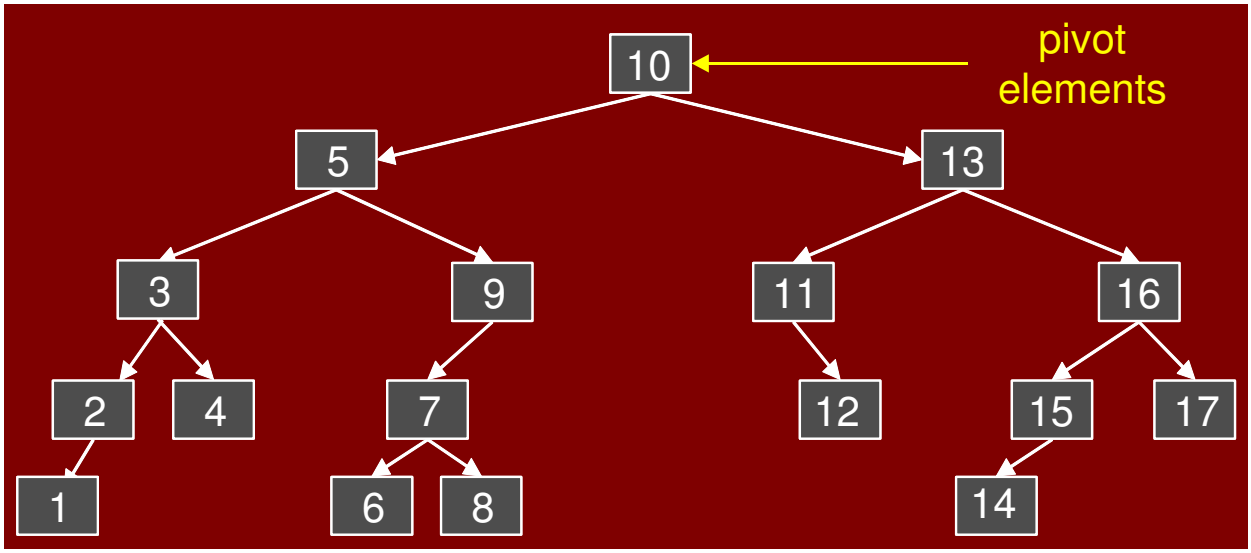


Figure 4.6: Quicksort BST

4.3.3 Analysis of Quicksort

The running time of quicksort depends heavily on the selection of the pivot. If the rank of the pivot is very large or very small then the partition (BST) will be unbalanced. Since the pivot is chosen randomly in our algorithm, the expected running time is $O(n \log n)$. The worst case time, however, is $O(n^2)$. Luckily, this happens rarely.

4.3.4 Worst Case Analysis of Quick Sort

Let's begin by considering the worst-case performance, because it is easier than the average case. Since this is a recursive program, it is natural to use a recurrence to describe its running time. But unlike MergeSort, where we had control over the sizes of the recursive calls, here we do not. It depends on how the pivot is chosen. Suppose that we are sorting an array of size n, $A[1 : n]$, and further suppose that the pivot that we select is of rank q, for some q in the range 1 to n. It takes $\Theta(n)$ time to do the partitioning and other overhead, and we make two recursive calls. The first is to the subarray $A[1 : q - 1]$ which has $q - 1$ elements, and the other is to the subarray $A[q + 1 : n]$ which has $n - q$ elements. So if we ignore the $\Theta(n)$ (as usual) we get the recurrence:

$$T(n) = T(q - 1) + T(n - q) + n$$

This depends on the value of q. To get the worst case, we maximize over all possible values of q. Putting is together, we get the recurrence

$$T(n) = \begin{cases} 1 & \text{if } n \leq 1 \\ \max\limits_{1 \leq q \leq n} (T(q-1) + T(n-q) + n) & \text{otherwise} \end{cases}$$

Recurrences that have max's and min's embedded in them are very messy to solve. The key is determining which value of q gives the maximum. (A rule of thumb of algorithm analysis is that the worst cases tends to happen either at the extremes or in the middle. So I would plug in the value $q = 1$, $q = n$, and $q = n/2$ and work each out.) In this case, the worst case happens at either of the extremes. If we expand the recurrence for $q = 1$, we get:

$$\begin{aligned} T(n) &\leq T(0) + T(n-1) + n \\ &= 1 + T(n-1) + n \\ &= T(n-1) + (n+1) \\ &= T(n-2) + n + (n+1) \\ &= T(n-3) + (n-1) + n + (n+1) \\ &= T(n-4) + (n-2) + (n-1) + n + (n+1) \\ &= T(n-k) + \sum_{i=-1}^{k-2} (n-i) \end{aligned}$$

For the basis $T(1) = 1$ we set $k = n - 1$ and get

$$\begin{aligned} T(n) &\leq T(1) + \sum_{i=-1}^{n-3} (n-i) \\ &= 1 + (3 + 4 + 5 + \cdots + (n-1) + n + (n+1)) \\ &\leq \sum_{i=1}^{n+1} i = \frac{(n+1)(n+2)}{2} \in \Theta(n^2) \end{aligned}$$

4.3.5 Average-case Analysis of Quicksort

We will now show that in the average case, quicksort runs in $\Theta(n \log n)$ time. Recall that when we talked about average case at the beginning of the semester, we said that it depends on some assumption about the distribution of inputs. However, in the case of quicksort, the analysis does not depend on the distribution of input at all. It only depends upon the random choices of pivots that the algorithm makes. This is good, because it means that the analysis of the algorithm's performance is the same for all inputs. In this case the average is computed over all possible random choices that the algorithm might make for the choice of the pivot index in the second step of the QuickSort procedure above.

To analyze the average running time, we let $T(n)$ denote the average running time of QuickSort on a list of size n. It will simplify the analysis to assume that all of the elements are distinct. The algorithm has n

random choices for the pivot element, and each choice has an equal probability of $1/n$ of occuring. So we can modify the above recurrence to compute an average rather than a max, giving:

$$T(n) = \begin{cases} 1 & \text{if } n \leq 1 \\ \frac{1}{n}\sum_{q=1}^{n}(T(q-1)+T(n-q)+n) & \text{otherwise} \end{cases}$$

The time $T(n)$ is the weighted sum of the times taken for various choices of q. I.e.,

$$\begin{aligned} T(n) = \Big[&\frac{1}{n}(T(0)+T(n-1)+n) \\ &+\frac{1}{n}(T(1)+T(n-2)+n) \\ &+\frac{1}{n}(T(2)+T(n-3)+n) \\ &+\cdots+\frac{1}{n}(T(n-1)+T(0)+n)\Big] \end{aligned}$$

We have not seen such a recurrence before. To solve it, expansion is possible but it is rather tricky. We will attempt a constructive induction to solve it. We know that we want a $\Theta(n \log n)$. Let us assume that $T(n) \leq cn \log n$ for $n \geq 2$ where c is a constant.

For the base case $n = 2$ we have

$$\begin{aligned} T(n) &= \frac{1}{2}\sum_{q=1}^{2}(T(q-1)+T(2-q)+2) \\ &= \frac{1}{2}\Big[(T(0)+T(1)+2)+(T(1)+T(0)+2)\Big] \\ &= \frac{8}{2} = 4. \end{aligned}$$

We want this to be at most $c2\log 2$, i.e.,

$$T(2) \leq c2\log 2$$

or

$$4 \leq c2\log 2$$

therefore

$$c \geq 4/(2\log 2) \approx 2.88.$$

For the induction step, we assume that $n \geq 3$ and The induction hypothesis is that for any $n' < n$, we have $T(n') \leq cn' \log n'$. We want to prove that it is true for $T(n)$. By expanding $T(n)$ and moving the

factor of n outside the sum we have

$$\begin{aligned} T(n) &= \frac{1}{n}\sum_{q=1}^{n}(T(q-1)+T(n-q)+n) \\ &= \frac{1}{n}\sum_{q=1}^{n}(T(q-1)+T(n-q))+n \\ &= \frac{1}{n}\sum_{q=1}^{n}T(q-1)+\frac{1}{n}\sum_{q=1}^{n}T(n-q)+n \end{aligned}$$

$$T(n) = \frac{1}{n}\sum_{q=1}^{n}T(q-1)+\frac{1}{n}\sum_{q=1}^{n}T(n-q)+n$$

Observe that the two sums add up the same values $T(0)+T(1)+\cdots+T(n-1)$. One counts up and other counts down. Thus we can replace them with $2\sum_{q=0}^{n-1}T(q)$. We will extract $T(0)$ and $T(1)$ and treat them specially. These two do not follow the formula.

$$\begin{aligned} T(n) &= \frac{2}{n}\Big(\sum_{q=0}^{n-1}T(q)\Big)+n \\ &= \frac{2}{n}\Big(T(0)+T(1)+\sum_{q=2}^{n-1}T(q)\Big)+n \end{aligned}$$

We will apply the induction hypothesis for $q < n$ we have

$$\begin{aligned} T(n) &= \frac{2}{n}\Big(T(0)+T(1)+\sum_{q=2}^{n-1}T(q)\Big)+n \\ &\leq \frac{2}{n}\Big(1+1+\sum_{q=2}^{n-1}(cq\log q)\Big)+n \\ &= \frac{2c}{n}\Big(\sum_{q=2}^{n-1}(cq\ln q)\Big)+n+\frac{4}{n} \end{aligned}$$

We have never seen this sum before:

$$S(n) = \sum_{q=2}^{n-1}(cq\ln q)$$

Recall from calculus that for any monotonically increasing function $f(x)$

$$\sum_{i=a}^{b-1}f(i) \leq \int_{a}^{b}f(x)dx$$

The function $f(x) = x\ln x$ is monotonically increasing, and so

$$S(n) = \sum_{q=2}^{n-1} (cq\ln q) \leq \int_2^n x\ln x\, dx \tag{4.1}$$

We can integrate this by parts:

$$\int_2^n x\ln x\, dx = \frac{x^2}{2}\ln x - \frac{x^2}{4}\Big|_{x=2}^{n}$$

$$\begin{aligned}\int_2^n x\ln x\, dx &= \frac{x^2}{2}\ln x - \frac{x^2}{4}\Big|_{x=2}^{n} \\ &= \Big(\frac{n^2}{2}\ln n - \frac{n^2}{4}\Big) - (2\ln 2 - 1) \\ &\leq \frac{n^2}{2}\ln n - \frac{n^2}{4}\end{aligned}$$

We thus have

$$S(n) = \sum_{q=2}^{n-1} (cq\ln q) \leq \frac{n^2}{2}\ln n - \frac{n^2}{4}$$

Plug this back into the expression for $T(n)$ to get

$$T(n) = \frac{2c}{n}\Big(\frac{n^2}{2}\ln n - \frac{n^2}{4}\Big) + n + \frac{4}{n}$$

$$\begin{aligned}T(n) &= \frac{2c}{n}\Big(\frac{n^2}{2}\ln n - \frac{n^2}{4}\Big) + n + \frac{4}{n} \\ &= cn\ln n - \frac{cn}{2} + n + \frac{4}{n} \\ &= cn\ln n + n\big(1 - \frac{c}{2}\big) + \frac{4}{n}\end{aligned}$$

$$T(n) = cn\ln n + n\big(1 - \frac{c}{2}\big) + \frac{4}{n}$$

To finish the proof,we want all of this to be at most $cn\ln n$. For this to happen, we will need to select c such that

$$n\big(1 - \frac{c}{2}\big) + \frac{4}{n} \leq 0$$

If we select $c = 3$, and use the fact that $n \geq 3$ we get

$$\begin{aligned}n\big(1 - \frac{c}{2}\big) + \frac{4}{n} &= \frac{3}{n} - \frac{n}{2} \\ &\leq 1 - \frac{3}{2} = -\frac{1}{2} \leq 0.\end{aligned}$$

From the basis case we had $c \geq 2.88$. Choosing $c = 3$ satisfies all the constraints. Thus $T(n) = 3n\ln n \in \Theta(n\log n)$.

4.4 In-place, Stable Sorting

An *in-place* sorting algorithm is one that uses no additional array for storage. A sorting algorithm is *stable* if duplicate elements remain in the same relative position after sorting.

$9 \mid 3 \mid 3' \mid 5 \mid 6 \mid 5' \mid 2 \mid 1 \mid 3''$	unsorted
$1 \mid 2 \mid 3 \mid 3' \mid 3'' \mid 5 \mid 5' \mid 6 \mid 9$	stable sort
$1 \mid 2 \mid 3' \mid 3 \mid 3'' \mid 5' \mid 5 \mid 6 \mid 9$	unstable

Bubble sort, insertion sort and selection sort are in-place sorting algorithms. Bubble sort and insertion sort can be implemented as stable algorithms but selection sort cannot (without significant modifications). Mergesort is a stable algorithm but not an in-place algorithm. It requires extra array storage. Quicksort is not stable but is an in-place algorithm. Heapsort is an in-place algorithm but is not stable.

4.5 Lower Bounds for Sorting

The best we have seen so far is $O(n \log n)$ algorithms for sorting. Is it possible to do better than $O(n \log n)$? If a sorting algorithm is solely based on comparison of keys in the array then it is *impossible* to sort more efficiently than $\Omega(n \log n)$ time. All algorithms we have seen so far are comparison-based sorting algorithms.

Consider sorting three numbers a_1, a_2, a_3. There are $3! = 6$ possible combinations:

$$(a_1, a_2, a_3), (a_1, a_3, a_2), (a_3, a_2, a_1)$$
$$(a_3, a_1, a_2), (a_2, a_1, a_3), (a_2, a_3, a_1)$$

One of these permutations leads to the numbers in sorted order.

The comparison based algorithm defines a *decision tree*. Here is the tree for the three numbers.

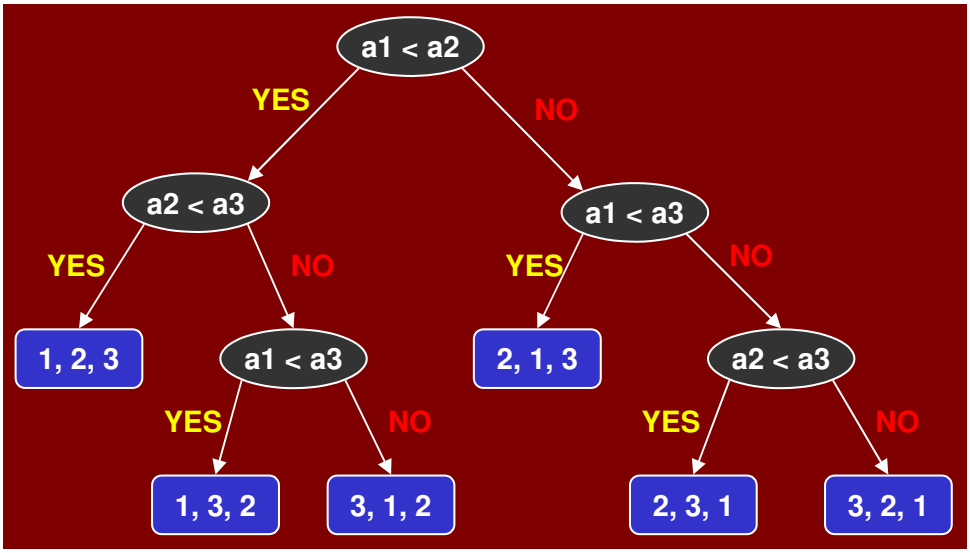


Figure 4.7: Decision Tree

For n elements, there will be $n!$ possible permutations. The height of the tree is exactly equal to $T(n)$, the running time of the algorithm. The height is $T(n)$ because any path from the root to a leaf corresponds to a sequence of comparisons made by the algorithm.

Any binary tree of height $T(n)$ has at most $2^{T(n)}$ leaves. Thus a comparison based sorting algorithm can distinguish between at most $2^{T(n)}$ different final outcomes. So we have

$$\begin{aligned} 2^{T(n)} &\geq n! \qquad \text{and therefore} \\ T(n) &\geq \log(n!) \end{aligned}$$

We can use *Stirling's approximation* for $n!$:

$$n! \geq \sqrt{2\pi n}\left(\frac{n}{e}\right)^n$$

Thereofore

$$\begin{aligned} T(n) &\geq \log\left(\sqrt{2\pi n}\left(\frac{n}{e}\right)^n\right) \\ &= \log(\sqrt{2\pi n}) + n\log n - n\log e \\ &\in \Omega(n\log n) \end{aligned}$$

We thus have the following theorem.

Theorem 1
Any comparison-based sorting algorithm has worst-case running time $\Omega(n\log n)$.

Chapter 5

Linear Time Sorting

The lower bound implies that if we hope to sort numbers faster than $O(n \log n)$, we cannot do it by making comparisons alone. Is it possible to sort without making comparisons? The answer is yes, but only under very restrictive circumstances. Many applications involve sorting small integers (e.g. sorting characters, exam scores, etc.). We present three algorithms based on the theme of speeding up sorting in special cases, by not making comparisons.

5.1 Counting Sort

We will consider three algorithms that are faster and work by not making comparisons. Counting sort assumes that the numbers to be sorted are in the range 1 to k where k is small. The basic idea is to determine the rank of each number in final sorted array.

Recall that the rank of an item is the number of elements that are less than or equal to it. Once we know the ranks, we simply copy numbers to their final position in an output array.

The question is how to find the rank of an element without comparing it to the other elements of the array?. The algorithm uses three arrays. As usual, $A[1..n]$ holds the initial input, $B[1..n]$ holds the sorted output and $C[1..k]$ is an array of integers. $C[x]$ is the rank of x in A, where $x \in [1..k]$. The algorithm is remarkably simple, but deceptively clever. The algorithm operates by first constructing C. This is done in two steps. First we set $C[x]$ to be the number of elements of $A[j]$ that are equal to x. We can do this initializing C to zero, and then for each j, from 1 to n, we increment $C[A[j]]$ by 1. Thus, if $A[j] = 5$, then the 5th element of C is incremented, indicating that we have seen one more 5. To determine the number of elements that are less than or equal to x, we replace $C[x]$ with the sum of elements in the sub array $R[1 : x]$. This is done by just keeping a running total of the elements of C.

$C[x]$ now contains the rank of x. This means that if $x = A[j]$ then the final position of $A[j]$ should be at position $C[x]$ in the final sorted array. Thus, we set $B[C[x]] = A[j]$. Notice We need to be careful if there are duplicates, since we do not want them to overwrite the same location of B. To do this, we decrement

C[i] after copying.

```
COUNTING-SORT( array A, array B, int k)
 1  for i ← 1 to k
 2  do C[i] ← 0          [k times]
 3  for j ← 1 to length[A]
 4  do C[A[j]] ← C[A[j]] + 1          [n times]
 5  // C[i] now contains the number of elements = i
 6  for i ← 2 to k
 7  do C[i] ← C[i] + C[i − 1]          [k times]
 8  // C[i] now contains the number of elements ≤ i
 9  for j ← length[A] downto 1
10  do B[C[A[j]]] ← A[j]
11     C[A[j]] ← C[A[j]] − 1          [n times]
```

There are four (unnested) loops, executed k times, n times, $k-1$ times, and n times, respectively, so the total running time is $\Theta(n+k)$ time. If $k = O(n)$, then the total running time is $\Theta(n)$.

Figure 5.1 through 5.19 shows an example of the algorithm. You should trace through the example to convince yourself how it works.

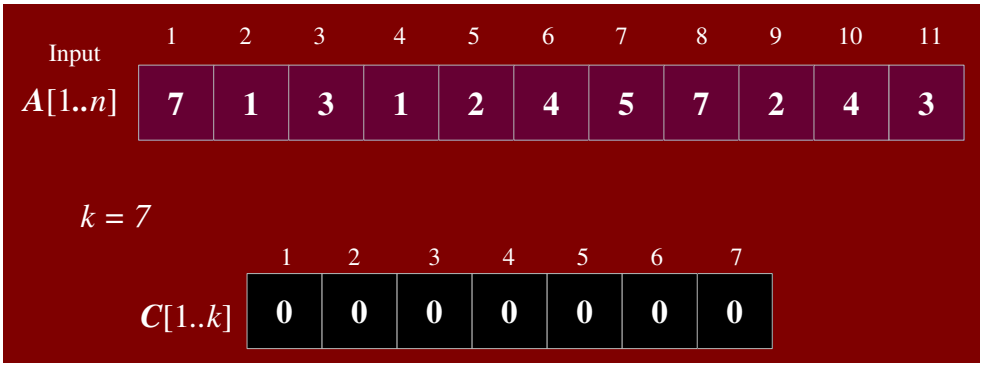


Figure 5.1: Initial A and C arrays.

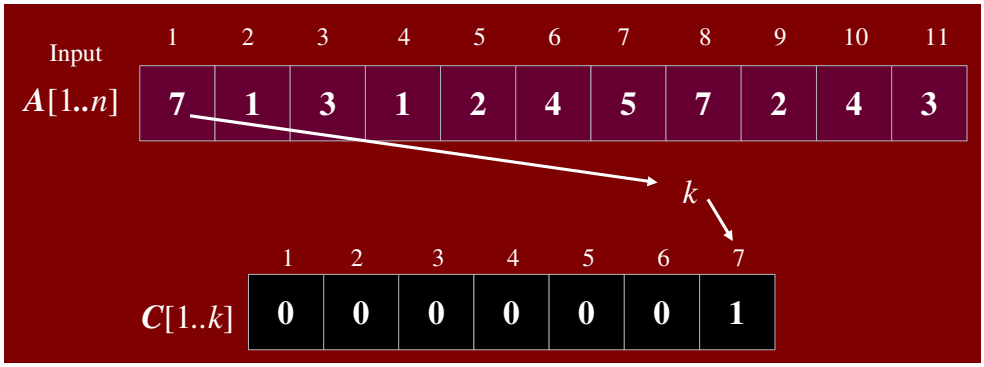


Figure 5.2: $A[1] = 7$ processed

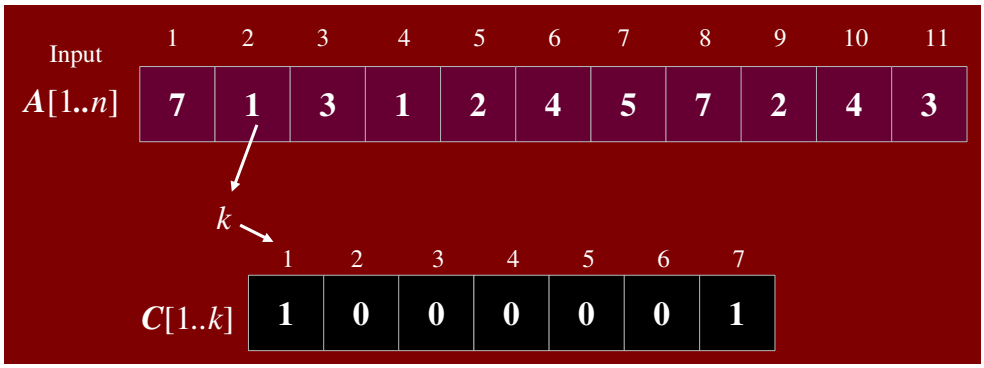


Figure 5.3: $A[2] = 1$ processed

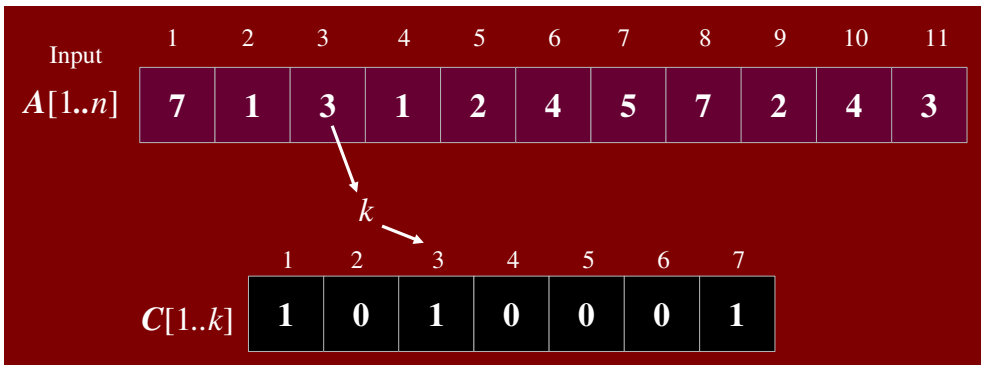


Figure 5.4: $A[3] = 3$ processed

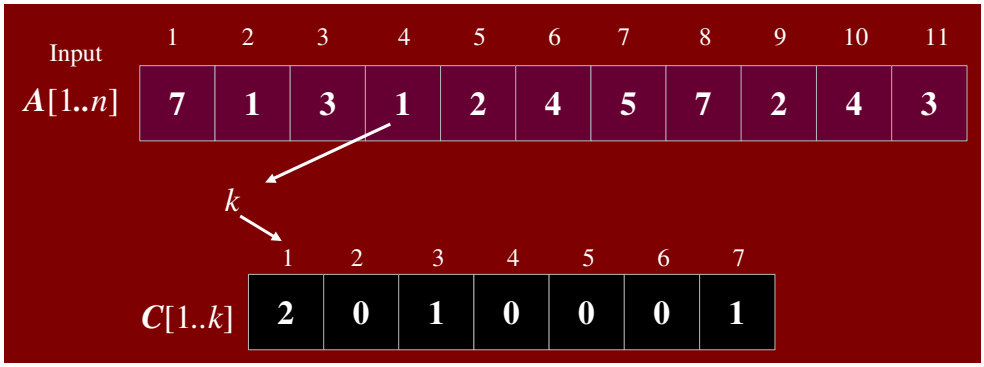


Figure 5.5: $A[4] = 1$ processed

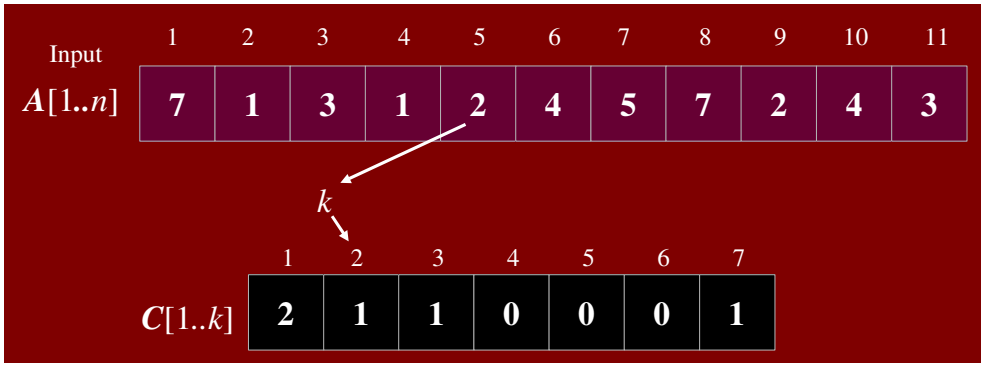


Figure 5.6: $A[5] = 2$ processed

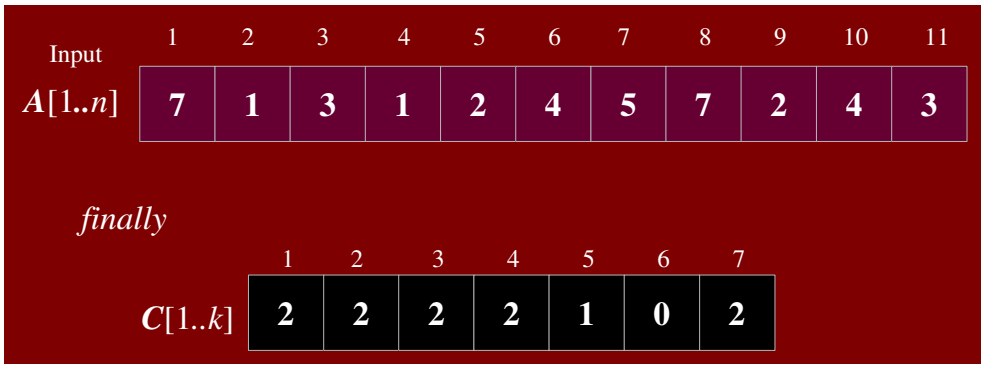


Figure 5.7: C now contains count of elements of A

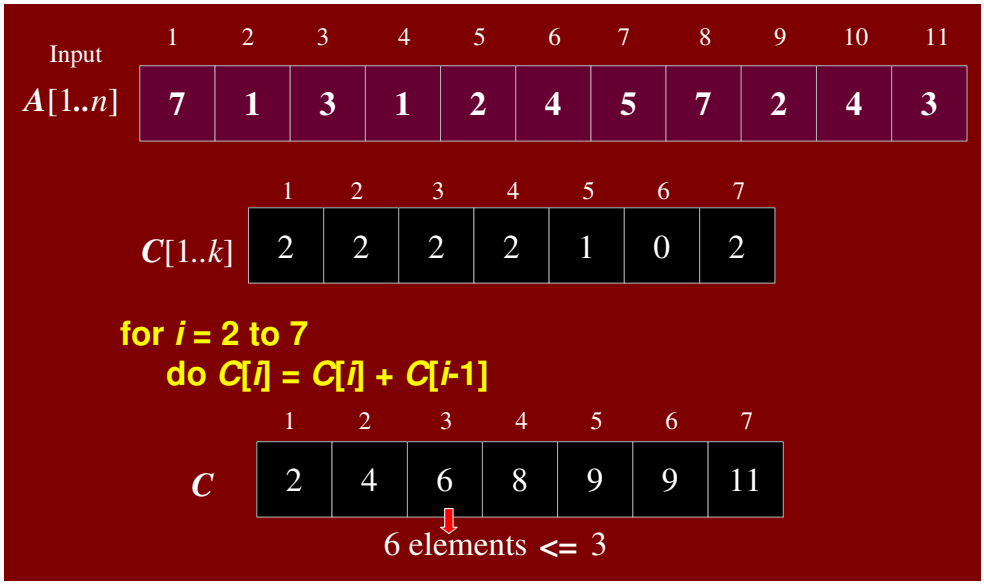


Figure 5.8: C set to rank each number of A

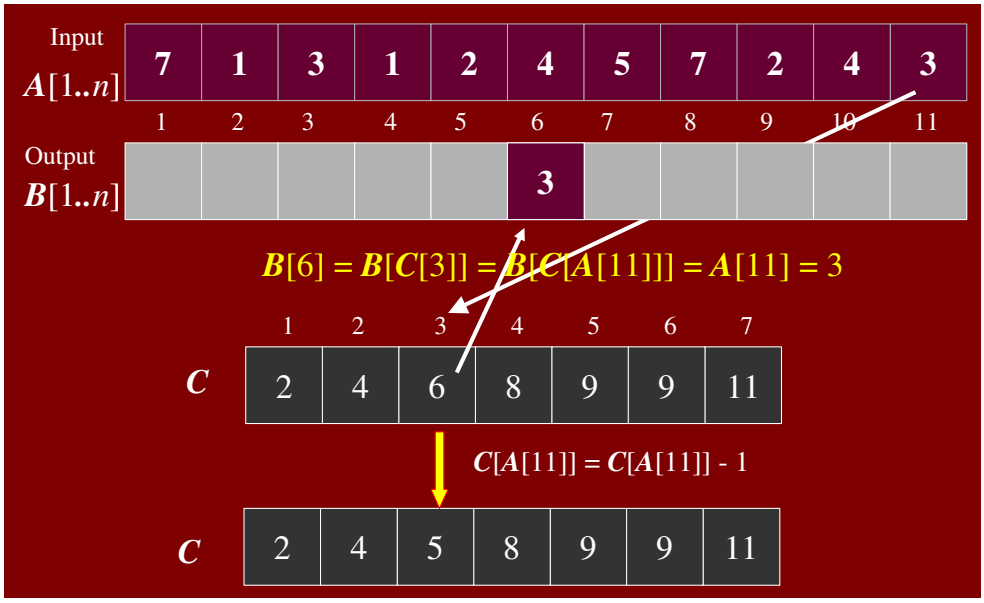


Figure 5.9: A[11] placed in output array B

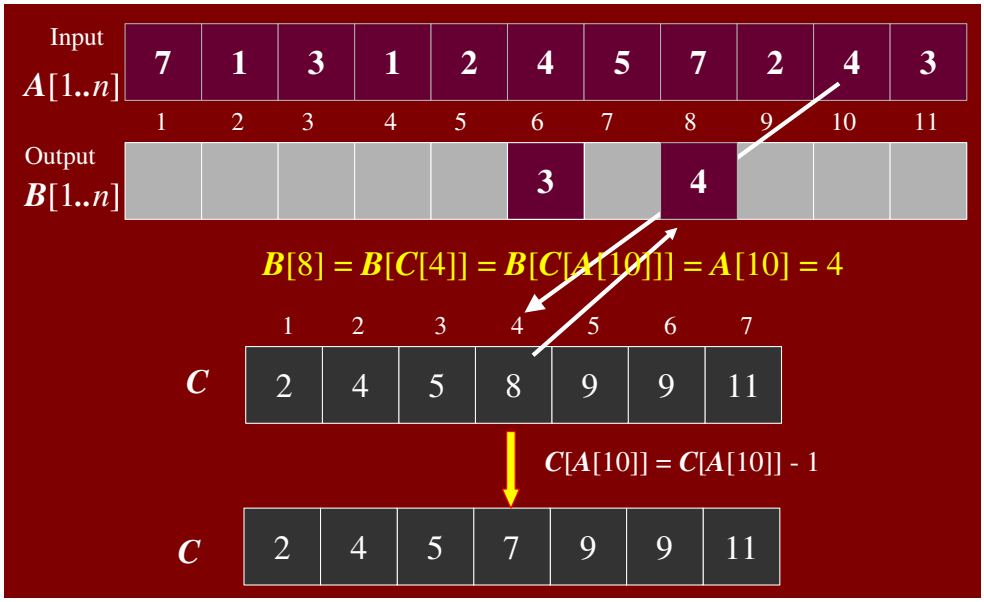


Figure 5.10: $A[10]$ placed in output array B

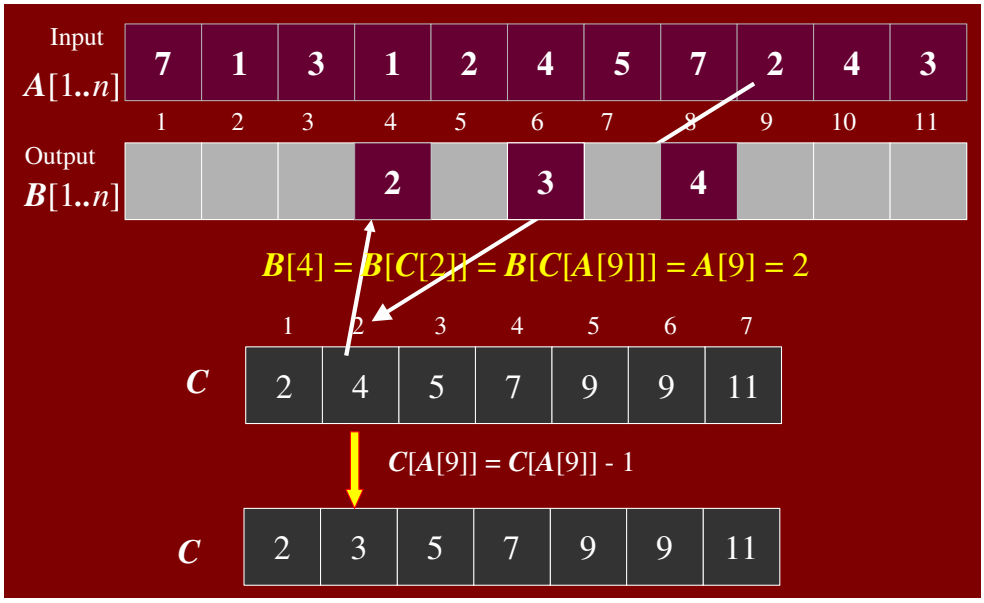


Figure 5.11: $A[9]$ placed in output array B

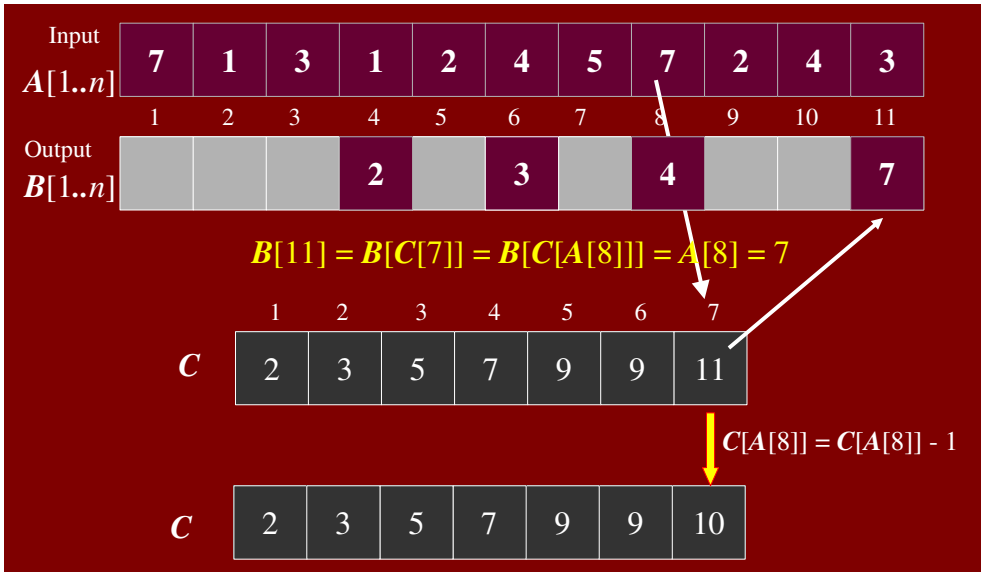


Figure 5.12: A[8] placed in output array B

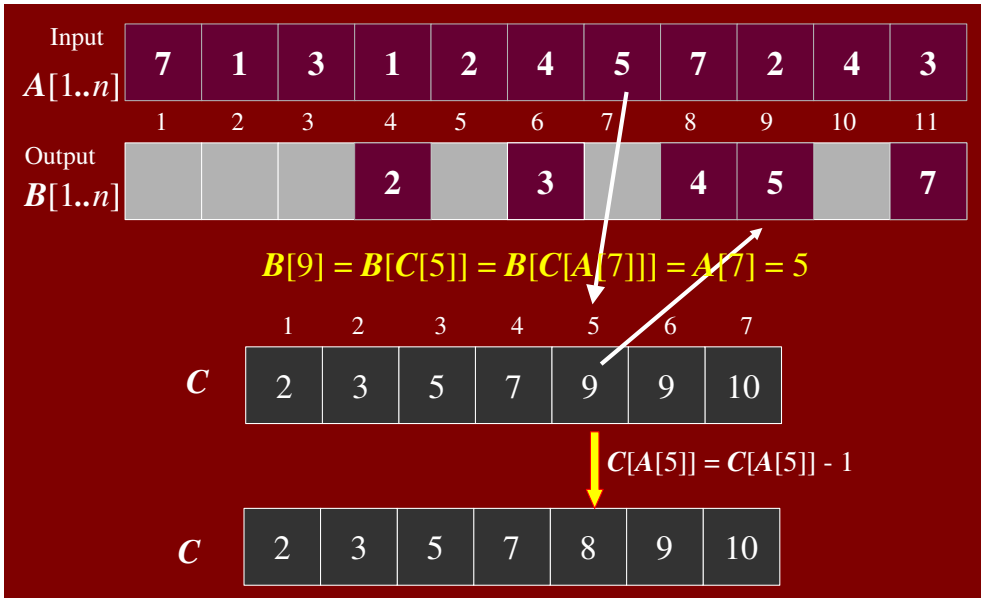


Figure 5.13: A[7] placed in output array B

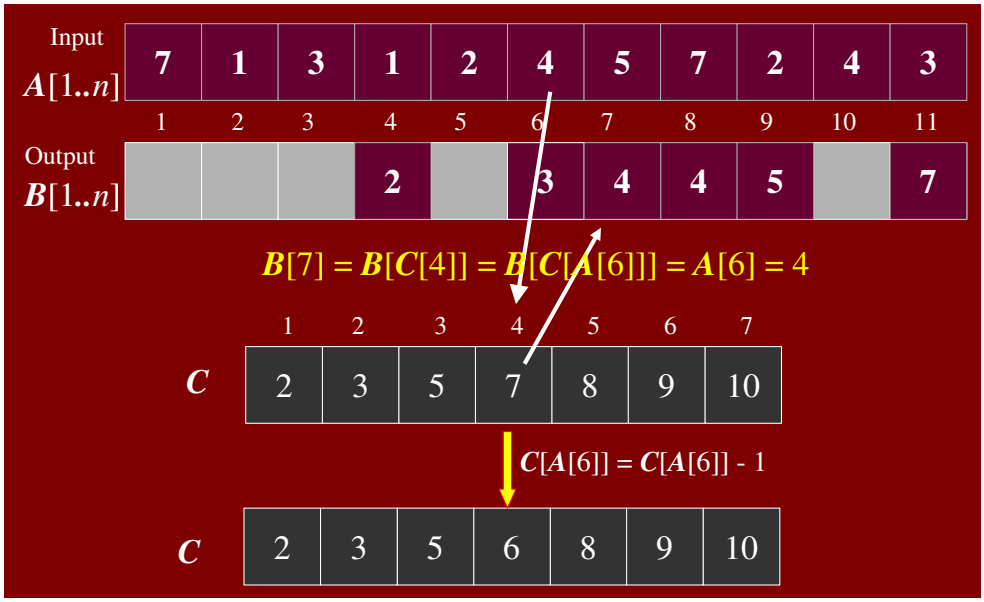


Figure 5.14: A[6] placed in output array B

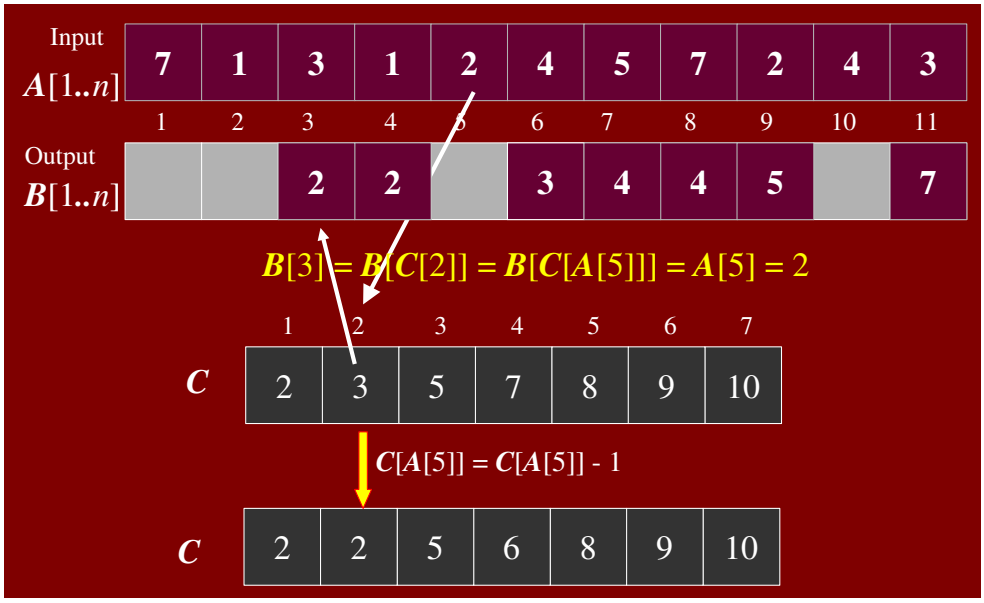


Figure 5.15: A[5] placed in output array B

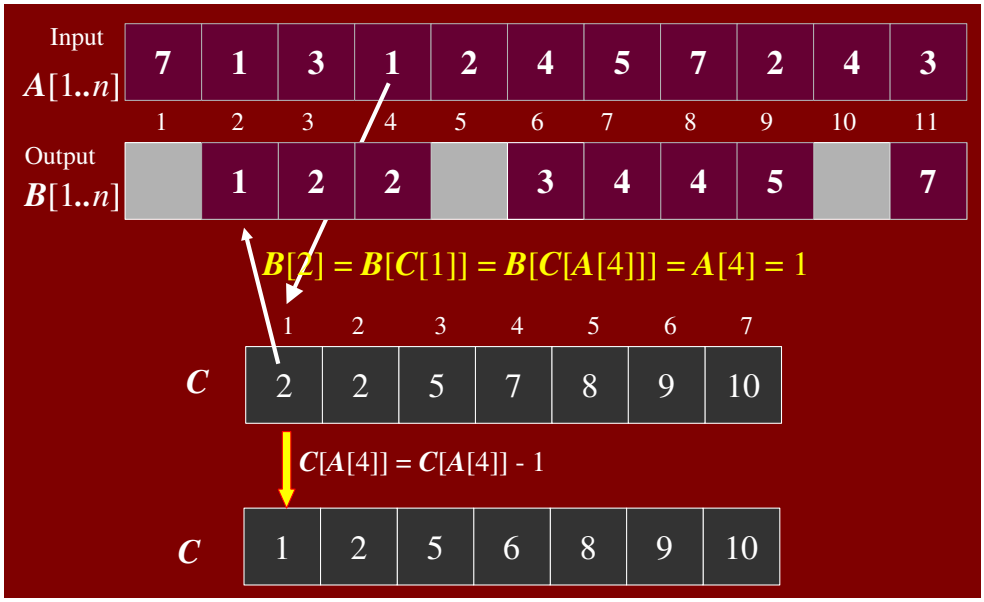


Figure 5.16: A[4] placed in output array B

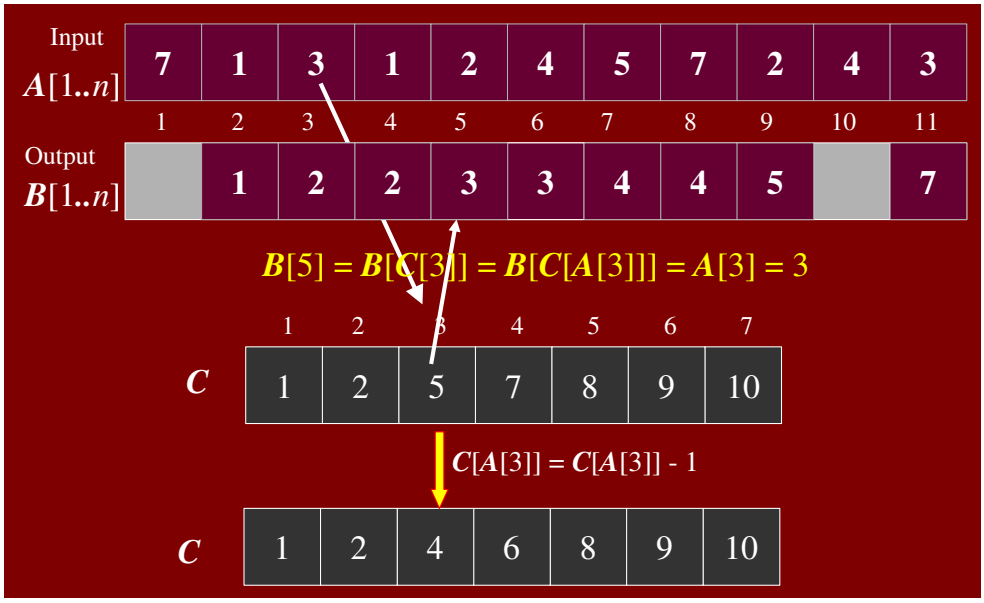


Figure 5.17: A[3] placed in output array B

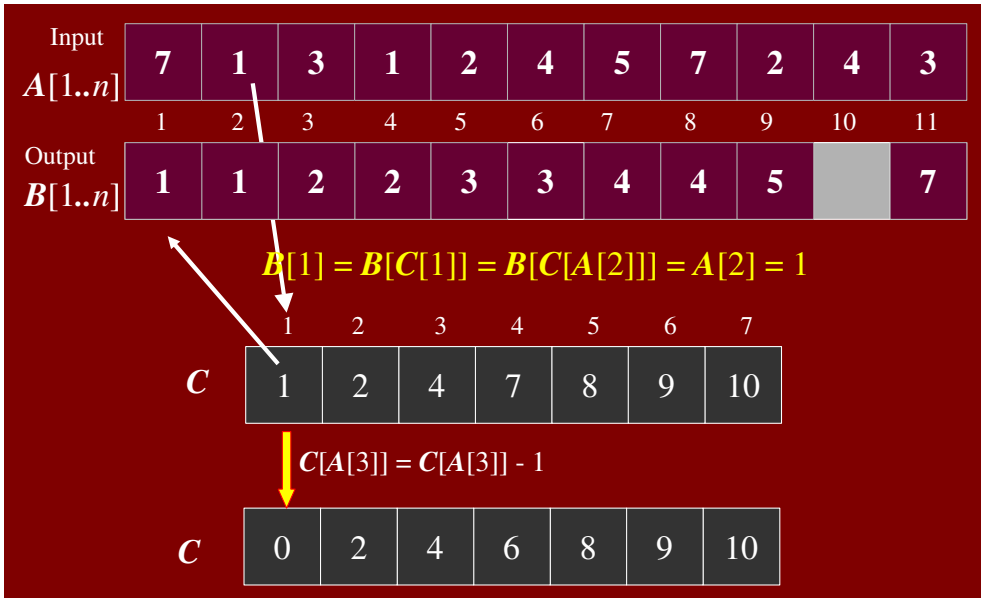


Figure 5.18: A[2] placed in output array B

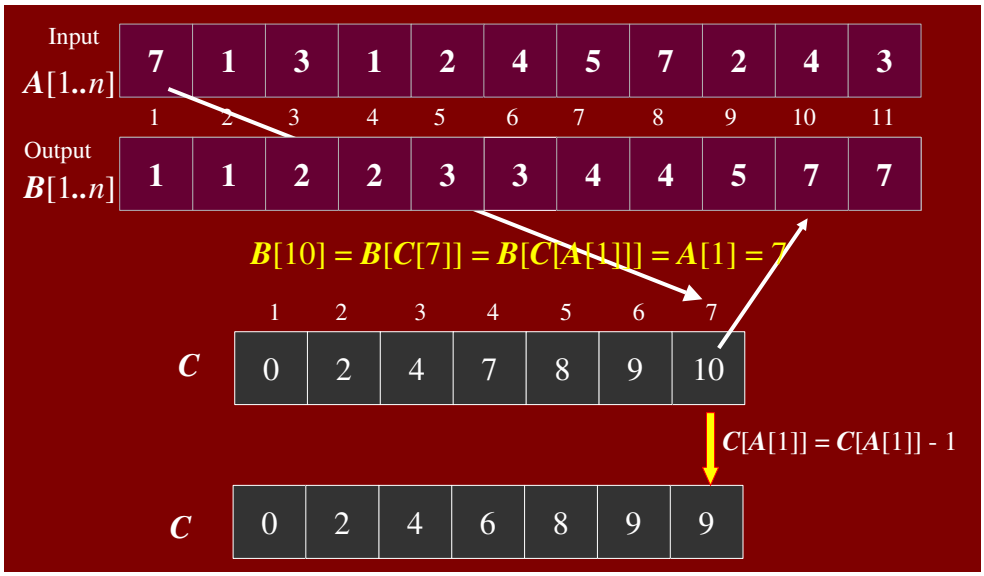


Figure 5.19: B now contains the final sorted data.

Counting sort is not an in-place sorting algorithm but it is stable. Stability is important because data are often carried with the keys being sorted. radix sort (which uses counting sort as a subroutine) relies on it to work correctly. Stability achieved by running the loop down from n to 1 and not the other way around:

```
COUNTING-SORT( array A, array B, int k)
1  ⋮
2  for j ← length[A] downto 1
3  do B[C[A[j]]] ← A[j]
4     C[A[j]] ← C[A[j]] − 1
```

Figure 5.20 illustrates the stability. The numbers 1, 2, 3, 4, and 7, each appear twice. The two 4's have been given the superscript "*****". Numbers are placed in the output B array starting from the right. The two 4's maintain their relative position in the B array. If the sorting algorithm had caused 4^{**} to end up on the left of 4^{*}, the algorithm would be termed unstable.

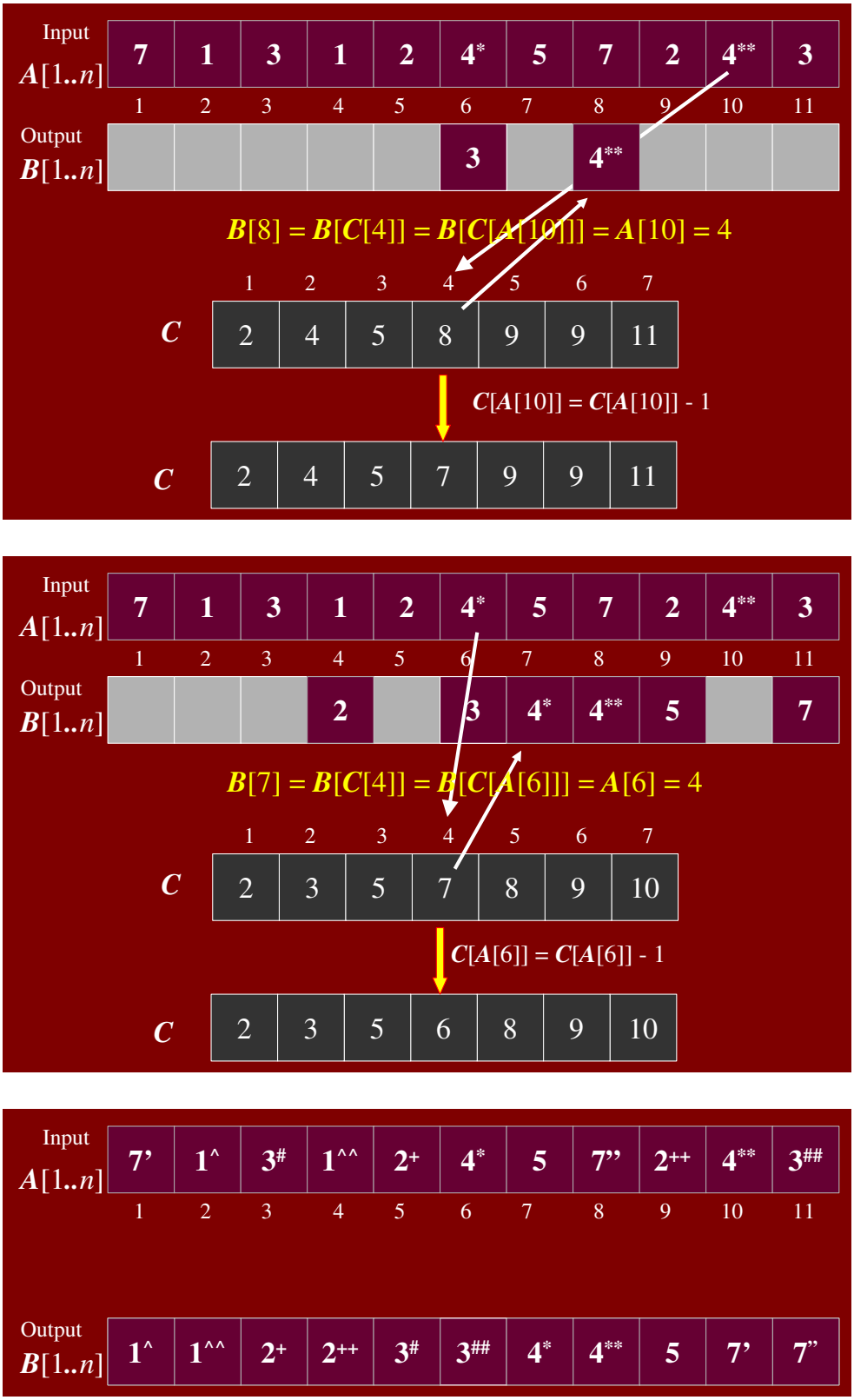


Figure 5.20: Stability of counting sort

5.2 Bucket or Bin Sort

Assume that the keys of the items that we wish to sort lie in a small fixed range and that there is only one item with each value of the key. Then we can sort with the following procedure:

1. Set up an array of "bins" - one for each value of the key - in order,

2. Examine each item and use the value of the key to place it in the appropriate bin.

Now our collection is sorted and it only took n operations, so this is an $O(n)$ operation. However, note that it will only work under very restricted conditions. To understand these restrictions, let's be a little more precise about the specification of the problem and assume that there are m values of the key. To recover our sorted collection, we need to examine each bin. This adds a third step to the algorithm above,

3. Examine each bin to see whether there's an item in it.

which requires m operations. So the algorithm's time becomes:

$$T(n) = c_1 n + c_2 m$$

and it is strictly $O(n + m)$. If $m \leq n$, this is clearly $O(n)$. However if $m >> n$, then it is $O(m)$. An implementation of bin sort might look like:

```
BUCEKTSORT( array A, int n, int M)
1  // Pre-condition: for 1 ≤ i ≤ n, 0 ≤ a[i] < M
2  // Mark all the bins empty
3  for i ← 1 to M
4  do bin[i] ← Empty
5  for i ← 1 to n
6  do bin[A[i]] ← A[i]
```

If there are *duplicates*, then each bin can be replaced by a *linked list*. The third step then becomes:

3. Link all the lists into one list.

We can add an item to a linked list in $O(1)$ time. There are n items requiring $O(n)$ time. Linking a list to another list simply involves making the tail of one list point to the other, so it is $O(1)$. Linking m such lists obviously takes $O(m)$ time, so the algorithm is still $O(n + m)$. Figures 5.21 through 5.23 show the algorithm in action using linked lists.

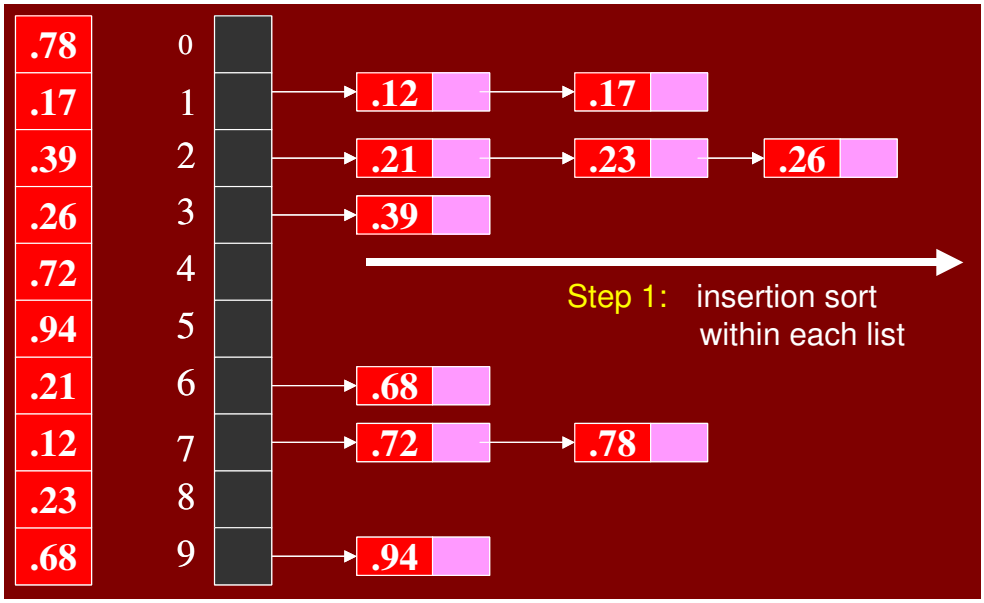


Figure 5.21: Bucket sort: step 1, placing keys in bins in sorted order

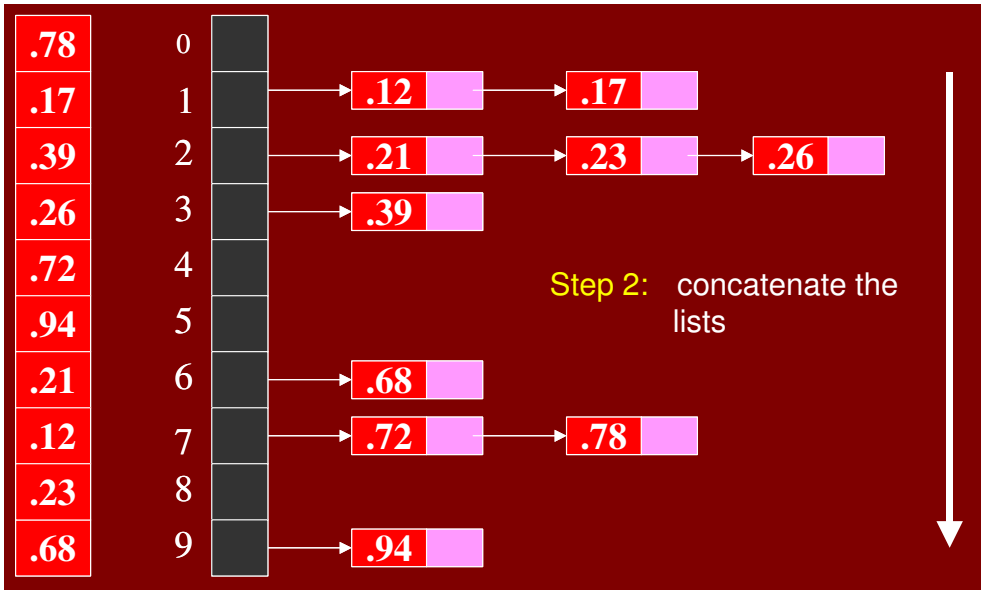


Figure 5.22: Bucket sort: step 2, concatenate the lists

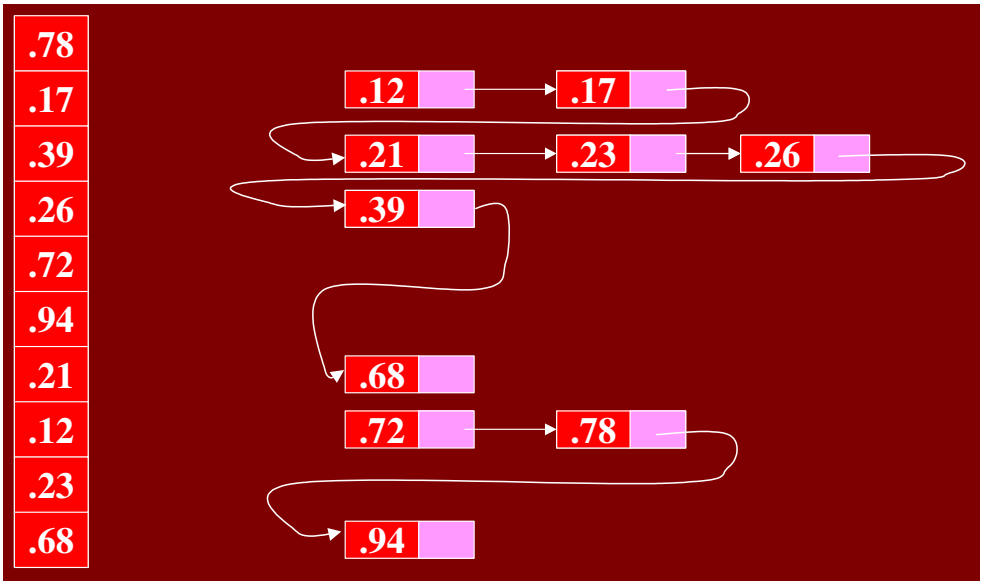


Figure 5.23: Bucket sort: the final sorted sequence

5.3 Radix Sort

The main shortcoming of counting sort is that it is useful for small integers, i.e., $1..k$ where k is small. If k were a million or more, the size of the rank array would also be a million. Radix sort provides a nice work around this limitation by sorting numbers one digit at a time.

576		49[4]		9[5]4		[1]76		176
494		19[4]		5[7]6		[1]94		194
194		95[4]		1[7]6		[2]78		278
296	$\Rightarrow$	57[6]	$\Rightarrow$	2[7]8	$\Rightarrow$	[2]96	$\Rightarrow$	296
278		29[6]		4[9]4		[4]94		494
176		17[6]		1[9]4		[5]76		576
954		27[8]		2[9]6		[9]54		954

Here is the algorithm that sorts $A[1..n]$ where each number is d digits long.

```
RADIX-SORT( array A, int n, int d)
1  for i ← 1 to d
2  do stably sort A w.r.t i^th lowest order digit
```

Chapter 6

Dynamic Programming

6.1 Fibonacci Sequence

Suppose we put a pair of rabbits in a place surrounded on all sides by a wall. How many pairs of rabbits can be produced from that pair in a year if it is supposed that every month each pair begets a new pair which from the second month on becomes productive? Resulting sequence is $1, 1, 2, 3, 5, 8, 13, 21, 34, 55, \ldots$ where each number is the sum of the two preceding numbers.

This problem was posed by Leonardo Pisano, better known by his nickname Fibonacci (son of Bonacci, born 1170, died 1250). This problem and many others were in posed in his book *Liber abaci*, published in 1202. The book was based on the arithmetic and algebra that Fibonacci had accumulated during his travels. The book, which went on to be widely copied and imitated, introduced the Hindu-Arabic place-valued decimal system and the use of Arabic numerals into Europe. The rabbits problem in the third section of Liber abaci led to the introduction of the Fibonacci numbers and the Fibonacci sequence for which Fibonacci is best remembered today.

This sequence, in which each number is the sum of the two preceding numbers, has proved extremely fruitful and appears in many different areas of mathematics and science. The *Fibonacci Quarterly* is a modern journal devoted to studying mathematics related to this sequence. The Fibonacci numbers F_n are defined as follows:

$$F_0 = 0$$
$$F_1 = 1$$
$$F_n = F_{n-1} + F_{n-2}$$

The recursive definition of Fibonacci numbers gives us a recursive algorithm for computing them:

```
FIB(n)
1  if (n < 2)
2     then return n
3     else  return FIB(n − 1) + FIB(n − 2)
```

Figure **??** shows four levels of recursion for the call $\mathtt{fib}(8)$:

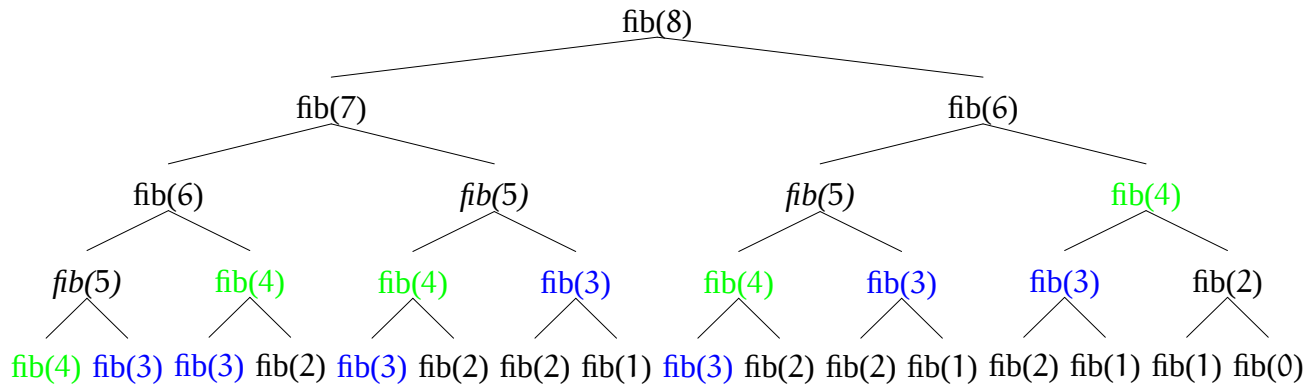


Figure 6.1: Recursive calls during computation of Fibonacci number

A single recursive call to fib(n) results in one recursive call to fib(n − 1), two recursive calls to fib(n − 2), three recursive calls to fib(n − 3), five recursive calls to fib(n − 4) and, in general, F_{k-1} recursive calls to fib(n − k) For each call, we're recomputing the same fibonacci number from scratch.

We can avoid this unnecessary repetitions by writing down the results of recursive calls and looking them up again if we need them later. This process is called *memoization*. Here is the algorithm with memoization.

```
MEMOFIB(n)
1  if (n < 2)
2     then return n
3  if (F[n] is undefined)
4     then F[n] ← MEMOFIB(n − 1) + MEMOFIB(n − 2)
5  return F[n]
```

If we trace through the recursive calls to MEMOFIB, we find that array F[] gets filled from bottom up. I.e., first F[2], then F[3], and so on, up to F[n]. We can replace recursion with a simple for-loop that just fills up the array F[] in that order.

This gives us our first explicit *dynamic programming* algorithm.

```
ITERFIB(n)
1  F[0] ← 0
2  F[1] ← 1
3  for i ← 2 to n
4  do
5      F[i] ← F[i − 1] + F[i − 2]
6  return F[n]
```

This algorithm clearly takes only $O(n)$ time to compute F_n. By contrast, the original recursive algorithm takes $\Theta(\phi^n)$, $\phi = \frac{1+\sqrt{5}}{2} \approx 1.618$. ITERFIB achieves an exponential speedup over the original recursive algorithm.

6.2 Dynamic Programming

Dynamic programming is essentially recursion without repetition. Developing a dynamic programming algorithm generally involves two separate steps:

- **Formulate problem recursively**. Write down a formula for the whole problem as a simple combination of answers to smaller subproblems.
- **Build solution to recurrence from bottom up**. Write an algorithm that starts with base cases and works its way up to the final solution.

Dynamic programming algorithms need to store the results of intermediate subproblems. This is often *but not always* done with some kind of table. We will now cover a number of examples of problems in which the solution is based on dynamic programming strategy.

6.3 Edit Distance

The words “computer” and “commuter” are very similar, and a change of just one letter, p-¿m, will change the first word into the second. The word “sport” can be changed into “sort” by the deletion of the ‘p’, or equivalently, ‘sort’ can be changed into ‘sport’ by the insertion of ‘p’. The edit distance of two strings, s1 and s2, is defined as the minimum number of point mutations required to change s1 into s2, where a point mutation is one of:

- change a letter,
- insert a letter or

- delete a letter

For example, the edit distance between *FOOD* and *MONEY* is at most four:

$$\underline{\mathrm{F}}\mathrm{OOD} \longrightarrow \mathrm{MO}\underline{\mathrm{O}}\mathrm{D} \longrightarrow \mathrm{MON}\underline{\ }\mathrm{D}$$
$$\longrightarrow \mathrm{MONE}\underline{\mathrm{D}} \longrightarrow \mathrm{MONEY}$$

6.3.1 Edit Distance: Applications

There are numerous applications of the Edit Distance algorithm. Here are some examples:

Spelling Correction

If a text contains a word that is not in the dictionary, a 'close' word, i.e. one with a small edit distance, may be suggested as a correction. Most word processing applications, such as Microsoft Word, have spelling checking and correction facility. When Word, for example, finds an incorrectly spelled word, it makes suggestions of possible replacements.

Plagiarism Detection

If someone copies, say, a C program and makes a few changes here and there, for example, change variable names, add a comment of two, the edit distance between the source and copy may be small. The edit distance provides an indication of similarity that might be too close in some situations.

Computational Molecular Biology DNA is a polymer. The monomer units of DNA are nucleotides, and the polymer is known as a "polynucleotide." Each nucleotide consists of a 5-carbon sugar (deoxyribose), a nitrogen containing base attached to the sugar, and a phosphate group. There are four different types of nucleotides found in DNA, differing only in the nitrogenous base. The four nucleotides are given one letter abbreviations as shorthand for the four bases.

- A-adenine
- G-guanine
- C-cytosine
- T-thymine

Double-helix of DNA molecule with nucleotides Figure of Double-helix of DNA molecule with nucleotides goes here

The edit distance like algorithms are used to compute a distance between DNA sequences (strings over A,C,G,T, or protein sequences (over an alphabet of 20 amino acids), for various purposes, e.g.:

- to find genes or proteins that may have shared functions or properties
- to infer family relationships and evolutionary trees over different organisms.

Speech Recognition

Algorithms similar to those for the edit-distance problem are used in some speech recognition systems. Find a close match between a new utterance and one in a library of classified utterances.

6.3.2 Edit Distance Algorithm

A better way to display this editing process is to place the words above the other:

S	*D*	*I*	*M*	*D*	*M*
M	A	_	T	H	S
A	_	R	T	_	S

The first word has a gap for every insertion (I) and the second word has a gap for every deletion (D). Columns with two different characters correspond to substitutions (S). Matches (M) do not count. The *Edit transcript* is defined as a string over the alphabet M, S, I, D that describes a transformation of one string into another. For example

S	*D*	*I*	*M*	*D*	*M*	
1+	1+	1+	0+	1+	0+	= 4

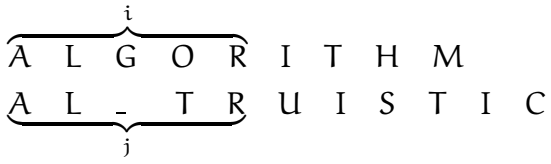
In general, it is not easy to determine the optimal edit distance. For example, the distance between *ALGORITHM* and *ALTRUISTIC* is at most 6.

A	L	G	O	R	_	I	_	T	H	M
A	L	_	T	R	U	I	S	T	I	C

Is this optimal?

6.3.3 Edit Distance: Dynamic Programming Algorithm

Suppose we have an m-character string A and an n-character string B. Define $E(i,j)$ to be the edit distance between the first i characters of A and the first j characters of B. For example,



The edit distance between entire strings A and B is $E(m,n)$. The gap representation for the edit sequences has a crucial "*optimal substructure*" property. If we remove the last column, the remaining columns must represent the shortest edit sequence for the remaining substrings. The edit distance is 6 for the following two words.

A L G O R _ I _ T H M
A L _ T R U I S T I C

If we remove the last column, the edit distance reduces to 5.

A L G O R _ I _ T H
A L _ T R U I S T I

We can use the optimal substructure property to devise a recursive formulation of the edit distance problem. There are a couple of obvious base cases:

- The only way to convert an empty string into a string of j characters is by doing j insertions. Thus

$$E(0, j) = j$$

- The only way to convert a string of i characters into the empty string is with i deletions:

$$E(i, 0) = i$$

There are four possibilities for the last column in the shortest possible edit sequence:

Deletion: Last entry in bottom row is empty.

$$\overbrace{\begin{matrix} A & L & G \end{matrix}}^{i=3} \begin{matrix} O & R & I & T & H & M \\ \underbrace{A \quad L \quad _}_{j=2} & T & R & U & I & S & T & I & C \end{matrix}$$

In this case

$$E(i, j) = E(i-1, j) + 1$$

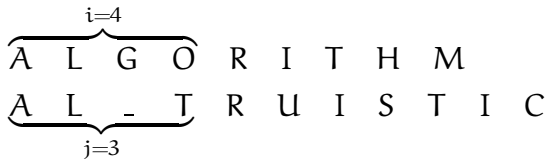
Insertion: The last entry in the top row is empty.

$$\overbrace{\begin{matrix} A & L & G & O & R & _ \end{matrix}}^{i=5} \begin{matrix} I & T & H & M \\ \underbrace{A \quad L \quad _ \quad T \quad R \quad U}_{j=5} & I & S & T & I & C \end{matrix}$$

In this case

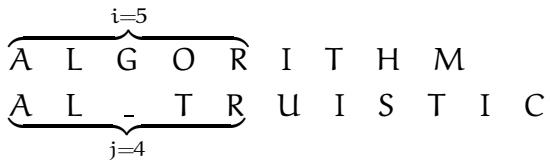
$$E(i, j) = E(i, j-1) + 1$$

Substitution: Both rows have characters in the last column.



If the characters are different, then

$$E(i,j) = E(i-1,j-1) + 1$$



If characters are same, no substitution is needed:

$$E(i,j) = E(i-1,j-1)$$

Thus the edit distance $E(i,j)$ is the smallest of the four possibilities:

$$E(i,j) = \min \begin{pmatrix} E(i-1,j)+1 & \\ E(i,j-1)+1 & \\ E(i-1,j-1)+1 & \text{if } A[i] \neq B[j] \\ E(i-1,j-1) & \text{if } A[i] = B[j] \end{pmatrix}$$

Consider the example of edit between the words “ARTS” and “MATHS”:

A	R	T	S	
M	A	T	H	S

The edit distance would be in $E(4,5)$. If we recursion to compute, we will have

$$E(4,5) = \min \begin{pmatrix} E(3,5)+1 & \\ E(4,4)+1 & \\ E(3,4)+1 & \text{if } A[4] \neq B[5] \\ E(3,4) & \text{if } A[4] = B[5] \end{pmatrix}$$

Recursion clearly leads to the same repetitive call pattern that we saw in Fibonnaci sequence. To avoid this, we will use the DP approach. We will build the solution bottom-up. We will use the base case $E(0,j)$ to fill first row and the base case $E(i,0)$ to fill first column. We will fill the remaining E matrix row by row.

		A	R	T	S
	0	→1	→2	→3	→4
M					
A					
T					
H					
S					

		A	R	T	S
	0	→1	→2	→3	→4
M	↓ 1				
A	↓ 2				
T	↓ 3				
H	↓ 4				
S	↓ 5				

Table 6.1: First row and first column entries using the base cases

We can now fill the second row. The table not only shows the values of the cells $E[i, j]$ but also arrows that indicate how it was computed using values in $E[i-1, j]$, $E[i, j-1]$ and $E[i-1, j-1]$. Thus, if a cell $E[i, j]$ has a down arrow from $E[i-1, j]$ then the minimum was found using $E[i-1, j]$. For a right arrow, the minimum was found using $E[i, j-1]$. For a diagonal down right arrow, the minimum was found using $E[i-1, j-1]$. There are certain cells that have two arrows pointed to it. In such a case, the minimum could be obtained from the diagonal $E[i-1, j-1]$ and either of $E[i-1, j]$ and $E[i, j-1]$. We will use these arrows later to determine the edit script.

		A	R	T	S
	0	→1	→2	→3	→4
M	↓ 1	↘ 1			
A	↓ 2				
T	↓ 3				
H	↓ 4				
S	↓ 5				

		A	R	T	S
	0	→1	→2	→3	→4
M	↓ 1	↘ 1	↘ →2		
A	↓ 2				
T	↓ 3				
H	↓ 4				
S	↓ 5				

Table 6.2: Computing $E[1,1]$ and $E[1,2]$

		A	R	T	S
	0	→1	→2	→3	→4
M	↓ 1	↘ 1	↘ →2	↘ →3	
A	↓ 2				
T	↓ 3				
H	↓ 4				
S	↓ 5				

		A	R	T	S
	0	→1	→2	→3	→4
M	↓ 1	↘ 1	↘ →2	↘ →3	↘ →4
A	↓ 2				
T	↓ 3				
H	↓ 4				
S	↓ 5				

Table 6.3: Computing $E[1,3]$ and $E[1,4]$

An edit script can be extracted by following a unique path from $E[0,0]$ to $E[4,5]$. There are three possible paths in the current example. Let us follow these paths and compute the edit script. In an actual implementation of the dynamic programming version of the edit distance algorithm, the arrows would be recorded using an appropriate data structure. For example, each cell in the matrix could be a record with fields for the value (numeric) and flags for the three incoming arrows.

		A	R	T	S
	0	→1	→2	→3	→4
M	↓ 1	↘ 1	↘ →2	↘ →3	↘ →4
A	↓ 2	↘ 1	↘ →2	↘ →3	↘ →4
T	↓ 3	↓ 2	↘ 2	↘ 2	→3
H	↓ 4	↓ 3	↘↓ 3	↘↓ 3	↘ 3
S	↓ 5	↓ 4	↘↓ 4	↘↓ 4	↘ 3

Table 6.4: The final table with all $E[i, j]$ entries computed

		A	R	T	S
	0	→1	→2	→3	→4
M	↓ 1	↘ 1	↘ →2	↘ →3	↘ →4
A	↓ 2	↘ 1	↘ →2	↘ →3	↘ →4
T	↓ 3	↓ 2	↘ 2	↘ 2	→3
H	↓ 4	↓ 3	↘↓ 3	↘↓ 3	↘ 3
S	↓ 5	↓ 4	↘↓ 4	↘↓ 4	↘ 3

Solution path 1:

1+	0+	1+	1+	0	= 3
D	**M**	**S**	**S**	**M**	
M	A	T	H	S	
_	A	R	T	S	

		A	R	T	S
	0	→1	→2	→3	→4
M	↓ 1	↘ 1	↘ →2	↘ →3	↘ →4
A	↓ 2	↘ 1	↘ →2	↘ →3	↘ →4
T	↓ 3	↓ 2	↘ 2	↘ 2	→3
H	↓ 4	↓ 3	↘↓ 3	↘↓ 3	↘ 3
S	↓ 5	↓ 4	↘↓ 4	↘↓ 4	↘ 3

Table 6.5: Possible edit scripts. The red arrows from $E[0,0]$ to $E[4,5]$ show the paths that can be followed to extract edit scripts.

		A	R	T	S
	0	→1	→2	→3	→4
M	↓ 1	↘ 1	↘ →2	↘ →3	↘ →4
A	↓ 2	↘ 1	↘ →2	↘ →3	↘ →4
T	↓ 3	↓ 2	↘ 2	↘ 2	→3
H	↓ 4	↓ 3	↘↓ 3	↘↓ 3	↘ 3
S	↓ 5	↓ 4	↘↓ 4	↘↓ 4	↘ 3

Solution path 2:

1+	1+	0+	1+	0	= 3
S	**S**	**M**	**D**	**M**	
M	A	T	H	S	
A	R	T	_	S	

		A	R	T	S
	0	→1	→2	→3	→4
M	↓ 1	↘ 1	↘ →2	↘ →3	↘ →4
A	↓ 2	↘ 1	↘ →2	↘ →3	↘ →4
T	↓ 3	↓ 2	↘ 2	↘ 2	→3
H	↓ 4	↓ 3	↘↓ 3	↘↓ 3	↘ 3
S	↓ 5	↓ 4	↘↓ 4	↘↓ 4	↘ 3

Solution path 3:

1+	0+	1+	0+	1+	0	= 3
D	**M**	**I**	**M**	**D**	**M**	
M	A	_	T	H	S	
_	A	R	T	_	S	

6.3.4 Analysis of DP Edit Distance

There are $\Theta(n^2)$ entries in the matrix. Each entry $E(i,j)$ takes $\Theta(1)$ time to compute. The total running time is $\Theta(n^2)$.

6.4 Chain Matrix Multiply

Suppose we wish to multiply a series of matrices:

$$A_1 A_2 \ldots A_n$$

In what order should the multiplication be done? A $p \times q$ matrix A can be multiplied with a $q \times r$ matrix B. The result will be a $p \times r$ matrix C. In particular, for $1 \leq i \leq p$ and $1 \leq j \leq r$,

$$C[i,j] = \sum_{k=1}^{q} A[i,k]B[k,j]$$

There are $(p \cdot r)$ total entries in C and each takes $O(q)$ to compute.

$$C[i,j] = \sum_{k=1}^{q} A[i,k]B[k,j]$$

Thus the total number of multiplications is $p \cdot q \cdot r$. Consider the case of 3 matrices: A_1 is 5×4, A_2 is 4×6 and A_3 is 6×2 The multiplication can be carried out either as $((A_1A_2)A_3)$ or $(A_1(A_2A_3))$. The cost of the two is

$$((A_1A_2)A_3) = (5 \cdot 4 \cdot 6) + (5 \cdot 6 \cdot 2) = 180$$
$$(A_1(A_2A_3)) = (4 \cdot 6 \cdot 2) + (5 \cdot 4 \cdot 2) = 88$$

There is considerable savings achieved even for this simple example. In general, however, in what order should we multiply a series of matrices $A_1A_2\ldots A_n$. Matrix multiplication is an associative but not commutative operation. We are free to add parenthesis the above multiplication but the order of matrices can not be changed. The *Chain Matrix Multiplication Problem* is stated as follows:

> Given a sequence $A_1, A_2, \ldots, A_n$ and dimensions $p_0, p_1, \ldots, p_n$ where A_i is of dimension $p_{i-1} \times p_i$, determine the order of multiplication that minimizes the number of operations.

We could write a procedure that tries all possible parenthesizations. Unfortunately, the number of ways of parenthesizing an expression is very large. If there are n items, there are $n-1$ ways in which outer most pair of parentheses can placed.

$$\begin{array}{ll} & (A_1)(A_2A_3A_4\ldots A_n) \\ \text{or} & (A_1A_2)(A_3A_4\ldots A_n) \\ \text{or} & (A_1A_2A_3)(A_4\ldots A_n) \\ \ldots & \ldots \\ \text{or} & (A_1A_2A_3A_4\ldots A_{n-1})(A_n) \end{array}$$

Once we split just after the k^{th} matrix, we create two sublists to be parethesized, one with k and other with $n-k$ matrices.

$$(A_1A_2\ldots A_k)\,(A_{k+1}\ldots A_n)$$

We could consider all the ways of parenthesizing these two. Since these are independent choices, if there are L ways of parenthesizing the left sublist and R ways to parenthesize the right sublist, then the total is $L \cdot R$. This suggests the following recurrence for $P(n)$, the number of different ways of parenthesizing n items:

$$P(n) = \begin{cases} 1 & \text{if } n = 1, \\ \sum_{k=1}^{n-1} P(k)P(n-k) & \text{if } n \geq 2 \end{cases}$$

This is related to a famous function in combinatorics called the *Catalan numbers*. Catalan numbers are related the number of different binary trees on n nodes. Catalan number is given by the formula:

$$C(n) = \frac{1}{n+1}\binom{2n}{n}$$

In particular, $P(n) = C(n-1)\ C(n) \in \Omega(4^n/n^{3/2})$ The dominating term is the exponential 4^n thus $P(n)$ will grow large very quickly. So this approach is not practical.

6.4.1 Chain Matrix Multiplication-Dynamic Programming Formulation

The dynamic programming solution involves breaking up the problem into subproblems whose solutions can be combined to solve the global problem. Let $A_{i..j}$ be the result of multiplying matrices i through j. It is easy to see that $A_{i..j}$ is a $p_{i-1} \times p_j$ matrix.

$$\underset{4\times5}{A_3}\quad \underset{5\times2}{A_4}\quad \underset{2\times8}{A_5}\quad \underset{8\times7}{A_6} = \underset{4\times7}{A_{3..6}}$$

At the highest level of parenthesization, we multiply two matrices

$$A_{1..n} = A_{1..k} \cdot A_{k+1..n} \quad 1 \leq k \leq n-1.$$

The question now is: what is the optimum value of k for the split and how do we parenthesis the sub-chains $A_{1..k}$ and $A_{k+1..n}$. We can not use divide and conquer because we do not know what is the optimum k. We will have to consider all possible values of k and take the best of them. We will apply this strategy to solve the subproblems optimally.

We will store the solutions to the subproblem in a table and build the table bottom-up (why)? For $1 \leq i \leq j \leq n$, let $m[i,j]$ denote the minimum number of multiplications needed to compute $A_{i..j}$. The optimum can be described by the following recursive formulation.

- If $i = j$, there is only one matrix and thus $m[i,i] = 0$ (the diagonal entries).
- If $i < j$, the we are asking for the product $A_{i..j}$.
- This can be split by considering each k, $i \leq k < j$, as $A_{i..k}$ times $A_{k+1..j}$.

The optimum time to compute $A_{i..k}$ is $m[i,k]$ and optimum time for $A_{k+1..j}$ is in $m[k+1,j]$. Since $A_{i..k}$ is a $p_{i-1} \times p_k$ matrix and $A_{k+1..j}$ is $p_k \times p_j$ matrix, the time to multiply them is $p_{i-1} \times p_k \times p_j$. This suggests the following recursive rule:

$$\begin{aligned} m[i,i] &= 0 \\ m[i,j] &= \min_{i \leq k < j} (m[i,k] + m[k+1,j] + p_{i-1}p_k p_j) \end{aligned}$$

We do not want to calculate m entries recursively. So how should we proceed? We will fill m along the diagonals. Here is how. Set all $m[i,i] = 0$ using the base condition. Compute cost for multiplication of a sequence of 2 matrices. These are $m[1,2], m[2,3], m[3,4], \ldots, m[n-1,n]$. $m[1,2]$, for example is

$$m[1,2] = m[1,1] + m[2,2] + p_0 \cdot p_1 \cdot p_2$$

For example, for m for product of 5 matrices at this stage would be:

$m[1,1]$	$\leftarrow m[1,2]$ $\downarrow$			
	$m[2,2]$	$\leftarrow m[2,3]$ $\downarrow$		
		$m[3,3]$	$\leftarrow m[3,4]$ $\downarrow$	
			$m[4,4]$	$\leftarrow m[4,5]$ $\downarrow$
				$m[5,5]$

Next, we compute cost of multiplication for sequences of three matrices. These are $m[1,3], m[2,4], m[3,5], \ldots, m[n-2,n]$. $m[1,3]$, for example is

$$m[1,3] = \min \begin{cases} m[1,1] + m[2,3] + p_0 \cdot p_1 \cdot p_3 \\ m[1,2] + m[3,3] + p_0 \cdot p_2 \cdot p_3 \end{cases}$$

We repeat the process for sequences of four, five and higher number of matrices. The final result will end up in $m[1,n]$.

Example: Let us go through an example. We want to find the optimal multiplication order for

$$\underset{(5\times4)}{A_1} \cdot \underset{(4\times6)}{A_2} \cdot \underset{(6\times2)}{A_3} \cdot \underset{(2\times7)}{A_4} \cdot \underset{(7\times3)}{A_5}$$

We will compute the entries of the m matrix starting with the base condition. We first fill that main diagonal:

0				
	0			
		0		
			0	
				0

Next, we compute the entries in the first super diagonal, i.e., the diagonal above the main diagonal:

$$m[1,2] = m[1,1] + m[2,2] + p_0 \cdot p_1 \cdot p_2 = 0 + 0 + 5 \cdot 4 \cdot 6 = 120$$
$$m[2,3] = m[2,2] + m[3,3] + p_1 \cdot p_2 \cdot p_3 = 0 + 0 + 4 \cdot 6 \cdot 2 = 48$$
$$m[3,4] = m[3,3] + m[4,4] + p_2 \cdot p_3 \cdot p_4 = 0 + 0 + 6 \cdot 2 \cdot 7 = 84$$
$$m[4,5] = m[4,4] + m[5,5] + p_3 \cdot p_4 \cdot p_5 = 0 + 0 + 2 \cdot 7 \cdot 3 = 42$$

The matrix m now looks as follows:

0	120			
	0	48		
		0	84	
			0	42
				0

We now proceed to the second super diagonal. This time, however, we will need to try two possible values for k. For example, there are two possible splits for computing $m[1,3]$; we will choose the split that yields the minimum:

$$m[1,3] = m[1,1] + m[2,3] + p_0 \cdot p_1 \cdot p_3 == 0 + 48 + 5 \cdot 4 \cdot 2 = 88$$
$$m[1,3] = m[1,2] + m[3,3] + p_0 \cdot p_2 \cdot p_3 = 120 + 0 + 5 \cdot 6 \cdot 2 = 180$$
$$\text{the minimum } m[1,3] = 88 \text{ occurs with } k = 1$$

Similarly, for $m[2,4]$ and $m[3,5]$:

$$m[2,4] = m[2,2] + m[3,4] + p_1 \cdot p_2 \cdot p_4 = 0 + 84 + 4 \cdot 6 \cdot 7 = 252$$
$$m[2,4] = m[2,3] + m[4,4] + p_1 \cdot p_3 \cdot p_4 = 48 + 0 + 4 \cdot 2 \cdot 7 = 104$$
$$\text{minimum } m[2,4] = 104 \text{ at } k = 3$$

$$m[3,5] = m[3,3] + m[4,5] + p_2 \cdot p_3 \cdot p_5 = 0 + 42 + 6 \cdot 2 \cdot 3 = 78$$
$$m[3,5] = m[3,4] + m[5,5] + p_2 \cdot p_4 \cdot p_5 = 84 + 0 + 6 \cdot 7 \cdot 3 = 210$$
$$\text{minimum } m[3,5] = 78 \text{ at } k = 3$$

With the second super diagonal computed, the m matrix looks as follow:

0	120	88		
	0	48	104	
		0	84	78
			0	42
				0

We repeat the process for the remaining diagonals. However, the number of possible splits (values of k) increases:

$$m[1,4] = m[1,1] + m[2,4] + p_0 \cdot p_1 \cdot p_4 = 0 + 104 + 5 \cdot 4 \cdot 7 = 244$$
$$m[1,4] = m[1,2] + m[3,4] + p_0 \cdot p_2 \cdot p_4 = 120 + 84 + 5 \cdot 6 \cdot 7 = 414$$
$$m[1,4] = m[1,3] + m[4,4] + p_0 \cdot p_3 \cdot p_4 = 88 + 0 + 5 \cdot 2 \cdot 7 = 158$$
$$\text{minimum } m[1,4] = 158 \text{ at } k = 3$$

$$m[2,5] = m[2,2] + m[3,5] + p_1 \cdot p_2 \cdot p_5 = 0 + 78 + 4 \cdot 6 \cdot 3 = 150$$
$$m[2,5] = m[2,3] + m[4,5] + p_1 \cdot p_3 \cdot p_5 = 48 + 42 + 4 \cdot 2 \cdot 3 = 114$$
$$m[2,5] = m[2,4] + m[5,5] + p_1 \cdot p_4 \cdot p_5 = 104 + 0 + 4 \cdot 7 \cdot 3 = 188$$
$$\text{minimum } m[2,5] = 114 \text{ at } k = 3$$

The matrix m at this stage is:

0	120	88	158	
	0	48	104	114
		0	84	78
			0	42
				0

That leaves the $m[1,5]$ which can now be computed:

$$
\begin{aligned}
m[1,5] &= m[1,1] + m[2,5] + p_0 \cdot p_1 \cdot p_5 = 0 + 114 + 5 \cdot 4 \cdot 3 = 174 \\
m[1,5] &= m[1,2] + m[3,5] + p_0 \cdot p_2 \cdot p_5 = 120 + 78 + 5 \cdot 6 \cdot 3 = 288 \\
m[1,5] &= m[1,3] + m[4,5] + p_0 \cdot p_3 \cdot p_5 = 88 + 42 + 5 \cdot 2 \cdot 3 = 160 \\
m[1,5] &= m[1,4] + m[5,5] + p_0 \cdot p_4 \cdot p_5 = 158 + 0 + 5 \cdot 7 \cdot 3 = 263 \\
&\text{minimum } m[1,5] = 160 \text{ at } k = 3
\end{aligned}
$$

We thus have the final cost matrix.

0	120	88	158	***160***
0	0	48	104	114
0	0	0	84	78
0	0	0	0	42
0	0	0	0	0

Here is the order in which m entries are calculated

0	1	5	8	10
0	0	2	6	9
0	0	0	3	7
0	0	0	0	4
0	0	0	0	0

and the split k values that led to a minimum $m[i,j]$ value

0	1	1	3	3
	0	2	3	3
		0	3	3
			0	4
				0

Based on the computation, the minimum cost for multiplying the five matrices is 160 and the optimal order for multiplication is

$$((A_1(A_2A_3))(A_4A_5))$$

This can be represented as a binary tree

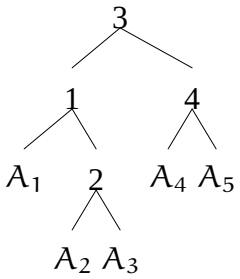


Figure 6.2: Optimum matrix multiplication order for the five matrices example

Here is the dynamic programming based algorithm for computing the minimum cost of chain matrix multiplication.

```
MATRIX-CHAIN(p, N)
 1  for i = 1, N
 2  do m[i, i] ← 0
 3  for L = 2, N
 4  do
 5     for i = 1, n − L + 1
 6     do j ← i + L − 1
 7        m[i, j] ← ∞
 8        for k = 1, j − 1
 9        do t ← m[i, k] + m[k + 1, j] + p_{i−1} · p_k · p_j
10           if (t < m[i, j])
11              then m[i, j] ← t;   s[i, j] ← k
```

Analysis: There are three nested loops. Each loop executes a maximum n times. Total time is thus $\Theta(n^3)$.

The s matrix stores the values k. The s matrix can be used to extracting the order in which matrices are to be multiplied. Here is the algorithm that caries out the matrix multiplication to compute $A_{i..j}$:

```
MULTIPLY(i, j)
1  if (i = j)
2     then return A[i]
3     else k ← s[i, j]
4          X ← MULTIPLY(i, k)
5          Y ← MULTIPLY(k + 1, j)
6          return X · Y
```

6.5 0/1 Knapsack Problem

A thief goes into a jewelry store to steal jewelry items. He has a knapsack (a bag) that he would like to fill up. The bag has a limit on the total weight of the objects placed in it. If the total weight exceeds the limit, the bag would tear open. The value of of the jewelry items varies for cheap to expensive. The thief's goal is to put items in the bag such that the value of the items is maximized and the weight of the items does not exceed the weight limit of the bag. Another limitation is that an item can either be put in the bag or not - fractional items are not allowed. The problem is: what jewelry should the thief choose that satisfy the constraints?

Formally, the problem can be stated as follows: Given a knapsack with maximum capacity W, and a set S consisting of n items Each item i has some weight w_i and value value v_i (all w_i , v_i and W are integer values) How to pack the knapsack to achieve maximum total value of packed items? For example, consider the following scenario:

Item i	Weight w_i	Value v_i
1	2	3
2	3	4
3	4	5
4	5	8
5	9	10

Figure 6.3: Knapsack can hold $W = 20$

The knapsack problem belongs to the domain of optimization problems. Mathematically, the problem is

$$\text{maximize} \sum_{i \in T} v_i$$

$$\text{subject to} \sum_{i \in T} w_i \leq W$$

The problem is called a *"0-1"* problem, because each item must be entirely accepted or rejected. How do we solve the problem. We could try the brute-force solution:

- Since there are n items, there are 2^n possible combinations of the items (an item either chosen or not).
- We go through all combinations and find the one with the most total value and with total weight less or equal to W

Clearly, the running time of such a brute-force algorithm will be $O(2^n)$. Can we do better? The answer is "yes", with an algorithm based on dynamic programming Let us recap the steps in the dynamic programming strategy:

1. **Simple Subproblems**: We should be able to break the original problem to smaller subproblems that have the same structure
2. **Principle of Optimality**: Recursively define the value of an optimal solution. Express the solution of the original problem in terms of optimal solutions for smaller problems.
3. **Bottom-up computation**: Compute the value of an optimal solution in a bottom-up fashion by using a table structure.
4. **Construction of optimal solution**: Construct an optimal solution from computed information.

Let us try this: If items are labelled $1, 2, \ldots, n$, then a subproblem would be to find an optimal solution for

$$S_k = \text{items labelled } 1, 2, \ldots, k$$

This is a valid subproblem definition. The question is: can we describe the final solution S_n in terms of subproblems S_k? Unfortunately, we cannot do that. Here is why. Consider the optimal solution if we can choose items 1 through 4 only.

Solution S_4

- Items chosen are $1, 2, 3, 4$
- Total weight: $2 + 3 + 4 + 5 = 14$
- Total value: $3 + 4 + 5 + 8 = 20$

Item	w_i	v_i
1	2	3
2	3	4
3	4	5
4	5	8
5	9	10

Now consider the optimal solution when items 1 through 5 are available.

Solution S_5

- Items chosen are $1, 3, 4, 5$
- Total weight: $2 + 4 + 5 + 9 = 20$
- Total value: $3 + 5 + 8 + 10 = 26$

S_4 is not part of S_5!!

Item	w_i	v_i
1	2	3
2	3	4
3	4	5
4	5	8
5	9	10

The solution for S_4 ***is not part of the solution for*** S_5. So our definition of a subproblem is flawed and we need another one.

6.5.1 0/1 Knapsack Problem: Dynamic Programming Approach

For each $i \leq n$ and each $w \leq W$, solve the knapsack problem for the first i objects when the capacity is w. Why will this work? Because solutions to larger subproblems can be built up easily from solutions to smaller ones. We construct a matrix $V[0 \ldots n, 0 \ldots W]$. For $1 \leq i \leq n$, and $0 \leq j \leq W$, $V[i, j]$ will store the maximum value of any set of objects $\{1, 2, \ldots, i\}$ that can fit into a knapsack of weight j. $V[n, W]$ will contain the maximum value of all n objects that can fit into the entire knapsack of weight W.

To compute entries of V we will imply an inductive approach. As a basis, $V[0, j] = 0$ for $0 \leq j \leq W$ since if we have no items then we have no value. We consider two cases:

Leave object i**:** If we choose to not take object i, then the optimal value will come about by considering how to fill a knapsack of size j with the remaining objects $\{1, 2, \ldots, i-1\}$. This is just $V[i-1, j]$.

Take object i**:** If we take object i, then we gain a value of v_i. But we use up w_i of our capacity. With the remaining $j - w_i$ capacity in the knapsack, we can fill it in the best possible way with objects $\{1, 2, \ldots, i-1\}$. This is $v_i + V[i-1, j-w_i]$. This is only possible if $w_i \leq j$.

This leads to the following recursive formulation:

$$
\begin{aligned}
V[i, j] &= -\infty \qquad \text{if } j < 0 \\
V[0, j] &= 0 \qquad \text{if } j \geq 0 \\
V[i, j] &= \begin{cases} V[i-1, j] & \text{if } w_i > j \\ \max\{V[i-1, j],\ \ v_i + V[i-1, j-w_i]\} & \text{if } w_i \leq j \end{cases}
\end{aligned}
$$

A naive evaluation of this recursive definition is exponential. So, as usual, we avoid re-computation by making a table.

Example: The maximum weight the knapsack can hold is W is 11. There are five items to choose from. Their weights and values are presented in the following table:

Weight limit (j):	0	1	2	3	4	5	6	7	8	9	10	11
$w_1 = 1\ v_1 = 1$												
$w_2 = 2\ v_2 = 6$												
$w_3 = 5\ v_3 = 18$												
$w_4 = 6\ v_4 = 22$												
$w_5 = 7\ v_5 = 28$												

The $[i, j]$ entry here will be $V[i, j]$, the best value obtainable using the first i rows of items if the maximum capacity were j. We begin by initializating and first row.

Weight limit:	0	1	2	3	4	5	6	7	8	9	10	11
$w_1 = 1\ v_1 = 1$	0	1	1	1	1	1	1	1	1	1	1	1
$w_2 = 2\ v_2 = 6$	0											
$w_3 = 5\ v_3 = 18$	0											
$w_4 = 6\ v_4 = 22$	0											
$w_5 = 7\ v_5 = 28$	0											

Recall that we take $V[i, j]$ to be 0 if either i or j is ≤ 0. We then proceed to fill in top-down, left-to-right always using

$$V[i, j] = \max\{V[i-1, j],\ \ v_i + V[i-1, j-w_i]\}$$

Weight limit:	0	1	2	3	4	5	6	7	8	9	10	11
$w_1 = 1\ v_1 = 1$	0	1	1	1	1	1	1	1	1	1	1	1
$w_2 = 2\ v_2 = 6$	0	1	6	7	7	7	7	7	7	7	7	7
$w_3 = 5\ v_3 = 18$	0											
$w_4 = 6\ v_4 = 22$	0											
$w_5 = 7\ v_5 = 28$	0											

Weight limit:	0	1	2	3	4	5	6	7	8	9	10	11
$w_1 = 1\ v_1 = 1$	0	1	1	1	1	1	1	1	1	1	1	1
$w_2 = 2\ v_2 = 6$	0	1	6	7	7	7	7	7	7	7	7	7
$w_3 = 5\ v_3 = 18$	0	1	6	7	7	18	19	*24*	25	25	25	25
$w_4 = 6\ v_4 = 22$	0											
$w_5 = 7\ v_5 = 28$	0											

As an illustration, the value of $V[3, 7]$ was computed as follows:

$$\begin{aligned} V[3,7] &= \max\{V[3-1,7],\ \ v_3 + V[3-1,7-w_3]\} \\ &= \max\{V[2,7],\ \ 18 + V[2,7-5]\} \\ &= \max\{7,\ \ 18+6\} \\ &= 24 \end{aligned}$$

Weight limit:	0	1	2	3	4	5	6	7	8	9	10	11
$w_1 = 1\ v_1 = 1$	0	1	1	1	1	1	1	1	1	1	1	1
$w_2 = 2\ v_2 = 6$	0	1	6	7	7	7	7	7	7	7	7	7
$w_3 = 5\ v_3 = 18$	0	1	6	7	7	18	19	24	25	25	25	25
$w_4 = 6\ v_4 = 22$	0	1	6	7	7	18	22	24	28	29	29	40
$w_5 = 7\ v_5 = 28$	0											

Finally, we have

Weight limit:	0	1	2	3	4	5	6	7	8	9	10	11
$w_1 = 1\ v_1 = 1$	0	1	1	1	1	1	1	1	1	1	1	1
$w_2 = 2\ v_2 = 6$	0	1	6	7	7	7	7	7	7	7	7	7
$w_3 = 5\ v_3 = 18$	0	1	6	7	7	18	19	24	25	25	25	25
$w_4 = 6\ v_4 = 22$	0	1	6	7	7	18	22	24	28	29	29	40
$w_5 = 7\ v_5 = 28$	0	1	6	7	7	18	22	28	29	34	35	*40*

The maximum value of items in the knapsack is 40, the bottom-right entry). The dynamic programming approach can now be coded as the following algorithm:

```
KNAPSACK(n, W)
1  for w = 0, W
2  do V[0, w] ← 0
3  for i = 0, n
4  do V[i, 0] ← 0
5     for w = 0, W
6     do if (w_i ≤ w & v_i + V[i − 1, w − w_i] > V[i − 1, w])
7           then V[i, w] ← v_i + V[i − 1, w − w_i]
8           else  V[i, w] ← V[i − 1, w]
```

The time complexity is clearly $O(n \cdot W)$. It must be cautioned that as n and W get large, both time and space complexity become significant.

Constructing the Optimal Solution

The algorithm for computing $V[i, j]$ does not keep record of which subset of items gives the optimal solution. To compute the actual subset, we can add an auxiliary boolean array $keep[i, j]$ which is 1 if we decide to take the i^{th} item and 0 otherwise. We will use all the values $keep[i, j]$ to determine the optimal subset T of items to put in the knapsack as follows:

- If $keep[n, W]$ is 1, then $n \in T$. We can now repeat this argument for $keep[n - 1, W - w_n]$.
- If $kee[n, W]$ is 0, the $n \notin T$ and we repeat the argument for $keep[n - 1, W]$.

We will add this to the knapsack algorithm:

```
KNAPSACK(n, W)
 1  for w = 0, W
 2  do V[0, w] ← 0
 3  for i = 0, n
 4  do V[i, 0] ← 0
 5     for w = 0, W
 6     do if (w_i ≤ w & v_i + V[i − 1, w − w_i] > V[i − 1, w])
 7           then V[i, w] ← v_i + V[i − 1, w − w_i]; keep[i, w] ← 1
 8           else V[i, w] ← V[i − 1, w]; keep[i, w] ← 0
 9  // output the selected items
10  k ← W
11  for i = n downto 1
12  do if (keep[i, k] = 1)
13        then output i
14             k ← k − w_i
```

Here is the keep matrix for the example problem.

Weight limit:	0	1	2	3	4	5	6	7	8	9	10	11
$w_1 = 1\ v_1 = 1$	0	1	1	1	1	1	1	1	1	1	1	1
$w_2 = 2\ v_2 = 6$	0	0	1	1	1	1	1	1	1	1	1	1
$w_3 = 5\ v_3 = 18$	0	0	0	0	0	**1**	1	1	1	1	1	1
$w_4 = 6\ v_4 = 22$	0	0	0	0	0	0	1	0	1	1	1	**1**
$w_5 = 7\ v_5 = 28$	0	0	0	0	0	0	0	1	1	1	1	0

When the item selection algorithm is applied, the selected items are 4 and 3. This is indicated by the boxed entries in the table above.

Chapter 7

Greedy Algorithms

An *optimization problem* is one in which you want to find, not just a solution, but the best solution. Search techniques look at many possible solutions. E.g. dynamic programming or backtrack search. A " greedy algorithm" sometimes works well for optimization problems

A greedy algorithm works in phases. At each phase:

- You take the best you can get right now, without regard for future consequences.
- You hope that by choosing a local optimum at each step, you will end up at a global optimum.

For some problems, greedy approach always gets optimum. For others, greedy finds good, but not always best. If so, it is called a greedy heuristic, or approximation. For still others, greedy approach can do very poorly.

7.1 Example: Counting Money

Suppose you want to count out a certain amount of money, using the fewest possible bills (notes) and coins. A greedy algorithm to do this would be: at each step, take the largest possible note or coin that does not overshoot.

```
while (N > 0){
    give largest denomination coin ≤ N
    reduce N by value of that coin
}
```

Consider the currency in U.S.A. There are paper notes for one dollar, five dollars, ten dollars, twenty dollars, fifty dollars and hundred dollars. The notes are also called "bills". The coins are one cent, five cents (called a "nickle"), ten cents (called a "dime") and twenty five cents (a "quarter"). In Pakistan, the currency notes are five rupees, ten rupees, fifty rupees, hundred rupees, five hundred rupees and thousand

rupees. The coins are one rupee and two rupees. Suppose you are asked to give change of $6.39 (six dollars and thirty nine cents), you can choose:

- a $5 note
- a $1 note to make $6
- a 25 cents coin (quarter), to make $6.25
- a 10 cents coin (dime), to make $6.35
- four 1 cents coins, to make $6.39

Notice how we started with the highest note, $5, before moving to the next lower denomination.

Formally, the Coin Change problem is: Given k denominations $d_1, d_2, \ldots, d_k$ and given N, find a way of writing

$$N = i_1 d_1 + i_2 d_2 + \cdots + i_k d_k$$

such that

$$i_1 + i_2 + \cdots + i_k \text{ is minimized.}$$

The "size" of problem is k.

The greedy strategy works for the coin change problem but not always. Here is an example where it fails. Suppose, in some (fictional) monetary system, " krons" come in 1 kron, 7 kron, and 10 kron coins Using a greedy algorithm to count out 15 krons, you would get A 10 kron piece Five 1 kron pieces, for a total of 15 krons This requires six coins. A better solution, however, would be to use two 7 kron pieces and one 1 kron piece This only requires three coins The greedy algorithm results in a solution, but not in an optimal solution

The greedy approach gives us an optimal solution when the coins are all powers of a fixed denomination.

$$N = i_0 D^0 + i_1 D^1 + i_2 D^2 + \cdots + i_k D^k$$

Note that this is N represented in based D. U.S.A coins are multiples of 5: 5 cents, 10 cents and 25 cents.

7.1.1 Making Change: Dynamic Programming Solution

The general coin change problem can be solved using Dynamic Programming. Set up a Table, $C[1..k, 0..N]$ in which the rows denote available denominations, d_i; $(1 \leq i \leq k)$ and columns denote the amount from $0 \ldots N$ units, $(0 \leq j \leq N)$. $C[i, j]$ denotes the minimum number of coins, required to pay an amount j using only coins of denominations 1 to i. C[k, N] is the solution required.

To pay an amount j units, using coins of denominations 1 to i, we have two choices:

1. either chose NOT to use any coins of denomination i,
2. or chose at least one coin of denomination i, and also pay the amount $(j - d_i)$.

To pay $(j - d_i)$ units it takes $C[i, j - d_i]$ coins. Thus,

$$C[i, j] = 1 + C[i, j - d_i]$$

Since we want to minimize the number of coins used,

$$C[i, j] = \min(C[i - 1, j], 1 + C[i, j - d_i])$$

Here is the dynamic programming based algorithm for the coin change problem.

```
COINS(N)
 1  d[1..n] = {1, 4, 6}   // (coinage, for example)
 2  for i = 1 to k
 3  do c[i, 0] ← 0
 4  for i = 1 to k
 5  do for j = 1 to N
 6      do if (i = 1 & j < d[i])
 7            then c[i, j] ← ∞
 8            else if (i = 1)
 9                    then c[i, j] ← 1 + c[1, j − d[1]]
10                    else if (j < d[i])
11                            then c[i, j] ← c[i − 1, j]
12                            else c[i, j] ← min (c[i − 1, j], 1 + c[i, j − d[i]])
13  return c[k, N]
```

7.1.2 Complexity of Coin Change Algorithm

Greedy algorithm (non-optimal) takes $O(k)$ time. Dynamic Programming takes $O(kN)$ time. Note that N can be as large as 2^k so the dynamic programming algorithm is really exponential in k.

7.2 Greedy Algorithm: Huffman Encoding

The Huffman codes provide a method of encoding data efficiently. Normally, when characters are coded using standard codes like ASCII. Each character is represented by a fixed-length codeword of bits, e.g., 8 bits per character. Fixed-length codes are popular because it is very easy to break up a string into its individual characters, and to access individual characters and substrings by direct indexing. However, fixed-length codes may not be he most efficient from the perspective of minimizing the total quantity of data.

Consider the string " abacdaacac". if the string is coded with ASCII codes, the message length would be $10 \times 8 = 80$ bits. We will see shortly that the same string encoded with a variable length Huffman encoding scheme will produce a shorter message.

7.2.1 Huffman Encoding Algorithm

Here is how the Huffman encoding algorithm works. Given a message string, determine the frequency of occurrence (relative probability) of each character in the message. This can be done by parsing the message and counting how many time each character (or symbol) appears. The probability is the number of occurrence of a character divided by the total characters in the message. The frequencies and probabilities for the example string " abacdaacac" are

character	a	b	c	d
frequency	5	1	3	1
probability	0.5	0.1	0.3	0.1

Next, create binary tree (leaf) node for each symbol (character) that occurs with nonzero frequency Set node weight equal to the frequency of the symbol. Now comes the greedy part: Find two nodes with smallest frequency. Create a new node with these two nodes as children, and with weight equal to the sum of the weights of the two children. Continue until we have a single tree.

Finding two nodes with the smallest frequency can be done efficiently by placing the nodes in a heap-based priority queue. The min-heap is maintained using the frequencies. When a new node is created by combining two nodes, the new node is placed in the priority queue. Here is the Huffman tree building algorithm.

```
HUFFMAN(N, symbol[1..N], freq[1..N])
 1  for i = 1 to N
 2  do t ← TreeNode(symbol[i], freq[i])
 3     pq.insert(t, freq[i]) // priority queue
 4  for i = 1 to N − 1
 5  do x = pq.remove(); y = pq.remove()
 6     z ← new TreeNode
 7     z.left ← x; z.right ← y
 8     z.freq ← x.freq + y.freq
 9     pq.insert(z, z.freq);
10  return pq.remove(); // root
```

Figure 7.1 shows the tree built for the example message "abacdaacac"

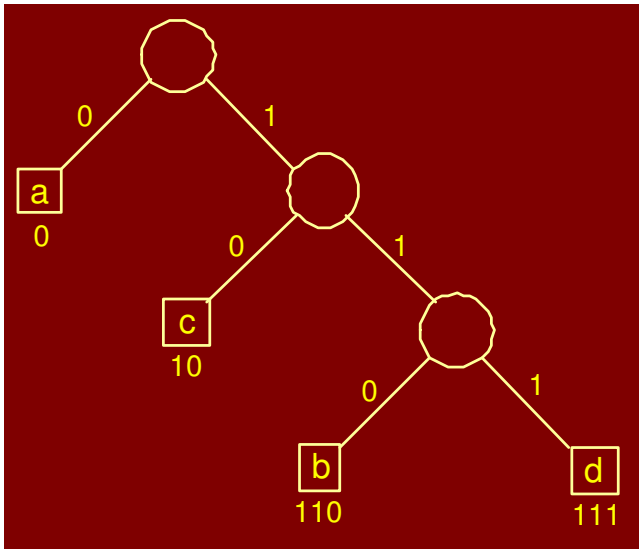


Figure 7.1: Huffman binary tree for the string "abacdaacac"

Prefix Property:

The codewords assigned to characters by the Huffman algorithm have the property that no codeword is a prefix of any other:

character	a	b	c	d
frequency	5	1	3	1
probability	0.5	0.1	0.3	0.1
codeword	0	110	10	111

The prefix property is evident by the fact that codewords are leaves of the binary tree. Decoding a prefix code is simple. We traverse the root to the leaf letting the input 0 or 1 tell us which branch to take.

Expected encoding length:

If a string of n characters over the alphabet $C = \{a, b, c, d\}$ is encoded using 8-bit ASCII, the length of encoded string is $8n$. For example, the string "abacdaacac" will require $8 \times 10 = 80$ bits. The same string encoded with Huffman codes will yield

a	b	a	c	d	a	a	c	a	c
0	110	0	10	111	0	0	10	0	10

This is just 17 bits, a significant saving!. For a string of n characters over this alphabet, the expected encoded string length is

$$n(0.5 \cdot 1 + 0.1 \cdot 3 + 0.3 \cdot 2 + 0.1 \cdot 3) = 1.7n$$

In general, let $p(x)$ be the probability of occurrence of a character, and let $d_T(x)$ denote the length of the codeword relative to some prefix tree T. The expected number of bits needed to encode a text with n characters is given by

$$B(T) = n \sum_{x \in C} p(x) d_T(x)$$

7.2.2 Huffman Encoding: Correctness

Huffman algorithm uses a greedy approach to generate a prefix code T that minimizes the expected length $B(T)$ of the encoded string. In other words, Huffman algorithm generates an optimum prefix code. The question that remains is that *why is the algorithm correct?*

Recall that the cost of any encoding tree T is

$$B(T) = n \sum_{x \in C} p(x) d_T(x)$$

Our approach to prove the correctness of Huffman Encoding will be to show that any tree that differs from the one constructed by Huffman algorithm can be converted into one that is equal to Huffman's tree without increasing its costs. Note that the binary tree constructed by Huffman algorithm is a full binary tree.

Claim:

Consider two characters x and y with the smallest probabilities. Then there is optimal code tree in which these two characters are siblings at the maximum depth in the tree.

Proof:

Let T be any optimal prefix code tree with two siblings b and c at the maximum depth of the tree. Such a tree is shown in Figure 7.2Assume without loss of generality that

$$p(b) \leq p(c) \qquad \text{and} \qquad p(x) \leq p(y)$$

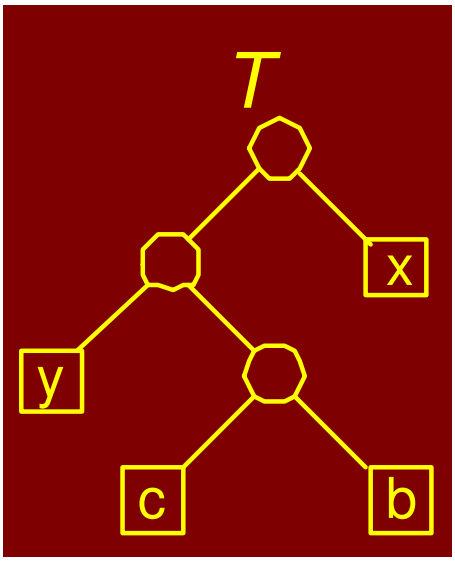


Figure 7.2: Optimal prefix code tree T

Since x and y have the two smallest probabilities (we claimed this), it follows that

$$p(x) \leq p(b) \qquad \text{and} \qquad p(y) \leq p(c)$$

Since b and c are at the deepest level of the tree, we know that

$$d(b) \geq d(x) \qquad \text{and} \qquad d(c) \geq d(y) \qquad (d \text{ is the depth})$$

Thus we have

$$p(b) - p(x) \geq 0$$

and

$$d(b) - d(x) \geq 0$$

Hence their product is non-negative. That is,

$$(p(b) - p(x)) \cdot (d(b) - d(x)) \geq 0$$

Now swap the positions of x and b in the tree

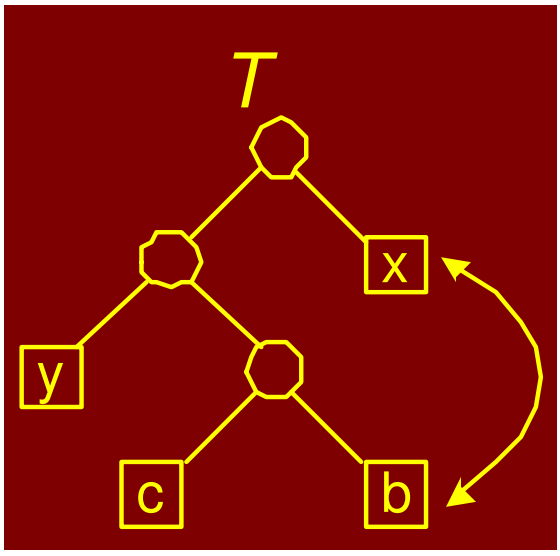


Figure 7.3: Swap x and b in tree prefix tree T

This results in a new tree T'

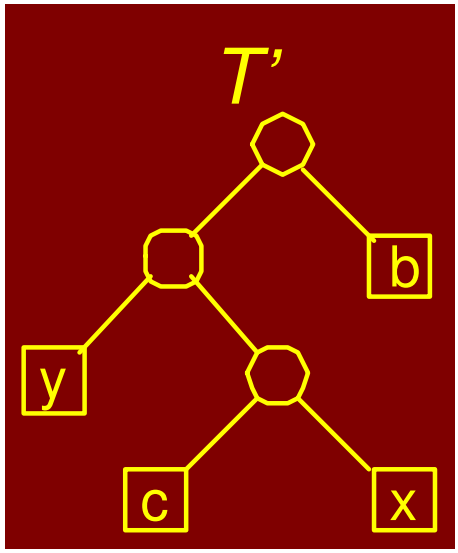


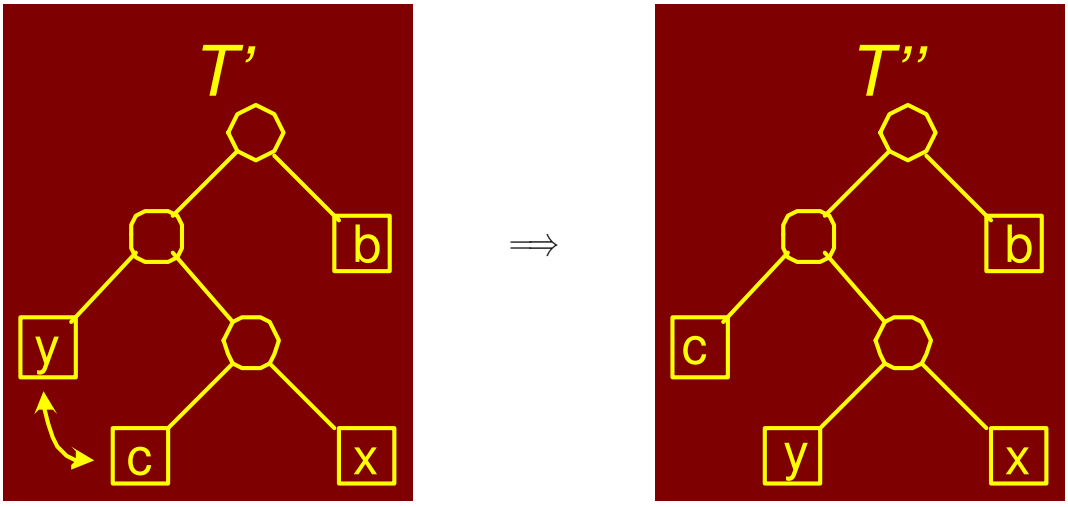
Figure 7.4: Prefix tree T' after x and b are swapped

Let's see how the cost changes. The cost of T' is

$$\begin{aligned} B(T') &= B(T) - p(x)d(x) + p(x)d(b) - p(b)d(b) + p(b)d(x) \\ &= B(T) + p(x)[d(b) - d(x)] - p(b)[d(b) - d(x)] \\ &= B(T) - (p(b) - p(x))(d(b) - d(x)) \\ &\leq B(T) \quad \text{because } (p(b) - p(x))(d(b) - d(x)) \geq 0 \end{aligned}$$

Thus the cost does not increase, implying that T' is an optimal tree.

By switching y with c we get the tree T''. Using a similar argument, we can show that T'' is also optimal.



The final tree T'' satisfies the claim we made earlier, i.e., consider two characters x and y with the

smallest probabilities. Then there is optimal code tree in which these two characters are siblings at the maximum depth in the tree.

The claim we just proved asserts that the first step of Huffman algorithm is the proper one to perform (the greedy step). The complete proof of correctness for Huffman algorithm follows by induction on n.

Claim: Huffman algorithm produces the optimal prefix code tree.

Proof: The proof is by induction on n, the number of characters. For the basis case, $n = 1$, the tree consists of a single leaf node, which is obviously optimal. We want to show it is true with exactly n characters.

Suppose we have exactly n characters. The previous claim states that two characters x and y with the lowest probability will be siblings at the lowest level of the tree. Remove x and y and replace them with a new character z whose probability is $p(z) = p(x) + p(y)$. Thus $n - 1$ characters remain.

Consider any prefix code tree T made with this new set of $n - 1$ characters. We can convert T into prefix code tree T' for the original set of n characters by replacing z with nodes x and y. This is essentially undoing the operation where x and y were removed an replaced by z. The cost of the new tree T' is

$$\begin{aligned} B(T') &= B(T) - p(z)d(z) + p(x)[d(z) + 1] + p(y)[d(z) + 1] \\ &= B(T) - (p(x) + p(y))d(z) + (p(x) + p(y))[d(z) + 1] \\ &= B(T) + (p(x) + p(y))[d(z) + 1 - d(z)] \\ &= B(T) + p(x) + p(y) \end{aligned}$$

The cost changes but the change depends in no way on the structure of the tree T (T is for $n - 1$ characters). Therefore, to minimize the cost of the final tree T', we need to build the tree T on $n - 1$ characters optimally. By induction, this is exactly what Huffman algorithm does. Thus the final tree is optimal.

7.3 Activity Selection

The activity scheduling is a simple scheduling problem for which the greedy algorithm approach provides an optimal solution. We are given a set $S = \{a_1, a_2, \ldots, a_n\}$ of n activities that are to be scheduled to use some resource. Each activity a_i must be started at a given start time s_i and ends at a given finish time f_i.

An example is that a number of lectures are to be given in a single lecture hall. The start and end times have be set up in advance. The lectures are to be scheduled. There is only one resource (e.g., lecture hall) Some start and finish times may overlap. Therefore, not all requests can be honored. We say that two activities a_i and a_j are non-interfering if their start-finish intervals do not overlap. I.e, $(s_i, f_i) \cap (s_j, f_j) = \varnothing$. The activity selection problem is to select a maximum-size set of mutually non-interfering activities for use of the resource.

So how do we schedule the largest number of activities on the resource? Intuitively, we do not like long

activities Because they occupy the resource and keep us from honoring other requests. This suggests the greedy strategy: Repeatedly select the activity with the smallest duration $(f_i - s_i)$ and schedule it, provided that it does not interfere with any previously scheduled activities. Unfortunately, this turns out to be non-optimal

Here is a simple greedy algorithm that works: Sort the activities by their finish times. Select the activity that finishes first and schedule it. Then, among all activities that do not interfere with this first job, schedule the one that finishes first, and so on.

```
SCHEDULE(a[1..N])
1   sort a[1..N] by finish times
2   A ← {a[1]};   // schedule activity 1 first
3   prev ← 1;   // most recently scheduled
4   for i = 2 to N
5   do if (a[i].start ≥ a[prev].finish)
6         then A ← A ∪ a[i];  prev ← i
```

Figure 7.5 shows an example of the activity scheduling algorithm. There are eight activities to be scheduled. Each is represented by a rectangle. The width of a rectangle indicates the duration of an activity. The eight activities are sorted by their finish times. The eight rectangles are arranged to show the sorted order. Activity a_1 is scheduled first. Activities a_2 and a_3 interfere with a_1 so they ar not selected. The next to be selected is a_4. Activities a_5 and a_6 interfere with a_4 so are not chosen. The last one to be chosen is a_7. Eventually, only three out of the eight are scheduled.

Timing analysis: Time is dominated by sorting of the activities by finish times. Thus the complexity is $O(N \log N)$.

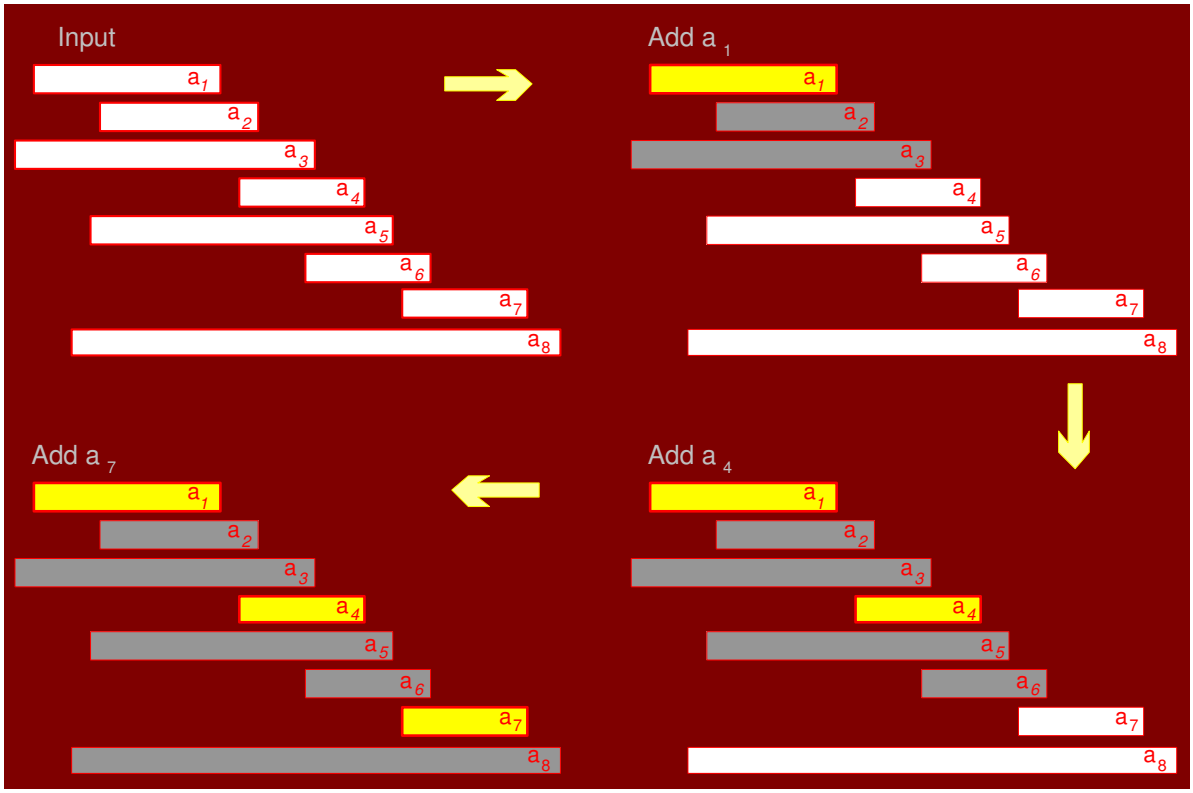


Figure 7.5: Example of greedy activity scheduling algorithm

7.3.1 Correctness of Greedy Activity Selection

Our proof of correctness is based on showing that the first choice made by the algorithm is the best possible. And then using induction to show that the algorithm is globally optimal. The proof structure is noteworthy because many greedy correctness proofs are based on the same idea: Show that any other solution can be converted into the greedy solution without increasing the cost.

Claim:

Let $S = \{a_1, a_2, \ldots, a_n\}$ of n activities, sorted by increasing finish times, that are to be scheduled to use some resource. Then there is an optimal schedule in which activity a_1 is scheduled first.

Proof:

Let A be an optimal schedule. Let x be the activity in A with the smallest finish time. If $x = a_1$ then we are done. Otherwise, we form a new schedule A' by replacing x with activity a_1.

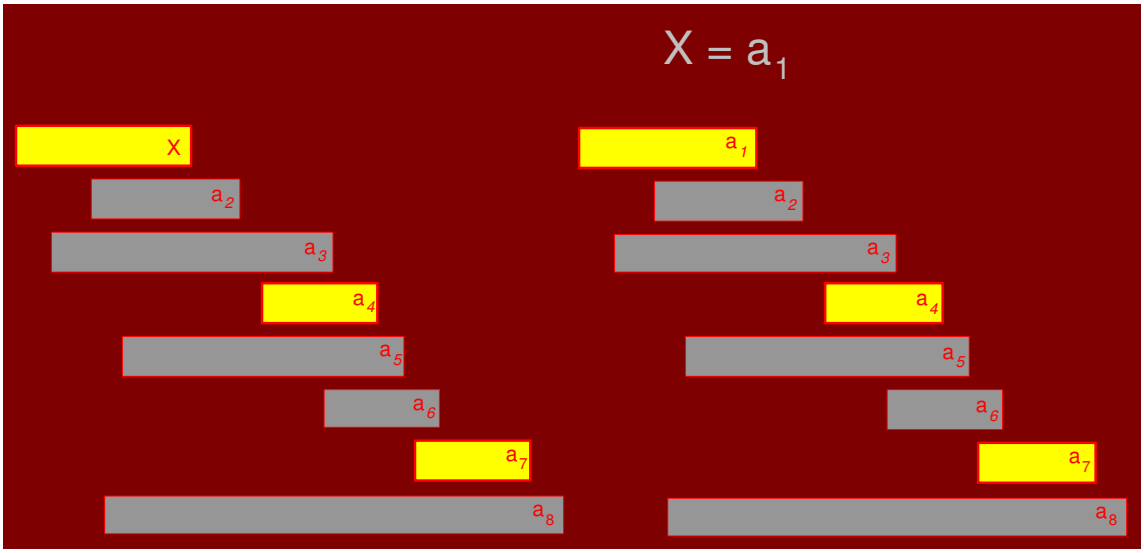


Figure 7.6: Activity $X = a_1$

We claim that $A' = A - \{x\} \cup \{a_1\}$ is a feasible schedule, i.e., it has no interfering activities. This because $A - \{x\}$ cannot have any other activities that start before x finishes, since otherwise, these activities will interfere with x.

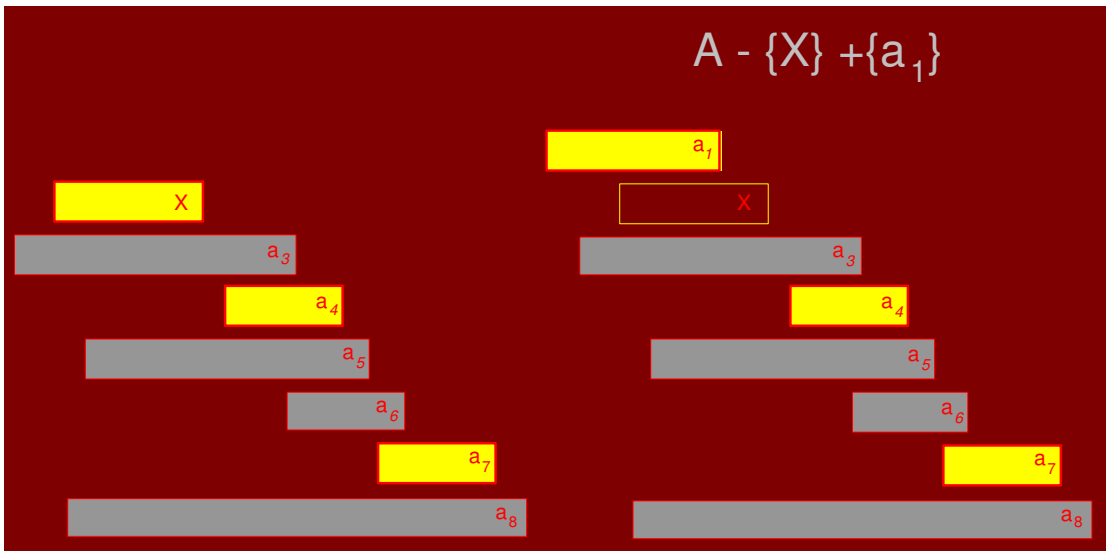


Figure 7.7: New schedule A' by replacing x with ctivity a_1.

Since a_1 is by definition the first activity to finish, it has an earlier finish time than x. Thus a_1 cannot interfere with any of the activities in $A - \{x\}$. Thus, A' is a feasible schedule. Clearly A and A' contain the same number of activities implying that A' is also optimal.

Claim:

The greedy algorithm gives an optimal solution to the activity scheduling problem.

Proof:

The proof is by induction on the number of activities. For the basis case, if there are no activities, then the greedy algorithm is trivially optimal. For the induction step, let us assume that the greedy algorithm is optimal on any set of activities of size strictly smaller than $|S|$ and we prove the result for S. Let S' be the set of activities that do not interfere with activity a_1, That is

$$S' = \{a_i \in S | s_i \geq f_1\}$$

Any solution for S' can be made into a solution for S by simply adding activity a_1, and vice versa. Activity a_1 is in the optimal schedule (by the above previous claim). It follows that to produce an optimal schedule for the overall problem, we should first schedule a_1 and then append the optimal schedule for S'. But by induction (since $|S'| < |S|$), this exactly what the greedy algorithm does.

7.4 Fractional Knapsack Problem

Earlier we saw the 0-1 knapsack problem. A knapsack can only carry W total weight. There are n items; the i^{th} item is worth v_i and weighs w_i. Items can either be put in the knapsack or not. The goal was to maximize the value of items without exceeding the total weight limit of W. In contrast, in the fractional knapsack problem, the setup is exactly the same. But, one is allowed to take *fraction* of an item for a fraction of the weight and fraction of value. The 0-1 knapsack problem is hard to solve. However, there is a simple and efficient greedy algorithm for the fractional knapsack problem.

Let $\rho_i = v_i/w_i$ denote the *value per unit weight* ratio for item i. Sort the items in decreasing order of ρ_i. Add items in decreasing order of ρ_i. If the item fits, we take it all. At some point there is an item that does not fit in the remaining space. We take as much of this item as possible thus filling the knapsack completely.

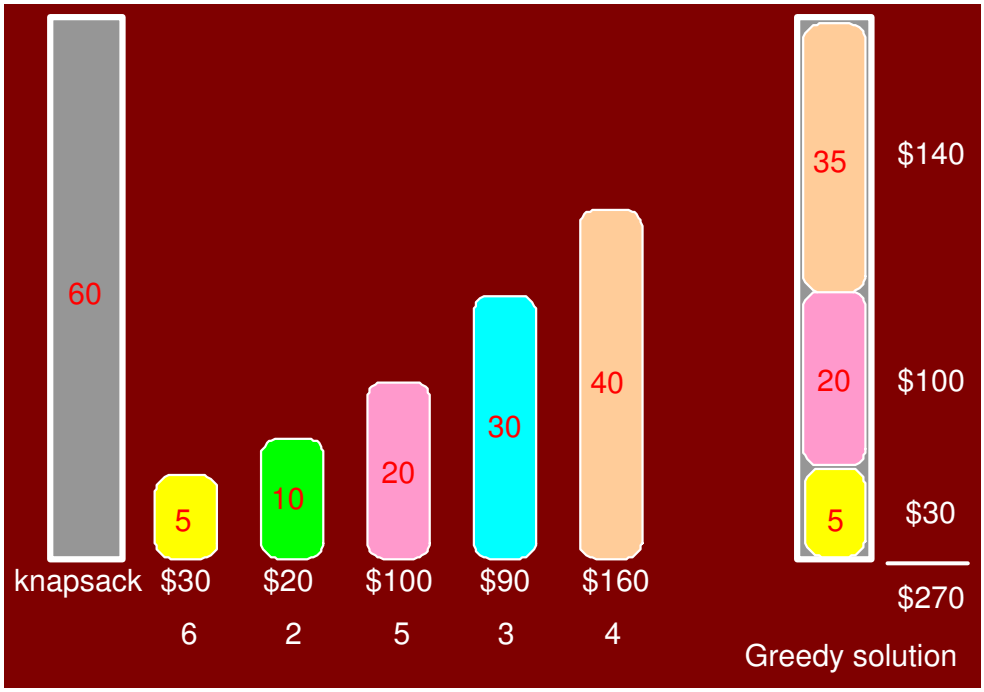


Figure 7.8: Greedy solution to the fractional knapsack problem

It is easy to see that the greedy algorithm is optimal for the fractional knapsack problem. Given a room with sacks of gold, silver and bronze, one (thief?) would probably take as much gold as possible. Then take as much silver as possible and finally as much bronze as possible. It would never benefit to take a little less gold so that one could replace it with an equal weight of bronze.

We can also observe that the greedy algorithm is not optimal for the 0-1 knapsack problem. Consider the example shown in the Figure 7.9. If you were to sort the items by ρ_i , then you would first take the items of weight 5, then 20, and then (since the item of weight 40 does not fit) you would settle for the item of weight 30, for a total value of \$30 + \$100 + \$90 = \$220. On the other hand, if you had been less greedy, and ignored the item of weight 5, then you could take the items of weights 20 and 40 for a total value of \$100+\$160 = \$260. This is shown in Figure 7.10.

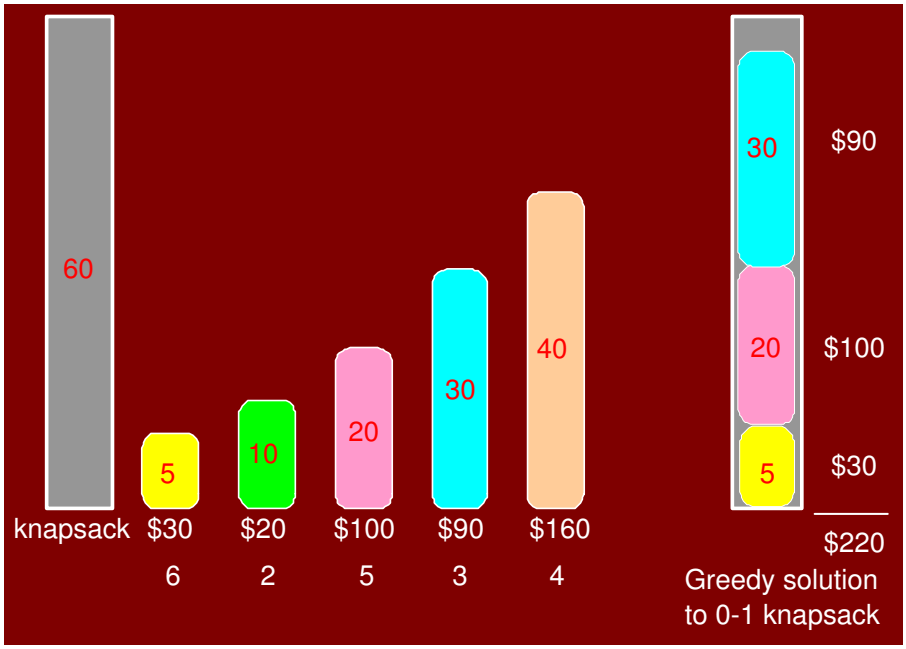


Figure 7.9: Greedy solution for the 0-1 knapsack problem (non-optimal)

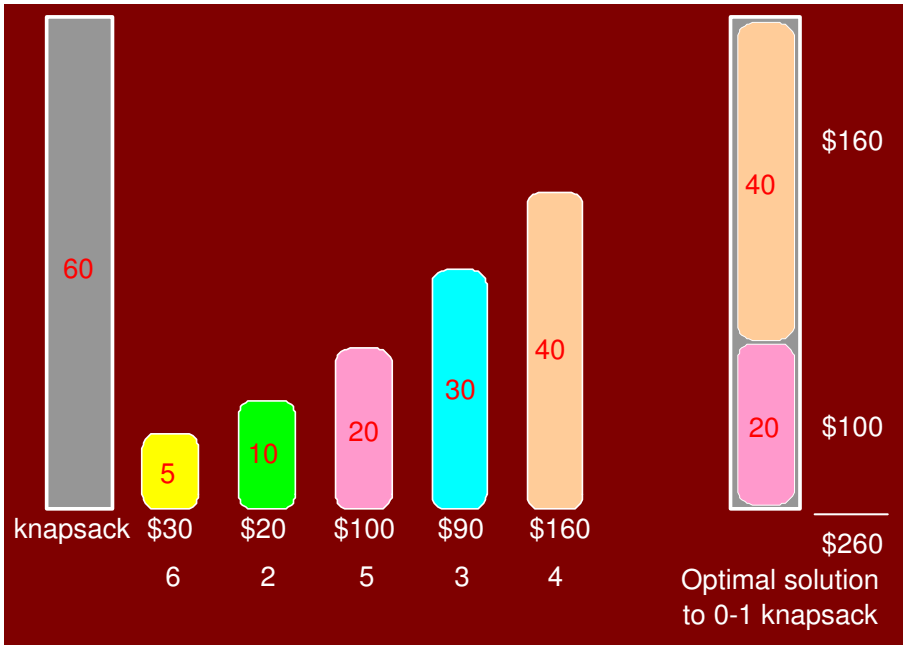


Figure 7.10: Optimal solution for the 0-1 knapsack problem

Chapter 8

Graphs

We begin a major new topic: Graphs. Graphs are important discrete structures because they are a flexible mathematical model for many application problems. Any time there is a set of objects and there is some sort of "connection" or "relationship" or "interaction" between pairs of objects, a graph is a good way to model this. Examples of this can be found in computer and communication networks transportation networks, e.g., roads VLSI, logic circuits surface meshes for shape description in computer-aided design and GIS precedence constraints in scheduling systems.

A *graph* $G = (V, E)$ consists of a finite set of *vertices* V (or nodes) and E, a binary relation on V called *edges*. E is a set of pairs from V. If a pair is *ordered*, we have a *directed* graph. For *unordered* pair, we have an *undirected* graph.

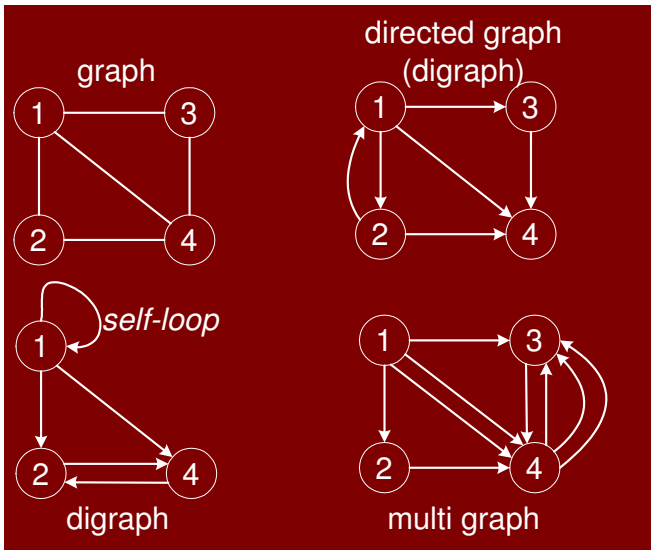


Figure 8.1: Types of graphs

A vertex w is *adjacent* to vertex v if there is an edge from v to w.

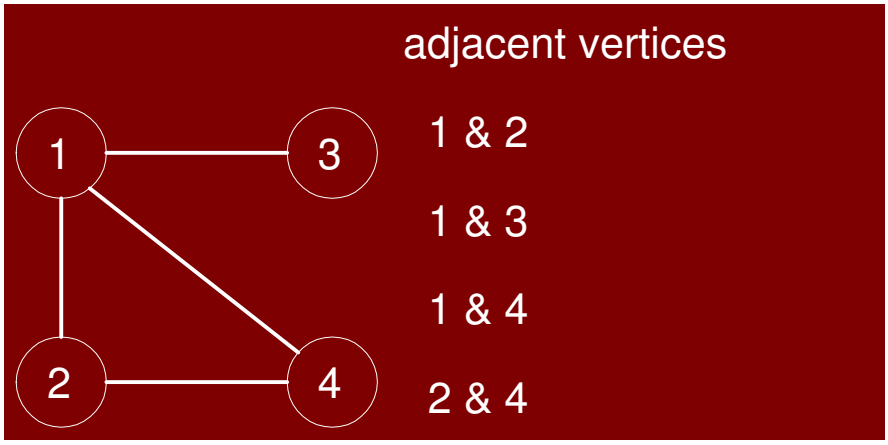


Figure 8.2: Adjacent vertices

In an undirected graph, we say that an edge is *incident* on a vertex if the vertex is an endpoint of the edge. of the edge

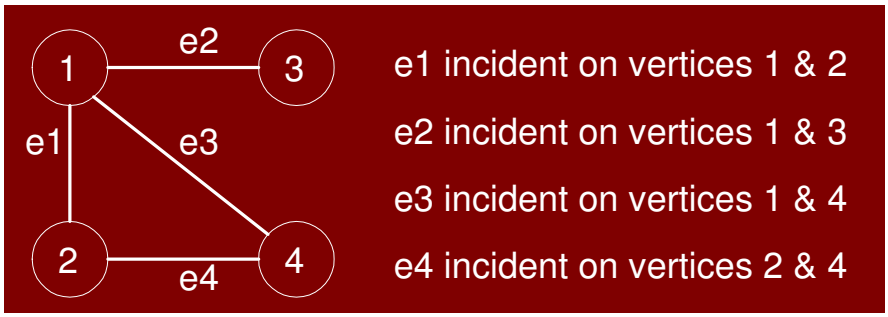


Figure 8.3: Incidence of edges on vertices

In a digraph, the number of edges coming out of a vertex is called the *out-degree* of that vertex. Number of edges coming in is the *in-degree*. In an undirected graph, we just talk of degree of a vertex. It is the number of edges incident on the vertex.

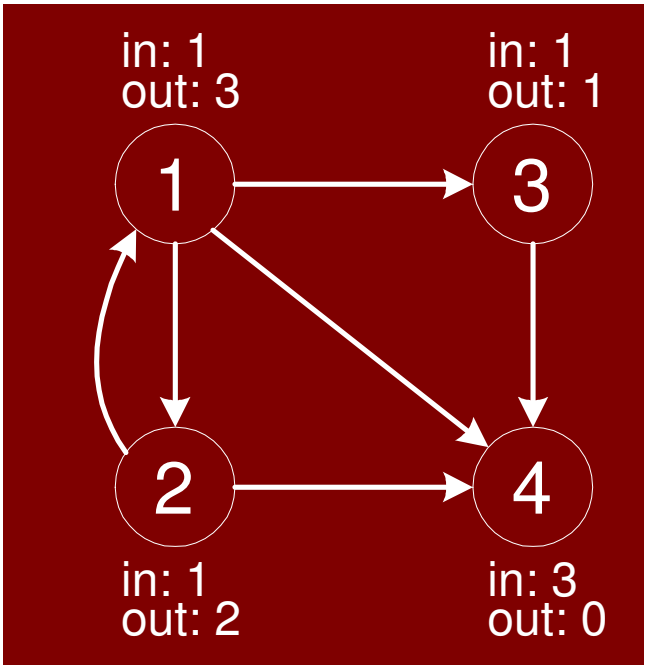


Figure 8.4: In and out degrees of vertices of a graph

For a digraph $G = (V, E)$,

$$\sum_{v \in V} \texttt{in-degree}(v) = \sum_{v \in V} \texttt{out-degree}(v) = |E|$$

where $|E|$ means the cardinality of the set E, i.e., the number of edges.

For an undirected graph $G = (V, E)$,

$$\sum_{v \in V} \texttt{degree}(v) = 2|E|$$

where $|E|$ means the cardinality of the set E, i.e., the number of edges.

A *path* in a directed graphs is a sequence of vertices $\langle v_0, v_1, \ldots, v_k \rangle$ such that (v_{i-1}, v_i) is an edge for $i = 1, 2, \ldots, k$. The *length* of the paths is the number of edges, k. A vertex w is *reachable* from vertex u is there is a path from u to w. A path is simple if all vertices (except possibly the fist and last) are distinct.

A *cycle* in a digraph is a path containing at least one edge and for which $v_0 = v_k$. A *Hamiltonian* cycle is a cycle that visits every vertex in a graph exactly once. A *Eulerian* cycle is a cycle that visits every edge of the graph exactly once. There are also "path" versions in which you do not need return to the starting vertex.

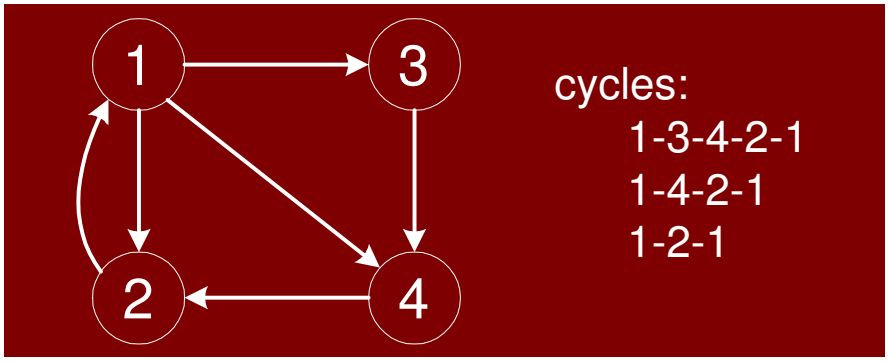


Figure 8.5: Cycles in a directed graph

A graph is said to be *acyclic* if it contains no cycles. A graph is *connected* if every vertex can reach every other vertex. A directed graph that is acyclic is called a *directed acyclic graph (DAG)*.

There are two ways of representing graphs: using an adjacency matrix and using an adjacency list. Let $G = (V, E)$ be a digraph with $n = |V|$ and let $e = |E|$. We will assume that the vertices of G are indexed $\{1, 2, \ldots, n\}$.

An *adjacency matrix* is a $n \times n$ matrix defined for $1 \leq v, w \leq n$.

$$A[v, w] = \begin{cases} 1 & \text{if } (v, w) \in E \\ 0 & \text{otherwise} \end{cases}$$

An *adjacency list* is an array $Adj[1..n]$ of pointers where for $1 \leq v \leq n$, $Adj[v]$ points to a linked list containing the vertices which are adjacent to v

Adjacency matrix requires $\Theta(n^2)$ storage and adjacency list requires $\Theta(n + e)$ storage.

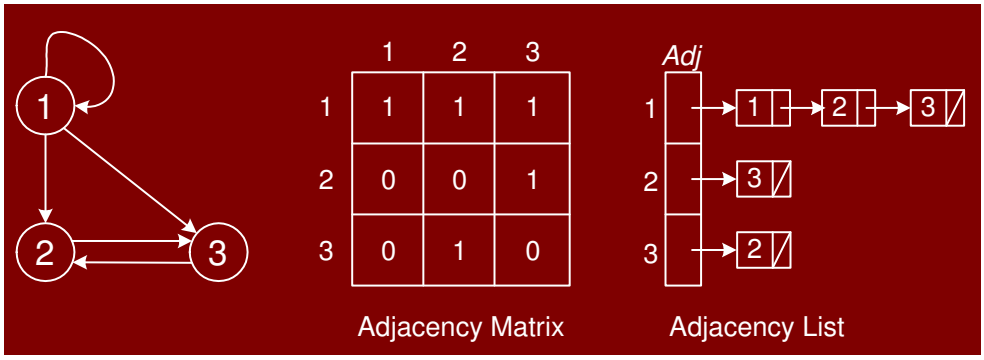


Figure 8.6: Graph Representations

8.1 Graph Traversal

To motivate our first algorithm on graphs, consider the following problem. We are given an undirected graph $G = (V, E)$ and a *source vertex* $s \in V$. The *length* of a path in a graph is the number of edges on

the path. We would like to find the shortest path from s to each other vertex in the graph. The final result will be represented in the following way. For each vertex $v \in V$, we will store $d[v]$ which is the *distance* (length of the shortest path) from s to v. Note that $d[s] = 0$. We will also store a predecessor (or parent) pointer $\pi[v]$ which is the first vertex along the shortest path if we walk from v backwards to s. We will set $\pi[s] = \mathrm{Nil}$.

There is a simple brute-force strategy for computing shortest paths. We could simply start enumerating all simple paths starting at s, and keep track of the shortest path arriving at each vertex. However, there can be as many as $n!$ simple paths in a graph. To see this, consider a fully connected graph shown in Figure 8.7

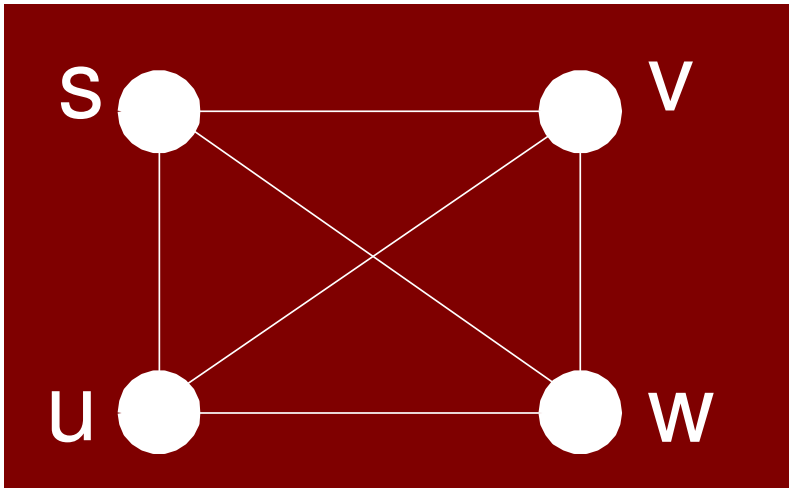


Figure 8.7: Fully connected graph

There n choices for source node s, $(n-1)$ choices for destination node, $(n-2)$ for first hop (edge) in the path, $(n-3)$ for second, $(n-4)$ for third down to $(n-(n-1))$ for last leg. This leads to $n!$ simple paths. Clearly this is not feasible.

8.1.1 Breadth-first Search

Here is a more efficient algorithm called the *breadth-first search* (BFS) Start with s and visit its adjacent nodes. Label them with distance 1. Now consider the neighbors of neighbors of s. These would be at distance 2. Now consider the neighbors of neighbors of neighbors of s. These would be at distance 3. Repeat this until no more unvisited neighbors left to visit. The algorithm can be visualized as a *wave front* propagating outwards from s visiting the vertices in bands at ever increasing distances from s.

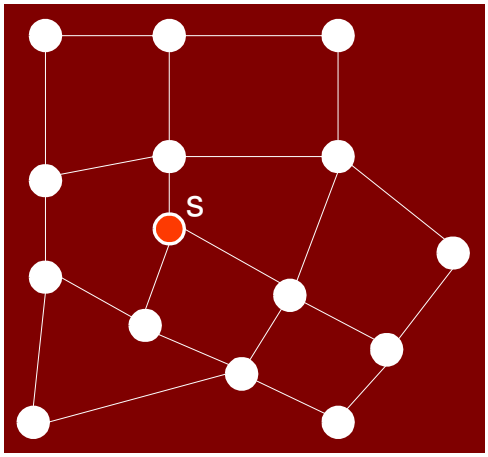


Figure 8.8: Source vertex for breadth-first-search (BFS)

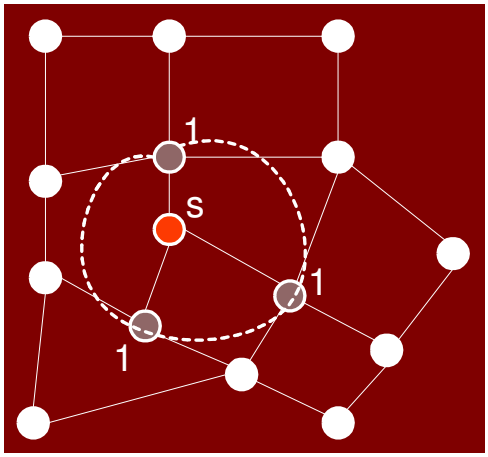


Figure 8.9: Wave reaching distance 1 vertices during BFS

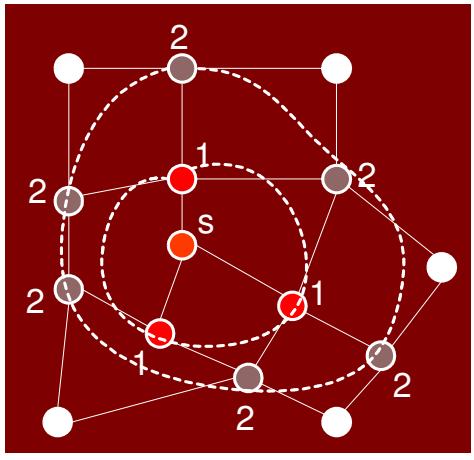


Figure 8.10: Wave reaching distance 2 vertices during BFS

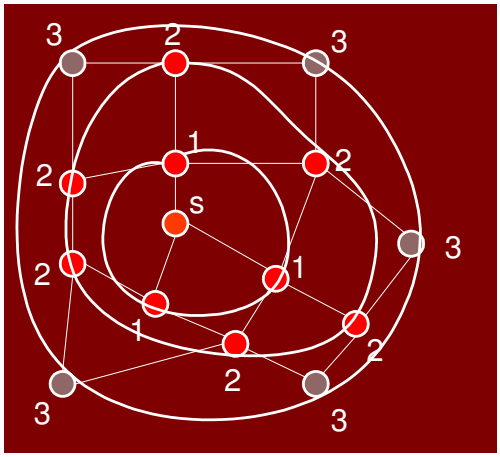


Figure 8.11: Wave reaching distance 3 vertices during BFS

8.1.2 Depth-first Search

Breadth-first search is one instance of a general family of *graph traversal algorithms*. Traversing a graph means visiting every node in the graph. Another traversal strategy is *depth-first search* (DFS). DFS procedure can be written recursively or non-recursively. Both versions are passed s initially.

```
RECURSIVEDFS(v)
1  if (v is unmarked )
2    then mark v
3         for each edge (v,w)
4         do RECURSIVEDFS(w)
```

```
ITERATIVEDFS(s)
1  PUSH(s)
2  while stack not empty
3  do v ← POP()
4     if v is unmarked
5       then mark v
6            for each edge (v,w)
7            do PUSH(w)
```

8.1.3 Generic Graph Traversal Algorithm

The *generic graph traversal* algorithm stores a set of candidate edges in some data structures we'll call a "*bag*". The only important properties of the "bag" are that we can put stuff into it and then later take stuff

back out. Here is the generic traversal algorithm.

```
TRAVERSE(s)
1  put (∅, s) in bag
2  while bag not empty
3  do take (p, v) from bag
4     if (v is unmarked)
5        then mark v
6             parent (v) ← p
7             for each edge (v, w)
8             do put (v, w) in bag
```

Notice that we are keeping edges in the bag instead of vertices. This is because we want to remember, whenever we visit v for the first time, which previously-visited vertex p put v into the bag. The vertex p is call the *parent of* v.

The running time of the traversal algorithm depends on how the graph is represented and what data structure is used for the bag. But we can make a few general observations.

- Since each vertex is visited at most once, the for loop in line 7 is executed at most V times.
- Each edge is put into the bag exactly twice; once as (u, v) and once as (v, u), so line 8 is executed at most $2E$ times.
- Finally, since we can't take out more things out of the bag than we put in, line 3 is executed at most $2E + 1$ times.
- Assume that the graph is represented by an adjacency list so the overhead of the for loop in line 7 is constant per edge.

If we implement the bag by using a *stack*, we have *depth-first* search (DFS) or traversal.

```
TRAVERSE(s)
1  push(∅, s)
2  while stack not empty
3  do pop(p, v)
4     if (v is unmarked)
5        then mark v
6             parent (v) ← p
7             for each edge (v, w)
8             do push(v, w)
```

Figures 8.12 to 8.20 show a trace of the DFS algorithm applied to a graph. The figures show the content of the stack during the execution of the algorithm.

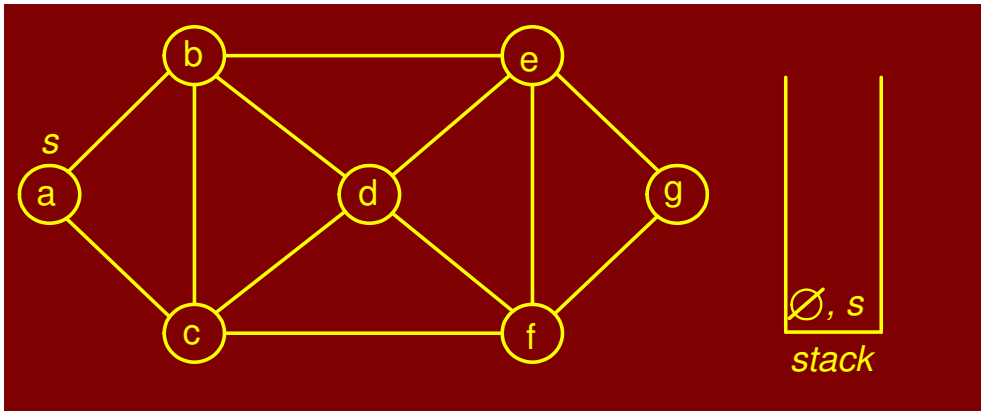


Figure 8.12: Trace of Depth-first-search algorithm: source vertex ‘s’

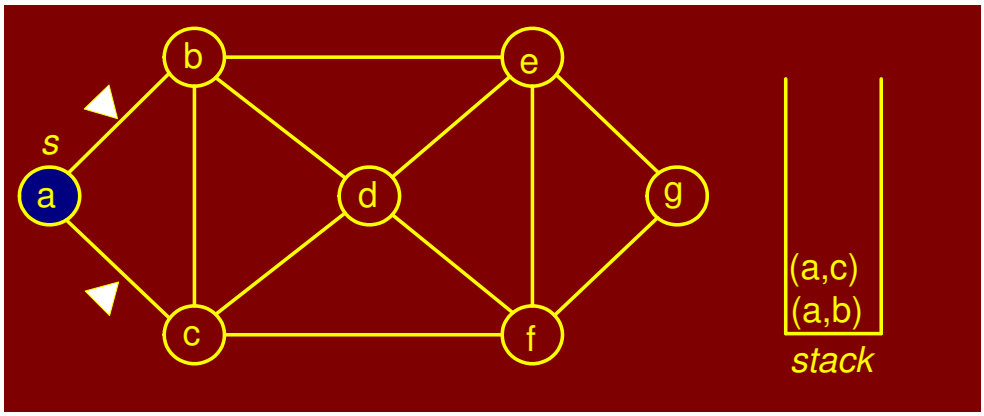


Figure 8.13: Trace of DFS algorithm: vertex ‘a’ popped

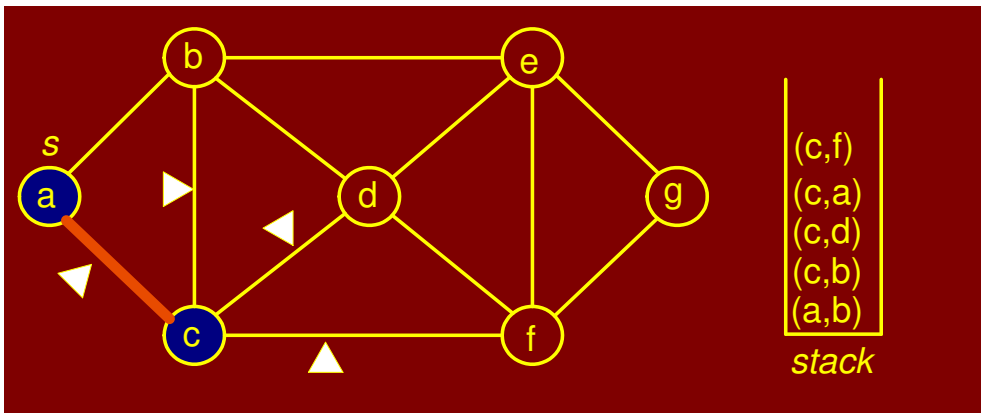


Figure 8.14: Trace of DFS algorithm: vertex ‘c’ popped

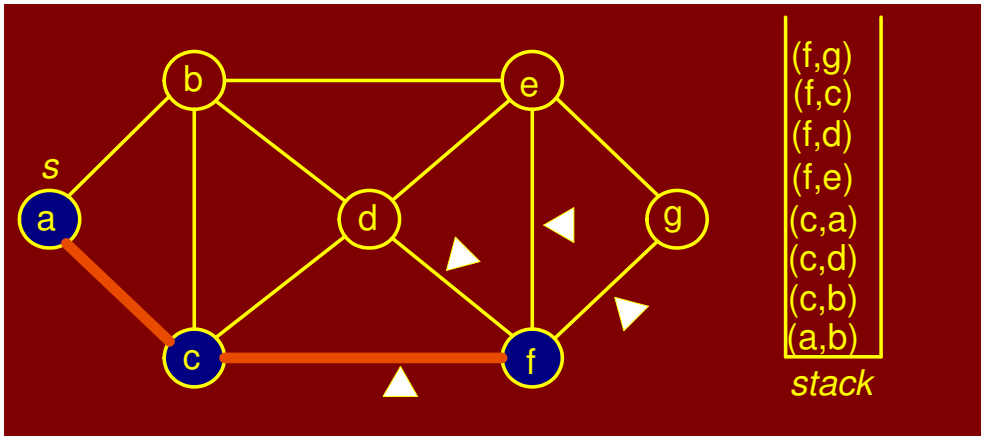


Figure 8.15: Trace of DFS algorithm: vertex ‘f’ popped

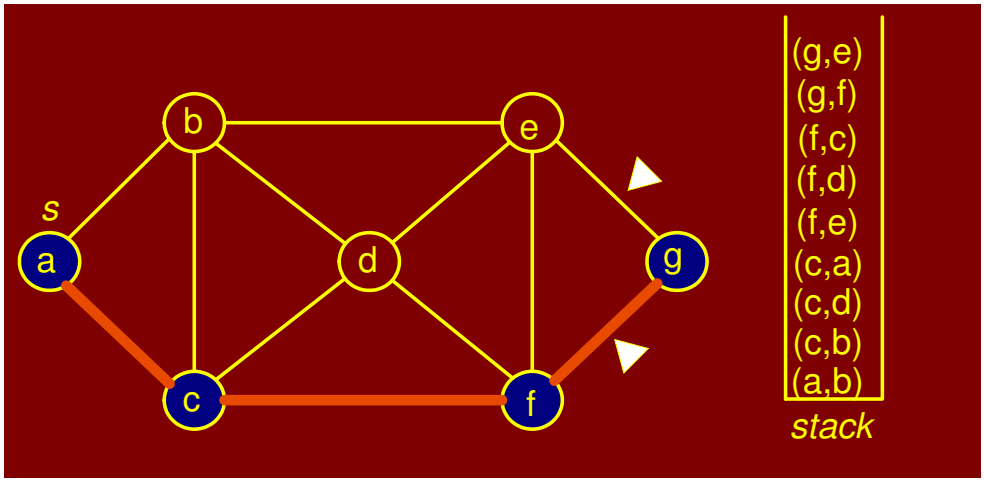


Figure 8.16: Trace of DFS algorithm: vertex ‘g’ popped

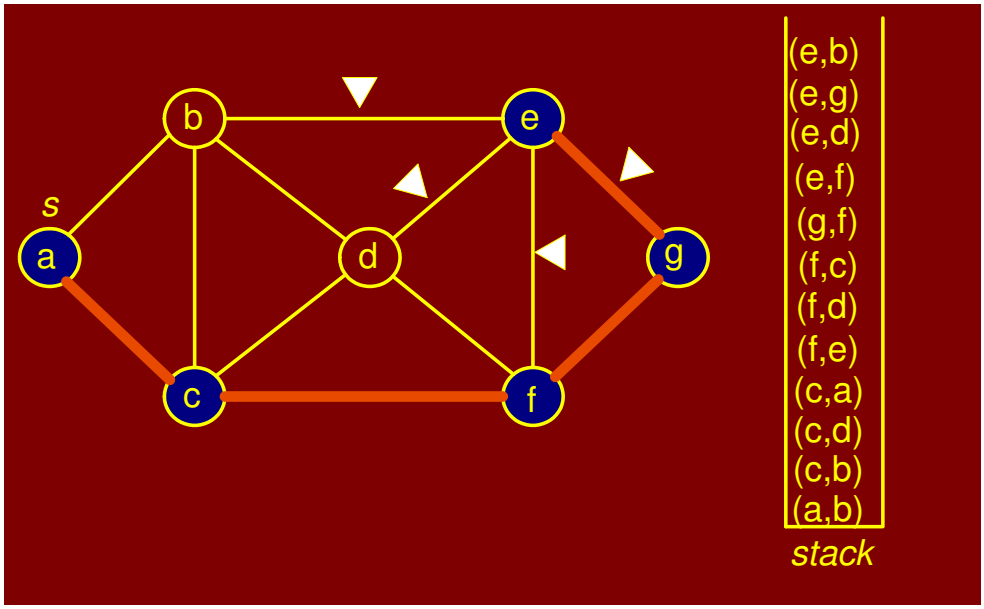


Figure 8.17: Trace of DFS algorithm: vertex ‘e’ popped

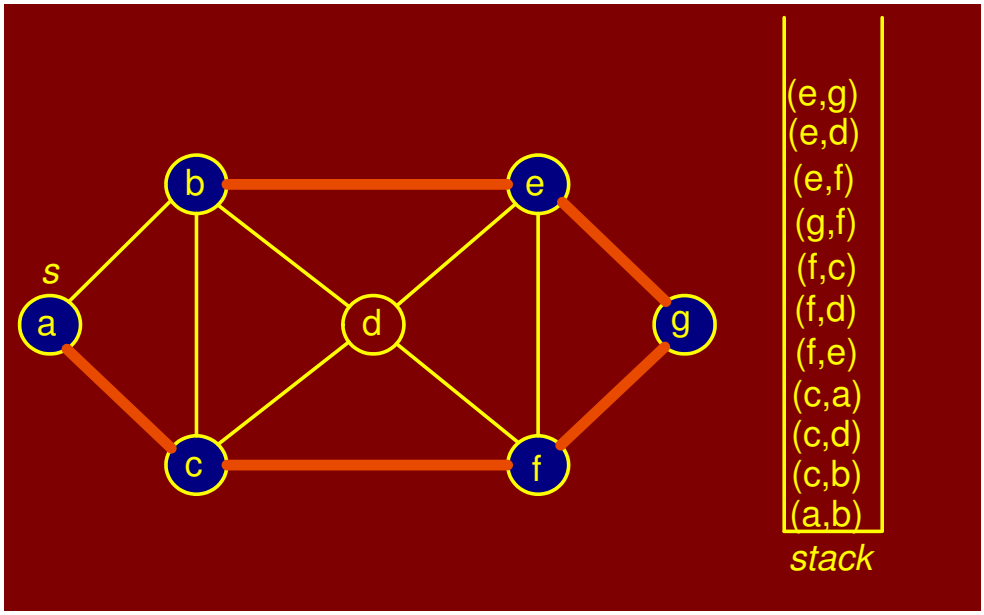


Figure 8.18: Trace of DFS algorithm: vertex ‘b’ popped

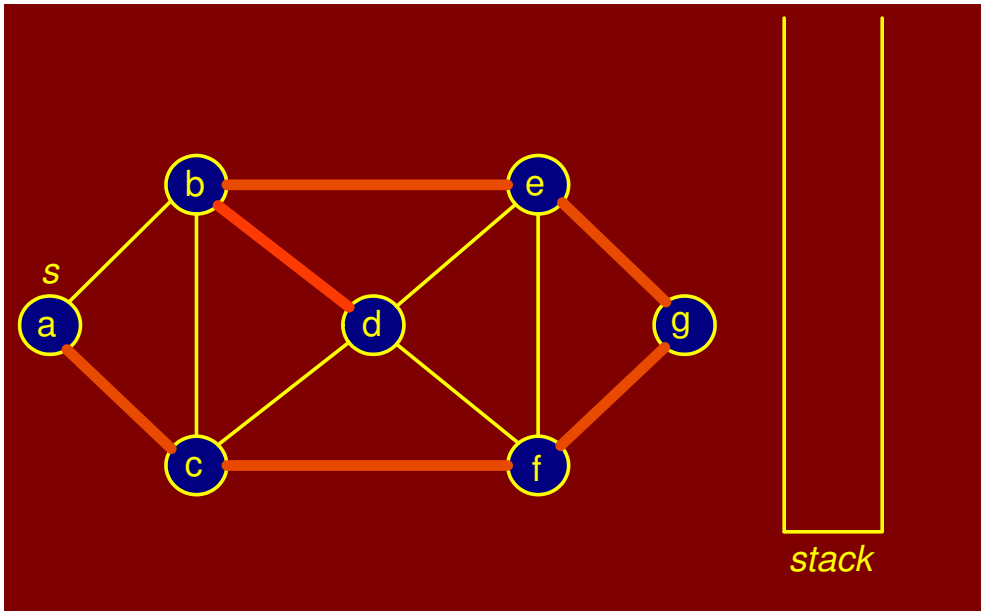


Figure 8.19: Trace of DFS algorithm: vertex ‘d’ popped

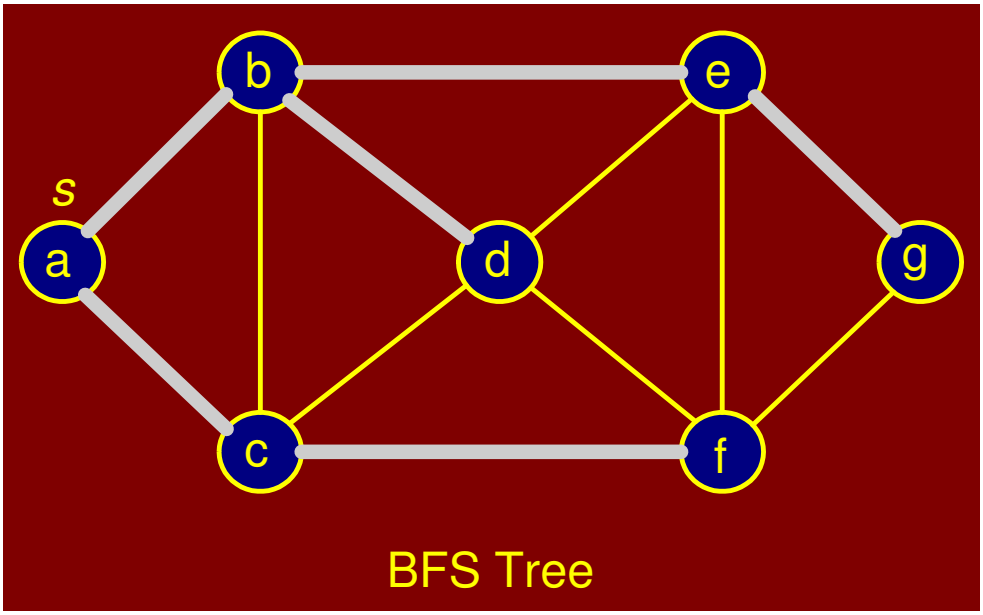


Figure 8.20: Trace of DFS algorithm: the final DFS tree

Each execution of line 3 or line 8 in the TRAVERSE-DFS algorithm takes constant time. So the overall running time is $O(V + E)$. Since the graph is connected, $V \leq E + 1$, this is $O(E)$.

If we implement the bag by using a *queue*, we have *breadth-first* search (BFS). Each execution of line 3

or line 8 still takes constant time. So overall running time is still $O(E)$.

```
TRAVERSE(s)
1  enqueue(∅, s)
2  while queue not empty
3  do dequeue(p, v)
4     if (v is unmarked)
5        then mark v
6             parent (v) ← p
7             for each edge (v, w)
8             do enqueue(v, w)
```

If the graph is represented using an *adjacency matrix*, the finding of all the neighbors of vertex in line 7 takes $O(V)$ time. Thus depth-first and breadth-first take $O(V^2)$ time overall.

Either DFS or BFS yields a spanning tree of the graph. The tree visits every vertex in the graph. This fact is established by the following lemma:

Lemma:

The generic TRAVERSE(S) marks every vertex in any connected graph exactly once and the set of edges $(v, parent(v))$ with $parent(v) \neq \varnothing$ form a spanning tree of the graph.

Proof:

First, it should be obvious that no vertex is marked more than once. Clearly, the algorithm marks s. Let $v \neq s$ be a vertex and let $s \rightarrow \cdots \rightarrow u \rightarrow v$ be a path from s to v with the minimum number of edges.

Since the graph is connected, such a path always exists. If the algorithm marks u, then it must put (u, v) into the bag, so it must take (u, v) out of the bag at which point v must be marked. Thus, by induction on the shortest-path distance from s, the algorithm marks every vertex in the graph.

Call an edge $(v, parent(v))$ with $parent(v) \neq \varnothing$, a *parent edge*. For any node v, the path of parent edges $v \rightarrow parent(v) \rightarrow parent(parent(v)) \rightarrow \ldots$ eventually leads back to s. So the set of parent edges form a connected graph.

Clearly, both end points of every parent edge are marked, and the number of edges is exactly one less than the number of vertices. Thus, the parent edges form a *spanning tree*.

8.1.4 DFS - Timestamp Structure

As we traverse the graph in DFS order, we will associate two numbers with each vertex. When we first discover a vertex u, store a counter in $d[u]$. When we are finished processing a vertex, we store a counter in $f[u]$. These two numbers are *time stamps*.

Consider the *recursive* version of depth-first traversal

```
DFS(G)
1  for (each u ∈ V)
2  do color[u] ← white
3     pred[u] ← nil
4  time ← 0
5  for each u ∈ V
6  do if (color[u] = white)
7        then DFSVISIT(u)
```

The DFSVISIT routine is as follows:

```
DFSVISIT(u)
1  color[u] ← gray;  // mark u visited
2  d[u] ←++ time
3  for (each v ∈ Adj[u])
4  do if (color[v] = white)
5        then pred[v] ← u
6             DFSVISIT(v)
7  color[u] ← black;  // we are done with u
8  f[u] ←++ time;
```

Figures 8.21 through 8.25 present a trace of the execution of the time stamping algorithm. Terms like "2/5" indicate the value of the counter (time). The number before the "/" is the time when a vertex was discovered (colored gray) and the number after the "/" is the time when the processing of the vertex finished (colored black).

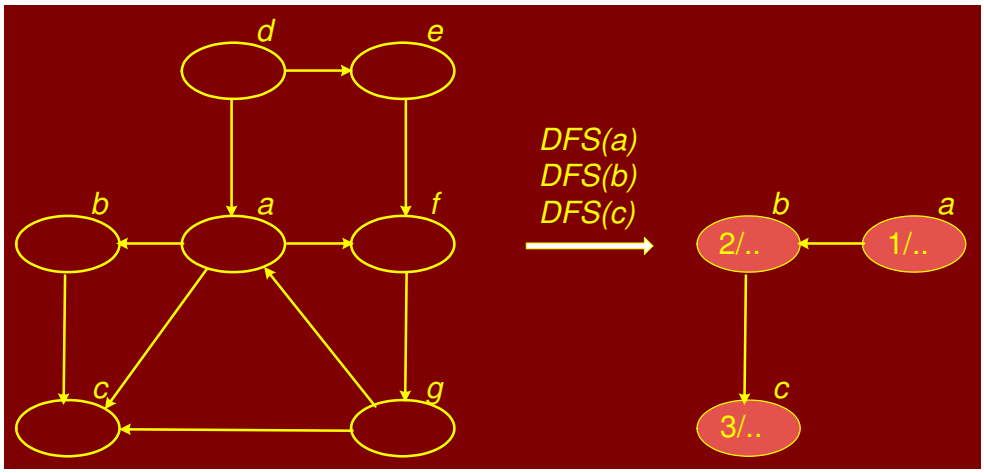


Figure 8.21: DFS with time stamps: recursive calls initiated at vertex 'a'

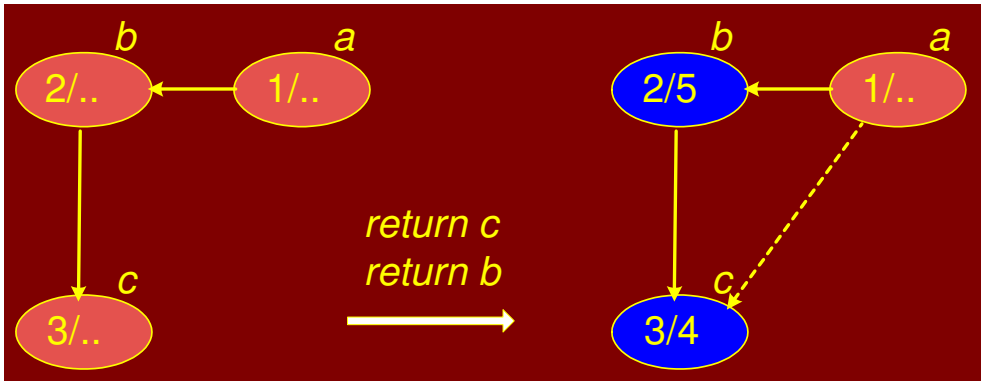


Figure 8.22: DFS with time stamps: processing of ‘b’ and ‘c’ completed

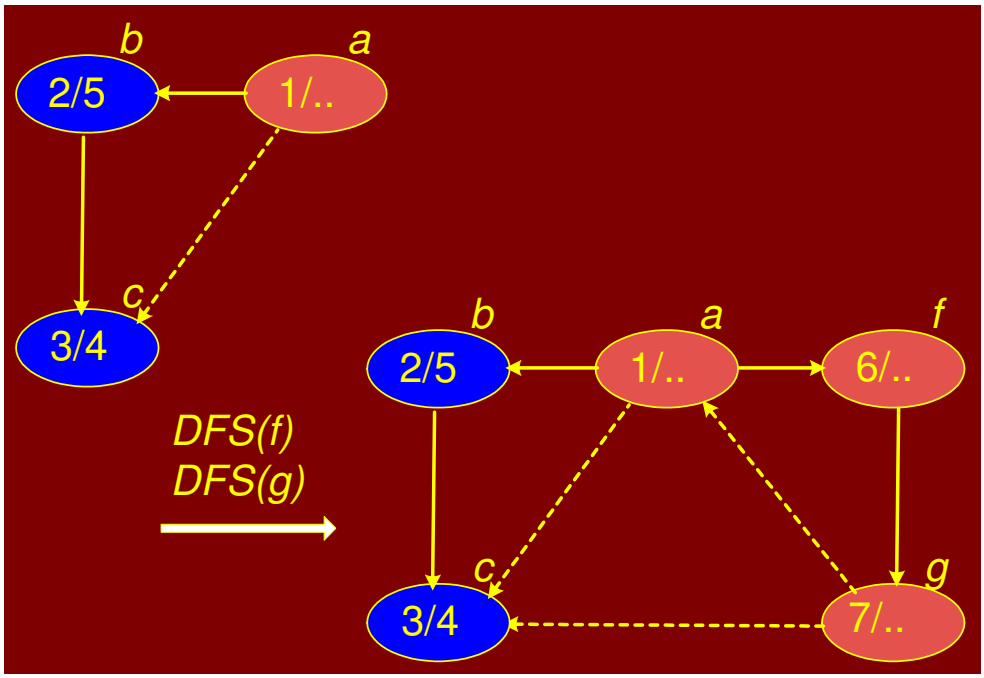


Figure 8.23: DFS with time stamps: recursive processing of ‘f’ and ‘g’

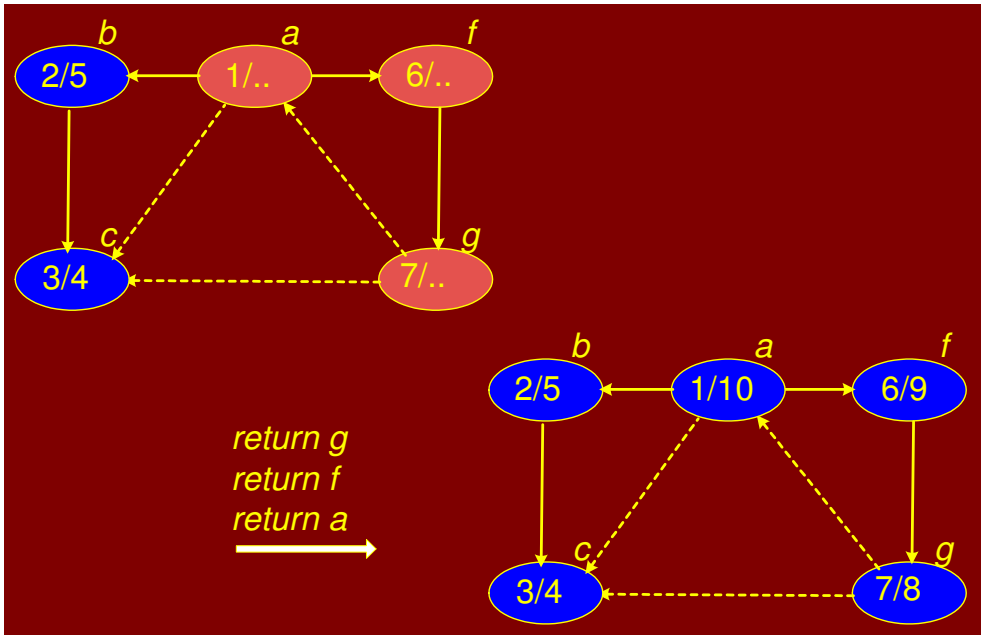


Figure 8.24: DFS with time stamps: processing of ‘f’ and ‘g’ completed

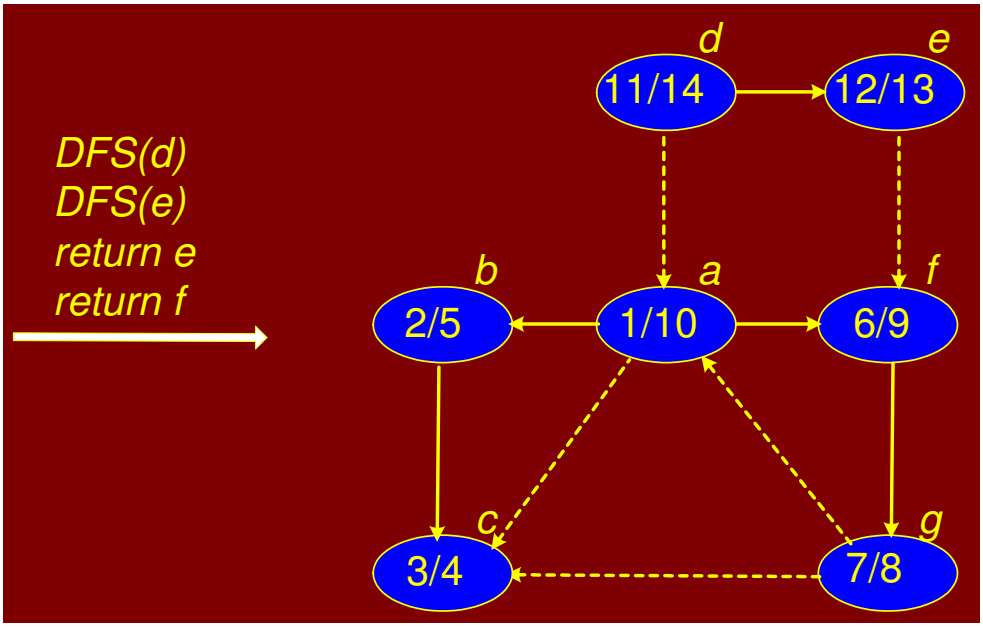


Figure 8.25: DFS with time stamps: processing of ‘d’ and ‘e’

Notice that the DFS tree structure (actually a collection of trees, or a forest) on the structure of the graph is just the recursion tree, where the edge (u, v) arises when processing vertex u we call DFSVISIT(V) for some neighbor v. For *directed graphs* the edges that are not part of the tree (indicated as dashed edges in Figures 8.21 through 8.25) edges of the graph can be classified as follows:

Back edge: (u, v) where v is an ancestor of u in the tree.

Forward edge: (u, v) where v is a proper descendent of u in the tree.

Cross edge: (u, v) where u and v are not ancestor or descendent of one another. In fact, the edge may go between different trees of the forest.

The ancestor and descendent relation can be nicely inferred by the *parenthesis* lemma. u is a descendent of v if and only if $[d[u], f[u]] \subseteq [d[v], f[v]]$. u is a ancestor of v if and only if $[d[u], f[u]] \supseteq [d[v], f[v]]$. u is unrelated to v if and only if $[d[u], f[u]]$ and $[d[v], f[v]]$ are disjoint. The is shown in Figure 8.26. The width of the rectangle associated with a vertex is equal to the time the vertex was discovered till the time the vertex was completely processed (colored black). Imagine an opening parenthesis '(' at the start of the rectangle and and closing parenthesis ')' at the end of the rectangle. The rectangle (parentheses) for vertex 'b' is completely enclosed by the rectangle for 'a'. Rectangle for 'c' is completely enclosed by vertex 'b' rectangle.

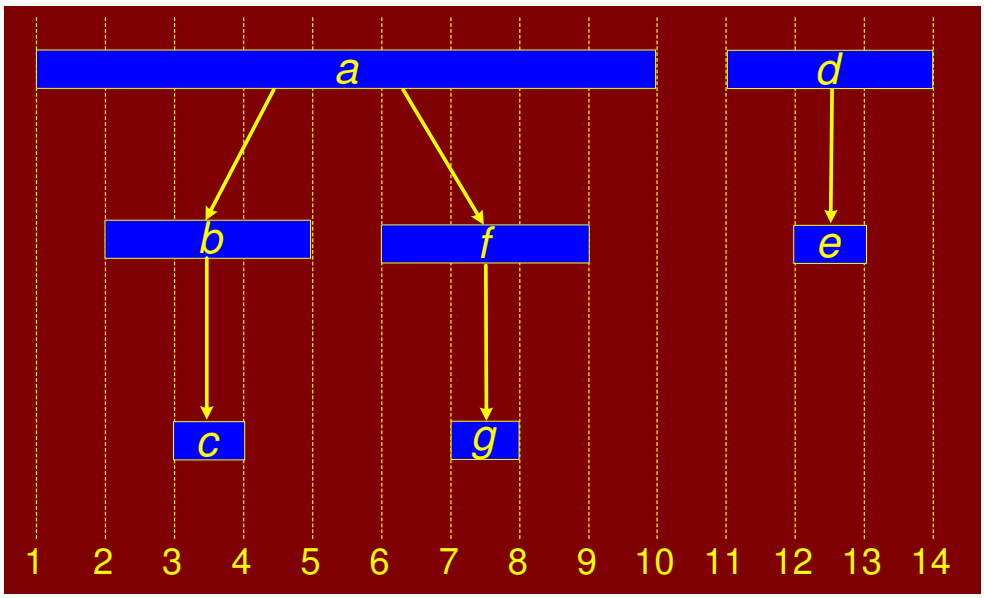


Figure 8.26: Parenthesis lemma

Figure 8.27 shows the classification of the non-tree edges based on the parenthesis lemma. Edges are labelled 'F', 'B' and 'C' for forward, back and cross edge respectively.

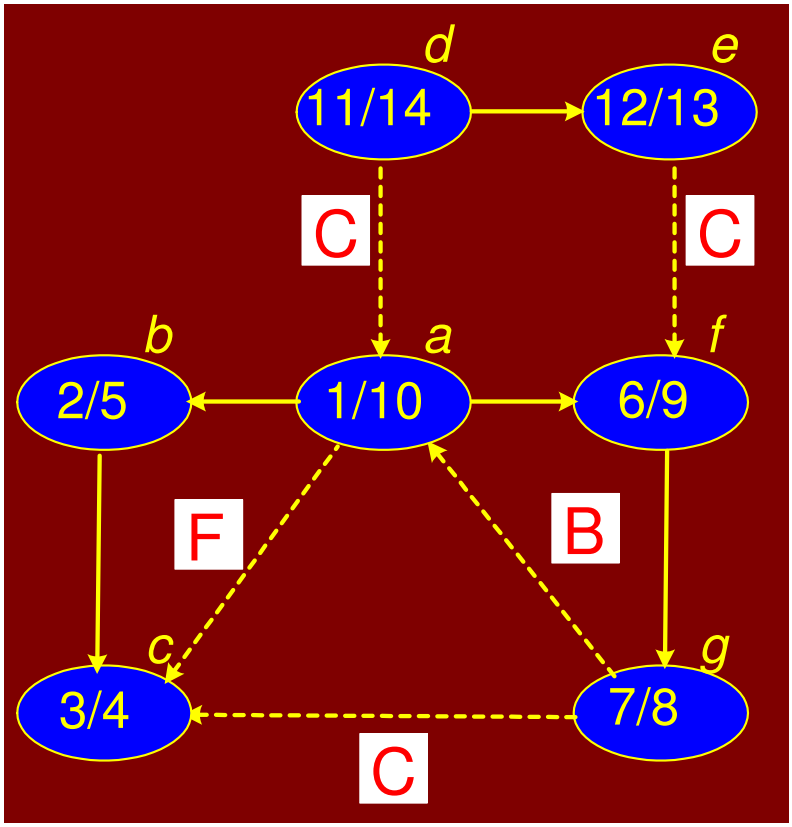


Figure 8.27: Classfication of non-tree edges in the DFS tree for a graph

For *undirected* graphs, there is no distinction between forward and back edges. By convention they are all called back edges. Furthermore, there are no cross edges (can you see why not?)

8.1.5 DFS - Cycles

The time stamps given by DFS allow us to determine a number of things about a graph or digraph. For example, we can determine whether the graph contains any *cycles*. We do this with the help of the following two lemmas.

Lemma: Given a digraph $G = (V, E)$, consider any DFS forest of G and consider any edge $(u, v) \in E$. If this edge is a tree, forward or cross edge, then $f[u] > f[v]$. If this edge is a back edge, then $f[u] \leq f[v]$.

Proof: For the non-tree forward and back edges the proof follows directly from the parenthesis lemma. For example, for a forward edge (u, v), v is a descendent of u and so v's start-finish interval is contained within u's implying that v has an earlier finish time. For a cross edge (u, v) we know that the two time intervals are disjoint. When we were processing u, v was not white (otherwise (u, v) would be a tree edge), implying that v was started before u. Because the intervals are disjoint, v must have also finished before u.

Lemma: Consider a digraph $G = (V, E)$ and any DFS forest for G. G has a cycle if and only if the DFS forest has a *back* edge.

Proof: If there is a back edge (u, v) then v is an ancestor of u and by following tree edge from v to u, we get a cycle.

We show the contrapositive: suppose there are no back edges. By the lemma above, each of the remaining types of edges, tree, forward, and cross all have the property that they go from vertices with higher finishing time to vertices with lower finishing time. Thus along any path, finish times decrease monotonically, implying there can be no cycle.

The DFS forest in Figure 8.27 has a back edge from vertex 'g' to vertex 'a'. The cycle is 'a-g-f'.

Beware: No back edges means no cycles. But you should not infer that there is some simple relationship between the number of back edges and the number of cycles. For example, a DFS tree may only have a single back edge, and there may anywhere from one up to an exponential number of simple cycles in the graph.

A similar theorem applies to undirected graphs, and is not hard to prove.

8.2 Precedence Constraint Graph

A *directed acyclic graph* (DAG) arise in many applications where there are precedence or ordering constraints. There are a series of tasks to be performed and certain tasks must precede other tasks. For example, in construction, you have to build the first floor before the second floor but you can do electrical work while doors and windows are being installed. In general, a *precedence constraint graph* is a DAG in which vertices are tasks and the edge (u, v) means that task u must be completed before task v begins.

For example, consider the sequence followed when one wants to dress up in a suit. One possible order and its DAG are shown in Figure 8.28. Figure 8.29 shows the DFS with time stamps of the DAG.

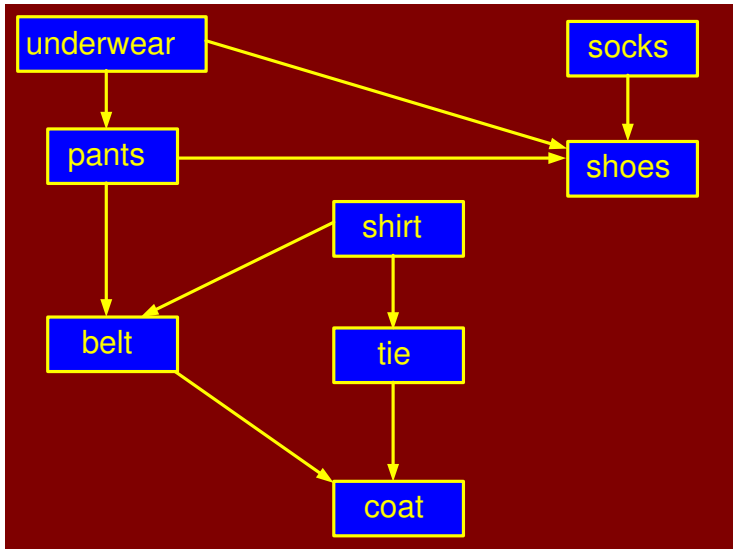


Figure 8.28: Order of dressing up in a suit

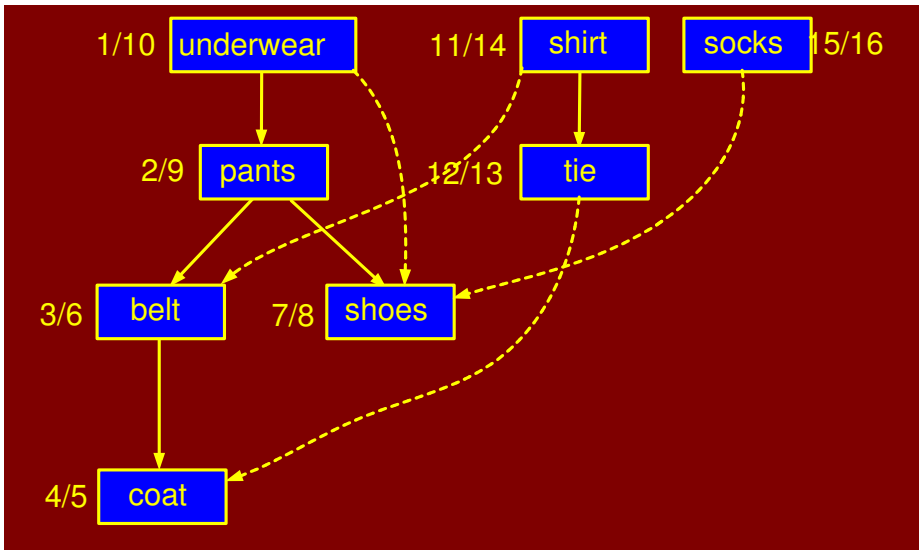


Figure 8.29: DFS of dressing up DAG with time stamps

Another example of precedence constraint graph is the sets of prerequisites for CS courses in a typical undergraduate program.

C1	Introduction to Computers	
C2	Introduction to Computer Programming	
C3	Discrete Mathematics	
C4	Data Structures	C2
C5	Digital Logic Design	C2
C6	Automata Theory	C3
C7	Analysis of Algorithms	C3, C4
C8	Computer Organization and Assembly	C2
C9	Data Base Systems	C4, C7
C10	Computer Architecture	C4, C5,C8
C11	Computer Graphics	C4,C7
C12	Software Engineering	C7,C11
C13	Operating System	C4,C7,C11
C14	Compiler Construction	C4,C6,C8
C15	Computer Networks	C4,C7,C10

Table 8.1: Prerequisites for CS courses

The prerequisites can be represented with a precedence constraint graph which is shown in Figure 8.30

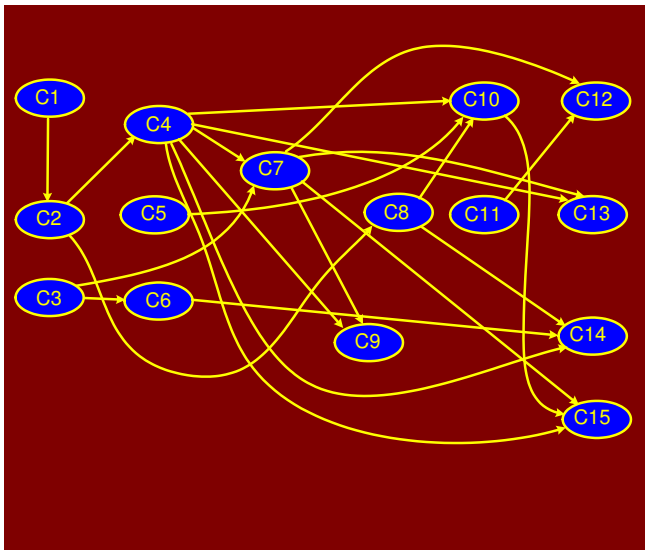


Figure 8.30: Precedence constraint graph for CS courses

8.3 Topological Sort

A topological sort of a DAG is a linear ordering of the vertices of the DAG such that for each edge (u, v), u appears before v in the ordering.

Computing a topological ordering is actually quite easy, given a DFS of the DAG. For every edge (u, v) in a DAG, the finish time of u is greater than the finish time of v (by the lemma). Thus, it suffices to output the vertices in the reverse order of finish times.

We run DFS on the DAG and when each vertex is finished, we add it to the front of a linked. Note that in general, there may be many legal topological orders for a given DAG.

```
TOPOLOGICALSORT(G)
1  for (each u ∈ V)
2  do color[u] ← white
3  L ← new LinkedList()
4  for each u ∈ V
5  do if (color[u] = white)
6        then TOPVISIT(u)
7  return L
```

```
TOPVISIT(u)
1  color[u] ← gray;  // mark u visited
2  for (each v ∈ Adj[u])
3  do if (color[v] = white)
4        then TOPVISIT(v)
5  Append u to the front of L
```

Figure 8.31 shows the linear order obtained by the topological sort of the sequence of putting on a suit. The DAG is still the same; it is only that the order in which the vertices of the graph have been laid out is special. As a result, all directed edges go from left to right.

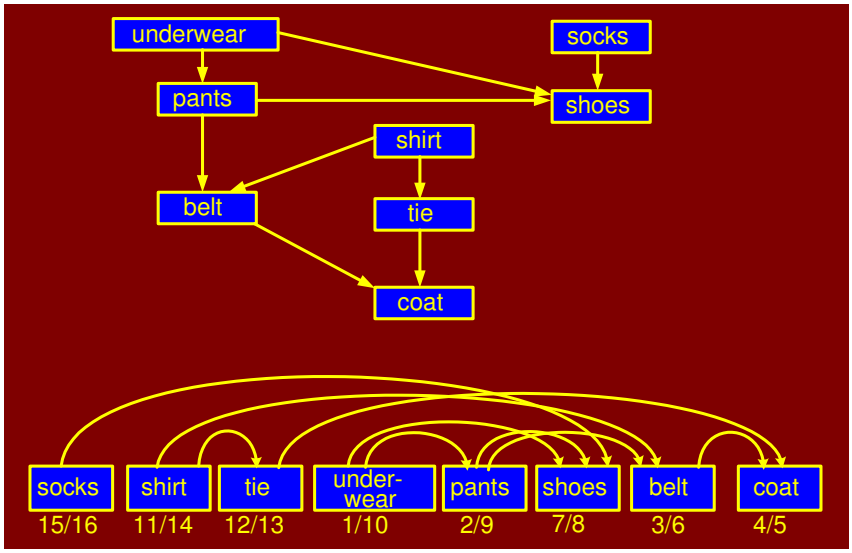


Figure 8.31: Topological sort of the dressing up sequence

This is a typical example of how DFS used in applications. The running time is $\Theta(V + E)$.

8.4 Strong Components

We consider an important connectivity problem with digraphs. When diagraphs are used in communication and transportation networks, people want to know that their networks are *complete*. Complete in the sense that that it is possible from any location in the network to reach any other location in the digraph.

A digraph is *strongly connected* if for every pair of vertices $u, v \in V$, u can reach v and vice versa. We would like to write an algorithm that determines whether a digraph is strongly connected. In fact, we will solve a generalization of this problem, of computing the *strongly connected components* of a digraph.

We partition the vertices of the digraph into subsets such that the induced subgraph of each subset is strongly connected. We say that two vertices u and v are *mutually reachable* if u can reach v and vice versa. Consider the directed graph in Figure 8.32. The strong components are illustrated in Figure 8.33.

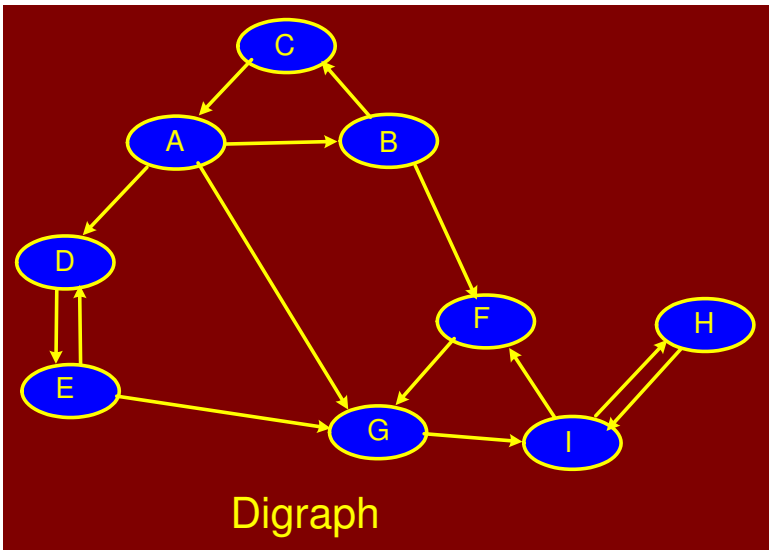


Figure 8.32: A directed graph

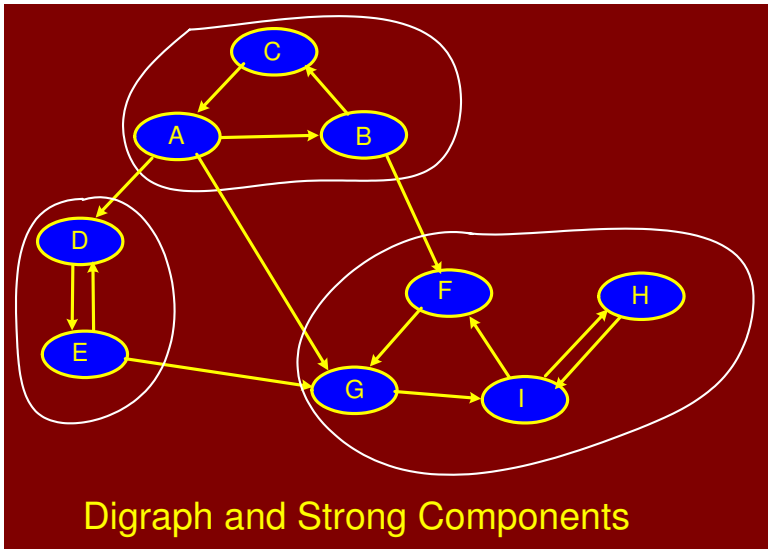


Figure 8.33: Digraph with strong components

It is easy to see that mutual reachability is an *equivalence relation*. This equivalence relation partitions the vertices into equivalence classes of mutually reachable vertices and these are the strong components.

If we merge the vertices in each strong component into a single *super vertex*, and join two super vertices (A, B) if and only if there are vertices $u \in A$ and $v \in B$ such that $(u, v) \in E$, then the resulting digraph is called the *component digraph*. The component digraph is necessarily acyclic. The is illustrated in Figure 8.34.

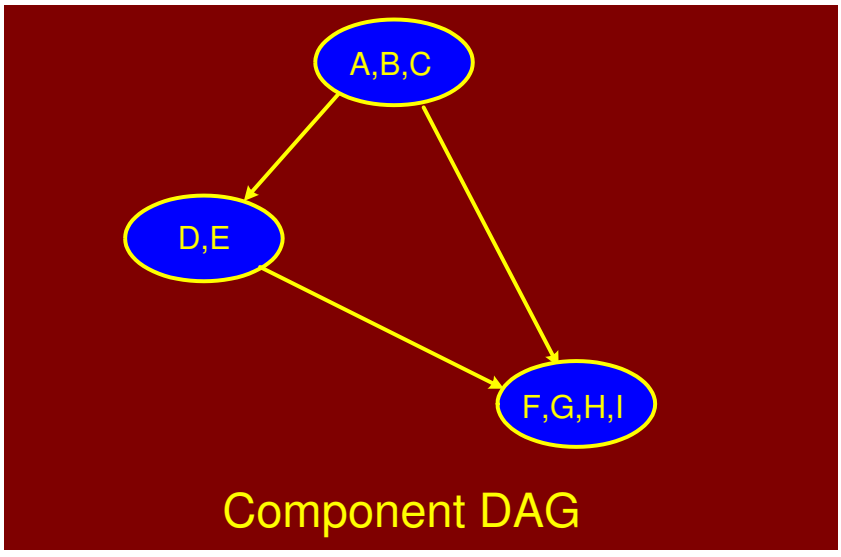


Figure 8.34: Component DAG of super vertices

8.4.1 Strong Components and DFS

Consider DFS of a digraph given in Figure **??**. Once you enter a strong component, every vertex in the component is reachable. So the DFS does not terminate until all the vertices in the component have been visited. Thus all vertices in a strong component must appear in the same tree of the DFS forest.

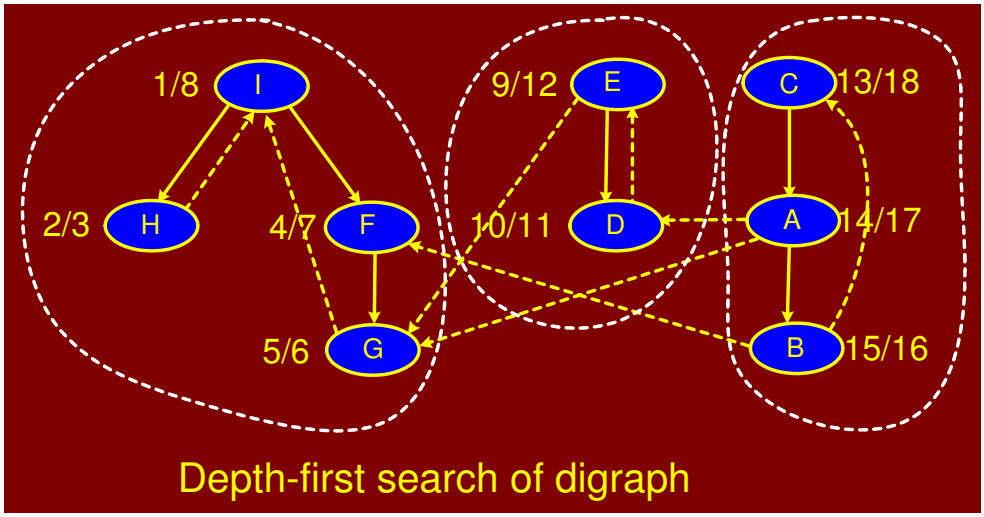


Figure 8.35: DFS of a digraph

fig:dfsofdigraph

Observe that each strong component is a subtree in the DFS forest. Is it always true for any DFS? The answer is “no”. In general, many strong components may appear in the same DFS tree as illustrated in Figure 8.36

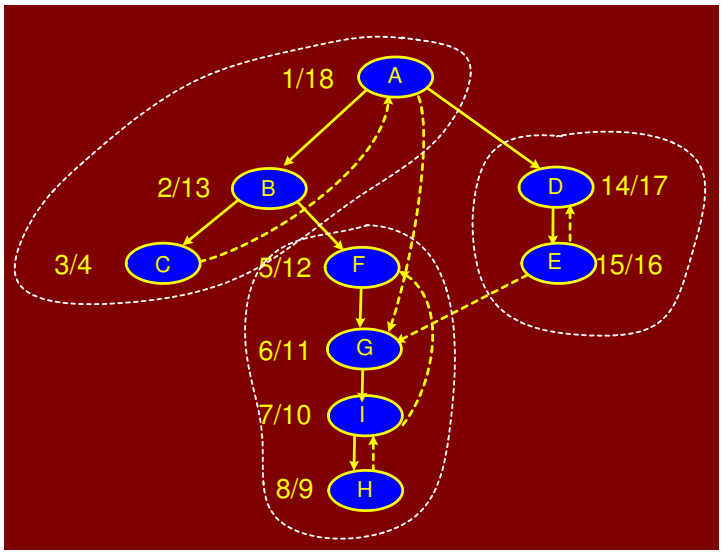


Figure 8.36: Another DFS tree of the digraph

Is there a way to order the DFS such that it true? Fortunately, the answer is "yes". Suppose that you knew the component DAG in advance. (This is ridiculous, because you would need to know the strong components and this is the problem we are trying to solve.) Further, suppose that you computed a *reversed topological order* on the component DAG. That is, for edge (u, v) in the component DAG, then v comes before u. This is presented in Figure 8.37. Recall that the component DAG consists of super vertices.

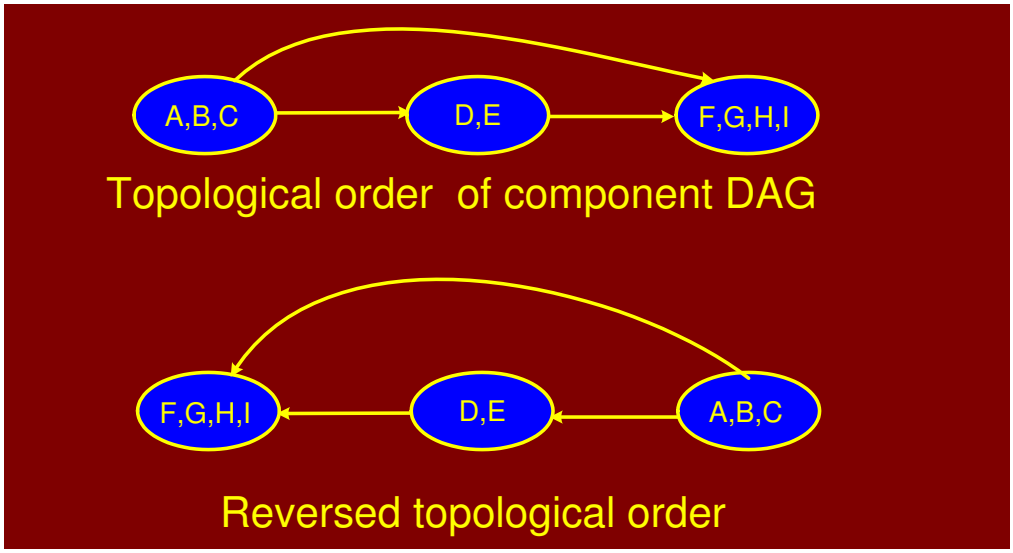


Figure 8.37: Reversed topological sort of component DAG

Now, run DFS, but every time you need a new vertex to start the search from, select the next available vertex according to this reverse topological order of the component digraph. Here is an informal justification. Clearly once the DFS starts within a given strong component, it must visit every vertex within the component (and possibly some others) before finishing. If we do not start in reverse topological, then the search may "leak out" into other strong components, and put them in the same DFS tree. For example, in the Figure 8.36, when the search is started at vertex 'a', not only does it visit its component with 'b' and 'c', but it also visits the other components as well. However, by visiting components in reverse topological order of the component tree, each search cannot "leak out" into other components, because other components would have already have been visited earlier in the search.

This leaves us with the intuition that if we could somehow order the DFS, so that it hits the strong components according to a reverse topological order, then we would have an easy algorithm for computing strong components. However, we do not know what the component DAG looks like. (After all, we are trying to solve the strong component problem in the first place). The trick behind the strong component algorithm is that we can find an ordering of the vertices that has essentially the necessary property, without actually computing the component DAG.

We will discuss the algorithm without proof. Define G^T to be the digraph with the same vertex set at G but in which all edges have been reversed in direction. This is shown in Figure 8.38. Given an adjacency list for G, it is possible to compute G^T in $\Theta(V + E)$ time.

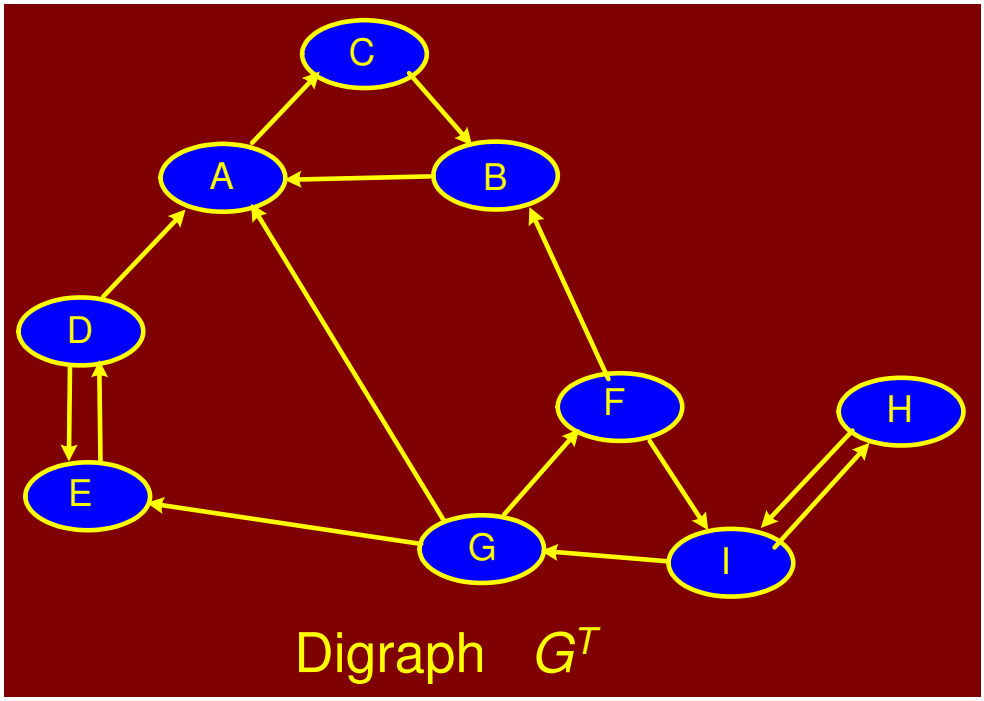


Figure 8.38: The digraph G^T

Observe that the strongly connected components are not affected by reversal of all edges. If u and v are mutually reachable in G, then this is certainly true in G^T. All that changes is that the component DAG is completely reversed. The ordering trick is to order the vertices of G according to their finish times in a DFS. Then visit the nodes of G^T in decreasing order of finish times. All the steps of the algorithm are quite easy to implement, and all operate in $\Theta(V + E)$ time. Here is the algorithm:

```
STRONGCOMPONENTS(G)
1  Run DFS(G) computing finish times f[u]
2  Compute G^T
3  Sort vertices of G^T in decreasing f[u]
4  Run DFS(G^T) using this order
5  Each DFS tree is a strong component
```

The execution of the algorithm is illustrated in Figures 8.39, 8.40 and 8.41.

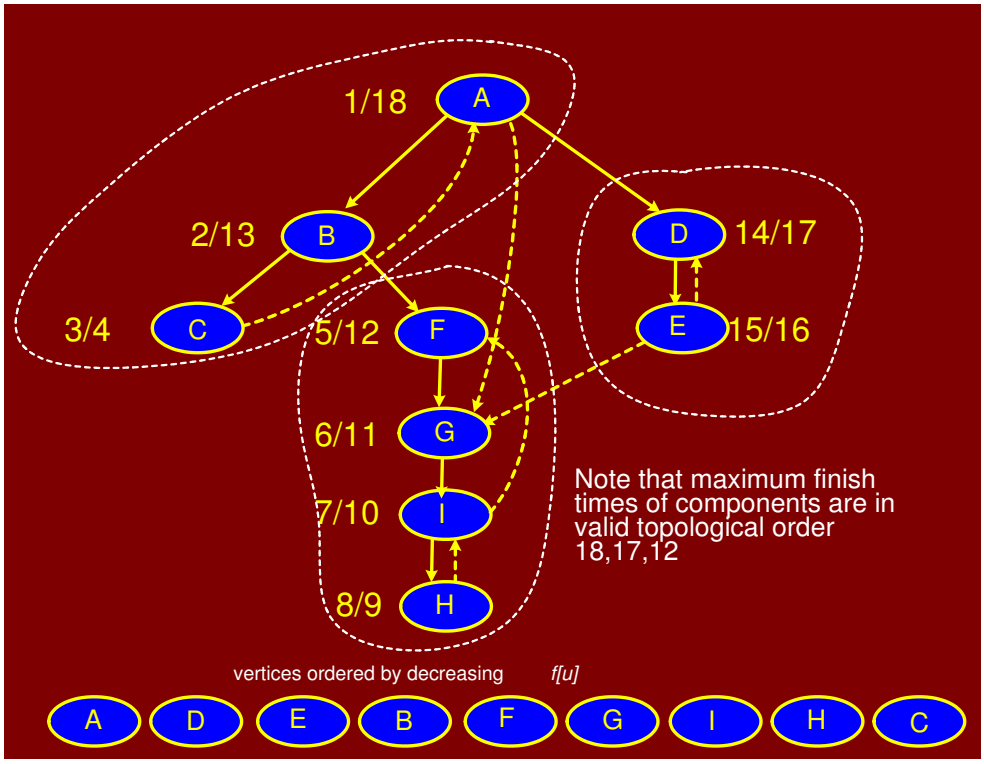


Figure 8.39: DFS of digraph with vertices in descending order by finish times

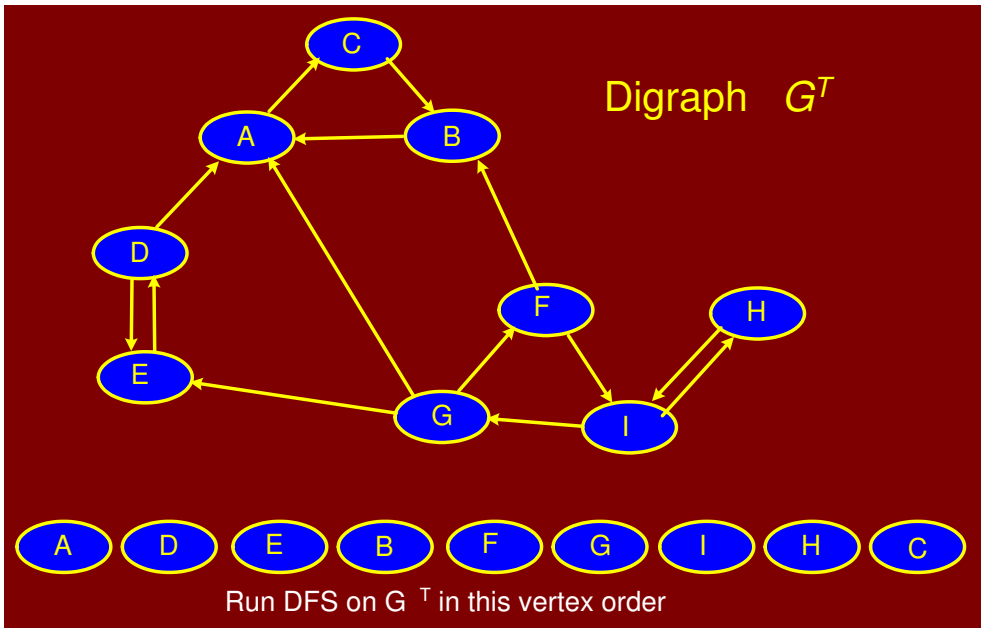


Figure 8.40: Digraph G^T and the vertex order for DFS

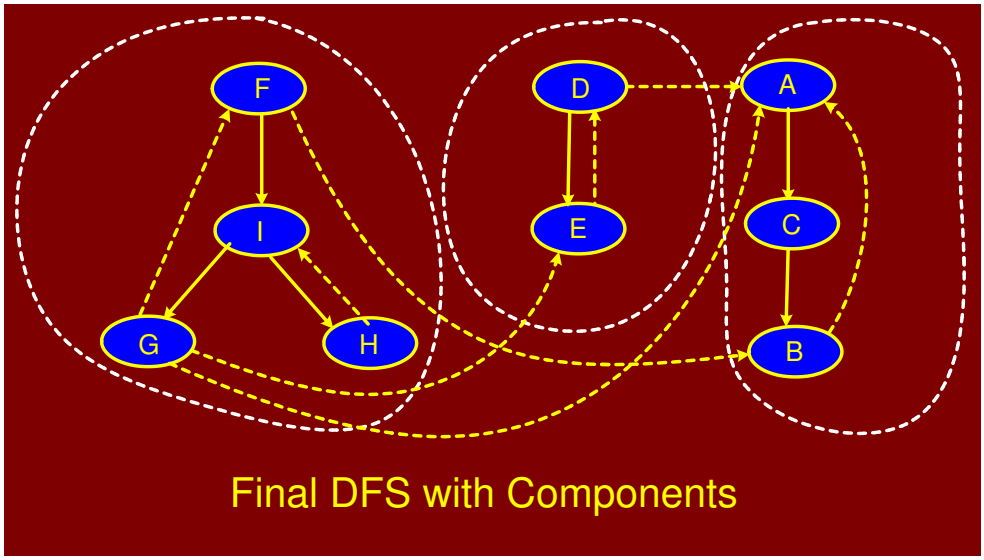


Figure 8.41: Final DFS with strong components of G^T

The complete proof for why this algorithm works is in CLR. We will discuss the intuition behind why the algorithm visits vertices in decreasing order of finish times and why the graph is revered. Recall that the main intent is to visit the strong components in a reverse topological order. The problem is how to order the vertices so that this is true. Recall from the topological sorting algorithm, that in a DAG, finish times occur in reverse topological order (i.e., the first vertex in the topological order is the one with the highest finish time). So, if we wanted to visit the components in reverse topological order, this suggests that we should visit the vertices in increasing order of finish time, starting with the lowest finishing time.

This is a good starting idea, but it turns out that it doesn't work. The reason is that there are many vertices in each strong component, and they all have different finish times. For example, in Figure 8.36, observe that in the first DFS, the lowest finish time (of 4) is achieved by vertex 'c', and its strong component is first, not last, in topological order.

However, there is something to notice about the finish times. If we consider the *maximum finish time* in each component, then these are related to the topological order of the component graph. In fact it is possible to prove the following (but we won't).

Lemma: Consider a digraph G on which DFS has been run. Label each component with the maximum finish time of all the vertices in the component, and sort these in decreasing order. Then this order is a topological order for the component digraph.

For example, in Figure 8.36, the maximum finish times for each component are 18 (for $\{a, b, c\}$), 17 (for $\{d,e\}$), and 12 (for $\{f,g,h,i\}$). The order (18, 17, 12) is a valid topological order for the component digraph. The problem is that this is not what we wanted. We wanted a reverse topological order for the component digraph. So, the final trick is to reverse the digraph. This does not change the component graph, but it reverses the topological order, as desired.

8.5 Minimum Spanning Trees

A common problem is communications networks and circuit design is that of connecting together a set of nodes by a network of total minimum length. The length is the sum of lengths of connecting wires. Consider, for example, laying cable in a city for cable t.v.

The computational problem is called the *minimum spanning tree* (MST) problem. Formally, we are given a connected, undirected graph $G = (V, E)$ Each edge (u, v) has numeric weight of cost. We define the cost of a spanning tree T to be the sum of the costs of edges in the spanning tree

$$w(T) = \sum_{(u,v)\in T} w(u, v)$$

A minimum spanning tree is a tree of minimum weight.

Figures **??**, **??** and **??** show three spanning trees for the same graph. The first is a spanning tree but is not a MST; the other two are.

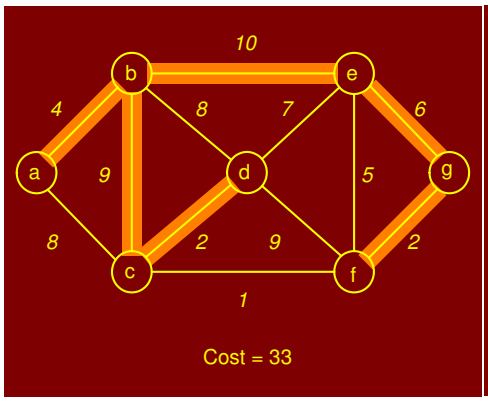


Figure 8.42: A spanning tree that is *not* MST

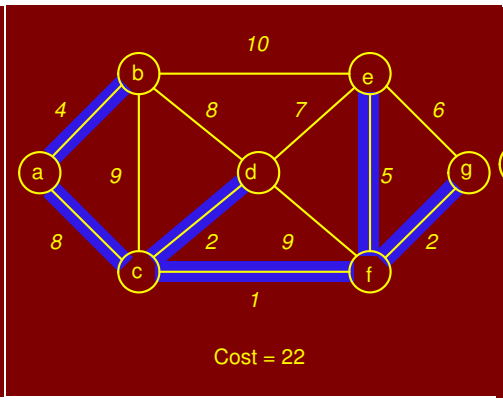


Figure 8.43: A minimum spanning tree

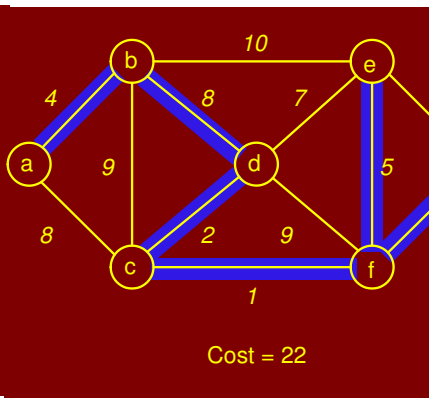


Figure 8.44: Another minimum spanning tree

We will present two *greedy* algorithms (Kruskal's and Prim's) for computing MST. Recall that a greedy algorithm is one that builds a solution by repeatedly selecting the cheapest among all options at each stage. Once the choice is made, it is never undone.

Before presenting the two algorithms, let us review facts about *free trees*. A free tree is a tree with no vertex designated as the root vertex. A free tree with n vertices has exactly $n - 1$ edges. There exists a unique path between any two vertices of a free tree. Adding any edge to a free tree creates a unique cycle. Breaking any edge on this cycle restores the free tree. This is illustrated in Figure 8.45. When the edges (b, e) or (b, d) are added to the free tree, the result is a cycle.

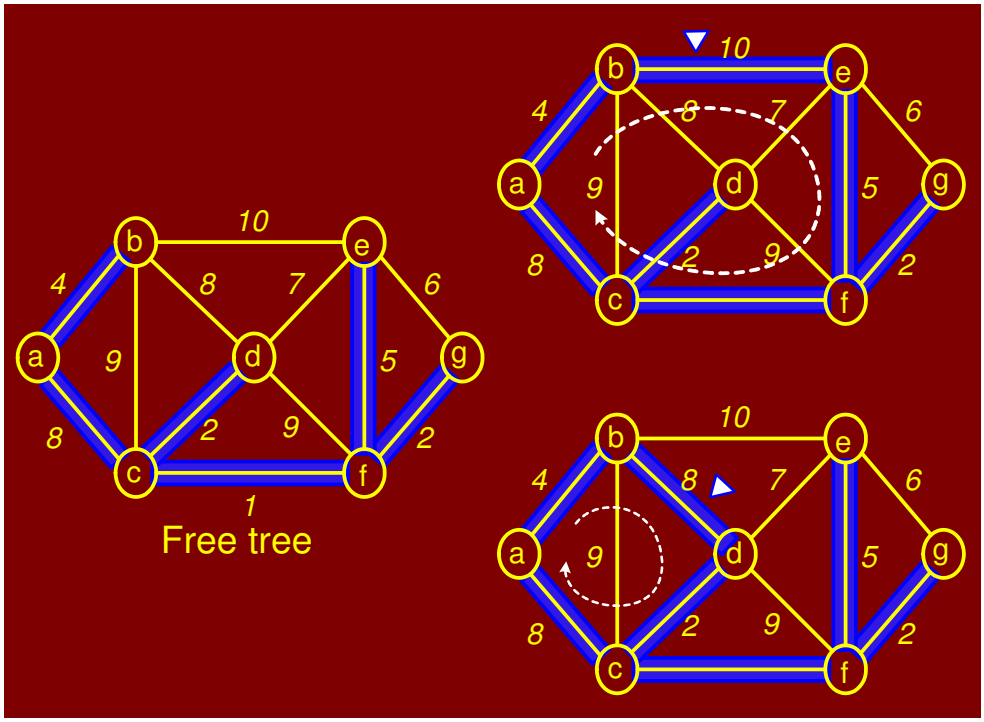


Figure 8.45: Free tree facts

8.5.1 Computing MST: Generic Approach

Let $G = (V, E)$ be an undirected, connected graph whose edges have numeric weights. The intuition behind greedy MST algorithm is simple: we maintain a subset of edges E of the graph . Call this subset A. Initially, A is empty. We will add edges one at a time until A equals the MST.

A subset $A \subseteq E$ is *viable* if A is a subset of edges of *some* MST. An edge $(u, v) \in E - A$ is *safe* if $A \cup \{(u, v)\}$ is viable. In other words, the choice (u, v) is a safe choice to add so that A can still be extended to form a MST.

Note that if A is viable, it cannot contain a cycle. A generic greedy algorithm operates by repeatedly adding any *safe* edge to the current spanning tree.

When is an edge safe? Consider the theoretical issues behind determining whether an edge is safe or not. Let S be a subset of vertices $S \subseteq V$. A *cut* $(S, V - S)$ is just a partition of vertices into two disjoint subsets. An edge (u, v) *crosses* the cut if one endpoint is in S and the other is in $V - S$.

Given a subset of edges A, a cut *respects* A if no edge in A crosses the cut. It is not hard to see why respecting cuts are important to this problem. If we have computed a partial MST and we wish to know which edges can be added that *do not* induce a cycle in the current MST, any edge that crosses a respecting cut is a possible candidate.

8.5.2 Greedy MST

An edge of E is a *light edge* crossing a cut if among all edges crossing the cut, it has the minimum weight. Intuition says that since all the edges that cross a respecting cut do not induce a cycle, then the lightest edge crossing a cut is a natural choice. The main theorem which drives both algorithms is the following:

MST Lemma: Let $G = (V, E)$ be a connected, undirected graph with real-valued weights on the edges. Let A be a viable subset of E (i.e., a subset of some MST). Let $(S, V - S)$ be any cut that respects A and let (u, v) be a light edge crossing the cut. Then the edge (u, v) is *safe* for A. This is illustrated in Figure 8.46.

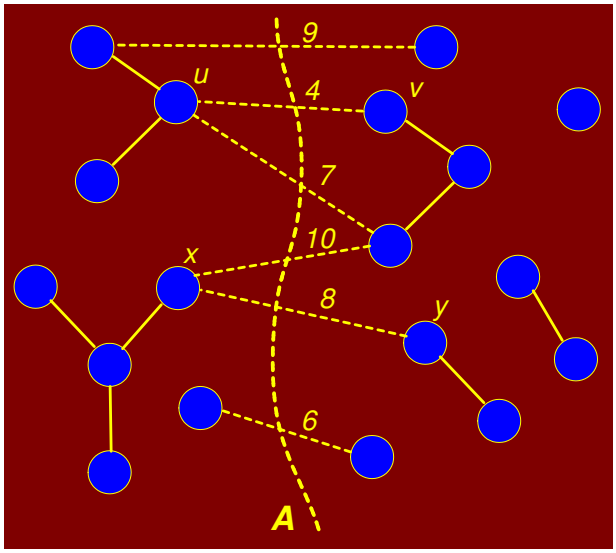


Figure 8.46: Subset A with a cut (wavy line) that respects A

MST Proof: It would simplify the proof if we assume that all edge weights are distinct. Let T be any MST for G. If T contains (u, v) then we are done. This is shown in Figure 8.47 where the lightest edge (u, v) with cost 4 has been chosen.

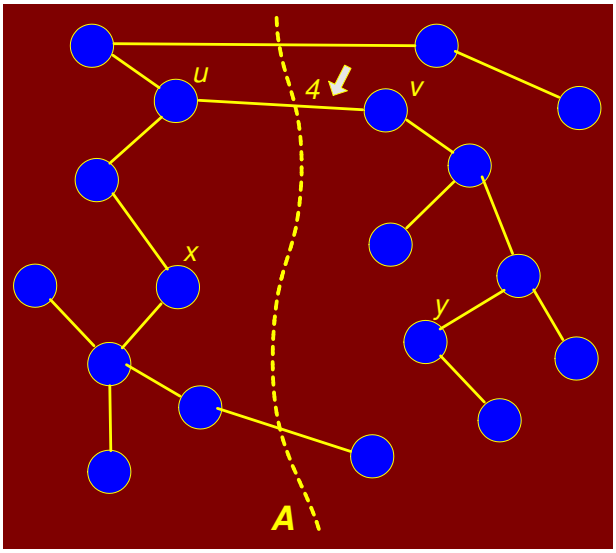


Figure 8.47: MST T which contains light edge (u, v)

Suppose no MST contains (u, v). Such a tree is shown in Figure 8.48. We will derive a contradiction.

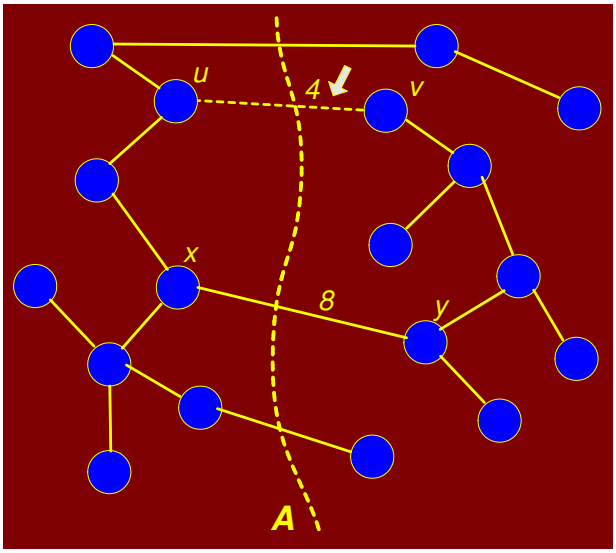


Figure 8.48: MST T which *does not* contains light edge (u, v)

Add (u, v) to T thus creating a cycle as illustrated in Figure 8.49.

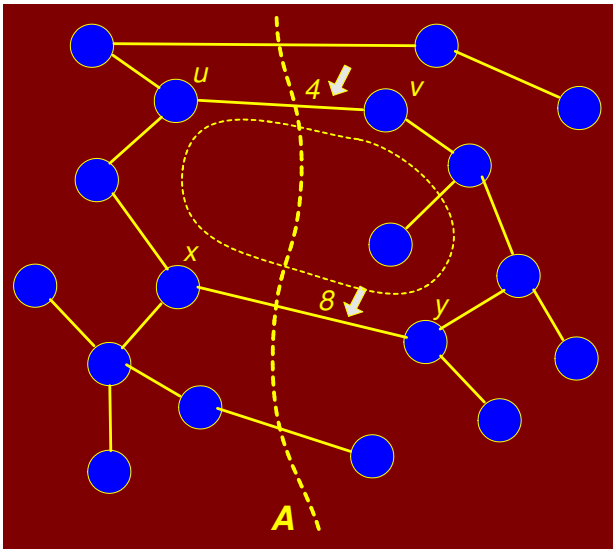


Figure 8.49: Cycle created due to $T + (u, v)$

Since u and v are on opposite sides of the cut, and any cycle must cross the cut an even number of times, there must be at least one other edge (x, y) in T that crosses the cut. The edge (x, y) is not in A because the cut respects A. By removing (x, y) we restore a spanning tree, call it T'. This is shown in Figure 8.50

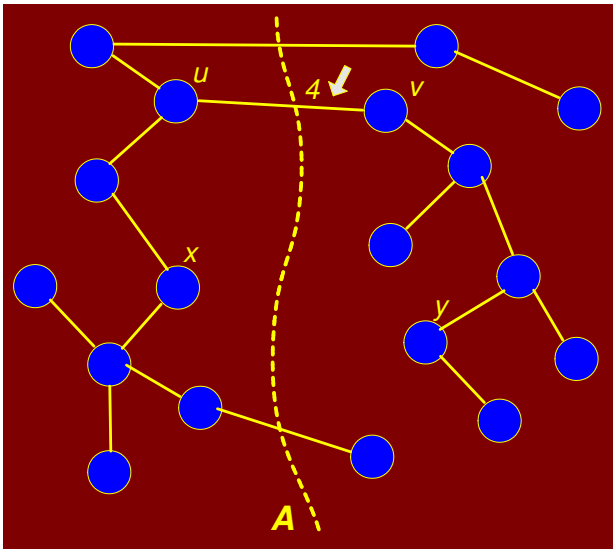


Figure 8.50: Tree $T' = T - (x, y) + (u, v)$

We have $w(T') = w(T) - w(x, y) + w(u, v)$. Since (u, v) is the lightest edge crossing the cut we have $w(u, v) < w(x, y)$. Thus $w(T') < w(T)$ which contradicts the assumption that T was an MST.

8.5.3 Kruskal's Algorithm

Kruskal's algorithm works by adding edges in increasing order of weight (lightest edge first). If the next edge does not induce a cycle among the current set of edges, then it is added to A. If it does, we skip it and consider the next in order. As the algorithm runs, the edges in A induce a *forest* on the vertices. The trees of this forest are eventually merged until a single tree forms containing all vertices.

The tricky part of the algorithm is how to detect whether the addition of an edge will create a cycle in A. Suppose the edge being considered has vertices (u, v). We want a fast test that tells us whether u and v are in the same tree of A. This can be done using the *Union-Find* data structure which supports the following $O(\log n)$ operations:

Create-set(u): Create a set containing a single item u.

Find-set(u): Find the set that contains u

Union(u,v): merge the set containing u and set containing v into a common set.

In Kruskal's algorithm, the vertices will be stored in sets. The vertices in each tree of A will be a set. The edges in A can be stored as a simple list. Here is the algorithm: Figures 8.51 through **??** demonstrate the algorithm applied to a graph.

```
KRUSKAL(G = (V, E))
1  A ← {}
2  for ( each  u ∈ V)
3  do create_set(u)
4  sort E in increasing order by weight w
5  for ( each (u, v) in sorted edge list)
6  do if (find(u) ≠ find(v))
7        then add (u, v) to A
8             union(u, v)
9  return A
```

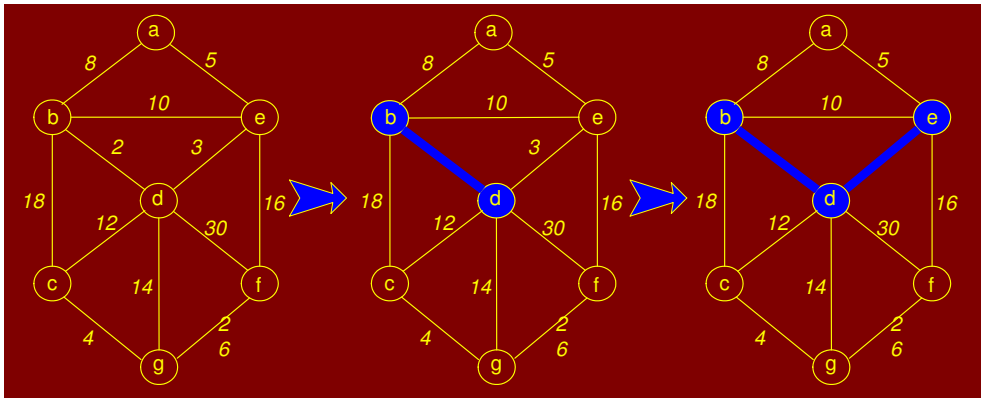


Figure 8.51: Kruskal algorithm: (b, d) and (d, e) added

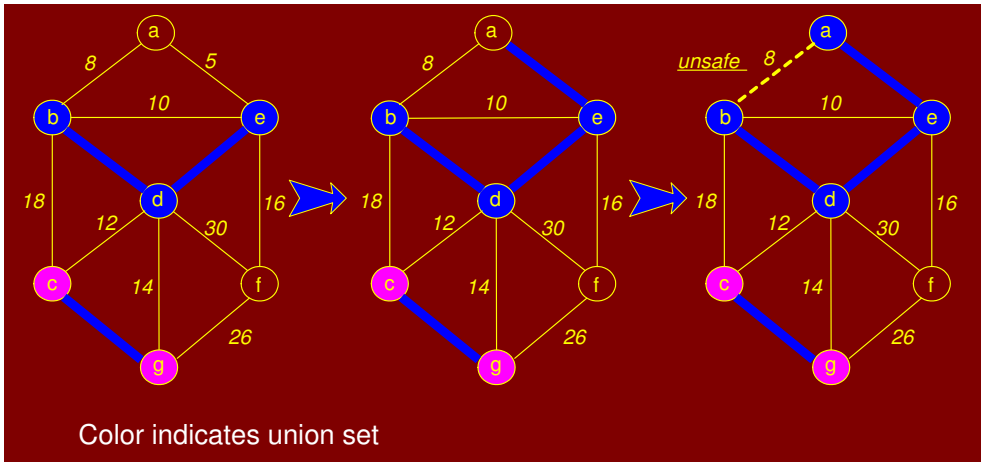


Figure 8.52: Kruskal algorithm: (c, g) and (a, e) added

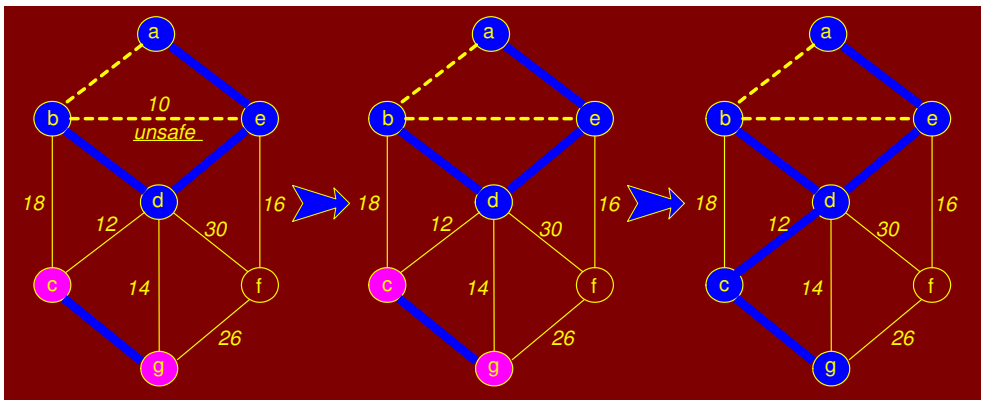


Figure 8.53: Kruskal algorithm: unsafe edges

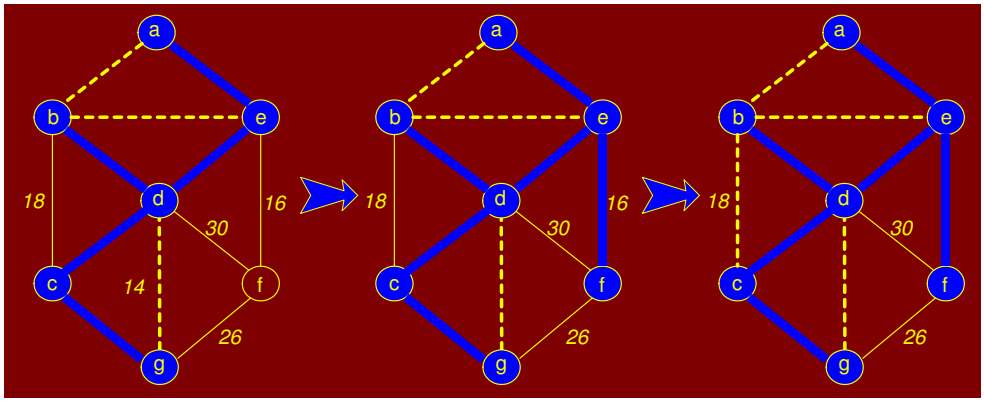


Figure 8.54: Kruskal algorithm: (e, f) added

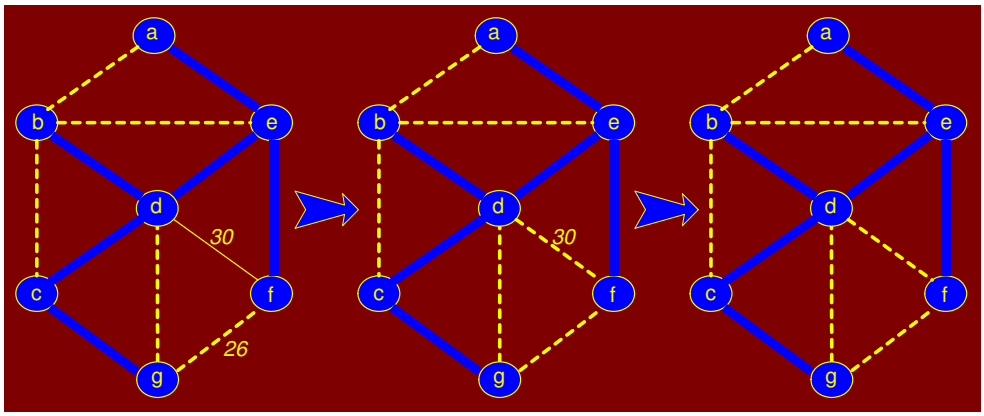


Figure 8.55: Kruskal algorithm: more unsafe edges and final MST

Analysis:

Since the graph is connected, we may assume that $E \geq V - 1$. Sorting edges (*line 4*) takes $\Theta(E \log E)$. The for loop (*line 5*) performs $O(E)$ find and $O(V)$ union operations. Total time for union $-$ find is $O(E\alpha(V))$ where $\alpha(V)$ is the inverse Ackerman function. $\alpha(V) < 4$ for V less the number of atoms in the entire universe. Thus the time is dominated by sorting. Overall time for Kruskal is $\Theta(E \log E) = \Theta(E \log V)$ if the graph is sparse.

8.5.4 Prim's Algorithm

Kruskal's algorithm worked by ordering the edges, and inserting them one by one into the spanning tree, taking care never to introduce a cycle. Intuitively Kruskal's works by merging or splicing two trees together, until all the vertices are in the same tree.

In contrast, Prim's algorithm builds the MST by adding leaves one at a time to the current tree. We start with a root vertex r; it can be any vertex. At any time, the subset of edges A forms a single tree (in Kruskal's, it formed a forest). We look to add a single vertex as a leaf to the tree.

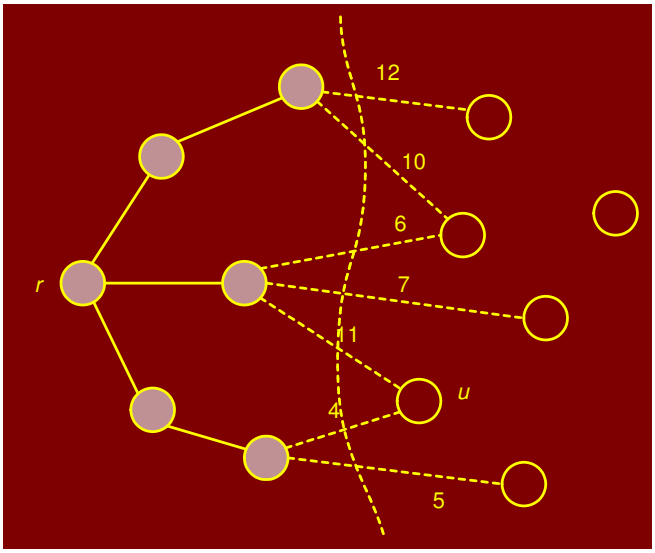


Figure 8.56: Prim's algorithm: a *cut* of the graph

Consider the set of vertices S currently part of the tree and its complement $(V - S)$ as shown in Figure 8.56. We have *cut* of the graph. Which edge should be added next? The greedy strategy would be to add the lightest edge which in the figure is edge to 'u'. Once u is added, Some edges that crossed the cut are no longer crossing it and others that were not crossing the cut are as shown in Figure 8.57

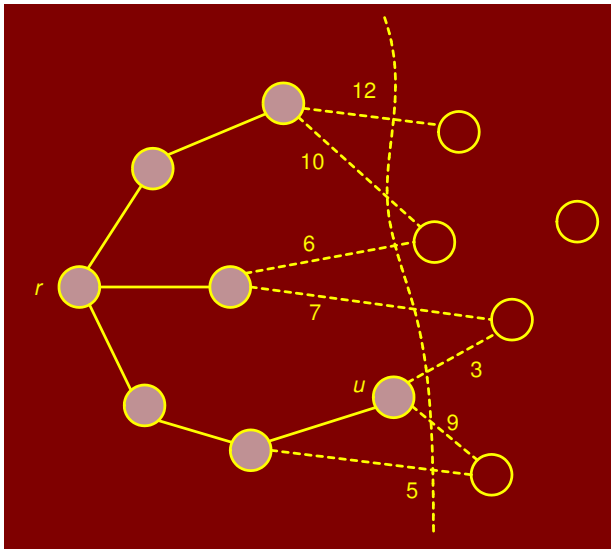


Figure 8.57: Prim's algorithm: u selected

We need an efficient way to update the cut and determine the light edge quickly. To do this, we will make use of a *priority queue*. The question is what do we store in the priority queue? It may seem logical that edges that cross the cut should be stored since we choose light edges from these. Although possible, there is more elegant solution which leads to a simpler algorithm.

For each vertex $u \in (V - S)$ (not part of the current spanning tree), we associate a key $key[u]$. The $key[u]$ is the weight of the lightest edge going from u to any vertex in S. If there is no edge from u to a vertex in S, we set the key value to ∞. We also store in $pred[u]$ the end vertex of this edge in S. We will also need to know which vertices are in S and which are not. To do this, we will assign a color to each vertex. If the color of a vertex is black then it is in S otherwise not. Here is the algorithm:

```
PRIM((G, w, r))
 1  for ( each  u ∈ V)
 2  do  key [u] ← ∞;   pq.insert (u, key[u])
 3      color [u] ←  white
 4   key [r] ← 0;   pred [r] ← nil;  pq.decrease_key (r,  key [r]);
 5  while ( pq.not_empty ())
 6  do u ←  pq.extract_min ()
 7     for ( each  u ∈  adj [u])
 8     do if ( color [v] ==  white )and( w (u, v) <  key [v])
 9           then  key [v] =  w (u, v)
10                 pq.decrease_key (v,  key [v])
11                 pred [v] = u
12      color [u] =  black
```

Figures 8.58 through 8.60 illustrate the algorithm applied to a graph. The contents of the priority queue are shown as the algorithm progresses. The arrows indicate the predecessor pointers and the numeric labels in each vertex is its key value.

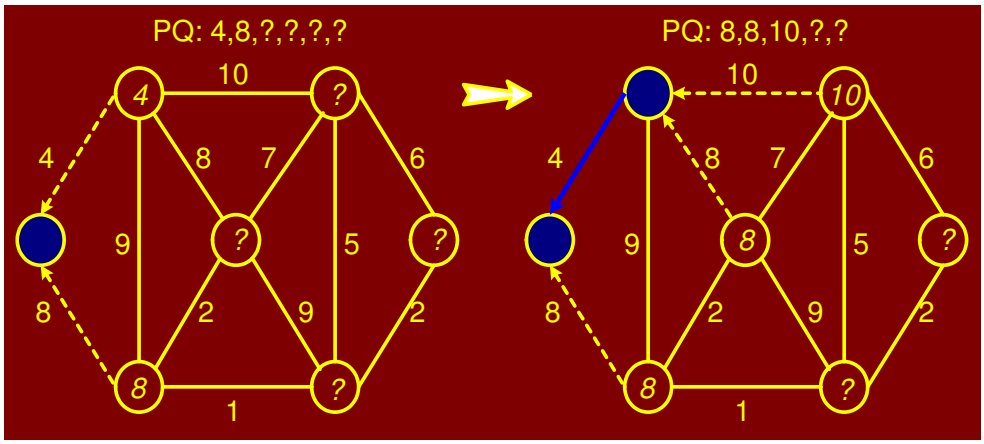


Figure 8.58: Prim's algorithm: edge with weight 4 selected

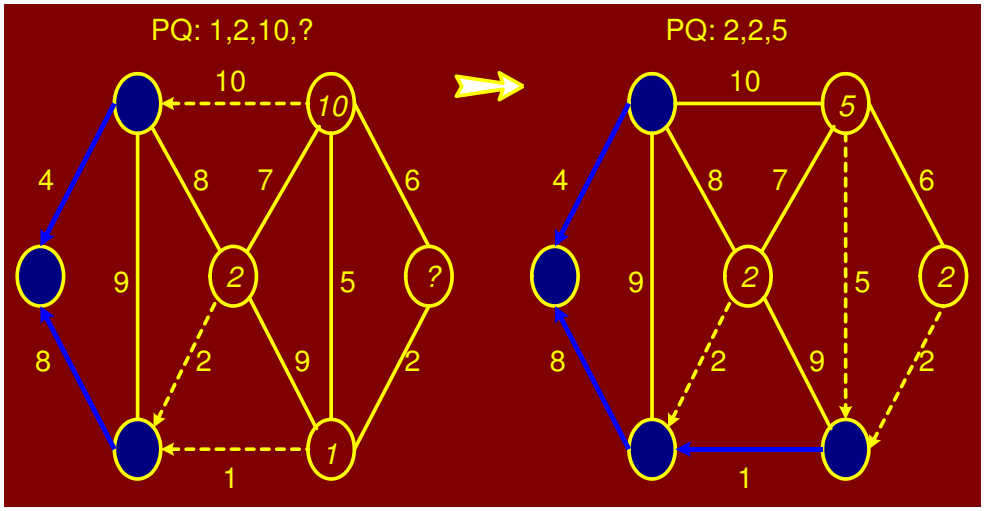


Figure 8.59: Prim's algorithm: edges with weights 8 and 1 selected

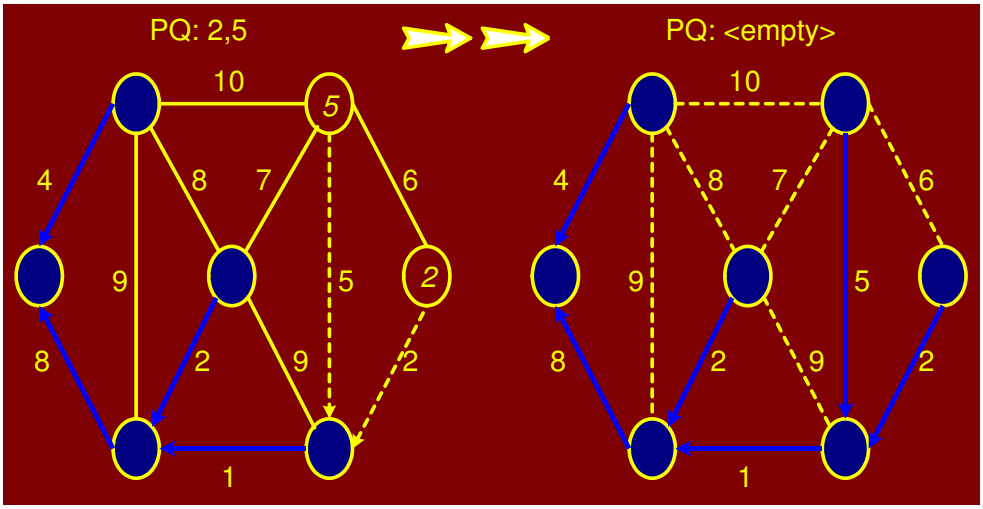


Figure 8.60: Prim's algorithm: the final MST

Analysis:

It takes $O(\log V)$ to extract a vertex from the priority queue. For each incident edge, we spend potentially $O(\log V)$ time decreasing the key of the neighboring vertex. Thus the total time is $O(\log V + \deg(u) \log V)$. The other steps of update are constant time.

So the overall running time is

$$\begin{aligned} T(V,E) &= \sum_{u \in V} (\log V + \deg(u) \log V) \\ &= \log V \sum_{u \in V} (1 + \deg(u)) \\ &= (\log V)(V + 2E) = \Theta((V+E) \log V) \end{aligned}$$

Since G is connected, V is asymptotically no greater than E so this is $\Theta(E \log V)$, same as Kruskal's algorithm.

8.6 Shortest Paths

A motorist wishes to find the shortest possible route between Peshawar and Karachi. Given a road map of Pakistan on which the distance between each pair of adjacent cities is marked Can the motorist determine the shortest route?

In the *shortest-paths problem* We are given a weighted, directed graph $G = (V, E)$ The weight of path $p =< v_0, v_1, \ldots, v_k >$ is the sum of the constituent edges:

$$w(p) = \sum_{i=1}^{k} w(v_{i-1}, v_i)$$

We define the *shortest-path weight* from u to v by

$$\delta(u, v) = \begin{cases} \min\{w(p) : u \overset{p}{\rightsquigarrow} v\} & \text{if there is a path from } u \text{ to } v \\ \infty & \text{otherwise} \end{cases}$$

Problems such as shortest route between cities can be solved efficiently by modelling the road map as a graph. The nodes or vertices represent cities and edges represent roads. Edge weights can be interpreted as distances. Other metrics can also be used, e.g., time, cost, penalties and loss.

Similar scenarios occur in computer networks like the Internet where data packets have to be routed. The vertices are *routers*. Edges are communication links which may be be wire or wireless. Edge weights can be distance, link speed, link capacity link delays, and link utilization.

The breadth-first-search algorithm we discussed earlier is a shortest-path algorithm that works on un-weighted graphs. An un-weighted graph can be considered as a graph in which every edge has weight one unit.

There are a few variants of the shortest path problem. We will cover their definitions and then discuss algorithms for some.

Single-source shortest-path problem: Find shortest paths from a given (single) *source* vertex $s \in V$ to every other vertex $v \in V$ in the graph G.

Single-destination shortest-paths problem: Find a shortest path to a given destination vertex t from each vertex v. We can reduce the this problem to a single-source problem by reversing the direction of each edge in the graph.

Single-pair shortest-path problem: Find a shortest path from u to v for given vertices u and v. If we solve the single-source problem with source vertex u, we solve this problem also. No algorithms for this problem are known to run asymptotically faster than the best single-source algorithms in the worst case.

All-pairs shortest-paths problem: Find a shortest path from u to v for *every pair* of vertices u and v. Although this problem can be solved by running a single-source algorithm once from each vertex, it can usually be solved faster.

8.6.1 Dijkstra's Algorithm

Dijkstra's algorithm is a simple *greedy* algorithm for computing the **single-source shortest-paths** to all other vertices. Dijkstra's algorithm works on a weighted directed graph $G = (V, E)$ in which all edge weights are non-negative, i.e., $w(u, v) \geq 0$ for each edge $(u, v) \in E$.

Negative edges weights maybe counter to intuition but this can occur in real life problems. However, we will *not allow negative cycles* because then there is no shortest path. If there is a negative cycle between, say, s and t, then we can always find a shorter path by going around the cycle one more time.

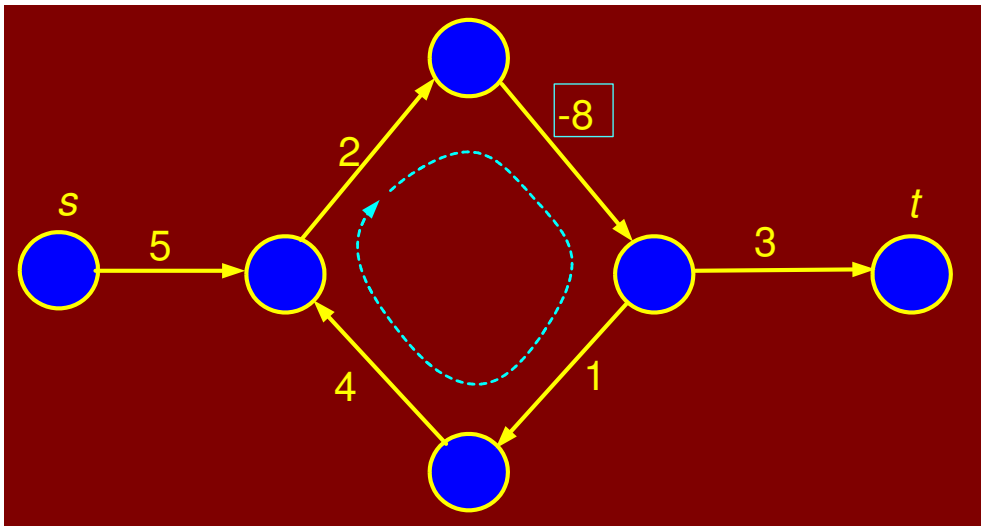


Figure 8.61: Negative weight cycle

The basic structure of Dijkstra's algorithm is to maintain an *estimate* of the shortest path from the source vertex to each vertex in the graph. Call this estimate $d[v]$. Intuitively, $d[v]$ will the length of the shortest path *that the algorithm knows of* from s to v. This value will always be greater than or equal to the true shortest path distance from s to v. I.e., $d[v] \geq \delta(u, v)$. Initially, we know of no paths, so $d[v] = \infty$. Moreover, $d[s] = 0$ for the source vertex.

As the algorithm goes on and sees more and more vertices, it attempts to update $d[v]$ for each vertex in the graph. The process of updating estimates is called *relaxation*. Here is how relaxation works.

Intuitively, if you can see that your solution is not yet reached an optimum value, then push it a little closer to the optimum. In particular, if you discover a path from s to v shorter than $d[v]$, then you need to update $d[v]$. This notion is common to many optimization algorithms.

Consider an edge from a vertex u to v whose weight is $w(u, v)$. Suppose that we have already computed current estimates on $d[u]$ and $d[v]$. We know that there is a path from s to u of weight $d[u]$. By taking

this path and following it with the edge (u, v) we get a path to v of length $d[u] + w(u, v)$. If this path is better than the existing path of length $d[v]$ to v, we should take it. The relaxation process is illustrated in the following figure. We should also remember that the shortest way back to the source is through u by updating the predecessor pointer.

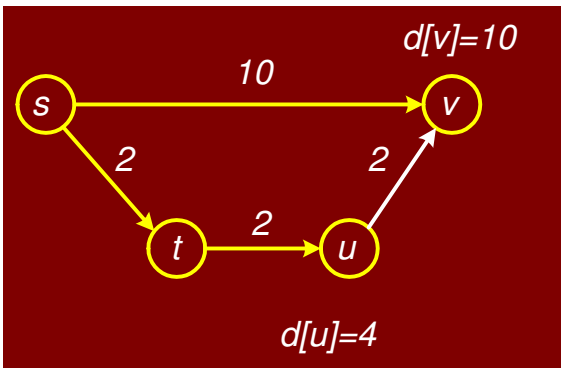


Figure 8.62: Vertex u relaxed

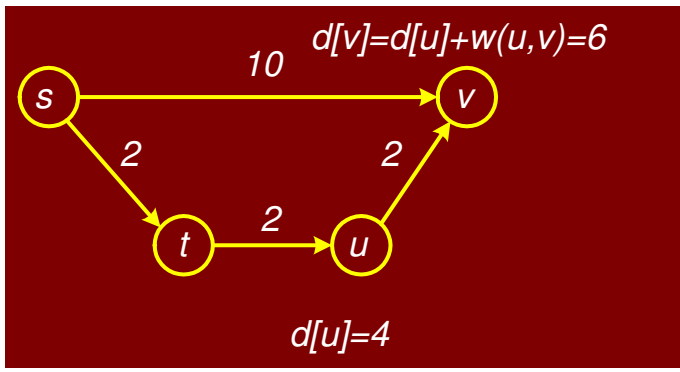


Figure 8.63: Vertex v relaxed

```
RELAX((u, v))
1  if (d[u] + w(u, v) < d[v])
2     then d[v] ← d[u] + w(u, v)
3          pred[v] = u
```

Observe that whenever we set $d[v]$ to a finite value, there is always evidence of a path of that length. Therefore $d[v] \geq \delta(s, v)$. If $d[v] = \delta(s, v)$, then further relaxations cannot change its value.

It is not hard to see that if we perform RELAX(U,V) repeatedly over all edges of the graph, the $d[v]$ values will eventually converge to the final true distance value from s. The cleverness of any shortest path algorithm is to perform the updates in a judicious manner, so the convergence is as fast as possible.

Dijkstra's algorithm is based on the notion of performing repeated relaxations. The algorithm operates by maintaining a subset of vertices, $S \subseteq V$, for which we claim we *know* the true distance, $d[v] = \delta(s, v)$.

Initially $S = \emptyset$, the empty set. We set $d[u] = 0$ and all others to ∞. One by one we select vertices from $V - S$ to add to S.

How do we select which vertex among the vertices of $V - S$ to add next to S? Here is *greediness* comes in. For each vertex $u \in (V - S)$, we have computed a distance estimate $d[u]$.

The greedy thing to do is to take the vertex for which $d[u]$ is minimum, i.e., take the unprocessed vertex that is closest by our estimate to s. Later, we justify why this is the proper choice. In order to perform

this selection efficiently, we store the vertices of $V - S$ in a *priority queue*.

```
DIJKSTRA((G, w, s))
 1  for ( each  u ∈ V)
 2  do d[u] ← ∞
 3      pq.insert (u, d[u])
 4  d[s] ← 0;  pred [s] ← nil;  pq.decrease_key (s, d[s]);
 5  while ( pq.not_empty ())
 6  do u ←  pq.extract_min ()
 7      for ( each  v ∈ adj[u])
 8      do if (d[u] + w(u, v) < d[v])
 9            then d[v] = d[u] + w(u, v)
10                 pq.decrease_key (v, d[v])
11                 pred [v] = u
```

Note the similarity with Prim's algorithm, although a different key is used here. Therefore the running time is the same, i.e., $\Theta(E \log V)$.

Figures 8.64 through **??** demonstrate the algorithm applied to a directed graph with no negative weight edges.

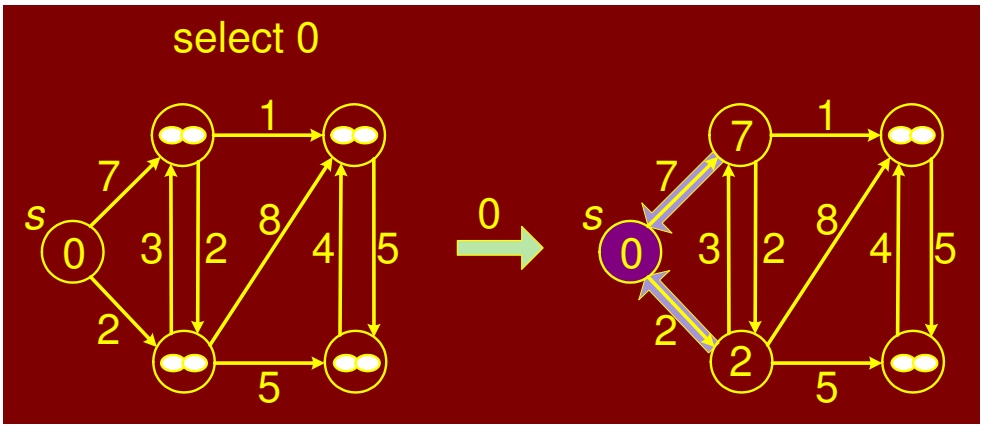


Figure 8.64: Dijkstra's algorithm: select 0

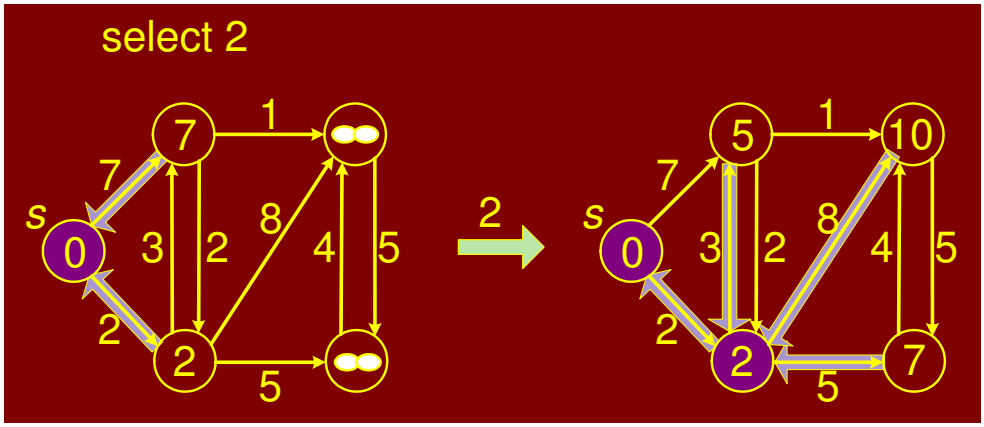


Figure 8.65: Dijkstra's algorithm: select 2

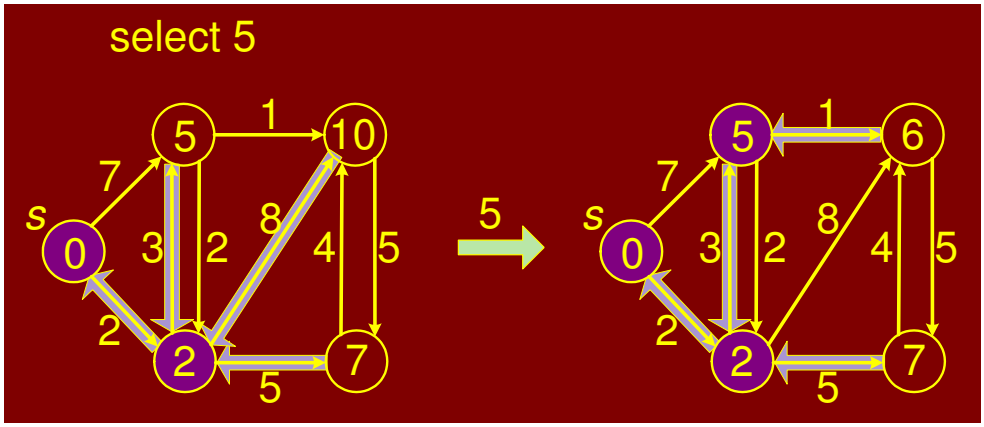


Figure 8.66: Dijkstra's algorithm: select 5

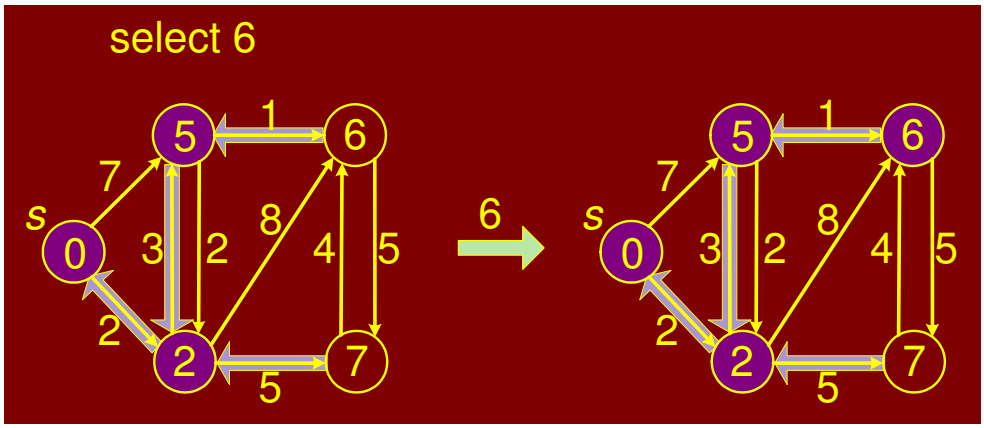


Figure 8.67: Dijkstra's algorithm: select 6

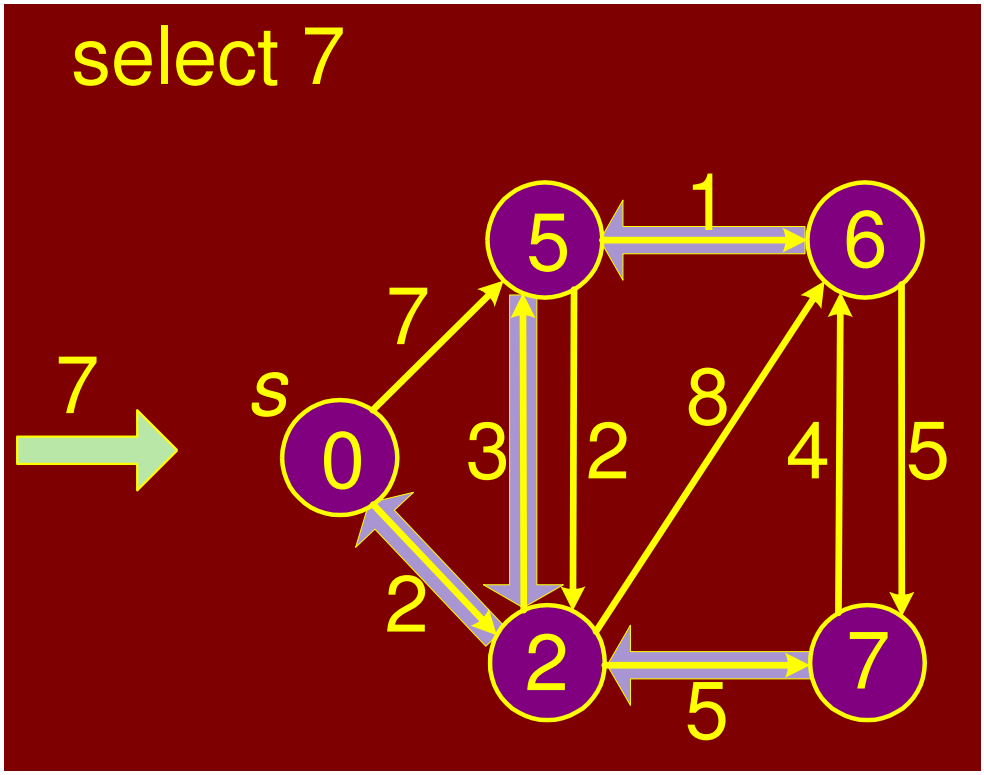


Figure 8.68: Dijkstra's algorithm: select 7

fig:dijlast

8.6.2 Correctness of Dijkstra's Algorithm

We will prove the correctness of Dijkstr's algorithm by Induction. We will use the definition that $\delta(s, v)$ denotes the minimal distance from s to v.

For the base case

1. $S = \{s\}$
2. $d(s) = 0$, which is $\delta(s, s)$

Assume that $d(v) = \delta(s, v)$ for every $v \in S$ and all neighbors of v have been relaxed. If $d(u) \leq d(u')$ for every $u' \in V$ then $d(u) = \delta(s, u)$, and we can transfer u from V to S, after which $d(v) = \delta(s, v)$ for every $v \in S$.

We do this as a proof by contradiction. Suppose that $d(u) > \delta(s, u)$. The shortest path from s to u, $p(s, u)$, must pass through one or more vertices exterior to S. Let x be the last vertex inside S and y be the first vertex outside S on this path to u. Then $p(s, u) = p(s, x) \cup \{(x, y)\} \cup p(y, u)$.

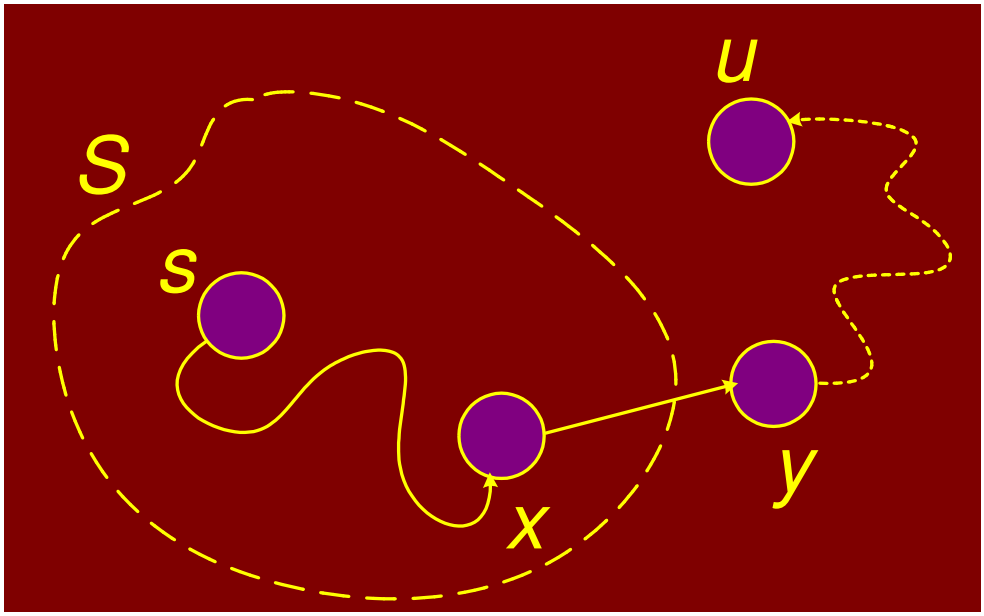


Figure 8.69: Correctness of Dijkstra's algorithm

The length of $p(s,u)$ is $\delta(s,u) = \delta(s,y) + \delta(y,u)$. Because y was relaxed when x was put into S, $d(y) = \delta(s,y)$ by the convergence property. Thus $d(y) \leq \delta(s,u) \leq d(u)$. But, because $d(u)$ is the smallest among vertices not in S, $d(u) \leq d(y)$. The only possibility is $d(u) = d(y)$, which requires $d(u) = \delta(s,u)$ contradicting the assumption.

By the upper bound property, $d(u) \geq \delta(s,u)$. Since $d(u) > \delta(s,u)$ is false, $d(u) = \delta(s,u)$, which is what we wanted to prove. Thus, if we follow the algorithm's procedure, transferring from V to S, the vertex in V with the smallest value of $d(u)$ then all vertices in S have $d(v) = \delta(s,v)$

8.6.3 Bellman-Ford Algorithm

Dijkstra's single-source shortest path algorithm works if all edges weights are non-negative and there are no negative cost cycles. Bellman-Ford *allows* negative weights edges and no negative cost cycles. The algorithm is slower than Dijkstra's, running in $\Theta(VE)$ time.

Like Dijkstra's algorithm, Bellman-Ford is based on performing repeated relaxations. Bellman-Ford applies relaxation to *every edge* of the graph and repeats this $V - 1$ times. Here is the algorithm; its is

illustrated in Figure 8.70.

```
BELLMAN-FORD(G, w, s)
1  for ( each u ∈ V)
2  do d[u] ← ∞
3     pred[u] = nil
4
5  d[s] ← 0;
6  for i = 1 to V − 1
7  do for ( each (u, v) in E )
8     do RELAX(u, v)
```

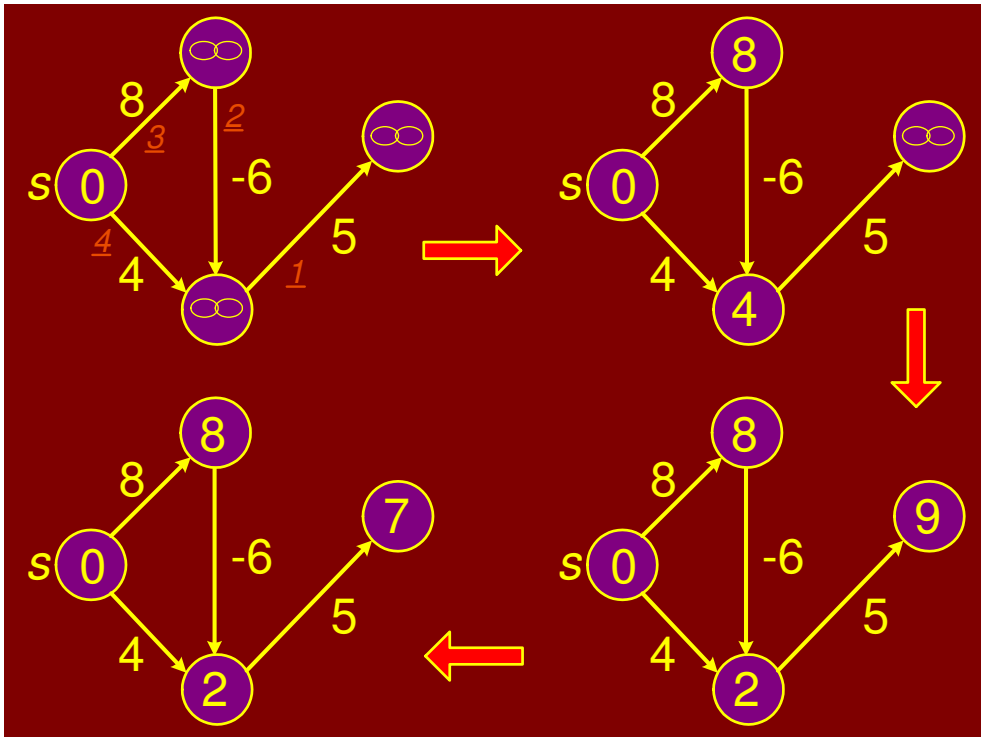


Figure 8.70: The Bellman-Ford algorithm

8.6.4 Correctness of Bellman-Ford

Think of Bellman-Ford as a sort of bubble-sort analog for shortest path. The shortest path information is propagated sequentially along each shortest path in the graph. Consider any shortest path from s to some other vertex u: $\langle v_0, v_1, \ldots, v_k \rangle$ where $v_0 = s$ and $v_k = u$.

Since a shortest path will never visit the same vertex twice, we know that $k \leq V - 1$. Hence the path consists of at most $V - 1$ edges. Since this a shortest path, it is $\delta(s, v_i)$, the true shortest path cost from s

to v_i that satisfies the equation:

$$\delta(s, v_i) = \delta(s, v_{i-1}) + w(v_{i-1}, v_i)$$

Claim: We assert that after the i^{th} pass of the "for-i" loop, $d[v_i] = \delta(s, v_i)$.

Proof: The proof is by induction on i. Observe that after the initialization (pass 0), $d[v_1] = d[s] = 0$.

In general, prior to the i^{th} pass through the loop, the induction hypothesis tells us that $d[v_{i-1}] = \delta(s, v_{i-1})$. After the i^{th} pass, we have done relaxation on the edge (v_{i-1}, v_i) (since we do relaxation along all edges). Thus after the i^{th} pass we have

$$\begin{aligned} d[v_i] &\leq d[v_{i-1}] + w(v_{i-1}, v_i) \\ &= \delta(s, v_{i-1}) + w(v_{i-1}, v_i) \\ &= \delta(s, v_i) \end{aligned}$$

Recall from Dijkstra's algorithm that $d[v_i]$ is never less than $\delta(s, v_i)$. Thus, $d[v_i]$ is in fact equal to $\delta(s, v_i)$. This completes the induction proof.

In summary, after i passes through the for loop, all vertices that are i edges away along the shortest path tree from the source have the correct values stored in $d[u]$. Thus, after the $(V-1)^{st}$ iteration of the for loop, all vertices u have correct distance values stored in $d[u]$.

8.6.5 Floyd-Warshall Algorithm

We consider the generalization of the shortest path problem: to compute the shortest paths between all pairs of vertices. This is called the all-pairs shortest paths problem.

Let $G = (V, E)$ be a directed graph with edge weights. If $(u, v) \in E$ is an edge then $w(u, v)$ denotes its weight. $\delta(u, v)$ is the distance of the minimum cost path between u and v. We will allow G to have negative edges weights but will not allow G to have negative cost cycles. We will present an $\Theta(n^3)$ algorithm for the all pairs shortest path. The algorithm is called the *Floyd-Warshall algorithm* and is based on *dynamic programming*.

We will use an adjacency matrix to represent the digraph. Because the algorithm is matrix based, we will employ the common matrix notation, using i, j and k to denote vertices rather than u, v and w.

The input is an $n \times n$ matrix of edge weights:

$$w_{ij} = \begin{cases} 0 & \text{if } i = j \\ w(i, j) & \text{if } i \neq j \text{ and } (i, j) \in E \\ \infty & \text{if } i \neq j \text{ and } (i, j) \notin E \end{cases}$$

The output will be an $n \times n$ distance matrix $D = d_{ij}$, where $d_{ij} = \delta(i, j)$, the shortest path cost from vertex i to j.

The algorithm dates back to the early 60's. As with other dynamic programming algorithms, the genius of the algorithm is in the clever recursive formulation of the shortest path problem. For a path $p = \langle v_1, v_2, \ldots, v_l$, we say that the vertices $v_2, v_3, \ldots, v_{l-1}$ are the *intermediate vertices* of this path.

Formulation: Define $d_{ij}^{(k)}$ to be the shortest path from i to j such that any intermediate vertices on the path are chosen from the set $\{1, 2, \ldots, k\}$. The path is free to visit any subset of these vertices and in any order. How do we compute $d_{ij}^{(k)}$ assuming we already have the previous matrix $d^{(k-1)}$? There are two basic cases:

1. Don't go through vertex k at all.

2. Do go through vertex k.

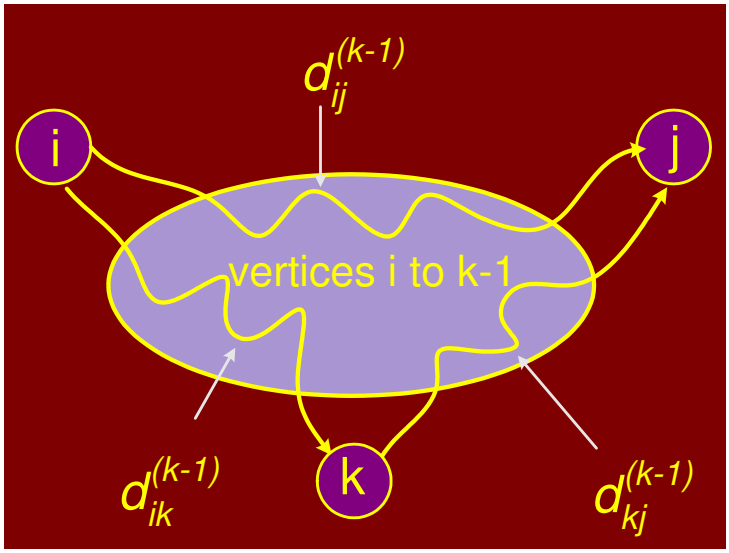


Figure 8.71: Two cases for all-pairs shortest path

Don't go through k at all

Then the shortest path from i to j uses only intermediate vertices $\{1, 2, \ldots, k-1\}$. Hence the length of the shortest is $d_{ij}^{(k-1)}$

Do go through k

First observe that a shortest path does not go through the same vertex twice, so we can assume that we pass through k exactly once. That is, we go from i to k and then from k to j. In order for the overall path to be as short as possible, we should take the shortest path from i to k and the shortest path from k to j. Since each of these paths uses intermediate vertices $\{1, 2, \ldots, k-1\}$, the length of the path is $d_{ik}^{(k-1)} + d_{kj}^{(k-1)}$.

The following illustrate the process in which the value of $d_{3,2}^{k}$ is updated as k goes from 0 to 4.

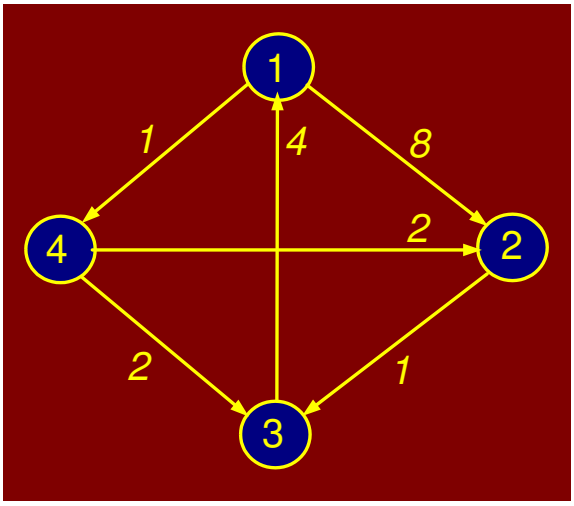


Figure 8.72: $k = 0,\ d^{(0)}_{3,2} = \infty$ (no path)

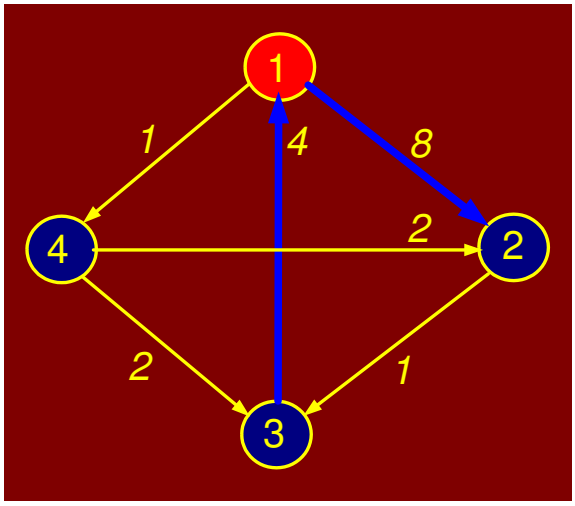


Figure 8.73: $k = 1,\ d^{(1)}_{3,2} = 12\ \ (3 \to 1 \to 2)$

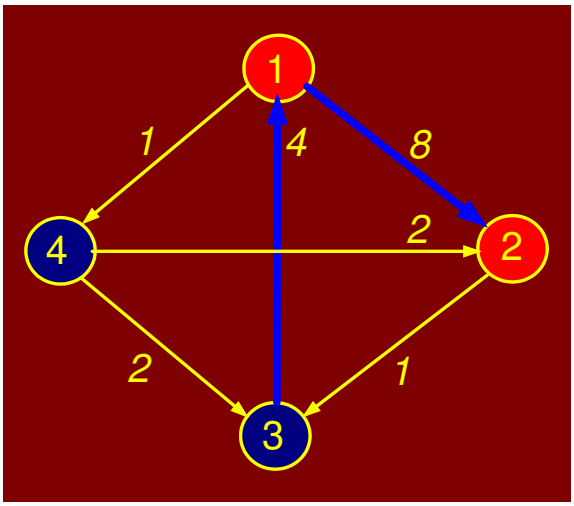


Figure 8.74: $k = 2,\ d^{(2)}_{3,2} = 12\ \ (3 \to 1 \to 2)$

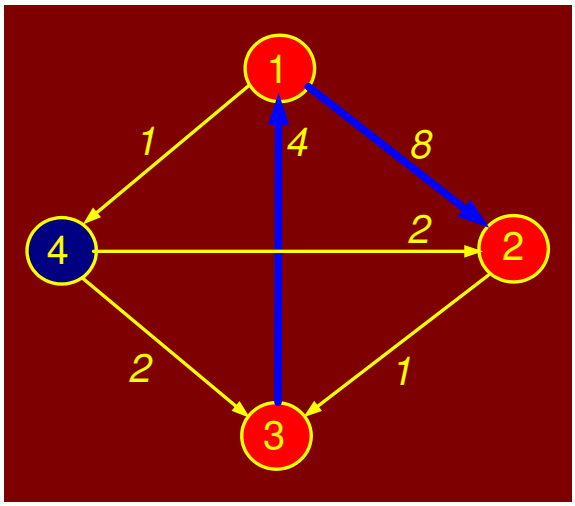


Figure 8.75: $k = 3,\ d^{(3)}_{3,2} = 12\ \ (3 \to 1 \to 2)$

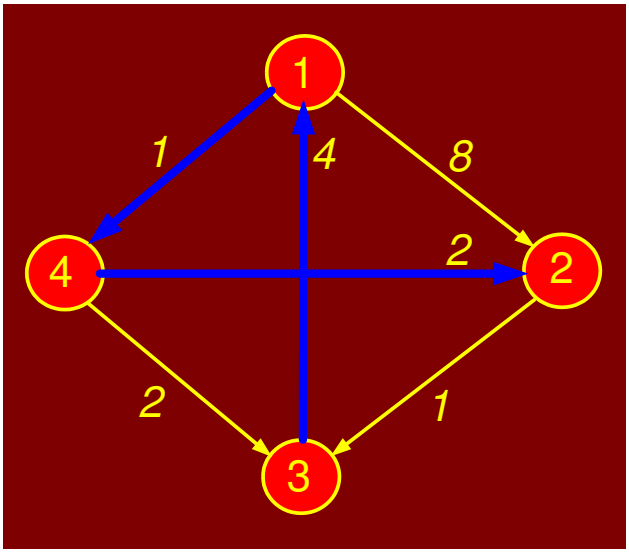


Figure 8.76: $k = 4,\ d^{(4)}_{3,2} = 7\ \ (3 \to 1 \to 4 \to 2)$

This suggests the following recursive (DP) formulation:

$$d^{(0)}_{ij} = w_{ij}$$
$$d^{(k)}_{ij} = \min\left(d^{(k-1)}_{ij}, d^{(k-1)}_{ik} + d^{(k-1)}_{kj}\right)$$

The final answer is $d^{(n)}_{ij}$ because this allows all possible vertices as intermediate vertices.

As is the case with DP algorithms, we will avoid recursive evaluation by generating a table for $d^{(k)}_{ij}$. The algorithm also includes mid-vertex pointers stored in $mid[i, j]$ for extracting the final path.

```
FLOYD-WARSHALL(n, w[1..n, 1..n])
1  for (i = 1, n)
2  do for (j = 1, n)
3      do d[i, j] ← w[i, j]; mid[i, j] ← null
4  for (k = 1, n)
5  do for (i = 1, n)
6      do for (j = 1, n)
7          do if (d[i, k] + d[k, j] < d[i, j])
8                then d[i, j] = d[i, k] + d[k, j]
9                     mid[i, j] = k
```

Clearly, the running time is $\Theta(n^3)$. The space used by the algorithm is $\Theta(n^2)$.

Figure 8.77 through 8.81 demonstrate the algorithm when applied to a graph. The matrix to left of the graph contains the matrix d entries. A circle around an entry $d_{i,k}$ indicates that it was updated in the current k iteration.

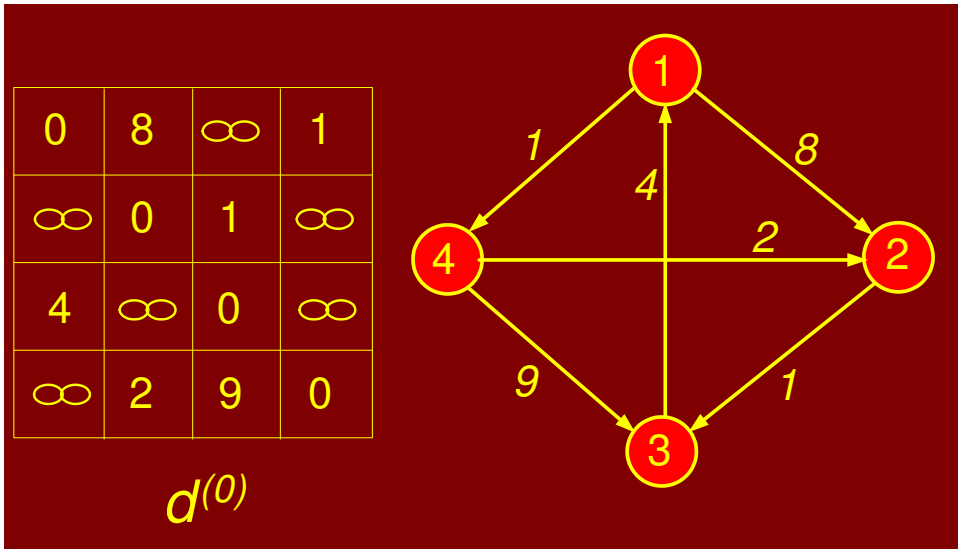


Figure 8.77: Floyd-Warshall Algorithm example: $d^{(0)}$

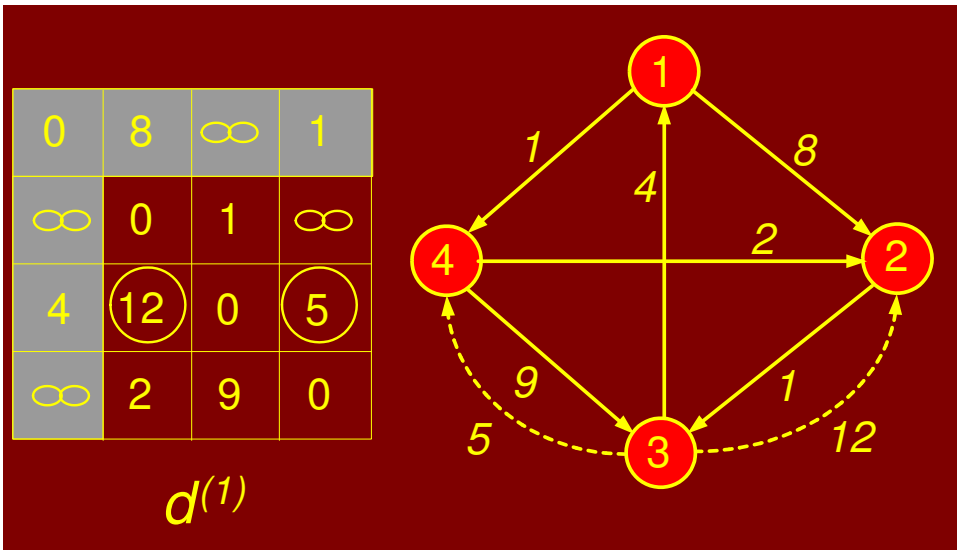


Figure 8.78: Floyd-Warshall Algorithm example: $d^{(1)}$

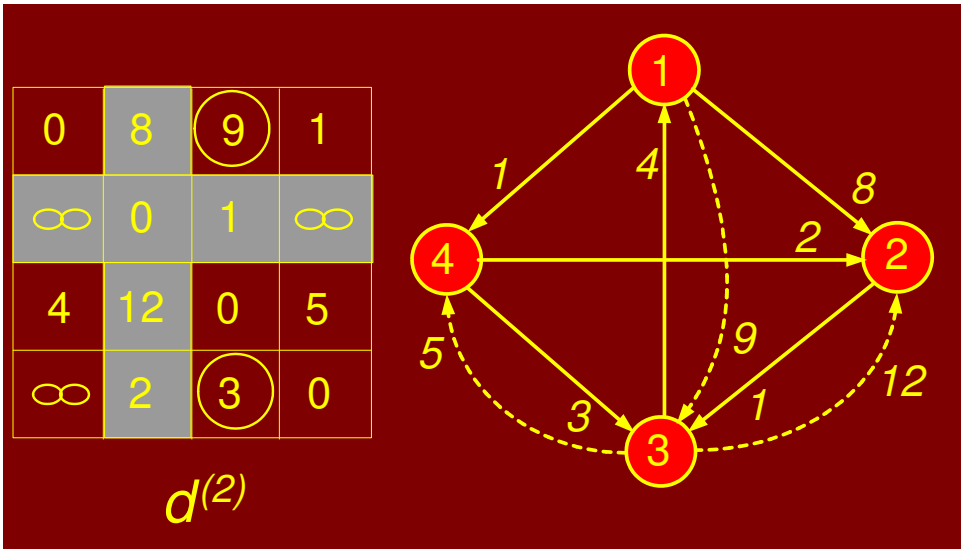


Figure 8.79: Floyd-Warshall Algorithm example: $d^{(2)}$

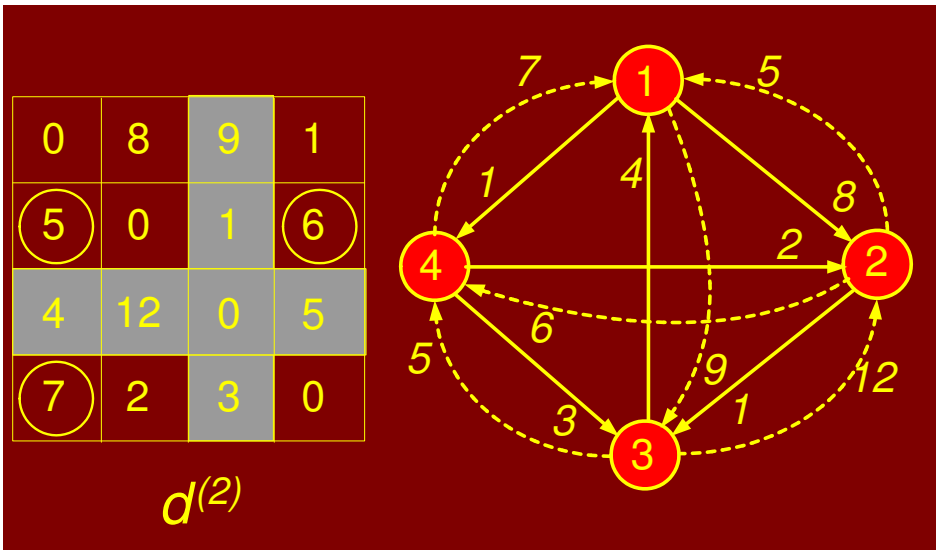


Figure 8.80: Floyd-Warshall Algorithm example: $d^{(3)}$

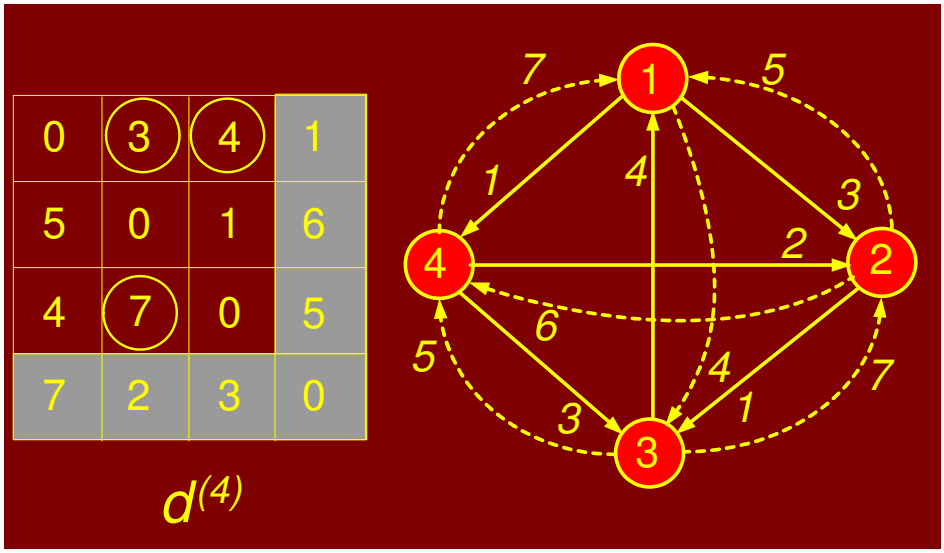


Figure 8.81: Floyd-Warshall Algorithm example: $d^{(4)}$

Extracting Shortest Path:

The matrix d holds the final shortest distance between pairs of vertices. In order to compute the shortest path, the mid-vertex pointers $mid[i,j]$ can be used to extract the final path. Whenever we discovered that the shortest path from i to j passed through vertex k, we set $mid[i,j] = k$. If the shortest path did not pass through k then $mid[i,j] = null$.

To find the shortest path from i to j, we consult $mid[i,j]$. If it is null, then the shortest path is just the edge (i,j). Otherwise we recursively compute the shortest path from i to $mid[i,j]$ and the shortest path from $mid[i,j]$ to j.

```
PATH(i, j)
1  if (mid[i, j] == null)
2     then output(i, j)
3     else PATH(i, mid[i, j])
4          PATH(mid[i, j], j)
```

Chapter 9

Complexity Theory

So far in the course, we have been building up a "bag of tricks" for solving algorithmic problems. Hopefully you have a better idea of how to go about solving such problems. What sort of design paradigm should be used: divide-and-conquer, greedy, dynamic programming.

What sort of data structures might be relevant: trees, heaps, graphs. What is the running time of the algorithm. All of this is fine if it helps you discover an acceptably efficient algorithm to solve your problem.

The question that often arises in practice is that you have tried every trick in the book and nothing seems to work. Although your algorithm can solve small problems reasonably efficiently (e.g., $n \leq 20$), for the really large problems you want to solve, your algorithm never terminates. When you analyze its running time, you realize that it is running in exponential time, perhaps $n^{\sqrt{n}}$, or 2^n, or 2^{2^n}, or $n!$ or worse!.

By the end of 60's, there was great success in finding efficient solutions to many combinatorial problems. But there was also a growing list of problems for which there seemed to be no known efficient algorithmic solutions.

People began to wonder whether there was some unknown paradigm that would lead to a solution to there problems. Or perhaps some proof that these problems are inherently hard to solve and no algorithmic solutions exist that run under exponential time.

Near the end of the 1960's, a remarkable discovery was made. Many of these hard problems were interrelated in the sense that if you could solve any one of them in polynomial time, then you could solve all of them in polynomial time. this discovery gave rise to the notion of NP-completeness.

This area is a radical departure from what we have been doing because the emphasis will change. The goal is no longer to prove that a problem *can* be solved efficiently by presenting an algorithm for it. Instead we will be trying to show that a problem *cannot* be solved efficiently.

Up until now all algorithms we have seen had the property that their worst-case running time are bounded above by some *polynomial* in n. A *polynomial time algorithm* is any algorithm that runs in $O(n^k)$ time. A problem is solvable in polynomial time if there is a polynomial time algorithm for it.

Some functions that do not look like polynomials (such as $O(n \log n)$ are bounded above by polynomials (such as $O(n^2)$). Some functions that do look like polynomials are not. For example, suppose you have

an algorithm that takes as input a graph of size n and an integer k and run in $O(n^k)$ time.

Is this a polynomial time algorithm? No, because k is an input to the problem so the user is allowed to choose $k = n$, implying that the running time would be $O(n^n)$. $O(n^n)$ is surely not a polynomial in n. The important aspect is that the exponent must be a constant independent of n.

9.1 Decision Problems

Most of the problems we have discussed involve optimization of one form of another. Find the shortest path, find the minimum cost spanning tree, maximize the knapsack value. For rather technical reasons, the NP-complete problems we will discuss will be phrased as *decision problems*.

A problem is called a *decision problem* if its output is a simple "yes" or "no" (or you may this of this as true/false, 0/1, accept/reject.) We will phrase may optimization problems as decision problems. For example, the MST decision problem would be: Given a weighted graph G and an integer k, does G have a spanning tree whose weight is at most k?

This may seem like a less interesting formulation of the problem. It does not ask for the weight of the minimum spanning tree, and it does not even ask for the edges of the spanning tree that achieves this weight. However, our job will be to show that certain problems cannot be solved efficiently. If we show that the simple decision problem cannot be solved e¡ciently, then the more general optimization problem certainly cannot be solved efficiently either.

9.2 Complexity Classes

Before giving all the technical definitions, let us say a bit about what the general classes look like at an intuitive level.

Class P: This is the set of all decision problems that can be *solved* in polynomial time. We will generally refer to these problems as being "easy" or "efficiently solvable".

Class NP: This is the set of all decision problems that can be *verified* in polynomial time. This class contains P as a subset. It also contains a number of problems that are believed to be very " hard" to solve.

Class NP: The term "NP" does not mean "not polynomial". Originally, the term meant " non-deterministic polynomial" but it is a bit more intuitive to explain the concept from the perspective of verification.

Class NP-hard: In spite of its name, to say that a problem is NP-hard does not mean that it is hard to solve. Rather, it means that if we could solve this problem in polynomial time, then we could solve *all* NP problems in polynomial time. Note that for a problem to NP-hard, it does not have to be in the class NP.

Class NP-complete: A problem is NP-complete if (1) it is in NP and (2) it is NP-hard.

The Figure 9.1 illustrates one way that the sets P, NP, NP-hard, and NP-complete (NPC) might look. We say might because we do not know whether all of these complexity classes are distinct or whether they are all solvable in polynomial time. The Graph Isomorphism, which asks whether two graphs are identical up to a renaming of their vertices. It is known that this problem is in NP, but it is not known to be in P. The other is QBF, which stands for Quantified Boolean Formulas. In this problem you are given a boolean formula and you want to know whether the formula is true or false.

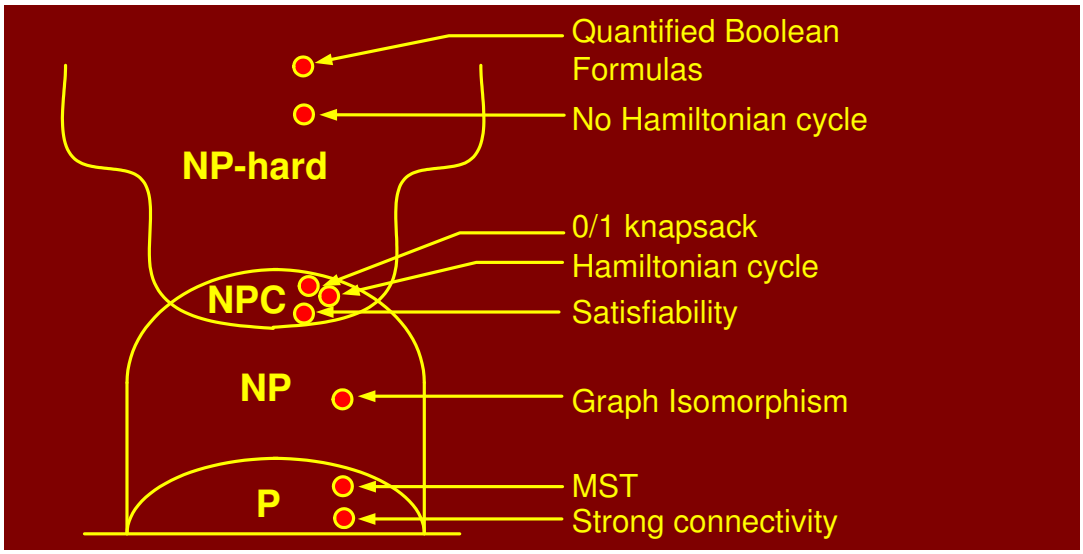


Figure 9.1: Complexity Classes

9.3 Polynomial Time Verification

Before talking about the class of NP-complete problems, it is important to introduce the notion of a *verification algorithm*. Many problems are hard to solve but they have the property that it easy to verify the solution if one is provided. Consider the Hamiltonian cycle problem.

Given an undirected graph G, does G have a cycle that visits every vertex exactly once? There is no known polynomial time algorithm for this problem.

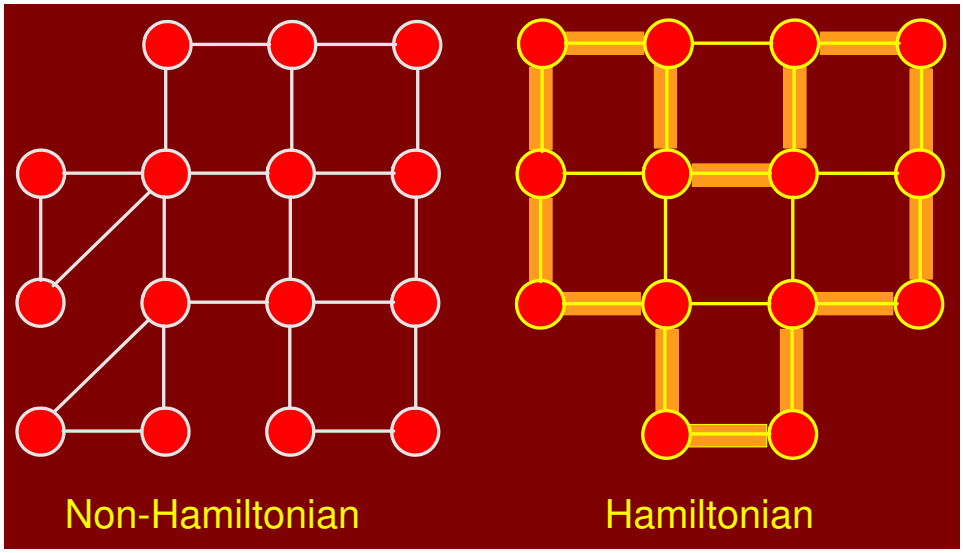


Figure 9.2: Hamiltonian Cycle

However, suppose that a graph did have a Hamiltonian cycle. It would be easy for someone to convince of this. They would simply say: "the cycle is $\langle v_3, v_7, v_1, \ldots, v_13\rangle$ We could then inspect the graph and check that this is indeed a legal cycle and that it visits all of the vertices of the graph exactly once. Thus, even though we know of no efficient way to *solve* the Hamiltonian cycle problem, there is a very efficient way to *verify* that a a given cycle is indeed a Hamiltonian cycle.

The piece of information that allows verification is called a *certificate*. Note that not all problems have the property that they are easy to verify. For example, consider the following two:

1. $\mathrm{UHC} = \{(G)|G \text{ has a unique Hamiltonian cycle}\}$
2. $\overline{\mathrm{HC}} = \{(G)|G \text{ has no Hamiltonian cycle}\}$

Suppose that a graph G is in UHC. What information would someone give us that would allow us to verify this? They could give us an example of the unique Hamiltonian cycle and we could verify that it is a Hamiltonian cycle. But what sort of certificate could they give us to convince us that this is the *only* one?

They could give another cycle that is *not* Hamiltonian. But this does not mean that there is not another cycle somewhere that is Hamiltonian. They could try to list every other cycle of length n, but this is not efficient at all since there are $n!$ possible cycles in general. Thus it is hard to imagine that someone could give us some information that would allow us to efficiently verify that the graph is in UHC.

9.4 The Class NP

The class NP is a set of all problems that can be verified by a polynomial time algorithm. Why is the set called "NP" and not "VP"? The original term NP stood for *non-deterministic polynomial time*. This

referred to a program running on a non-deterministic computer that can make guesses. Such a computer could non-deterministically guess the value of the certificate. and then verify it in polynomial time. We have avoided introducing non-determinism here; it is covered in other courses such as automata or complexity theory.

Observe that $\mathsf{P} \subseteq \mathsf{NP}$. In other words, if we can solve a problem in polynomial time, we can certainly verify the solution in polynomial time. More formally, we do not need to see a certificate to solve the problem; we can solve it in polynomial time anyway.

However, it is not known whether $\mathsf{P} = \mathsf{NP}$. It seems unreasonable to think that this should be so. Being able to verify that you have a correct solution does not help you in finding the actual solution. The belief is that $\mathsf{P} \neq \mathsf{NP}$ but no one has a proof for this.

9.5 Reductions

The class NP-complete (NPC) problems consists of a set of decision problems (a subset of class NP) that no one knows how to solve efficiently. But if there were a polynomial solution for even a single NP-complete problem, then ever problem in NPC will be solvable in polynomial time. For this, we need the concept of *reductions*.

Consider the question: Suppose there are two problems, A and B. You know (or you strongly believe at least) that it is impossible to solve problem A in polynomial time. You want to prove that B cannot be solved in polynomial time. We want to show that

$$(\mathsf{A} \notin \mathsf{P}) \Rightarrow (\mathsf{B} \notin \mathsf{P})$$

How would you do this? Consider an example to illustrate reduction: The following problem is well-known to be NPC:

3-color: Given a graph G, can each of its vertices be labelled with one of 3 different colors such that two adjacent vertices have the same label (color).

Coloring arises in various partitioning problems where there is a constraint that two objects cannot be assigned to the same set of partitions. The term “coloring” comes from the original application which was in map drawing. Two countries that share a common border should be colored with different colors.

It is well known that planar graphs can be colored (maps) with *four colors*. There exists a polynomial time algorithm for this. But determining whether this can be done with 3 colors is hard and there is no polynomial time algorithm for it. In Figure 9.3, the graph on the left can be colored with 3 colors while the graph on the right cannot be colored.

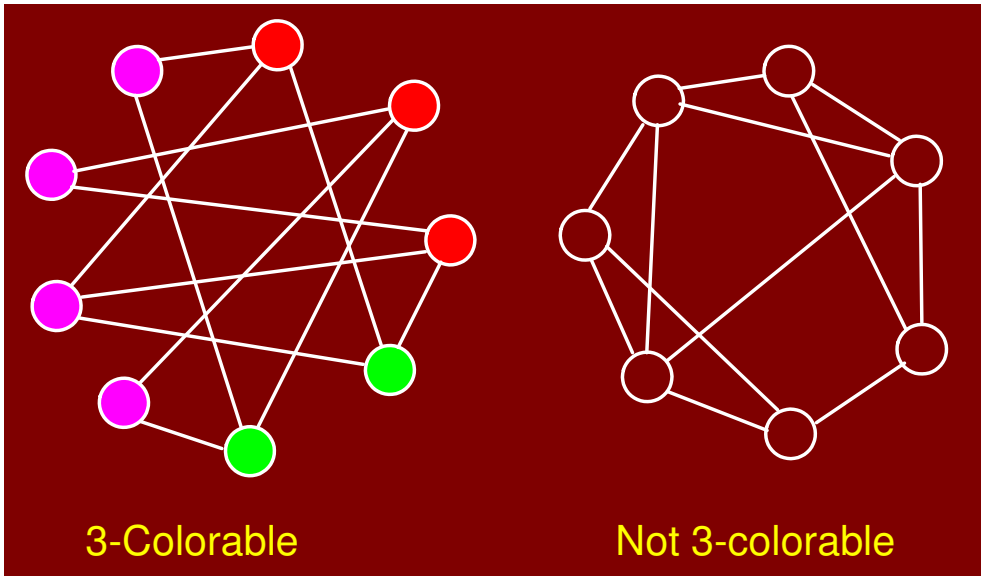


Figure 9.3: Examples of 3-colorable and non-3-colorable graphs

Example1: Fish tank problem

Consider the following problem than can be solved with the graph coloring approach. A tropical fish hobbyist has six different types of fish designated by A, B, C, D, E, and F, respectively. Because of predator-prey relationships, water conditions and size, some fish can be kept in the same tank. The following table shows which fish cannot be together:

Type	Cannot be with
A	B, C
B	A ,C, E
C	A, B, D, E
D	C, F
E	B, C, F
F	D, E

These constraints can be displayed as a graph where an edge between two vertices exists if the two species cannot be together. This is shown in Figure 9.4. For example, A cannot be with B and C; there is an edge between A and B and between A and C.

Given these constraints, What is the *smallest* number of tanks needed to keep all the fish? The answer can be found by coloring the vertices in the graph such that no two adjacent vertices have the same color. This particular graph is 3-colorable and therefore, 3 fish tanks are enough. This is depicted in Figure 9.5. The 3 fish tanks will hold fish as follows:

Tank 1	Tank 2	Tank 3
A, D	F, C	B, E

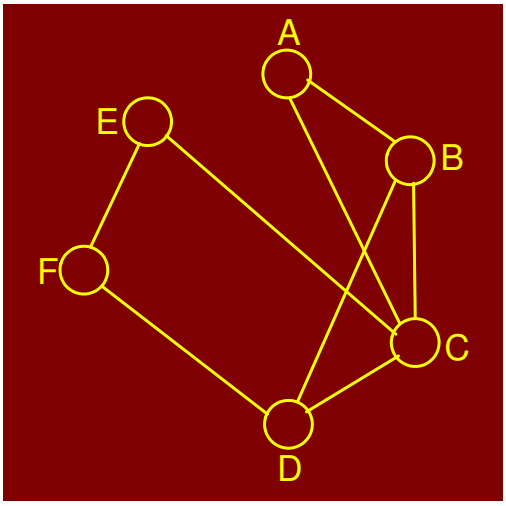


Figure 9.4: Graph representing constraints between fish species

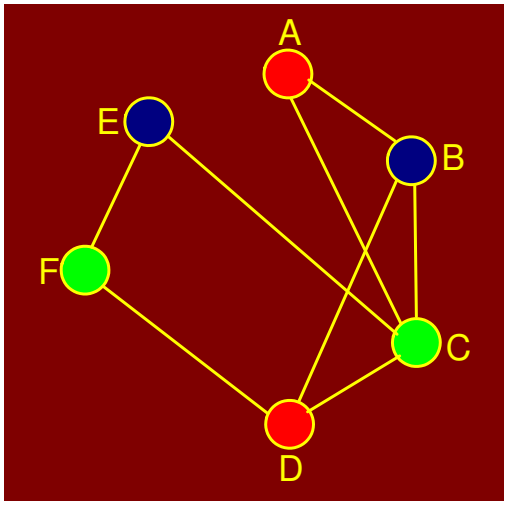


Figure 9.5: Fish tank graph colored with 3 colors

The 3-color (3Col) problem will the play the role of A, which we strongly suspect to not be solvable in polynomial time. For our problem B, consider the following problem: Given a graph $G = (V, E)$, we say that a subset of vertices $V' \subseteq V$ forms a *clique* if for every pair of vertices $u, v \in V'$, the edge $(u, v) \in V'$ That is, the subgraph induced by V' is a complete graph.

Clique Cover: Given a graph G and an integer k, can we find k subsets of vertices $V_1, V_2, \ldots, V_k$, such that $\bigcup_i V_i = V$, and that each V_i is a clique of G.

The following figure shows a graph that has a clique cover of size 3. There are three subgraphs that are complete.

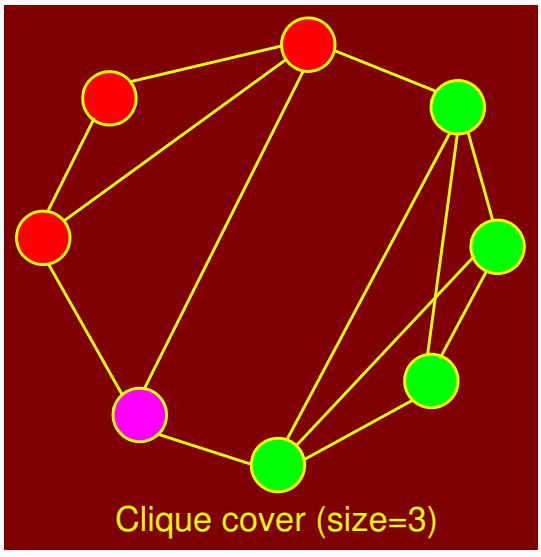


Figure 9.6: Graph with clique cover of size 3

The clique cover problem arises in applications of clustering. We put an edge between two nodes if they are similar enough to be clustered in the same group. We want to know whether it is possible to cluster all the vertices into k groups.

Suppose that you want to solve the CCov problem. But after a while of fruitless effort, you still cannot find a polynomial time algorithm for the CCov problem. How can you prove that CCov is likely to not have a polynomial time solution?

You know that 3Col is NP-complete and hence, experts believe that $3\text{Col} \notin P$. You feel that there is some connection between the CCov problem and the 3Col problem. Thus, you want to show that

$$(3\text{Col} \notin P) \Rightarrow (\text{CCov} \notin P)$$

Both problems involve partitioning the vertices into groups. In the clique cover problem, for two vertices to be in the same group, they must be adjacent to each other. In the 3-coloring problem, for two vertices to be in the same color group, they must not be adjacent. In some sense, the problems are almost the same but the adjacency requirements are exactly reversed.

We claim that we can reduce the 3-coloring problem into the clique cover problem as follows: Given a graph G for which we want to determine its 3-colorability, output the pair $(\overline{G}, 3)$ where $\overline{G}$ denotes the complement of G. Feed the pair $(\overline{G}, 3)$ into a routine for clique cover.

For example, the graph G in Figure 9.7 is 3-colorable and its complement $(\overline{G}, 3)$ is coverable by 3 cliques. The graph G in Figure 9.8 is not 3-colorable; it is also not coverable by cliques.

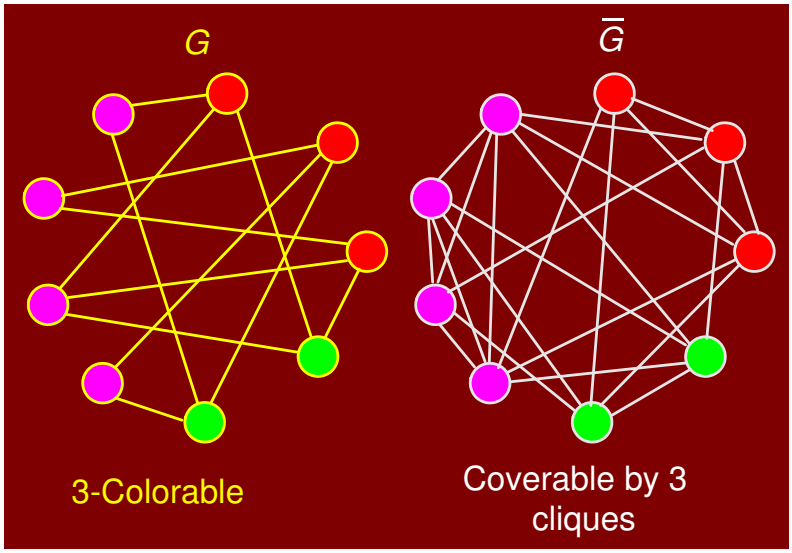


Figure 9.7: 3-colorable G and clique coverable $(\overline{G}, 3)$

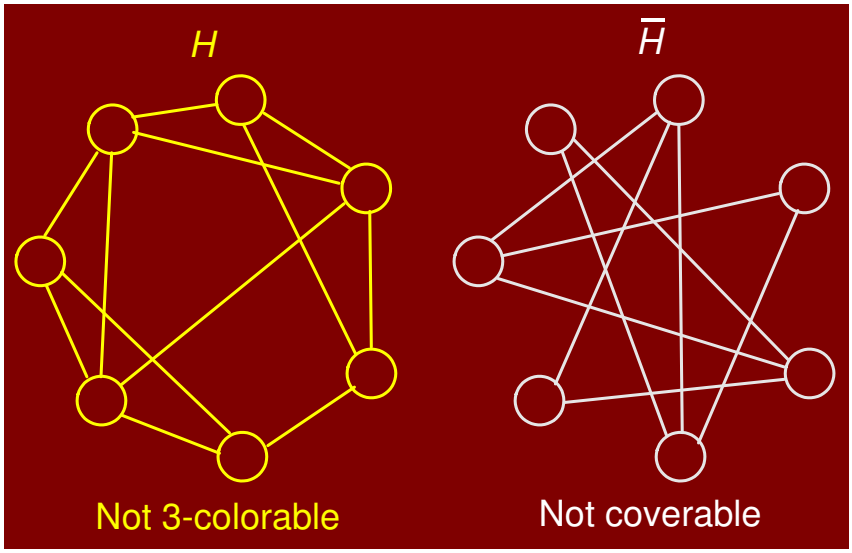


Figure 9.8: G is not 3-colorable and $(\overline{G}, 3)$ is not clique coverable

9.6 Polynomial Time Reduction

Definition: We say that a decision problem L_1 is polynomial-time reducible to decision problem L_2 (written $L_1 \leq_p L_2$) if there is polynomial time computable function f such that for all x, $x \in L_1$ if and only if $f(x) \in L_2$.

In the previous example we showed that

$$3\text{Col} \leq_P \text{CCov}$$

In particular, we have $f(G) = (\overline{G}, 3)$. It is easy to complement a graph in $O(n^2)$ (i.e., polynomial time). For example, flip the 0's and 1's in the adjacency matrix.

The way this is used in NP-completeness is that we have strong evidence that L_1 is not solvable in polynomial time. Hence, the reduction is effectively equivalent to saying that "since L_1 is not likely to be solvable in polynomial time, then L_2 is also not likely to be solvable in polynomial time.

9.7 NP-Completeness

The set of NP-complete problems is all problems in the complexity class NP for which it is known that if any one is solvable in polynomial time, then they all are. Conversely, if any one is not solvable in polynomial time, then none are.

Definition: A decision problem L is NP-Hard if

$$L' \leq_P L \quad \text{for all } L' \in \text{NP}.$$

Definition: L is NP-complete if

1. $L \in$ NP and
2. $L' \leq_P L$ for some known NP-complete problem L'.

Given this formal definition, the complexity classes are:

P: is the set of decision problems that are solvable in polynomial time.

NP: is the set of decision problems that can be verified in polynomial time.

NP-Hard: L is NP-hard if for all $L' \in$ NP, $L' \leq_P L$. Thus if we could solve L in polynomial time, we could solve all NP problems in polynomial time.

NP-Complete L is NP-complete if

1. $L \in$ NP and
2. L is NP-hard.

The importance of NP-complete problems should now be clear. If any NP-complete problem is solvable in polynomial time, then every NP-complete problem is also solvable in polynomial time. Conversely, if we can prove that any NP-complete problem cannot be solved in polynomial time, the every NP-complete problem cannot be solvable in polynomial time.

9.8 Boolean Satisfiability Problem: Cook's Theorem

We need to have at least one NP-complete problem to start the ball rolling. Stephen Cook showed that such a problem existed. He proved that the *boolean satisfiability problem* is NP-complete. A boolean formula is a logical formulation which consists of variables x_i. These variables appear in a logical expression using logical operations

1. negation of x: $\overline{x}$
2. boolean or: $(x \vee y)$
3. boolean and: $(x \wedge y)$

For a problem to be in NP, it must have an efficient verification procedure. Thus virtually all NP problems can be stated in the form, "does there exists X such that $P(X)$", where X is some structure (e.g. a set, a path, a partition, an assignment, etc.) and $P(X)$ is some property that X must satisfy (e.g. the set of objects must fill the knapsack, or the path must visit every vertex, or you may use at most k colors and no two adjacent vertices can have the same color). In showing that such a problem is in NP, the certificate consists of giving X, and the verification involves testing that $P(X)$ holds.

In general, any set X can be described by choosing a set of objects, which in turn can be described as choosing the values of some boolean variables. Similarly, the property $P(X)$ that you need to satisfy, can be described as a boolean formula. Stephen Cook was looking for the most general possible property he could, since this should represent the hardest problem in NP to solve. He reasoned that computers (which represent the most general type of computational devices known) could be described entirely in terms of boolean circuits, and hence in terms of boolean formulas. If any problem were hard to solve, it would be one in which X is an assignment of boolean values (true/false, 0/1) and $P(X)$ could be any boolean formula. This suggests the following problem, called the *boolean satisfiability problem*.

SAT: Given a boolean formula, is there some way to assign truth values (0/1, true/false) to the variables of the formula, so that the formula evaluates to true?

A boolean formula is a logical formula which consists of variables x_i , and the logical operations $\overline{x}$ meaning the *negation* of x, *boolean-or* $(x \vee y)$ and *boolean-and* $(x \wedge y)$. Given a boolean formula, we say that it is satisfiable if there is a way to assign truth values (0 or 1) to the variables such that the final result is 1. (As opposed to the case where no matter how you assign truth values the result is always 0.) For example

$$(x_1 \wedge (x_2 \vee \overline{x_3})) \wedge ((\overline{x_2} \wedge \overline{x_3}) \vee \overline{x_1})$$

is satisfiable, by the assignment $x_1 = 1$, $x_2 = 0$ and $x_3 = 0$. On the other hand,

$$(\overline{x_1} \vee (x_2 \wedge x_3)) \wedge (x_1 \vee (\overline{x_2} \wedge \overline{x_3})) \wedge (x_2 \vee x_3) \wedge (\overline{x_2} \vee \overline{x_3})$$

is not satisfiable. Such a boolean formula can be represented by a logical circuit made up of OR, AND and NOT gates. For example, Figure 9.9 shows the circuit for the boolean formula

$$((x_1 \wedge x_4) \vee x_2) \wedge ((x_3 \wedge \overline{x_4}) \vee \overline{x_2}) \wedge \overline{x_5}$$

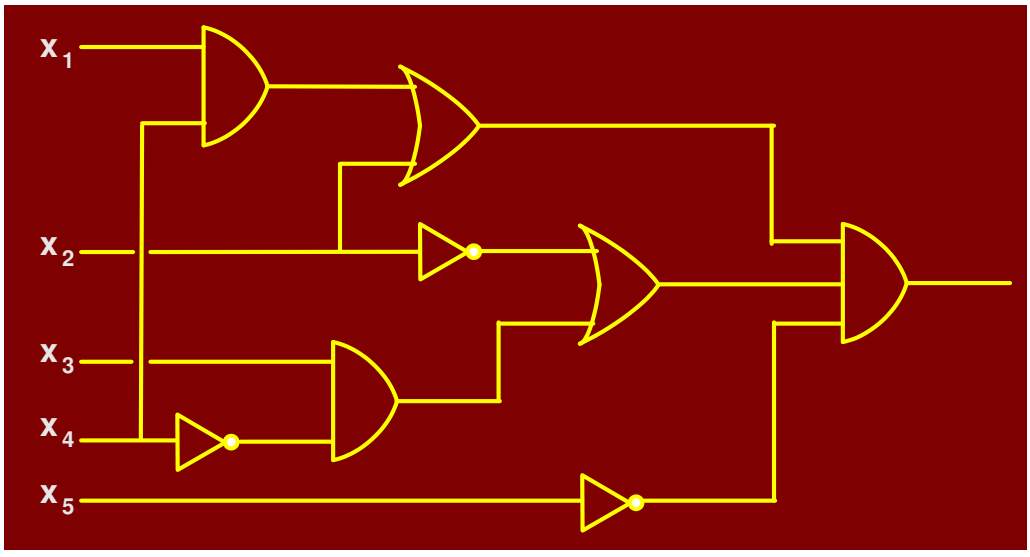


Figure 9.9: Logical circuit for a boolean formula

Cook's Theorem: SAT is NP-complete

We will not prove the theorem; it is quite complicated. In fact, it turns out that a even more restricted version of the satisfiability problem is NP-complete.

A *literal* is a variable x or its negation $\overline{x}$. A boolean formula is in *3-Conjunctive Normal Form* (3-CNF) if it is the boolean-and of clauses where each clause is the boolean-or of exactly three literals. For example,

$$(x_1 \vee x_2 \vee \overline{x_3}) \wedge (\overline{x_1} \vee x_3 \vee x_4) \wedge (x_2 \vee \overline{x_3} \vee \overline{x_4})$$

is in 3-CNF form. 3SAT is the problem of determining whether a formula is 3-CNF is satisfiable. 3SAT is NP-complete. We can use this fact to prove that other problems are NP-complete. We will do this with the *independent set problem*.

Independent Set Problem: Given an undirected graph $G = (V, E)$ and an integer k, does G contain a subset V' of k vertices such that no two vertices in V' are adjacent to each other.

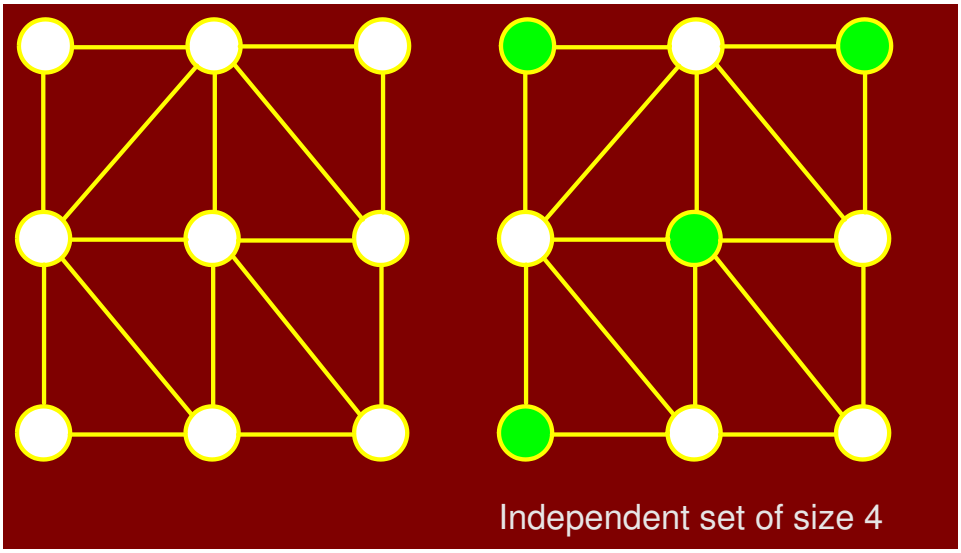


Figure 9.10:

The independent set problem arises when there is some sort of selection problem where there are mutual restrictions pairs that cannot both be selected. For example, a company dinner where an employee and his or her immediate supervisor cannot both be invited.

Claim: IS is NP-complete

The proof involves two parts. First, we need to show that IS $\in$ NP. The certificate consists of k vertices of V'. We simply verify that for each pair of vertices $u, v \in V'$, there is no edge between them. Clearly, this can be done in polynomial time, by an inspection of the adjacency matrix.

Second, we need to establish that IS is NP-hard This can be done by showing that some known NP-compete (3SAT) is polynomial-time reducible to IS. That is, 3SAT $\leq_P$ IS.

An important aspect to reductions is that we do not attempt to solve the satisfiability problem. Remember: It is NP-complete, and there is not likely to be any polynomial time solution. The idea is to translate the similar elements of the satisfiable problem to corresponding elements of the independent set problem.

What is to be selected?

3SAT: Which variables are to be assigned the value true, or equivalently, which literals will be true and which will be false.

IS: Which vertices will be placed in V'.

Requirements:

3SAT: Each clause must contain at least one true valued literal.

IS: V' must contain at least k vertices.

Restrictions:

3SAT: If x_i is assigned true, then $\overline{x_i}$ must be false and vice versa.

IS: If u is selected to be in V' and v is a neighbor of u then v cannot be in V'.

We want a function which given any 3-CNF boolean formula F, converts it into a pair (G, k) such that the above elements are translated properly. Our strategy will be to turn each literal into a vertex. The vertices will be in clause clusters of three, one for each clause. Selecting a true literal from some clause will correspond to selecting a vertex to add to V'. We will set k equal to the number of clauses, to force the independent set subroutine to select one true literal from each clause. To keep the IS subroutine from selecting two literals from one clause and none from some other, we will connect all the vertices in each clause cluster with edges. To keep the IS subroutine from selecting a literal and its complement to be true, we will put an edge between each literal and its complement.

A formal description of the reduction is given below. The input is a boolean formula F in 3-CNF, and the output is a graph G and integer k.

3SAT-TO-IS(F)
1 $k \leftarrow$ number of clauses in F
2 **for** (each clause C in F)
3 **do** create a clause cluster of
4 3 vertices from literals of C
5 **for** (each clause cluster (x_1, x_2, x_3))
6 **do** create an edge (x_i, x_j) between
7 all pairs of vertices in the cluster
8 **for** (each vertex x_i)
9 **do** create an edge between x_i and
10 all its complement vertices $\overline{x_i}$

11 **return** (G, k) *// output is graph* G *and integer* k

If F has k clauses, then G has exactly 3k vertices. Given any reasonable encoding of F , it is an easy programming exercise to create G (say as an adjacency matrix) in polynomial time. We claim that F is satisfiable if and only if G has an independent set of size k.

Example: Suppose that we are given the 3-CNF formula:

$$(x_1 \vee \overline{x_2} \vee \overline{x_3}) \wedge (\overline{x_1} \vee x_2 \vee x_3) \wedge (\overline{x_1} \vee x_2 \vee \overline{x_3}) \wedge (x_1 \vee \overline{x_2} \vee x_3)$$

The following series of figures show the reduction which produces the graph and sets k = 4. First, each of the four literals is converted into a three-vertices graph. This is shown in Figure 9.11

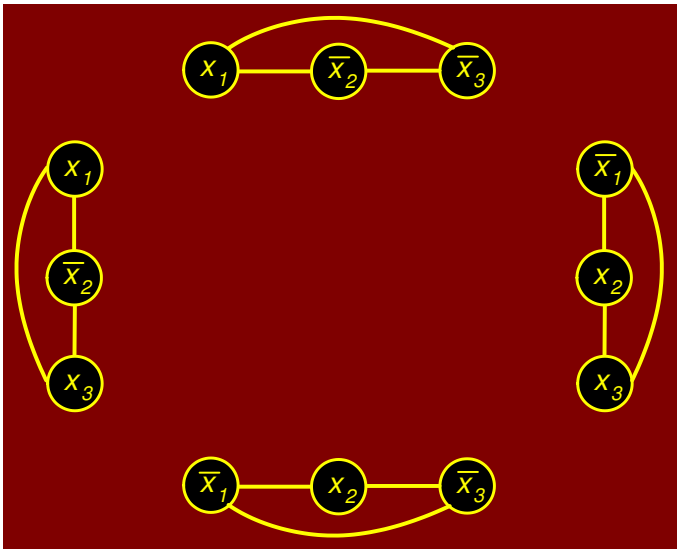


Figure 9.11: Four graphs, one for each of the 3-terms literal

Next, each term is connected to its complement. This is shown in Figure 9.12.

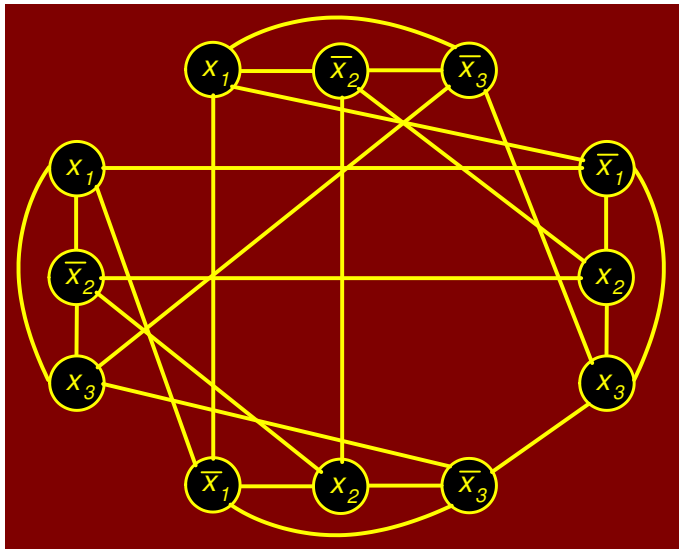


Figure 9.12: Each term is connected to its complement

The boolean formula is satisfied by the assignment $x_1 = 1,\ x_2 = 1\ x_3 = 0$. This implies that the first literal of the first and last clauses are 1, the second literal of the second clause is 1, and the third literal of the third clause is 1. By selecting vertices corresponding to true literals in each clause, we get an independent set of size $k = 4$. This is shown in Figure 9.13.

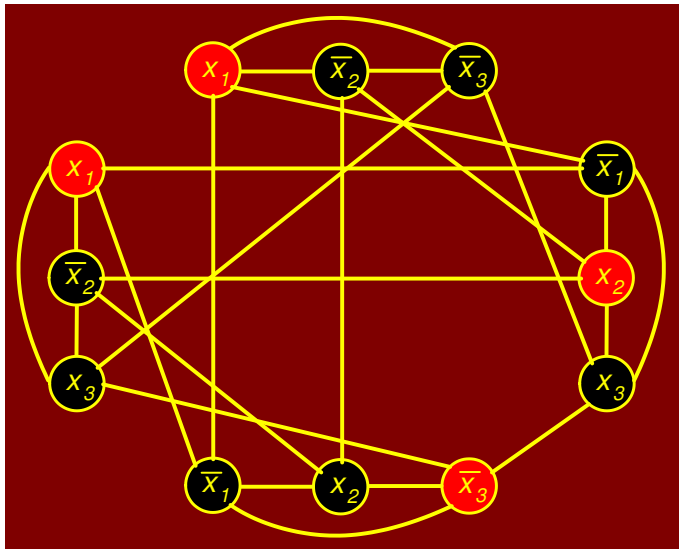


Figure 9.13: Independent set corresponding to $x_1 = 1,\ x_2 = 1\ x_3 = 0$

Correctness Proof:

We claim that F is satisfiable if and only if G has an independent set of size k. If F is satisfiable, then each of the k clauses of F must have at least one true literal. Let V' denote the corresponding vertices from each of the clause clusters (one from each cluster). Because we take vertices from each cluster, there are

no inter-cluster edges between them, and because we cannot set a variable and its complement to both be true, there can be no edge of the form $(x_i, \overline{x_i})$ between the vertices of V'. Thus, V' is an independent set of size k.

Conversely, G has an independent set V' of size k. First observe that we must select a vertex from each clause cluster, because there are k clusters, and we cannot take two vertices from the same cluster (because they are all interconnected). Consider the assignment in which we set all of these literals to 1. This assignment is logically consistent, because we cannot have two vertices labelled x_i and $\overline{x_i}$ in the same cluster.

Finally the transformation clearly runs in polynomial time. This completes the NP-completeness proof.

Also observe that the the reduction had no knowledge of the solution to either problem. Computing the solution to either will require exponential time. Instead, the reduction simple translated the input from one problem into an equivalent input to the other problem.

9.9 Coping with NP-Completeness

With NP-completeness we have seen that there are many important optimization problems that are likely to be quite hard to solve exactly. Since these are important problems, we cannot simply give up at this point, since people do need solutions to these problems. Here are some strategies that are used to cope with NP-completeness:

Use brute-force search: Even on the fastest parallel computers this approach is viable only for the smallest instance of these problems.

Heuristics: A heuristic is a strategy for producing a valid solution but there are no guarantees how close it to optimal. This is worthwhile if all else fails.

General search methods: Powerful techniques for solving general combinatorial optimization problems. Branch-and-bound, A*-search, simulated annealing, and genetic algorithms

Approximation algorithm: This is an algorithm that runs in polynomial time (ideally) and produces a solution that is within a guaranteed factor of the optimal solution.

CS502: QUIZ 1

SOLVED BY: ARSL SHANI, AINA MALIK&MCS FAMILY GROUP

1) For a Diagraph G= (V.E), Sum of in-degree (v) ----------.
   - Not equal to sum of out-degree(v)
   - = sum of out-degree(v) pg#115
   - < sum of out-degree(v)
   - > sum of out-degree(v)

For a digraph $G = (V, E)$,

$$\sum_{v \in V} in\text{-}degree(v) = \sum_{v \in V} out\text{-}degree(v) = |E|$$

2) DFS or BFS yields a----------of the graph.
   - Traversed tree
   - Spanning tree pg#125
   - Simple tree
   - Free tree

Either DFS or BFS yields a spanning tree of the graph. The tree visits every vertex in the graph. This fact is established by the following lemma:

3) Using ASCII code, each character is represented by a fixed-length code of----------bits per character.
   - 4
   - 6
   - 8 pg#100
   - 10

The Huffman codes provide a method of encoding data efficiently. Normally, when characters are coded using standard codes like ASCII. Each character is represented by a fixed-length codeword of bits, e.g., 8 bits per character. Fixed-length codes are popular because it is very easy to break up a string into its

4) In Knapsack Problem, the goal is to put items in the Knapsack such that the value of the items is----------subject to weight limit of the Knapsack.
   - Minimized
   - Decreased
   - Maximized pg#109
   - None of the above given

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7.4 Fractional Knapsack Problem

Earlier we saw the 0-1 knapsack problem. A knapsack can only carry W total weight. There are n items; the i^{th} item is worth v_i and weighs w_i. Items can either be put in the knapsack or not. The goal was to maximize the value of items without exceeding the total weight limit of W. In contrast, in the fractional

5) Consider the string"abcdaacac",if the string is coded with ASCII codes, the message length would be---
   - 70 bits
   - 60 bits
   - 90 bits
   - 72 bits conceptual

Consider the string " abacdaacac". if the string is coded with ASCII codes, the message length would be $10 \times 8 = 80$ bits. We will see shortly that the same string encoded with a variable length Huffman encoding scheme will produce a shorter message.

6) A graph is said to be acyclic if it contains---------.
   - At least one cycle
   - Exactly one cycle
   - Always more than one cycle
   - No cycles pg#116

A graph is said to be *acyclic* if it contains no cycles. A graph is *connected* if every vertex can reach every other vertex. A directed graph that is acyclic is called a *directed acyclic graph (DAG)*.

7) The number of edges that come out of a vertex is called the----------of that vertex in the digraph.
   - Post-degree
   - in-degree
   - out-degree pg#114
   - pre-degree

In a digraph, the number of edges coming out of a vertex is called the *out-degree* of that vertex. Number of edges coming in is the *in-degree*. In an undirected graph, we just talk of degree of a vertex. It is the number of edges incident on the vertex.

8) If Matrix-A has dimensions "3x2" and Matrix-B has dimensions "2x3", then multiplication of Matrix-A and Matrix-B will result a new Matrix-C having dimensions.
   - 3x2
   - 2x3
   - 2x2
   - 3x3 pg#87

The dynamic programming solution involves breaking up the problem into subproblems whose solutions can be combined to solve the global problem. Let $A_{i..j}$ be the result of multiplying matrices i through j. It is easy to see that $A_{i..j}$ is a $p_{i-1} \times p_j$ matrix.

$$\underset{4\times5}{A_3}\ \underset{5\times2}{A_4}\ \underset{2\times8}{A_5}\ \underset{8\times7}{A_6} = \underset{4\times7}{A_{3..6}}$$

9) A/an----------is one in which you want to find, not just a solution, but the best solution.

- Optimization problem
- Divide and Conquer
- NP complete problem
- Best problem

An *optimization problem* is one in which you want to find, not just a solution, but the best solution. Search techniques look at many possible solutions. E.g. dynamic programming or backtrack search. A "

10) Fractional Knapsack is founded on----------method.

- Greedy
- Recursive
- Divide and Conquer
- Dynamic programming

fraction of the weight and fraction of value. The 0-1 knapsack problem is hard to solve. However, there is a simple and efficient greedy algorithm for the fractional knapsack problem.

11) If the graph is represented using an adjacency matrix, then Breadth-first search takes----------time.

- O(E+1)
- O(V^2)
- O(V)
- O(E)

If the graph is represented using an *adjacency matrix*, the finding of all the neighbors of vertex in line 7 takes $O(V)$ time. Thus depth-first and breadth-first take $O(V^2)$ time overall.

12) In inductive approach of Knapsack problem, we consider 2 cases, ----------or----------.

- Median, Mode
- Recursive, Iterative
- Leave object, Take object pg#93
- Sequentially, Parallel

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To compute entries of V we will imply an inductive approach. As a basis, $V[0, j] = 0$ for $0 \leq j \leq W$ since if we have no items then we have no value. We consider two cases:

Leave object i: If we choose to not take object i, then the optimal value will come about by considering how to fill a knapsack of size j with the remaining objects $\{1, 2, \ldots, i-1\}$. This is just $V[i-1, j]$.

Take object i: If we take object i, then we gain a value of v_i. But we use up w_i of our capacity. With the remaining $j - w_i$ capacity in the knapsack, we can fill it in the best possible way with objects $\{1, 2, \ldots, i-1\}$. This is $v_i + V[i-1, j-w_i]$. This is only possible if $w_i \leq j$.

13) A Greedy algorithm can NOT be used to solve all the----------problems.

- Dynamic programming (Google)
- Memorization programming
- Edit-distance programming
- Storing value programming

Explanation: **A greedy algorithm** gives optimal **solution** for **all** subproblems, but when these locally optimal solutions are combined it **may NOT** result into a globally optimal **solution**. Hence, **a greedy algorithm CANNOT be used to solve all** the dynamic programming **problems**.

14) In Huffman encoding, the---------is the number of occurrences of a character divided by the total characters in the message.

- Counting
- Parsing
- Relative Probability pg#100
- Weight

Here is how the Huffman encoding algorithm works. Given a message string, determine the frequency of occurrence (relative probability) of each character in the message. This can be done by parsing the message and counting how many time each character (or symbol) appears. The probability is the number of occurrence of a character divided by the total characters in the message. The frequencies and probabilities for the example string " abacdaacac" are

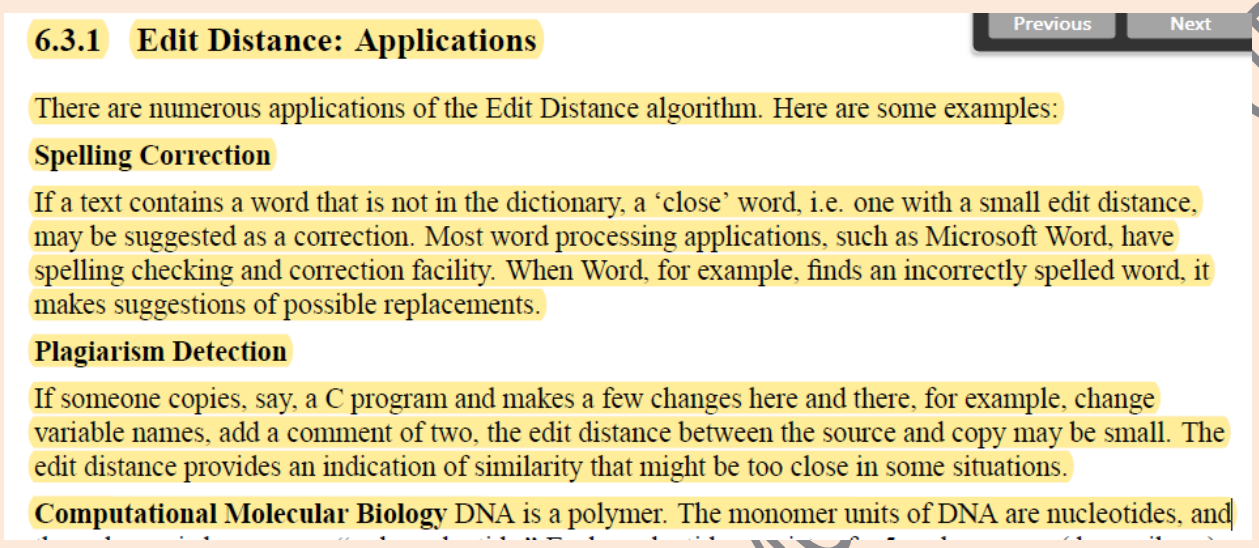
15) The Binary Tree constructed by a Huffman Encoding is a:

- Full Binary Tree pg#102
- Partial Binary Tree
- Incomplete Binary Tree
- None of the given option

without increasing its costs. Note that the binary tree constructed by Huffman algorithm is a full binary tree.

16) Following is not the application of Edit Distance Problem.

- Speech recognition pg#76
- Spelling correction
- Ascending order pg#76
- Computational Molecular Biology



6.3.1 Edit Distance: Applications

Previous Next

There are numerous applications of the Edit Distance algorithm. Here are some examples:

Spelling Correction

If a text contains a word that is not in the dictionary, a 'close' word, i.e. one with a small edit distance, may be suggested as a correction. Most word processing applications, such as Microsoft Word, have spelling checking and correction facility. When Word, for example, finds an incorrectly spelled word, it makes suggestions of possible replacements.

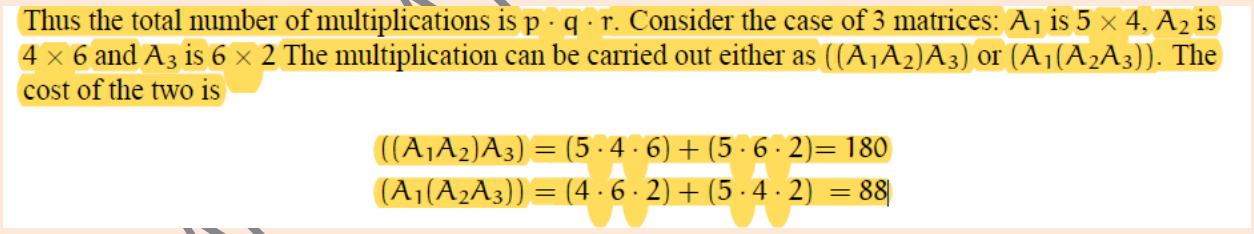
Plagiarism Detection

If someone copies, say, a C program and makes a few changes here and there, for example, change variable names, add a comment of two, the edit distance between the source and copy may be small. The edit distance provides an indication of similarity that might be too close in some situations.

Computational Molecular Biology DNA is a polymer. The monomer units of DNA are nucleotides, and

17) Consider three Matrices X, Y, Z of dimensions 1x2, 2x3, 3x4 respectively. The number of multiplication of(XYZ) is:

- 18
- 32
- 24
- 30



Thus the total number of multiplications is $p \cdot q \cdot r$. Consider the case of 3 matrices: A_1 is 5×4, A_2 is 4×6 and A_3 is 6×2 The multiplication can be carried out either as $((A_1A_2)A_3)$ or $(A_1(A_2A_3))$. The cost of the two is

$$((A_1A_2)A_3) = (5 \cdot 4 \cdot 6) + (5 \cdot 6 \cdot 2) = 180$$
$$(A_1(A_2A_3)) = (4 \cdot 6 \cdot 2) + (5 \cdot 4 \cdot 2) = 88$$

18) In ---------- Knapsack Problem, limitation is that an item can either be put in the bag or not. Fractional items are not allowed.

- 0
- 1
- 0/1 pg#91
- Fractional

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6.5 0/1 Knapsack Problem

A thief goes into a jewelry store to steal jewelry items. He has a knapsack (a bag) that he would like to fill up. The bag has a limit on the total weight of the objects placed in it. If the total weight exceeds the limit, the bag would tear open. The value of of the jewelry items varies for cheap to expensive. The thief's goal is to put items in the bag such that the value of the items is maximized and the weight of the items does not exceed the weight limit of the bag. Another limitation is that an item can either be put in the bag or not - fractional items are not allowed. The problem is: what jewelry should the thief choose

19) An in-place sorting algorithm is one that----------uses additional array for storage.

- Always
- Permanently
- Does not pg#54
- Sometime

An *in-place* sorting algorithm is one that uses no additional array for storage.

20) If Matrix-A has dimensions "pxq" and Matrix-B has dimensions "qxr", then multiplication of Matrix-A and Matrix-B will result a new Matrix-C having dimensions.

- P x q
- P x r page#84
- q x r
- q x p

In what order should the multiplication be done? A $p \times q$ matrix A can be multiplied with a $q \times r$ matrix B. The result will be a $p \times r$ matrix C. In particular, for $1 \le i \le p$ and $1 \le j \le r$,

$$C[i, j] = \sum_{k=1}^{q} A[i, k]B[k, j]$$

21) Counting sort is suitable to sort the elements in range 1 to K.

- K is large
- K is small pg#71
- K may be large or small
- None

The main shortcoming of counting sort is that it is useful for small integers, i.e., 1..k where k is small.

22) When matrix A of 5x 3 is multiply with matrix B of 3x 4 then the multiplication required is:

- 15
- 12
- 36
- 60

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Thus the total number of multiplications is $p \cdot q \cdot r$. Consider the case of 3 matrices: A_1 is 5×4, A_2 is 4×6 and A_3 is 6×2 The multiplication can be carried out either as $((A_1A_2)A_3)$ or $(A_1(A_2A_3))$. The cost of the two is

$$((A_1A_2)A_3) = (5 \cdot 4 \cdot 6) + (5 \cdot 6 \cdot 2) = 180$$
$$(A_1(A_2A_3)) = (4 \cdot 6 \cdot 2) + (5 \cdot 4 \cdot 2) = 88$$

23) ----------is a linear time sorting algorithm.

- Merge sort
- Quick sort
- Bubble sort
- Radix sort

5 Linear Time Sorting

5.1 Counting Sort . . . . .

5.2 Bucket or Bin Sort . . . . .

5.3 Radix Sort . . . . .

24) In Dynamic Programming approach, we do not store the solution to each sub problem in case if it reappears.

- True page#75
- False

Dynamic programming algorithms need to store the results of intermediate subproblems. This is often *but not always* done with some kind of table. We will now cover a number of examples of problems in

25) Dynamic Programming approach is usually useful in solving optimization problem.

- True page#97
- False

An *optimization problem* is one in which you want to find, not just a solution, but the best solution. Search techniques look at many possible solutions. E.g. dynamic programming or backtrack search. A "

26) Which of the following algorithm provides an optimal solution for the activity selection problem?

- Divide and Conquer
- Brute force
- Greedy pg#105
- Recursive

The activity scheduling is a simple scheduling problem for which the greedy algorithm approach provides an optimal solution. We are given a set $S = \{a_1, a_2, \ldots, a_n\}$ of n activities that are to be scheduled to use some resource. Each activity a_i must be started at a given start time s_i and ends at a given finish time f_i.

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27) A graph is---------- if every vertex can reach every other vertex.

- Connected pg#116
- Cycle
- Acyclic
- Loop

A graph is said to be *acyclic* if it contains no cycles. A graph is *connected* if every vertex can reach every other vertex. A directed graph that is acyclic is called a *directed acyclic graph (DAG)*.

28) In a Huffman encoding when a new node is created by combining two nodes, the new node is placed in the _____.

- Priority queue pg#100
- Linked list
- Min heap tree
- Graph traversal

heap-based priority queue. The min-heap is maintained using the frequencies. When a new node is created by combining two nodes, the new node is placed in the priority queue. Here is the Huffman tree building algorithm.

29) Huffman algorithm produces the _____ prefix code

- Optimal pg#105
- Best
- Worst
- Better

Claim: Huffman algorithm produces the optimal prefix code tree.

30) In _____ algorithm, you hope that by choosing a local optimum at each step, you will end up a global optimum.

- Simple
- Divide and conquer
- Greedy pg#97
- Brute Force

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Greedy Algorithms

choosing a

Previous Next

An *optimization problem* is one in which you want to find, not just a solution, but the best solution. Search techniques look at many possible solutions. E.g. dynamic programming or backtrack search. A "greedy algorithm" sometimes works well for optimization problems

A greedy algorithm works in phases. At each phase:

- You take the best you can get right now, without regard for future consequences.
- You hope that by choosing a local optimum at each step, you will end up at a global optimum.

31) The string "Imncde" is coded with ASCII code, the message length would be ____ bits.

- 24
- 36
- 48 conceptual (page 99)
- 60

Consider the string " abacdaacac". if the string is coded with ASCII codes, the message length would b
$10 \times 8 = 80$ bits. We will see shortly that the same string encoded with a variable length Huffman

32) For graph traversal, breadth-first search strategy _____

- Is always recursive
- Cannot br recursive
- Cannot be non-recursive
- Can be both recursive and non-recursive page 119

Breadth-first search is one instance of a general family of *graph traversal algorithms*. Traversing a graph means visiting every node in the graph. Another traversal strategy is *depth-first search* (DFS). DFS procedure can be written recursively or non-recursively. Both versions are passed s initially.

33) In activity scheduling algorithm , the width of a rectangle _____

- Is always ignored
- Directs towards recursion
- Should be maximized
- Indicates the duration of an activity pg#106

scheduled. Each is represented by a rectangle. The width of a rectangle indicates the duration of an activity. The eight activities are sorted by their finish times. The eight rectangles are arranged to show the

34) If the graph is represented using an adjacency list, then Breadth-first search takes--------- time

- O(V^2)
- O(V)

- O(V+E) pg#138
- O(E+1)

We will discuss the algorithm without proof. Define G^T to be the digraph with the same vertex set at G but in which all edges have been reversed in direction. This is shown in Figure 8.38. Given an adjacency list for G, it is possible to compute G^T in $\Theta(V + E)$ time.

35) Suppose you are given infinite coins of 1,2 ,3, and 4.Select the ways of the minimum number of coins that required to achieve a sum of 6:

- 1
- 2 conceptual
- 3
- 4

36) Using ASCII standard the string "greedy" will be encoded with

- 48 bits Conceptual
- 120 bits
- 44 bits
- 40 bits

Consider the string " abacdaacac". if the string is coded with ASCII codes, the message length would be $10 \times 8 = 80$ bits. We will see shortly that the same string encoded with a variable length Huffman

37) The Huffman codes provide a method of -------data efficiency.

- Reading/Writing
- Encoding/Decoding pg#99
- Divide/Conquer
- Inserting/Deleting

The Huffman codes provide a method of encoding data efficiently. Normally, when characters are coded using standard codes like ASCII. Each character is represented by a fixed-length codeword of bits, e.g., 8

38) In the context of activity selection algorithm, time s dominated by sorting of the activities by------

- Start Times
- Finish Times pg#106
- Average Times
- CPU Burst Times

Timing analysis: Time is dominated by sorting of the activities by finish times. Thus the complexity is $O(N \log N)$.

39) Time complexity of the "0-1" knapsack algorithm depends on----

- Number of items
- Capacity of the knapsack
- Size of the Table
- Number of items and capacity of knapsack (not sure)

Formally, the problem can be stated as follows: Given a knapsack with maximum capacity W, and a set S consisting of n items Each item i has some weight w_i and value value v_i (all w_i , v_i and W are integer values) How to pack the knapsack to achieve maximum total value of packed items? For example,

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40) The greedy approach gives us an optimal solution when the coins are all powers of a -------- denomination

- Fixed pg#98
- Variable
- Constant
- Static

The greedy approach gives us an optimal solution when the coins are all powers of a fixed denomination.

41) In Activity Selection, we say that two activities are non-interfering if their start-finish interval------- overlap

- Do
- Do not pg#105
- Sometimes
- Once

Some start and finish times may overlap. Therefore, not all requests can be honored. We say that two activities a_i and a_j are non-interfering if their start-finish intervals do not overlap. I.e,

42) How many steps are involved to design the dynamic programming strategy?

- 2
- 3
- 1
- 4 pg#92

"yes", with an algorithm based on dynamic programming Let us recap the steps in the dynamic programming strategy:

1. **Simple Subproblems**: We should be able to break the original problem to smaller subproblems that have the same structure
2. **Principle of Optimality**: Recursively define the value of an optimal solution. Express the solution of the original problem in terms of optimal solutions for smaller problems.
3. **Bottom-up computation**: Compute the value of an optimal solution in a bottom-up fashion by using a table structure.
4. **Construction of optimal solution**: Construct an optimal solution from computed information.

43) Bag is a----------

- type of algorithm
- data structure pg#119
- program
- compiler

The *generic graph traversal* algorithm stores a set of candidate edges in some data structures we'll call a "*bag*". The only important properties of the "bag" are that we can put stuff into it and then later take stuff

44) The running time of brute-force algorithm to solve Knapsack problem is------

- O(n)
- O(n3)
- O (n!)
- O(2^n) pg#92

Clearly, the running time of such a brute-force algorithm will be $O(2^n)$.

45) If a matrix has three rows and two columns, the dimension of matrix will be:

- 3x2 Conceptual
- 2x3
- 3x3
- 2x2

The dimensions of a matrix are the number of rows by the number of columns. If a matrix has a rows and b columns, it is an $a \times b$ matrix.

46) The Probability in Huffman encoding is the number of occurrences of a character divided by the total----------in the message.

- Numbers
- Frequencies
- Strings
- Characters pg#100

message and counting how many time each character (or symbol) appears. The probability is the number of occurrence of a character divided by the total characters in the message. The frequencies and

47) In recursive formulation of Knapsack Problem:

V [0, j] = ---------- for j>=0

- -1
- 0 pg#93
- 1
- 2

This leads to the following recursive formulation:

$$V[i, j] = -\infty \quad \text{if } j < 0$$
$$V[0, j] = 0 \quad \text{if } j \geq 0$$

48) The Knapsack Problem belongs to the domain of----------problem.

- Optimization pg#91
- Sorting
- Linear solution
- Searching

The knapsack problem belongs to the domain of optimization problems.

49) An optimization problem is one in which you want to find the----------solution.

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- Simple
- Good
- Best pg#97
- Worst

An *optimization problem* is one in which you want to find, not just a solution, but the best solution.

50) Huffman algorithm generates an optimum----------code.

- Prefix page 102
- Postfix
- Infix
- None of the above

length B(T) of the encoded string. In other words, Huffman algorithm generates an optimum prefix code. The question that remains is that *why is the algorithm correct?*

51) Which of the following algorithms solves the Fractional Knapsack Problem more effectively?

- Divide and Conquer
- Greedy algorithm page 109
- Dynamic programming
- Backtracking

fraction of the weight and fraction of value. The 0-1 knapsack problem is hard to solve. However, there is a simple and efficient greedy algorithm for the fractional knapsack problem.

52) If we implement the bag by using a queue, we have----------.

- BFS page 124
- DFS
- Graph
- Loop

If we implement the bag by using a *queue*, we have *breadth-first* search (BFS). Each execution of line

53) One of the limitation in 0/1 Knapsack is that an item can either be----------in the bag or not.

- Use
- Put page 91
- Move
- Store

items does not exceed the weight limit of the bag. Another limitation is that an item can either be put in the bag or not - fractional items are not allowed. The problem is: what jewelry should the thief choose

54) In Dynamic Programming based solution of Knapsack Problem, if we decide to take an object'i' then we gain----------.

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- W(Total weight of Knapsack)
- V(Total value of all items)
- vi(value of object i) page 93
- None of the given option

Take object i: If we take object i, then we gain a value of v_i. But we use up w_i of our capacity. With the

55) An activity scheduling algorithm, the width of a rectangle----------.

- Is always ignored
- Direct toward recursion
- Should be maximized
- Indicates the duration of an activity page 106

scheduled. Each is represented by a rectangle. The width of a rectangle indicates the duration of an activity. The eight activities are sorted by their finish times. The eight rectangles are arranged to show the

56) The prefix code generated by Huffman algorithm---------- the expected length of the encoded string.

- Minimizes page 102
- Balances
- Maximizes
- Keeps constant

Huffman algorithm uses a greedy approach to generate a prefix code T that minimizes the expected length B(T) of the encoded string. In other words, Huffman algorithm generates an optimum prefix code.

57) In Huffman encoding, the characters with smallest probabilities are placed at the----------depth of the tree.

- Minimum
- Average
- Maximum page 102
- Root

Consider two characters x and y with the smallest probabilities. Then there is optimal code tree in which these two characters are siblings at the maximum depth in the tree.

58) There are ---------- ways to representing graphs.

- 3
- 1
- 2 page 116
- 4

There are two ways of representing graphs: using an adjacency matrix and using an adjacency list. Let

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59) Each time we traverse graph by Breadth-first search algorithm, we count the distance from----------.

- Starting node
- Neighbours of the starting node page 117
- Right most node
- Left most node

Here is a more efficient algorithm called the *breadth-first search* (BFS) Start with s and visit its adjacent nodes. Label them with distance 1. Now consider the neighbors of neighbors of s. These would be at distance 2. Now consider the neighbors of neighbors of neighbors of s. These would be at distance 3.

60) In Activity Selection problem, intuitively----------.

- There are always short activities as input
- Short activities are not attractive
- Duration of the activities does not matter
- We do not like long activities page105

So how do we schedule the largest number of activities on the resource? Intuitively, we do not like long

activities Because they occupy the resource and keep us from honoring other requests. This suggests the

61) The activity scheduling is a simple scheduling problem for which the greedy algorithm approach provides a/an----------solution.

- Simple
- Sub-optimal
- Optimal page 105
- Non optimal

The activity scheduling is a simple scheduling problem for which the greedy algorithm approach provides an optimal solution. We are given a set $S = \{a_1, a_2, \ldots, a_n\}$ of n activities that are to be scheduled to use

62) The ----------is a problem for which the greedy algorithm approach provides an optimal solution.

- Activity selection page 105
- Dynamic programming
- Knapsack problem
- NP complete problem

The activity scheduling is a simple scheduling problem for which the greedy algorithm approach provides an optimal solution. We are given a set $S = \{a_1, a_2, \ldots, a_n\}$ of n activities that are to be scheduled to use

63) Which of the following ways can be used to represent a graph?

- Adjacency list
- Incidence matrix
- Adjacency list , Adjacency Matrix page 116
- No way to represent

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There are two ways of representing graphs: using an adjacency matrix and using an adjacency list. Let

64) Queue Data Structure work on ---------- principles.

- FIFO(first in first out) cs301
- LIFO(last in first out)
- JLO(just in last out)
- LOFI(last out first in)

Queues

A queue is a linear data structure into which items can only be inserted at one end and removed from the other. In contrast to the stack, which is a LIFO (Last In First Out) structure, a queue is a FIFO (First In First Out) structure.
The usage of queue in daily life is pretty common. For example, we queue up while

65) Graphs can be represented by an---------- and ----------.

- queue , stack
- adjacency list , adjacency matrix page 116
- adjacency right , adjacency left
- Binary , linear

There are two ways of representing graphs: using an adjacency matrix and using an adjacency list. Let

66) Identify a TRUE statement about Knapsack.

- The Knapsack Problem does not belongs to the domain of optimization problems
- The Knapsack Problem belongs to the domain of optimization problems page 91
- The Knapsack Problem cannot be solved by using Dynamic programming
- The Knapsack Problem is optimally solved by using Brute force algorithm

The knapsack problem belongs to the domain of optimization problems. Mathematically, the problem is

67) Which of the following is true about graph?

- A graph may contain no edges and many vertices
- A graph may contain many edges and no vertices (google)

68) Which type of algorithm is harder to prove the correctness?

- Greedy google
- Brute force
- Dynamic programming
- Divide and Conquer

ARSLAN ANJUM & MCS FAMILY GROUP

The difficult part is that for greedy algorithms you have to work much harder to understand correctness issues. Even with the correct algorithm, it is hard to prove why it is correct. Proving that a greedy algorithm is correct is more of an art than a science. It involves a lot of creativity. Usually, coming up with an algorithm might seem to be trivial, but proving that it is actually correct, is a whole different problem.

For instance, greedy algorithms may seem conceptually simpler, and usually, run faster, but they're much harder to prove correct because they require making a lot of implicit assumptions about the structure of the input.

ARSL SHANI,AINA MALIK& MCS

Mid

Quiz by Attiq Kundi

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CS-502

MIDTERM EXAMINATION

Fall 2011

CS502- Fundamentals of Algorithms

Question No: 1 (Marks: 1) - Please choose one

Due to left complete nature of binary tree, the heap can be stored in

► **Arrays (Page 40)**
► Structures
► Link Lis
►Stack

Question No: 1 (Marks: 1) - Please choose one

What type of instructions Random Access Machine (RAM) can execute?

►Algebraic and logic

►Geometric and arithmetic

►**Arithmetic and logic (Page 10)**

►Parallel and recursive

Question No: 1 (Marks: 1) - Please choose one

For Chain Matrix Multiplication we can not use divide and conquer approach because,

►**We do not know the optimum k (Page 86)**

►We use divide and conquer for sorting only

►We can easily perform it in linear time

►Size of data is not given

Question No: 1 (Marks: 1) - Please choose one

What is the total time to heapify?

► **O(log n) (Page 43)**
► O(n log n)
► O(n^2 log n)
► O($\log^2$ n)

Question No: 1 (Marks: 1) - Please choose one

word Algorithm comes from the name of the muslim author _____

►**Abu Ja'far Mohammad ibn Musa al-Khowarizmi**.

Question No: 1 (Marks: 1) - Please choose one al-Khwarizmi's work was written in a book titled _____

►**al Kitab al-mukhatasar fi hisab al-jabr wa'l-muqabalah**

MIDTERM EXAMINATION

Spring 2010

CS502- Fundamentals of Algorithms

Question No: 1 (Marks: 1) - Please choose one
Random access machine or RAM is a/an

► Machine build by Al-Khwarizmi
► Mechanical machine
► Electronics machine

► **Mathematical model (Page 10)**

Question No: 2 (Marks: 1) - Please choose one

_____ is a graphical representation of an algorithm

► Σ notation
► Θ notation
► **Flowchart** **Click here for detail**
► Asymptotic notation

Question No: 3 (Marks: 1) - Please choose one

A RAM is an idealized machine with _____ random-access memory.

► 256MB
► 512MB

► **an infinitely large (Page 10)**
► 100GB

Question No: 4 (Marks: 1) - Please choose one

What type of instructions Random Access Machine (RAM) can execute? Choose best answer

► Algebraic and logic
► Geometric and arithmetic

► **Arithmetic and logic (Rep)**
► Parallel and recursive

Question No: 5 (Marks: 1) - Please choose one

What will be the total number of max comparisons if we run brute-force maxima algorithm with n elements?

► n^2

n

► n^2

► n **(Page 14)**

► n^8

Question No: 6 (Marks: 1) - Please choose one
What is the solution to the recurrence $T(n) = T(n/2)+n$.

► $O(\log n)$

► ***O*(*n*) (Page 37)**
► $O(n\log n)$
► $O(n^2)$

Question No: 7 (Marks: 1) - Please choose one

Consider the following code:

```
For(j=1; j<n;j++)
        For(k=1; k<15;k++)
                For(l=5; l<n; l++)
                {
```

Do_something_constant();

}

What is the order of execution for this code.

- ► **O(n^3)**
- ► O(n_2)
- ► O(n^2 log n)
- ► O(n^2)

Question No: 8 (Marks: 1) - Please choose one

What is the total time to heapify?

- ► **O(log n) rep**
- ► O(n log n)
- ► O(n^2 log n)
- ► O($\log^2$ n)

Question No: 9 (Marks: 1) - Please choose one

Consider the following Algorithm:

```
Factorial (n){
  if (n=1)
    return 1
  else
     return (n * Factorial(n-1))
```

Recurrence for the following algorithm is:

- ► **T(n) = T(n-1) +1**
- ► **T(n) = nT(n-1) +1**
- ► **T(n)= T(n-1) +n**
- ► **T(n)=T(n(n-1)) +1**

Question No: 10 (Marks: 1) - Please choose one

When we call heapify then at each level the comparison performed takes time

► **It will take Θ (1)** **(Page 43)**

► Time will vary according to the nature of input data
► It can not be predicted
► It will take Θ (log n)

Question No: 11 (Marks: 1) - Please choose one

In Quick sort, we don't have the control over the sizes of recursive calls

► **True(Page 40)**
► False
► Less information to decide
► Either true or false

Question No: 12 (Marks: 1) - Please choose one Is it possible to sort without making comparisons?

► **Yes(Page 57)**
► No

Question No: 13 (Marks: 1) - Please choose one

If there are Θ (n^2) entries in edit distance matrix then the total running time is ►Θ(1)

► **Θ (n^2)** **Click here for detail**

► Θ (n)

► Θ (n log n)

Question No: 14 (Marks: 1) - Please choose one

For Chain Matrix Multiplication we can not use divide and conquer approach because,

► **We do not know the optimum k** **(Page 86)**

► We use divide and conquer for sorting only
► We can easily perform it in linear time

► Size of data is not given

Question No: 15 (Marks: 1) - Please choose one

The Knapsack problem belongs to the domain of _____ problems.

► **Optimization (Page 91)**

► NP Complete
► Linear Solution
► Sorting

Question No: 16 (Marks: 1) - Please choose one

Suppose we have three items as shown in the following table, and suppose the capacity of the knapsack is 50 i.e. W = 50.

Item	Value	Weight
1	60	10
2	100	20
3	120	30

The optimal solution is to pick

► Items 1 and 2
► Items 1 and 3

► **Items 2 and 3 (correct)**
► None of these

MIDTERM EXAMINATION

Spring 2010

CS502- Fundamentals of Algorithms

Question No: 1 (Marks: 1) - Please choose one

For the Sieve Technique we take time

► **T(nk) (Page 34)**

►T(n / 3)

►n^2

►n/3

Question No: 1 (Marks: 1) - Please choose one

Sieve Technique applies to problems where we are interested in finding a single item

from a larger set of _____

Select correct option:

►**n items (Page 34)**

►phases

►pointers

►constant

Question No: 1 (Marks: 1) - Please choose one

_____ graphical representation of algorithm.

►asymptotic

►**Flowchart** **(rep)**

Question No: 1 (Marks: 1) - Please choose one

who invented the quick sort

►**C.A.R. Hoare** Click here for detail

Question No: 1 (Marks: 1) - Please choose one

main elements to a divide-and-conquer

►**Divide, conquer, combine (Page 27)**

Question No: 1 (Marks: 1) - Please choose one Mergesort is a stable algorithm but not an in-place algorithm.

►**True (Page 54)**

►false

Question No: 1 (Marks: 1) - Please choose one

Counting sort the numbers to be sorted are in the range 1 to k where k is small.

►**True (Page 57)**

►False

MIDTERM EXAMINATION

Spring 2007

CS502- Fundamentals of Algorithms

Question No: 1 (Marks: 1) - Please choose one

Total time for heapify is:

►$O(\log^2 n)$

►O (n log n)

►$O(n^2 \log n)$

►**O (log n)** **Rep**

Question No: 1 (Marks: 1) - Please choose one

If an algorithm has a complexity of $\log_2 n + n\log_2 n + n$. we could say that it has complexity

►O(n)

►$O(n \log_2 n)$

►O(3)

►$O(\log_2 (\log_2 n))$

►$O(\log_2 n)$

Question No: 1 (Marks: 1) - Please choose one In RAM model instructions are executed

►**One after another (Page 10)**

►Parallel

►Concurrent

►Random

Question No: 1 (Marks: 1) - Please choose one

In selection algorithm, because we eliminate a constant fraction of the array with each phase, we get the

►**Convergent geometric series (Page 37)**

►Divergent geometric series

►None of these

Question No: 1 (Marks: 1) - Please choose one

Due to left-complete nature of binary tree, heaps can be stored in

►Link list

►Structure

►**Array (Page 40)**

►None of above

CS609- System Programming

Midterm Quizzes (Quiz No.1 & 2)

Quiz No.1 (04 – MAY - 2013)

Question No: 1 (Marks: 1) - Please choose one

The time assumed for each basic operation to execute on RAM model of computation is-----

Infinite

Continuous

Constant (Page 10)

Variable

Question No: 1 (Marks: 1) - Please choose one

If the indices passed to merge sort algorithm are not equal, the algorithm may return immediately.

True

False (Page 28)

Question No: 1 (Marks: 1) - Please choose one Brute-force algorithm
uses no intelligence in pruning out decisions.

True (Page 18)

False

Question No: 1 (Marks: 1) - Please choose one

In analysis, the Upper Bound means the function grows asymptotically no faster than its largest term.

True (Page 24)

False

Question No: 1 (Marks: 1) - Please choose one

For small values of n, any algorithm is fast enough. Running time does become an issue when n gets large.

True (Page 14)

Fast

Question No: 1 (Marks: 1) - Please choose one

The array to be sorted is not passed as argument to the merge sort algorithm.

True

False

Question No: 1 (Marks: 1) - Please choose one

In simple brute-force algorithm, we give no thought to efficiency.

True
False **(Page 11)**

Question No: 1 (Marks: 1) - Please choose one

The ancient Roman politicians understood an important principle of good algorithm design that is plan-sweep algorithm.

True

False (Page 27) [Divide and Conquer]

Question No: 1 (Marks: 1) - Please choose one

In 2d-space a point is said to be _____if it is not dominated by any other point in that space.

Member

Minimal

Maximal (Page 11)

Joint

Question No: 1 (Marks: 1) - Please choose one

An algorithm is a mathematical entity that is dependent on a specific programming language.

True **False**

Question No: 1 (Marks: 1) - Please choose one

The running time of an algorithm would not depend upon the optimization by the compiler but that of an implementation of the algorithm would depend on it.

True (Page 13)

False

Question No: 1 (Marks: 1) - Please choose one

F (n) and g (n) are asymptotically equivalent. This means that they have essentially the same _____ for large n.

Results

Variables

Size

Growth rates (Page 23)

Question No: 1 (Marks: 1) - Please choose one

8n2 + 2n - 3 will eventually exceed c2*(n) no matter how large we make c2.

True (Page 25)

False

Question No: 1 (Marks: 1) - Please choose one

If we associate (x, y) integers pair to cars where x is the speed of the car and y is the negation of the price. High y value for a car means a _____ car.

Fast

Slow

Expensive

Cheap (Page 11)

Question No: 1 (Marks: 1) - Please choose one

The function f(n)= n(logn+1)/2 is asymptotically equivalent to n log n. Here Upper Bound means the function f(n) grows asymptotically _____ faster than n log n.

More

Quiet

Not (Page 24)

At least

Question No: 1 (Marks: 1) - Please choose one

After sorting in merge sort algorithm, merging process is invoked.

Select correct option:

True (Page 28)

False

Question No: 1 (Marks: 1) - Please choose one

Asymptotic growth rate of the function is taken over_____ case running time.

Select correct option:

Best

Average

Worst (Page 14)

Normal

Question No: 1 (Marks: 1) - Please choose one

In analysis of f (n) =n (n/5) +n-10 log n, f (n) is asymptotically equivalent to _____.

n

2n

n+1

n2 (Page 23)

Question No: 1 (Marks: 1) - Please choose one

Algorithm is concerned with.....issues.

Macro

Micro

Both Macro & Micro (Page 8)

Normal

Question No: 1 (Marks: 1) - Please choose one

We cannot make any significant improvement in the running time which is better than that of brute-force algorithm.

True

False (Page 18)

Question No: 1 (Marks: 1) - Please choose one

In addition to passing in the array itself to Merge Sort algorithm, we will pass in _____other arguments

which are indices.

Two (Page 28)

Three

Four

Five

Question No: 1 (Marks: 1) - Please choose one

Consider the following Algorithm: Fun(n){ if (n=1) return 1 else return (n * Fun(n-1)) } Recurrence for the above algorithm is:

nT(n-1)+1

2T(n-1)+1

T(n-1)+cn

T(n-1)+1

Question No: 1 (Marks: 1) - Please choose one

In analysis, the Lower Bound means the function grows asymptotically at least as fast as its largest term.

True (Page 24)

False

Question No: 1 (Marks: 1) - Please choose one

Efficient algorithm requires less computational…….

Memory

Running Time

Memory and Running Time (Page 9)

Energy

Question No: 1 (Marks: 1) - Please choose one

The O-notation is used to state only the asymptotic _____bounds.

Two

Lower

Upper (Page 25)

Both lower & upper

Question No: 1 (Marks: 1) - Please choose one

For the worst-case running time analysis, the nested loop structure containing one “for” and one “while” loop, might be expressed as a pair of _____nested summations.

1

2 (Page 16)

3

4

Question No: 1 (Marks: 1) - Please choose one

Before sweeping a vertical line in plane sweep approach, in start sorting of the points is done in increasing order of their _____coordinates.

X (Page 18)

Y

Z

X & Y

Question No: 1 (Marks: 1) - Please choose one

Brute-force algorithm for 2D-Maxima is operated by comparing _____ pairs of points.

Two

Some

Most

All (Page 18)

Question No: 1 (Marks: 1) - Please choose one

The function f(n)=n(logn+1)/2 is asymptotically equivalent to nlog n. Here Lower Bound means function f(n) grows asymptotically at _____ as fast as nlog n.

Normal

Least (Page 23)

Most

All

Question No: 1 (Marks: 1) - Please choose one

The definition of Theta-notation relies on proving _____asymptotic bound.

One

Lower

Upper

Both lower & upper (Page 25) rep

Question No: 1 (Marks: 1) - Please choose one

In plane sweep approach, a vertical line is swept across the 2d-plane and _____structure is used for holding the maximal points lying to the left of the sweep line.

Array

Queue

Stack (Page 18)

Tree

Question No: 1 (Marks: 1) - Please choose one

Algorithm analysts know for sure about efficient solutions for NP-complete problems.

Select correct option:

True

False (Page 9)

Quiz No.1 (2012)

Question No: 1 of 10 (Marks: 1) - Please choose one The number of nodes in a complete binary tree of height h is

2^(h+1) – 1 (Page 40)

2 * (h+1) – 1

2 * (h+1)

((h+1) ^ 2) – 1

Question No: 1 of 10 (Marks: 1) - Please choose one

The analysis of Selection algorithm shows the total running time is indeed _____in n,

arithmetic

geometric

linear (Page 37)

orthogonal

Question No: 1 of 10 (Marks: 1) - Please choose one

A (an) _____ is a left-complete binary tree that conforms to the heap order

heap (Page 40)

binary tree

binary search tree

array

Question No: 1 of 10 (Marks: 1) - Please choose one Analysis of Selection algorithm ends up with,

T(n) (Page 37)

T(1 / 1 + n)

T(n / 2)

T((n / 2) + n)

Question No: 1 of 10 (Marks: 1) - Please choose one For the sieve technique we solve the problem,

recursively (Page 34)

mathematically

precisely

accurately

Question No: 1 of 10 (Marks: 1) - Please choose one

A heap is a left-complete binary tree that conforms to the _____

increasing order only

decreasing order only

heap order (Page 40)

(log n) order

Question No: 1 of 10 (Marks: 1) - Please choose one In which order we can sort?

increasing order only

decreasing order only

increasing order or decreasing order (Page 39)

both at the same time

Question No: 1 of 10 (Marks: 1) - Please choose one Divide-and-conquer as breaking the problem into a small number of

pivot

Sieve

smaller sub problems (Page 34)

Selection

Question No: 1 of 10 (Marks: 1) - Please choose one For the heap sort we store the tree nodes in

level-order traversal (Page 40)

in-order traversal

pre-order traversal

post-order traversal

Question No: 1 of 10 (Marks: 1) - Please choose one

The sieve technique works in _____ as follows

Phases (Page 34)

numbers

integers

routines

CS502 - Fundamentals of Algorithms

Quiz No.1 12-11-2012

Question No: 1 of 10 (Marks: 1) - Please choose one
We do sorting to,

keep elements in random positions

keep the algorithm run in linear order

keep the algorithm run in (log n) order

keep elements in increasing or decreasing order

Question No: 1 of 10 (Marks: 1) - Please choose one

Heaps can be stored in arrays without using any pointers; this is due to the _____ nature of the binary

tree,

left-complete(Page 40)

right-complete

tree nodes

tree leaves

Question No: 1 of 10 (Marks: 1) - Please choose one

Sieve Technique can be applied to selection problem?

True (Page 35)

False

Question No: 1 of 10 (Marks: 1) - Please choose one

In Sieve Technique we do not know which item is of interest

True (Page 34)

False

Question No: 1 of 10 (Marks: 1) - Please choose one

In the analysis of Selection algorithm, we eliminate a constant fraction of the array with each phase; we get the convergent _____ series in the analysis,

linear

arithmetic

geometric (Page 37)

exponent

Question No: 1 of 10 (Marks: 1) - Please choose one

For the heap sort, access to nodes involves simple _____ operations.

arithmetic (Page 41)

binary

algebraic

logarithmic

Question No: 1 of 10 (Marks: 1) - Please choose one Slow sorting algorithms run in,

T(n^2) (Page 39)

T(n)

T(log n)

Question No: 1 of 10 (Marks: 1) - Please choose one

In the analysis of Selection algorithm, we make a number of passes, in fact it could be as many as,

T(n)

T(n / 2)

log n (Page 37)

n / 2 + n / 4

Question No: 1 of 10 (Marks: 1) - Please choose one

The sieve technique is a special case, where the number of sub problems is just

5

many

1 (Page 34)

few

Question No: 1 of 10 (Marks: 1) - Please choose one

How many elements do we eliminate in each time for the Analysis of Selection algorithm?

(n / 2)+n elements

(n / 2) elements (Page 37)

n / 4 elements

2 n elements

Question No: 1 of 10 (Marks: 1) - Please choose one

One of the clever aspects of heaps is that they can be stored in arrays without using any _____.

pointers (Page 40)

constants

variables

functions

Question No: 1 of 10 (Marks: 1) - Please choose one

How much time merge sort takes for an array of numbers?

T(n^2)

T(n)

T(log n)

T(n log n) (Page 40)

Question No: 1 of 10 (Marks: 1) - Please choose one

The reason for introducing Sieve Technique algorithm is that it illustrates a very important special case of,

divide-and-conquer (Page 34)

decrease and conquer

greedy nature

2-dimension Maxima

Question No: 1 of 10 (Marks: 1) - Please choose one In Sieve Technique we do not know which item is of interest

True (Page 34) rep

False

Question No: 1 of 10 (Marks: 1) - Please choose one

Theta asymptotic notation for T (n) :

Set of functions described by: c1g(n)Set of functions described by c1g(n)>=f(n) for c1 s

Theta for T(n)is actually upper and worst case comp (Not sure)
Set of functions described by: c1g(n)

Question No: 1 of 10 (Marks: 1) - Please choose one

Memoization is?

To store previous results for future use

To avoid this unnecessary repetitions by writing down the results of recursive calls and looking them up again if we need them later (page 74)

To make the process accurate

None of the above

Question No: 1 of 10 (Marks: 1) - Please choose one
Which sorting algorithm is faster

O (n log n) Page 26

O n^
2 O
(n+k)
O
n^3

Question No: 1 of 10 (Marks: 1) - Please choose one Quick
sort is

Stable & in place

Not stable but in place (Page 54)

Stable but not in place

Some time stable & some times in place

Question No: 1 of 10 (Marks: 1) - Please choose one One
example of in place but not stable algorithm is

Merger Sort

Quick Sort (Page 54)

Continuation Sort

Bubble Sort

Question No: 1 of 10 (Marks: 1) - Please choose one

Cont sort is suitable to sort the elements in range 1 to k

K is Large

K is not known

K may be small or large

K is small (Page 57)

Question No: 1 of 10 (Marks: 1) - Please choose one

In place stable sorting algorithm.

If duplicate elements remain in the same relative position after sorting (Page 54)

One array is used

More than one arrays are required

Duplicating elements not handled

Question No: 1 of 10 (Marks: 1) - Please choose one

Which may be a stable sort?

Merger

Insertion (Page 54)

Both above

None of the above

Question No: 1 of 10 (Marks: 1) - Please choose one

An in place sorting algorithm is one that uses ___ arrays for storage

Two dimensional arrays

More than one array

No Additional Array (Page 54)

None of the above

Question No: 1 of 10 (Marks: 1) - Please choose one

Sieve Technique applies to problems where we are interested in finding a single item from a larger set of

_____

n items (Page 34)

phases

pointers

constant

Question No: 1 of 10 (Marks: 1) - Please choose one

Sorting is one of the few problems where provable _____ bonds exits on how fast we can sort,

upper

lower (Page 39)

average

log n

Question No: 1 of 10 (Marks: 1) - Please choose one

Counting sort has time complexity:

O(n) (Page 58)

O(n+k)

O(k)

O(nlogn)

Question No: 1 of 10 (Marks: 1) - Please choose one

The running time of quick sort depends heavily on the selection of

No of inputs

Arrangement of elements in array

Size o elements

Pivot elements (Page 49)

Question No: 1 of 10 (Marks: 1) - Please choose one

Which may be stable sort:

Bubble sort

Insertion sort

Both of above (Page 54)

Question No: 1 of 10 (Marks: 1) - Please choose one

One Example of in place but not stable sort is

Quick (Page 54)

Heap

Merge

Bubble

Question No: 1 of 10 (Marks: 1) - Please choose one
In Quick Sort Constants hidden in T(n log n) are

Large

Medium

Small Click here for detail

Not Known

Question No: 1 of 10 (Marks: 1) - Please choose one

Quick sort is based on divide and conquer paradigm; we divide the problem on base of pivot element and:

There is explicit combine process as well to conquer the solution.

No work is needed to combine the sub-arrays, the array is already sorted

Merging the sub arrays

None of above. (Page 51)

Ref: - random choices for the pivot element and each choice have an equal probability of 1/n of occurring. So we can modify the above recurrence to compute an average rather than a max

CS501 - Quiz No.2 (Spring 2013)

Question No: 1 of 10 (Marks: 1) - Please choose one

A point p in 2-dimensional space is usually given by its integer coordinate(s)_____

p.x only

p.y only

p.x & p.z

p.x & p.y (Page 10)

Question No: 1 of 10 (Marks: 1) - Please choose one

In _____ we have to find rank of an element from given input.

Merge sort algorithm

Selection problem (Page 34)

Brute force technique

Plane Sweep algorithm

Question No: 1 of 10 (Marks: 1) - Please choose one

In Heap Sort algorithm, if heap property is violated _____

We call Build heap procedure

We call Heapify procedure

We ignore

Heap property can never be violated

Question No: 1 of 10 (Marks: 1) - Please choose one

Upper bound requires that there exist positive constants c2 and n0 such that f(n) ____ c2n for all n <= n0(ye

question ghalat lag raha hai mujhae

Less than

Equal to or Less than Equal or Greater than Greater than **(Page 25)**

Question No: 1 of 10 (Marks: 1) - Please choose one

A RAM is an idealized algorithm with takes an infinitely large random-access memory.

True

False (Page 10)

Question No: 1 of 10 (Marks: 1) - Please choose one

_____ is one of the few problems, where provable lower bounds exist on how fast we can sort.

Searching

Sorting (Page)

Both Searching & Sorting

Graphing

Question No: 1 of 10 (Marks: 1) - Please choose one

Floor and ceiling are _____ to calculate while analyzing algorithms.

Very easy

Usually considered difficult (Page 31)

Question No: 1 of 10 (Marks: 1) - Please choose one

In Heap Sort algorithm, the maximum levels an element can move upward is _____

Theta (log n) (Page 43)

Order (log n)

Omega (log n)

O (1) i.e. Constant time

Question No: 1 of 10 (Marks: 1) - Please choose one

A point p in 2-dimensional space is usually given by its integer coordinate(s)_____

p.x only p.y

only p.x & p.z

p.x & p.y (Page 17)

Question No: 1 of 10 (Marks: 1) - Please choose one

In Heap Sort algorithm, the total running time for Heapify procedure is _____

Theta (log n) (Page 43)

Order (log n)

Omega (log n)

O (1) i.e. Constant time

Question No: 1 of 10 (Marks: 1) - Please choose one

Algorithm is a mathematical entity, which is independent of a specific machine and operating system.

True

False (Page 7)

Question No: 1 of 10 (Marks: 1) - Please choose one

While Sorting, the ordered domain means for any two input elements x and y _____ satisfies only.

x < y

x > y

x = y

All of the above (Page 39)

Question No: 1 of 10 (Marks: 1) - Please choose one Quick sort is best from the perspective of Locality of reference.

True
False **(Page 9)**

Question No: 1 of 10 (Marks: 1) - Please choose one

Sorting can be in _____

Increasing order only

Decreasing order only

Both Increasing and Decreasing order (Page 39)

Random order

Question No: 1 of 10 (Marks: 1) - Please choose one

In Heap Sort algorithm, we build _____ for ascending sort.

Max heap
Min heap **(Page 41)**

Question No: 1 of 10 (Marks: 1) - Please choose one In Sieve Technique, we know the item of interest.

True

False (Page 34)

Question No: 1 of 10 (Marks: 1) - Please choose one

While solving Selection problem, in Sieve technique we partition input data _____

In increasing order

In decreasing order

According to Pivot (Page 3

Randomly

Question No: 1 of 10 (Marks: 1) - Please choose one

In pseudo code, the level of details depends on intended audience of the algorithm.

True (Page 12)

False

Question No: 1 of 10 (Marks: 1) - Please choose one

The sieve technique works where we have to find _____ item(s) from a large input.

Single (Page 34)

Two

Three

Similar

Question No: 1 of 10 (Marks: 1) - Please choose one

If the indices passed to merge sort algorithm are _____,then this means that there is only one element to sort.

Small

Large

Equal (Page 28)

Not Equal

1) The sieve technique works where we have to find ---------- item(s) from a large input.

- **Single page 34**
- Two
- Three
- Similar

2) The only way to convert an empty string into a string of j characters is by doing j insertions , represented as

- E(i , j)=I
- E(i , 0)=i
- **E(0 , j)=j page 78**
- E(1 , j)=j

3) In Selection problem , the Sieve technique works in ----------

- Non-recursive manner
- Constant time
- **Phases page 34**
- One complete go

4) Algorithm is a sequence of computational steps that --------- the input into output.

- Merge
- Assign
- **Transform page 7**
- Integrate

5) In max heap (for Heap Sort algorithm), when every time maximum element is removed from top we replace it with ---------- in the leaf tree.

- Second last
- **Last page 41**
- First
- Any

6) If pj dominates pi and pi dominates ph then pj also dominates ph. It means dominance relation is

- **Transitive page 18**
- Non transitive
- Equation

Symbolic

7) In the following code the statement “cout<<j;” executes ---------- times

```
For(j=1;j<=5;j=j+2)
Cout<<j;
```

- 5 times
- 2 times
- **3 times (conceptual)**
- 0 times

9) For average-case time analysis of Quick sort algorithm, Pivot selection is on average basis from ----------.

- Half of the input values
- **All possible random values page 50**
- Pivot is input separately
- Values greater than 5

10) In the statement, “output P[i].x,P[i].y”, the number of times elements of P are accessed is ----------.

- 1
- **2 page 14**
- 3
- 4

11) Identify the TRUE statement.

- The Knapsack problem does not belong to the domain of optimization problems.
- **The Knapsack problem belongs to the domain of optimization problems Page 91**
- The Knapsack problem cannot be solved by using dynamic programming.
- The Knapsack problem is optimally solved by using brute force algorithm.

12) Matrix multiplication is a(n) ---------- operation.

- Commutative
- **Associative page 85**
- Neither commutative nor associative
- Commutative but not associative

13) ---------- provides us more accurate result when input values are not closer to each other.

- Average
- **Median page 34(conceptual)**

14) In merge sort algorithm, we split the array around the ---------- index q.

- Entering
- **Mid page 28**
- Exitin

Summing

14) In the analysis of Selection algorithm, we get the convergent ---------- series.

- Harmonic
- Linear
- Arithmetic
- **Geometric page 37**

16) In Selection problem, the Sieve technique ----------.

- Adds some more input items each time.
- Do not work recursively
- Do not uses Divide and Conquer approach
- **Eliminate undesired data items each time page 35**

17) If matrix A of dimension p x q is multiply with matrix B of dimension q x r, then each entry in resultant matrix takes ---------- time.

- **O(q) page 84**

- O(1)
- O(p x q)
- O(q x r)

18) In Sorting the key value or attribute ---------- form an ordered domain.

- Must be

Not always page 39

19) In Heap Sort algorithm, the total running time for Heapify procedure is ---------- .

Theta (log n) page 43

- Order (log n)
- Omega (log n)
- O (1) i.e. Constant time

20) Pseudo code of algorithm are to be read by ----------.

- **People page 12**
- RAM
- Computer
- Compiler

21) In Dynamic Programming based solution of Knapsack Problem, if we decide to take an object 'i', then we gain ----------.

- W (Total Weight of Knapsack)
- V (Total Value of all items)
- **vi(Value of object i) page 93**
- None of the given option

22) The worst case running time of Quick sort algorithm ----------.

- Cannot be quadratic

Is quadratic (google)

- Is always exponential
- Is linear

23) The only way to convert a string of i characters into the empty string is with i deletions , represented as

- E(0 , j)=j
- E(i , j)=I
- E(0 , i)=j
- **E(i , 0)=i page 78**

24) In Selection problem, the rank of an element will be its ---------- position if we sort the input data.

- First
- **Final page 34**
- Second last
- Last

25) Which one is not passed as parameter in Quick sort algorithm?

- End of array
- Middle of array

- **Array (containing input elements) (cs301)**

- Start of the array

26) While Sorting, the ordered domain means for any two input elements x and y --------- satisfies only.

- X<y
- X>y

- X=y
- **All of the above page 39**

27) Which of the following is calculated with Big O notation?

- Medium bounds
- **Upper bounds page 25**
- Lower bounds
- Both upper and lower bounds

28) For solving Selection problem, we introduced Sieve technique due to ---------

- Using Decrease and Conquer strategy
- Avoiding to sort all input data
- **Eliminating Rank of an element page 35**

Using Brute-Force approach

29) The worst-case running time of merge sort is ---------- in order to sort an array of n elements.

- O(log n)
- O(n)

O(n log n) page 30

-

30) Quick sort is a recursive algorithm.

- False

True (page49)

31) While analyzing Selection algorithm, we make a number of passes , in fact it could be as many as ----------.

- n(n+1)
- **log(n) page 37**

- n/3
- n/4

32) Approach for solving geometric problems by sweeping a line across the plane is called ---------- sweep.

- Line
- **Plane page 18**
- Cube
- Box

33) ---------- time is the maximum running time over all legal inputs.

- **Worst-case page 13**
- Average-case
- Best-case
- Good-case

34) The asymptotic growth of n(n+1)/2 is:

- **O(2) page 15**
- O(n)
- O(n+2)
- O(n log n)

35) One of the limitation in 0/1 Knapsack is that an item can either be ---------- in the bag or not.

- Use
- **Put page 91**
- Move
- Store

36) --------- is one of the few problems, where provable lower bounds exist on how fast we can sort.

- Searching
- **Sorting page 39**
- Both searching and sorting
- Graphing

37) Selection algorithm takes theta ----------.

- (n2)
- (**n) page 37**
- log(n)
- nlog(n)

38) The Sieve technique is a special case, where the numbers of sub-problems is just -----.

- **1 page 34**
- 2
- 3
- 4

39) Following is not the application of Edit Distance problem

- Speech recognition
- Spelling Correction
- **Ascending Sort page 76**
- Computational Molecular Biology

40) We can use the optimal substructure property to devise a ---------- formulation of the edit distance problem.

- Selective
- Optimum
- Iterative

Recursive (p78)

41) In plane sweep approach, a vertical line is swept across the 2d-plane form ----------

- Right to left
- **Left to right page 18**

- Top to bottom
- Bottom to top

42) In 3-Dimensional space, a point P has ---------- coordinate(s).

- (X,Y)
- (X,0)
- (0,Y)
- **(X,Y,Z) (google)**

43) ---------- items are not allowed in the 0/1 Knapsack.

- Lighter
- **Fractional page 91**

- Whole
- Weighty

44) Radix sort is not a non-comparative integer sorting algorithm.

- True
- **False (google)**

45) Fibonacci sequence was named on ----------, a famous mathematician in 12th Century.

- Fred Brooks
- Grady Booch
- **Leonardo Pisano page 73**

- Edgar F.Codd

46) In generating Fibonacci Sequence, we can avoid unnecessary repetitions by ---------- process.

- Tokenization
- **Memoization page 74**
- Randomization
- Memorization

47) The sequence of merge sort algorithm is:

- Divide-Combine-Conquer
- Conquer-Divide-Combine
- **Divide-Conquer-Combine page 27**
- Combine-Divide-Conquer

48) Sorting can be in ----------.

- Increasing order only
- Decreasing order only
- **Both Increasing and Decreasing order page 52**
- Random order

49) The process of --------- ends when you are left with such tiny pieces remaining that it is trivial to solve them.

- Brute-force
- Plan-sweep
- **Divide and Conquer page 27**
- Axis-sweep

50) Time complexity of the “0-1” knapsack algorithm depends on ----------.

- Number of items
- Capacity of the knapsack
- Size of the table
- **Number of items and capacity of knapsack (Google)**

51) In max-heap, largest element is stored at root node. Where is the smallest element stored?

- Right Node

Leaf Node (Google)

- Middle Node
- Left Node

52) For ---------- values of n, any----------algorithm is fast enough.

Small pg#14

- Medium
- Large
- Infinity

53) To predict the cost of an algorithm in terms of resources is called ----------.

- Key

Analysis (Google)

- Design
- Output

54) Dynamic programming comprises of ---------

- **Recursion only pg#75**
  - Repetition only
  - Recursion and Repetition
  - No Repetition but Recursion

55) For Quick sort algorithm, Partitioning takes theta ----------?

- (n)
- log(n)
- **nlog(n) (Google)**
- n2log(n)

56) An in-place sorting algorithm is one that ---------- uses additional array for storage.

- Always
- Permanently
- **Does not pg#54**
- Sometime

57) In Dynamic Programming approach, we do not store the solution to each sub-problem in case if it reappears.

- True
- **False pg#75**

58) Counting sort has time complexity:

- **O(n+k) pg#58**
- O(n)
- O(k)
- O(nlogn)

59) Due to left complete nature of binary tree, the heap ca be stored in

- **Arrays pg#40**
- Structures
- Link list
- Stack

60) For the heap sort we store the tree node in

- **Level-order traversal pg#40**

- In-order traversal
- Pre-order traversal
- Post-order traversal

61) n the clique cover problem, for two vertices to be in the same group, they must be ----------each other.

- Apart from
- Far from
- Near to
- **Adjacent to pg#176**

62) In in-place sorting algorithm is one that uses arrays for storage :

No additional array pg#54

- An additional array
- Both of above may be true according to algorithm
- More than 3 arrays of one dimension

63) Brute-force algorithm for 2D-Maxima is operated by comparing ---------- pairs of points.

- Two
- Some
- Most

All pg#18

64) F(n) and g(n) are asymptotically equivalent. This means they have essentially the same --------- for large n.

- Results

Growth rates pg#23

- Variables
- Size

65) Quick sort is

- Stable & in place
- **Not stable but in place pg#54**
- Stable but not in place
- Some time stable & some times in place

66) Heaps can be sorted in arrays without using any pointers; this is due to the ---------- nature of the binary tree.

- **Left-complete pg#40**
- Right-complete
- Tree nodes
- Tree leaves

67) Which may be a stable sort?

- **Merger**
- **Insertion**
- **Both above page 54**
- None of the above

68) For the Sieve Technique we take time

- **T(nk) page 34**
- IT(n/3)
- n^2
- n/3

69) One of clever aspects of heaps is that they can be stored in arrays without using any --------.

- **Pointers page 40**
- Constants
- Variables
- Functions

70) Counting sort is suitable to sort the elements in range 1 to k.

- K is large
- K is not known
- K may be small or large
- **K is small page 57**

71) Asymptotic growth rate of the function is taken over ---------- case running time.

- Best
- **Worst page 14**
- Average
- Normal

72) The Sieve technique is a special case, where the number of a sub-problems is just .

- 5
- **1 page 34**
- Few

73) Dynamic programming algorithms need to store the results of intermediate sub-problems.

- **True page 75**
- False

74) In counting sort, once we know the ranks, we simply ---------- numbers to their final positions in an output array.

- Delete
- **Copy page 57**
- Mark
- Arrange

75) In Quick sort, we don't have control over the sizes of recursive calls.

- **True page 49**
- False
- Less information to decide
- Either true or false

76) Before sweeping a vertical line in a plane sweep approach, in start sorting of the points is done in increasing order of their ---------- coordinates.

- **X (page #18)**
- Y
- Z
- X&Y

77) Random access machine or RAM is a/an

- Machine build by Al-Khwarizmi
- Mechanical machine
- **Mathematical model (page #10)**
- Electronics machine

78) The Huffman codes provide a method of encoding data inefficiently when coded using ASCII standard .

- True
- **False(page #99)**

79) A heap is a left-complete binary tree that conforms to the ----------.

- Increasing order only
- Decreasing order only
- **Heap order (page# 40)**
- (log n) order

80) For the sieve technique we solve the problem,

- **Recursively (page #34)**
- Mathematically
- Precisely
- Accurately

81) Median is not useful measure of central tendency of given input set especially when the distribution of values is highly skewed.

- True
- **False (page # 34)**

82) The omega-notation allows us to state only the asymptotic----------bounds.

- Middle
- **Lower (page #25)**
- Upper
- Both middle and upper

83) In plane sweep approach, a vertical line is swept across the 2-D plane and----------structure is used for holding the maximal points lying to the left of the sweep line.

- Array
- Queue
- **Stack (page #18)**
- Tree

84) When a heapify procedure is applied to the root node to restore the heap, then at each level, the comparison performed takes time.

- It will take O(log n)
- It cannot be predicted
- **It will takeO (1) (page #43)**
- Time will vary according to the nature of input data

85) Efficient algorithm requires less computational----------.

- Memory
- Running time
- **Memory and running time (page #9)**
- Energy

86) Recurrence can be described in terms of a tree.

- **Yes (page#31)**
- No

87) Time complexity of dynamic programming based algorithm for computing the minimum cost of Chain Matrix multiplication is----------.

- log n
- n
- n^2(n square)
- **n^3(n cube) (page#90)**

88) In order to say anything meaningful about our algorithms, it will be important for us to settle on a----------.

- Java programming
- C++ programming
- Pseudo program
- **Mathematical model of computation (page # 10)**

89) The iteration method is used for ----------.

- Comparing sorting algorithms only
- **Solving recurrence relation (page # 31)**
- Merging elements in merging sort
- Divide elements in merge sort

90) Chain matrix multiplication problem can be solved through----------strategy.

- **Dynamic programming (page #85)**
- Greedy
- Divide and conquer
- Sorting

91) For comparison-based sorting algorithms, it is ----------possible to sort more efficiently than Omega nlog (n) time.

- Always
- **Not (page #54)**
- Sometimes not
- Sometimes

92) Dynamic Programming approaches are usually useful in solving optimization problems.

- **True (page # 97)**
- False

93) In sorting the key value or attribute----------from an ordered domain.

- **Must be (page #39)**
- Not always

94) Result of asymptotical analysis of n (n-3) and 4n*n is that----------.

- n(n-1) is asymptotically less
- n(n-1) is asymptotically Greater
- both are asymptotically Not equivalent
- **both are asymptotically Equivalent (page #23)**

95) Floor and ceiling are----------to calculate while analyzing algorithms.

- Very easy
- **Usually consider difficult pg#31**

96) ---------- of reference is an important fact of current processor technology.

- Defining
- Assigning
- Formality
- **•Locality (page # 8)**

97) As per algorithm of dynamic programming , we need to store

- First sub- problem only
- Best solution only
- **Intermediate sub-problems (page #75)**
- Final solution only

8) In Sieve technique, we solve the problem----------.

- **•In recursive manner (page # 34)**
- Non recursively
- Using Merge sort algorithm
- Using Brute force technique

99) Upper bound requires that there exist positive constant c2and n0 such that f(n)---------c2n for all n<= n0.

- Less than
- **Equal to or less than (page # 24)**
- Equal to or greater than
- Greater than

100) In 2-D matrix problem, A point p is said to be dominated by point q if--------.

- P.x=q.x and p.y=q.y
- P.x≠q.x and p.y≠q.y
- P.x≥ q.x and p.y≥q.y
- **P.x≤q.x and p.y≤q.y (page # 11)**

101) A Random Access Machine (RAM) is an idealized machine with---------- random access.

- **Infinite large (page # 10)**
- 512 MB
- 256 MB
- 2 GBs

102) There are----------entries in the Edit Distance Matrix.

- Θ (n)
- **Θ (n2) (page # 84)**
- Θ (n+2)
- Θ (n+100)

103) Counting sort is suitable for sorting the elements within range 1 to P, where----------.

- P is large
- **P is small (page # 57)**
- P is very large
- P is undetermined

104) While analyzing searching algorithm, we make a number of passes, in fact it called----

- n(n+1)

log(n) (page #37)

- n/3

- n/4

105) It is not a Fibonacci Sequence.

1,1,1,2,3,5,8,13,21,34,

55,----

- **True (page #73)**
- False

106) In Random Access Machine (RAM), instructions are executed in----------.

- Parallel
- Batch

One by one (page # 10)

- Multiple times

107) Dynamic programming formulation of the matrix chain multiplication problem will store the solutions of each sub problem in a(n):

- Array

Table (page #86)

- Variable
- Class

108) An algorithm is a mathematical entity, which is independent of----------.

- Programming language
- Machine and programming language
- Compiler and programming language

Programming language Compiler and Machine (page #7)

109) In Fibonacci sequence, unnecessary repetitions do not exist at all.

True (page# 74)

- False

110) In Quick sort algorithm, pivots form----------.

- Stack

Queue

- Graph

111) In Heap Sort Algorithm, Heapify procedure is---------- in nature.

- **Recursive (page #43)**
- Non-recursive

112) In Bucket sort, if there are duplicates then each bin can be replaced by a :

- Stack

Link list (page #69)

- Hash table
- Heap

113) Counting sort assumes that the numbers to be sorted are in the range ----------.

- k to n where n is large

1 to k where k is small (page # 57)

- k to n where k is small
- k to n where n is small

114) In Dynamic Programming, our approach is to ----------.

- Develop the solution in a top-down fashion.
- Express the problem non-recursively.

o Build the solution in a bottom- up fashion. Pg#92

- Input several sub-problems simultaneously.

115) In knapsack Problem, the goal is to put items in the knapsack such that the value of the items is ---------- ------ subject to weight limit of knapsack.

- Minimized
- Decreased

Maximized

- None of the given option

116) If there are $\Theta(n^2)$ entries in edit distance matrix then each entry E(i,j) takes ---------- time to compute.

- $\Theta(n)$
- $\Theta(n^2)$

Θ (1) (page #84)

- $\Theta(n \log n)$

117) In ---------- knapsack Problem, limitation is that an item can either be put in the bag or not.
Fractional items are not allowed.

- 0
- 1

0/1 (page #91)

- Fractional

118) Heapsort is a/an ---------- and ---------- sorting algorithms.

- Not in-place, not stable one
- In-place , stable one

In-place, not stable one (page#54)

- Not in-place , stable one

119) Selection sort is not an in-place sorting algorithm.

- True

False (page #54)

120) Memoization is a part of Dynamic Programming strategy.

True (page 74)

- False

127) Quick sort algorithms divide the entire array into----------sub arrays.

- One

Two (Google)

- Three
- Four

121) Insertion sort is an in-place algorithm.

a) True (page #54)

b) False

122) When matrix A of 5x3 is multiply with matrix B of 3x4 then the number of multiplication required is :

- 15
- 12
- 36
- **60 page 84**

123) If matrix A of dimensions 2x4 is multiply with matrix B of dimensions 4x3, then the dimensions of resultant matrix is :

- 2x4
- 4x3
- 3x4
- **2x3 page 84(conceptual)**

124) Dynamic Programming strategy is useful when sub- problems are independent.

- True

False google

125) If Matrix-A of dimension 2x4 multiply with Matrix-B of dimension 4x3, then the dimension of resultant Matrix is:

- 2x4
- 4x3
- 3x4
- **2x3 page 84(conceptual)**

126) In the following code the statement “cout<< i” executes---------

----times. **for (int i=1; i<=n; i++)**

cout<< i;

- **n times (conceptual)**
- n+5 times
- Infinite times
- Zero times

127) Edit distance algorithm based on----------strategy.

- Greedy
- **Dynamic programming page 77**
- Divide and conquer
- Searching

128) If Matrix A has dimensions “3x2” and Matrix B has dimensions “2x3” , then multiplication of Matrix-A and Matrix-B will result a new Matrix-C having dimensions.

- 3x2
- 2x3
- 2x2
- **3x3 page 84(conceptual)**

129) The function f (n) =n (logn+1)/2 is asymptotically equivalent to nlogn. Here Lower Bound means function f

(n) grows asymptotically at ---------- as fast as nlogn.

- **Least page 24**
- Normal
- Most
- All

130) Single atom from a larger set of ----------.

- Constant
- Pointers
- Phases
- **n items page 34**

131) Fibonacci sequence was named on a----------, a famous mathematician in 12th century.

- Fred brooks
- Grady booch
- **Leonardo Pisano (page # 73)**
- Edgar F.Codd

132) In dynamic programming, our approach is to----------.

- Develop the solution in a top down fashion
- Express the problem non-recursively
- **Build the solution in a bottom up fashion (page #92)**
- Input several sub-programs simultaneously

133) In heap sort algorithm, we build----------for ascending order.

- **Max heap page 42(conceptual)**
- Min heap

134) Bubble sort is not an in-place sorting algorithm.

- True

False (page #54)

135) In partition algorithm, the subarray----------has elements which are greater than pivot element X.

- A[p….r]
- A[p….q-1]
- A[q]

A[q+1…..r] (page #46)

136) In Heap sort algorithm, if heap property is violated----------.

- We call build heap procedure

We call heapify procedure (page # 41)

- We ignore
- Heap property can never be violated

137) ----------is not a characteristics of Random Access Machine.

- Single-processor
- Assigning a value to a variable

Locality of reference (page #10)

- Executing an arithmetic instruction

138) Boolean operation is a----------operation on an idealized RAM model of computation.

- Starting

Basic (page #10)

- Advance
- Normal

139) How much time merge sort takes for an array of numbers?

- T(n^2)
- T(n)
- T(log n)
- **T(n log n) page 4**

140) Sorting is one of the---------problems in Computer Sciences.

Easiest page 39

- Solution-less
- Least popular
- Tony Hoare

142) We can make----------recursive calls in Fibonacci sequence.

- **Infinite google**
- Finite

140) Insertion sort is a ---------- sorting algorithm.

- Unstable
- **In-place page 54**
- Not in-place
- In-portion

141) We do not need to prove comparison- based sorting algorithms by mathematically , it always takes----------

----------:

- Big Oh nlog(n)
- **Omega nlog(n) page 54**
- Omega n(n^2)
- Theta nlog(n)

142) A Principal operation for maintaining the heap property is called heapify, it is also called :

- **Shifting Down page 43**
- Shifting Up
- Shifting Right
- Shifting Left

143) The number of nodes in a complete binary tree of height h is:

- 2*(h+1)-1
- 2^(h+1)-1
- **2^(h+1) page 45**
- ((h+1)^2)-1

144) After partitioning array in Quick sort, pivot is placed in a position such that:

- Pivot is the first element of array

- Pivot is the last element of array
- **Values smaller than pivot are on left and larger than pivot are on right page 35**
- Values larger than pivot are on left and smaller than pivot are on right

145) ---------- programming is essentially recursion without repetition:

- Fast
- **Dynamic page 75**
- Array
- N log n

146) We can use the _____ property to devise a recursive formulation of the edit distance problem.

- small substructure
- algorithmic

real

- **optimal substructure page 78**

147) Applying the sieve technique to selection problem,_____ element is picked from array.

- Output
- Total
- Input
- **Pivot page 35**

148) If matrix A of dimension p* q is multiply with matrix B of dimension q*r, then the dimension of the resultant matrix is:

- p*q
- **p*r page 84**
- q*r
- r*p

149) In knapsack problem, each item must be entirely accepted or rejected, is called _____ problem.

- Fractional
- **“0-1” page 92**
- Linear
- Optimal

150) In the case of _____ analysis does not depend upon on the distribution of input.

- Merge sort
- **Quick sort page 50**
- Insertion sort
- Heap sort

151) In Heapsort Algorithm, total time taken by heapify procedure is:

- **O(log n) page 43**
- O(log2 n)
- O(n log n)
- O(n2 log n)

152) In merge sort algorithm, we split the array _____ to find index q.

- from start
- **midway page 28**
- from end
- both from start or end

153) Al-Khwarizmi was a/an _____

- artist
- **mathematician page 7**
- astronomer
- khalifah

154) In Sieve Technique, we know the item of interest.

- True
- **False page 34**

155) Radix sort is a _____ integer sorting algorithm.

- Comparative
- **Non-comparative google**
- In-Place
- Unstable

156) Random Access Machine (RAM) can execute _____ instruction.

- only logical
- parallel
- only arithmetic
- **logical and arithmetic page 10**

157) Which of the following is calculated with Big Omega notation?

- Medium bounds
- **Upper bounds page 25**
- Lower bounds
- Both upper and lower bounds

158) Asymptotic growth of 8n^2 + 2n – 3 is:

- O(n^2+n)
- **O(n^2) page 23**
- O(Bn^2)
- O(Bn^2+2n)

159) _____ is the process of avoiding unnecessary repetitions by writing down the results of recursive calls and looking them up again if we need them later.

- Loop
- **Memoization page 74**
- Recursion
- Function

160) In Heap Sort algorithm, the first step is to_____.

- **Call Build-Heap procedure page 43**
- Sort the array in descending order
- Call Heapify procedure
- Find the number of input elements

161) The time assumed for each basic operation to execute on RAM model of computation is_____.

- Infinite
- Continuous
- **Constant page 10**
- Variable

162) If input “n” is odd, then median will be_____.

- **(n+1)/2 page 34**

- n/2
- (n-1)/2
- n+2

163) In quick sort algorithm, _____ decides nature of Binary Search Tree formed by pivots.

- middle element from input
- smallest element from input
- **rank of the pivot page 49**
- largest element from input

164) The brute-force algorithm for 2D-Maxima runs in order O(----------) time:

- n
- n(log n)
- **n*n page 18**
- n3

165) In average-case time analysis of quick sort algorithm, the most balanced case for partition is when we divide

the list of elements into----------

- Equal no. of pieces as input elements
- Single piece exactly
- **Two nearly equal pieces google**
- Three nearly equal pieces

166) The main purpose of mathematical analysis is measuring the ---------- required by the algorithm:

- Space
- **Execution time page 13**
- Inputs & outputs
- Execution time and memory

167) Dynamic Programming algorithms often use some kind of ---------- to store the results of intermediate Sub-problems:

- **Table page 75**
- Variable
- Stack
- Loop

168) The average case running time of quick sort algorithm is theta ----------:

- Log(n)
- (n)
- **nlog(n) page 53**
- nxn log(n)

169) What is the best case time complexity of merge sort?

- O((n^2logn)
- O((nlog^2)
- **O((nlogn) page 40**
- O((n^2)

170) While solving Selection problem, in Sieve technique we partition input data ----------:

- In increasing order
- In decreasing order
- **According to Pivot page 35**
- Randomly

171) In average-case time the probability of seeing input is denoted by----------:

- P{I}
- P[I}

- **P(I) page 13**
- P<I>

172) To check whether a function grows faster or slower than the other function, we use some asymptotic notations, which Is ----------:

- Big-oh notation
- Theta notation
- Omega notation
- **All of the given google**

173) Asymptotic notations are used to describe -------- of an algorithm:

- Length
- **Running time (google)**
- Size
- Compile time

174) In recursive formulation of Knapsack Problem

V[0,j] = ---------- for j>=0

- -1

0 (page #93)

- 1
- 2

175) If a matrix has three rows and two columns, then dimensions of matrix will be:

3x2 (google0

- 2x2
- 2x3
- 3x3

176) Selection sort takes theta --- in the worst case:

(n^2)

- (n)
- Log(n)
- nlog(n)

177) Which of the following problems can be solved using dynamic problem?

- Bubble sort problem
- **Matrix chain multiplication problem page 92**
- Greedy search problem
- Fractional knapsack problem

178) ---------- algorithm based on Dynamic Programming strategy:

- Heap sort
- Binary tree
- Quick sort
- **Edit distance page 77**

179) The definition of theta-notation relies on proving ------- asymptotic bound:

- One
- Lower
- Upper
- **Both lower and upper page 25**

180) In order to say anything meaningful about our algorithms, it will be important for us to settle on a --------:

- Java program
- C++ program
- Pseudo program
- **Mathematical model of computation page 10**

181) In asymptotical analysis of n(n-3) and 4n*n , as n becomes large, the dominant (fastest growing) term is some Constant times----------,

- n+1
- n-1
- n
- **n*n page 23**

182) ---------- is in- place sorting algorithm

- **Bubble sort page 54**
- Merge sort
- Linear sort
- Binary sort

183) In chain matrix multiplication, if there are --------- ways in which outer most pair of parentheses can placed:

- n^2
- 2n
- n+1
- **n-1 page 85**

184) In chain matrix multiplication, table is filled ----- to find the multiplication of matrix:

- row wise
- column wise
- **diagonally page 86**
- bottom-to-up

185) Which one sorting algorithm is best suited to sort an array of 2 million elements?

- Bubble sort
- Insertion sort
- Merge sort
- **Quick sort google**

186) The problem with the brute-force algorithm is that it uses ----- in pruning out decisions:

- Worst-case time
- **No intelligence page 18**

- Outside looping
- Artificial intelligence

187) Quicksort is a/an --------- and ---------- sorting algorithm:

- No in-place , not-stable one

In-place , not stable one page 54

- In-place , stable one
- Not in-place , stable one

191) In Heap Sort algorithm, the maximum levels an element can move upward is ----------:

- **Theta (log n) page 43**
- Big-oh(log n)
- Omega (log n)
- O (1) i.e. Constant time

192) If the time complexity of an algorithm is O(n) , then it is called ---------- time complexity:

- **Linear page 37**
- Constant
- Average
- Exponential

193) Dynamic Programming approach is usually useful in solving ---------- problems:

- Normal
- **Optimization page 97**
- Array
- Loop

194) What time does Merge Sort algorithm take in order to sort an array of ‘n’ numbers?

- Θ(n)
- Θ(log n)
- Θ(n^2)
- **Θ(n log n) page 30**

CS-502 (Fundamentals of Algorithms)

Grand Quiz - Fall 2020

If you found any error in the file, Please correct it accordingly.

163) The sequence of merge sort algorithm is:

- **Divide Combine-Conquer**
- **Conquer-Divide-Combine**

c. Divide-Conquer-Combine Page 27

- **Combine-Divide-Conquer**

- **In _____ Knapsack Problem, limitation is that an item can either be put in the bag or not. Fractional items are not allowed.**

0

1

c. 0/1 Page 91

Fractional

165) In Selection algorithm, we assume pivot selection takes theta _____ running time.

a. n Page - 36

n2

n3

log (n)

- In Heap Sort algorithm (using max heap), when every time maximum elements removed from top _____.

We call merge Sort Algorithm

it becomes Order n2 Algorithm

Divide and Conquer strategy helps us

d. We are left with a hole Page – 41

167) If matrix A of dimension p x q is multiply with matrix B of dimension q x r, then each entry in resultant matrix takes _____ time.

a. O (q) Page - 84

168) O (1)

169) O (p x q)

170) O (q x r)

169) _____ is a method of solving a problem in which we check all possible solutions to the problem to find the solution we need.

- Plane-Sweep Algorithm
- Sorting Algorithm

c. Brute-Force Algorithm google

- Greedy approach

171) The worst case running time of Quick sort algorithm _____.

- Cannot be quadratic

b. Is quadratic google QuickSort and its Analysis (codesdope.com) &

http://personal.kent.edu/~rmuhamma/Algorithms/MyAlgorithms/Sorting/quickSort.htm

Is always Exponential

Is linear

172) In max heap (for Heap Sort algorithm), when every time maximum element is removed from top we replace it with _____ leaf in the tree.

- second last

b. Last Page -41

- First
- Any

174) Quick sort algorithm was developed by -

AlferdAho

Sedgewick

John Vincent Atanasoff

d. Tony Hoare – Google wikipedia

- **If Matrix-A has dimensions "3x2" and Matrix-B has dimensions "2x3", then multiplication of Matrix-A and Matrix-B will result a new Matrix-C having dimensions.**

  3x2

  2x3

  2x2

  d. 3x3 **http://www.calcul.com/show/calculator/matrix-multiplication**

- **For comparison-based sorting algorithms, it is possible to sort more efficiently than Omega n log(n) time.**

  a. Always

  b. Not **P-54**

  c. Sometimes

Sometimes not

176) Dynamic Programming approach is usually useful in solving optimization problems.

a. True https://en.wikipedia.org/wiki/Dynamic_programming &

http://web.mit.edu/15.053/www/AMP-Chapter-11.pdf

178) False

179) In Sorting the key value or attribute_____ from an ordered domain.

a. Must be page 39

- Not always
- May be
- Occasionally

181) Result of asymptotical analysis of n(n -3) and 4n*n is that _____

- n(n-1) is asymptotically Less
- n(n-1) is asymptotically Greater
- Both are asymptotically Not equivalent

d. Both are asymptotically Equivalent page 23 (4n*n= 4n2)

182) Floor and ceiling are _____ to calculate while analyzing algorithms a. Very easy

b. Usually considered difficult P-31

3rd Option is missing

4th Option is missing

184) _____ of reference is an important fact of current processor technology.

- Defining
- Assigning

- **Formality**

d. Locality **P-8**

185) In max-heap, largest element is stored at root node. Where is the smallest element stored?

- **Right Node**

Leaf Node

- **Middle Node**
- **Left Node**

- **In average-case time analysis of Quick sort algorithm, the most balanced case for partition is when we divide the list of elements into _.**

Equal no. of pieces as of input elements

Single piece exactly

Two nearly equal pieces

Three nearly equal pieces

- **Which of the following is calculated with Big O notation?**

Medium bounds

b. Upper bounds **Page - 25**

- **Lower bounds**
- **Both upper and lower bounds**

187) Edit distance algorithm based on _____ strategy

- **Greedy**

b. Dynamic Programming **Page - 81**

Divide and Conquer

Searching

- **In Heapsort Algorithm, total time taken by heapify procedure is _____**

a. O (log n) **Page-43**

- **O (log2 n)**
- **O (n log n)**
- **O (n2 log n)**

193) Al-Khwarizmi was a/an _____

- **Artist**

b. Mathematician **P-7**

Astronomer

Khalifah

194) When matrix A of 5x3is multiply with metric B of 3x4 then the number of multiplication required is: Not found exactly

15

12

36

60Not Found exactly but as per formula at page 84,

24. Pseudo code of algorithms are to be read by _____.

a. People Page -12

RAM

Computer

Compiler

195) The sieve technique is a special case, where the number of sub-problems is Just

_____

a. 1 P-34

b. 2

c. 3

d. 4

26. When a recursive algorithm revisits the same problem over and over again, we say that the optimization problem has _____ sub-problems.

a. Overlapping – Google Search

b. Over costing

c. Optimized

d. Three

27. Sieve technique is very important special case of Divide-and-Conquer strategy.

a. True P-34

b. False

28. In order to say anything meaningful about our algorithms, it will be important for us to settle on a _____.

a. Java Program

b. C++ Program

c. Pseudo program

d. Mathematical model of computation P-10

29. Merge sort is based on _____.

a. Brute-force

b. Plan-sweep

c. Axis-sweep

d. Divide and Conquer P-27

30. What time does Merge Sort algorithm take in order to sort an array of 'n' numbers?

a. (n)

b. (log n)

c. (n^2)

d. (n log n) Google Search

31. In Heap Sort algorithm, the first step is to _____. a. Call Build-Heap

a. procedure Page - 46

b. Sort the array in descending order

c. Call Heapify procedure

d. Find the number of input elements

32. The definition of theta-notation relies on proving _____ asymptotic bound.

a. One

b. Lower

c. Upper

b. d. Both lower & upper Page - 25

33. In merge sort algorithm, to merge two lists of size n/2 to a list of size n, takes _____ time.

a. Theta (n) Page - 32

b. Theta log(n)

c. Theta log2(n)

d. Theta n log(n)

34. We can make _____ recursive calls in Fibonacci Sequence.

a. Infinite

b. Finite google

c. Only one

d. Zero

35. Following is NOT the application of Edit Distance problem.

a. Speech recognition

b. Spelling Correction

c. Ascending Sort Page - 76

d. Computational Molecular Biology

36. **In plane sweep approach, a vertical line is swept across the 2d-plane and structure is used for holding the maximal points lying to the left of the sweep line.**

    a. **Array**

    b. **Queue**

    c. **Stack** **Page - 18**

    d. **Tree**

37. **When a heapify procedure is applied to the root node to restore the heap, then at each level, the comparison performed takes time:**

    a. **It will take (log n).**

    b. **It can not be predicted**

    c. **It will take O (1).** **Page - 43**

    d. **Time will vary according to the nature of input data.**

38. **_____ time is the maximum running time over all legal inputs.**

    a. **Worst-case** **Page - 13**

    b. **Average-case**

    c. **Best-case**

    d. **Good-case**

39. **Efficient algorithm requires less computational...**

    a. **Memory**

    b. **Running Time**

    c. **Memory and Running Time** **Page - 9**

    d. **Energy**

40. For average-case time analysis of Quick sort algorithm, Pivot selection is on average basis from _____

    a. half of the input values

    b. all possible random values Page - 50

    c. Pivot is input separately

    d. values greater than 5

41. Selection algorithm takes theta _____

    a. (n2)

    b. (n) Page - 37

    c. log(n)

    d. n log(n)

42. Recurrence can be described in terms of a tree.

a. Yes Page - 31

b. No

43. Time complexity of Dynamic Programming based algorithm for computing the minimum cost of Chain Matrix Multiplication is _____

a. Log n

b. n

c. n^2 (n square)

d. n^3 (n cube) Page -90

44. The Iteration method is used for _____

a. Comparing sorting algorithms only

b. Solving Recurrence relations Page 31

c. Merging elements in Merge sort

d. Dividing elements in Merge sort

45. In 3-Dimensional space, a point P has _____ coordinate(s).

a. (X, Y)

b. (X, 0)

c. (0, Y)

d. (X,Y, Z)

46. Chain matrix multiplication problem can be solved through _____ strategy.

a. Dynamic programming Page - 85

b. Greedy

c. Divide and conquer

d. Sorting

47. Merge sort have running time....running time of Heap sort. Not found exactly

a. Greater than

b. Less than Google

c. Equal to

d. Different than

48. Median is not useful measure of central tendency of given input set especially when the distribution of values is highly skewed.

a. True

b. False Page – 34

49. We do not need to mathematically prove that for comparison-based sorting algorithms always takes Omega nlog (n) time.

a. True Google & VU Tech (pg 46 not very clear)

b. False

50. The Omega-notation allows us to state only the asymptotic _____ bounds.

a. Middle

b. Lower Page 25

c. Upper

d. Both lower & upper

51. Both lower & upperSorting can be in _____

a. Increasing order only

b. Decreasing order only

Both Increasing and Decreasing order – (My opinion)

c. Random order

52. Radix sort performs sorting the numbers _____ digit (s) at a time.

a. One Page - 71

b. Two

c. Three

d. All

53. Quicksort is a/an _____ and _____ sorting algorithm.

a. Not in place, not stable one

b. In place , not stable one Page - 54

c. In place , stable one

d. Not in place , stable one

54. Consider three matrices X,Y,Z of dimensions 1x2, 2x3,3x4 respectively. The number of multiplications of (XY) Z is:

a. 18 As per lecture slides

b. 32

c. 24

d. 30

55. In Fibonacci Sequence, unnecessary repetitions do not exist at all.

a. True

b. False Page – 74

56. It is not a Fibonacci sequence . 1,1,1,2,3,5,8,13,21,34,55,…..

a. True Page - 73

b. False

57. Heap sort is a/ an _____ and _____ sorting algorithem.

a. Not in place, not stable one

b. In place , not stable one Page - 54

c. In place , stable one

d. Not in place , stable one

58. Identify the True Statement

a. The knapsack problem does not belong to the domain of optimization problems.

b. The knapsack problem belongs to the domain of optimization problems. Page - 91

c. The Knapsack problem cannot be solved by using dynamic programming

d. The knapsack problem is optimally solved by using brute force algorithm.

59. In Dynamic Programming, our approach is to _____

a. Develop the solution in a top-down fashion

b. Express the problem non-recursively

c. Build the solution in a bottom-up fashion Page - 75

d. Input several sub-problems simultaneously

60. Counting sort is suitable to sort the elements in range 1 to K;

a. K is large

b. K is small Page - 57

c. K may be large or small

d. None

61. We can multiply two matrices A and B only when they are compatible which means

a. Number of columns in A must be equal to number of rows in B.

it seems Correct as per page 84

(https://www.varsitytutors.com/hotmath/hotmath_help/topics/matrix-multiplication#:~:text=You%20can%20only%20multiply%20two,an%20m%C3%97p%20mat rix.)

b. Number of rows and columns do not matter

c. Number of columns in A must be equal to number of columns in B

d. Number of rows in A must be equal to number of rows in B

62. Matrix multiplication is a (n) _____ operation.

a. Commutative

b.
c. b. Associative Page 85

c. Neither commutative nor associative

d. Commutative but not associative

63. In Dynamic Programming approach, solution is modified / changed

a. Always once

b. At each stage google and wikipedia

c. Only for specific problems

d. At 4th stage only

64. In Knapsack problem, the goal is to put items in the Knapsack such that the value of the items is _____ subject to weight limit of knapsack.

a. Minimized

b. Decreased

c. Maximized Page - 91

d. None of the given options

65. An in-place sorting algorithm is one that _____ uses additional array for storage.

a. Always

b. Permanently

c. Does not Page - 54

d. Sometime

66. Memoization is a part of Dynamic Programming Strategy.

a. True Page - 74

b. False

67. If matrix A of dimension 2x4 is multiply with matrix B of dimension 4x3, then the dimension of resultant matrix is Not found exactly

a. 2x4

b. 4x3

c. 3x4

d. 2x3 It seems correct as per second last Para of page 84

68. In Dynamic Programming approach, we do not store the solution to each sub-problem in case if it reappears.

a. True

b. False Page - 75

69. Dynamic Programming is a problem-solving approach in which___

a. Problem is solved in Zero time

b. Solution is developed only at final stage

c. Both are correct

d. Both are incorrect google

70. In Fibonacci sequence, each term is calculated by____ previous__ terms.

a. Subtracting, Two

b. Adding, Three

c. Adding, Two Page - 73

d. Multiplying, Two

71. Selection sort is not an in-place sorting algorithm.

a. True Page - 54

b. False

72. If there are θ (n^2) entries in edit distance matrix then the total running time is:

a. θ (n)

b. θ (1)

c. θ (n2) Page – 84

d. θ (n logn)

73. **The only way to convert a string of i characters into the empty string is with i deletions, represented as**

    a. **E(0.j) =j**

    b. **E(i.j) = 1**

    c. **E(0.i) = j**

    d. **E (i.0)=I** **Page - 78**

74. **Dynamic programming formulation of the matrix chain multiplication problem will store the solutions of each sub problem in an**

    a. **Array**

    b. **Table** **Page - 86**

    c. **Variable**

    d. **class**

75. We can use the optimal substructure property to devise a formulation of the edit distance problem.

    a. Selective

    b. Optimum

    c. Iterative

    d. Recursive Page - 78

76. Sorting is performed on the basis of _____.

    a. Computational resources

    b. Asymptotic notation

    c. Summation

    d. Some key value of attribute page- 39

77. In Heap Sort algorithm, we call Build-heap procedure _____.

    a. Only once page 46

    b. Twice

    c. Thrice

    d. As many times as we need

78. Radix sort is not a non-comparative integer sorting algorithm.

    a. True Google Search

    b. False

79. In the statement “output P[1].x, P[1].y”, the number of times elements of P are accessed is _____.

    a. 1

    b. 2 page 14

c. 3

d. 4

80. The main purpose of mathematical analysis is measuring the _____ required by the algorithm.

a. Space

b. Execution time P-13

c. Inputs & outputs

d. Execution time and memory

81. _____ provides us more accurate result when input values are not closer with each other

a. Average

Median (p-34)

c. Mode

d. Mean

82. The process of _____ ends when you are left with such tiny pieces remaining that it is trivial to solve them.

a. Brute-force

b. Plan-sweep

c. Divide and Conquer P-27

d. Axis-sweep

83. _____ overcomes the limitations of _____ by working as per positional notations of numbers.

a. Counting sort, Radix sort

b. Radix sort, Counting sort P-71

84. Memorization is a part of Dynamic Programming strategy.

a. True P-74

b. False

85. Rank of an element can be defined as _____.

a. One minus the number of elements that are smaller

b. Two plus the number of elements that are greater

One plus the number of elements that are smaller P-34

c. Two minus the number of elements that are smaller

86. If the time complexity of an algorithm is given by O (1), then its time complexity would be

a. Polynomial

b. Exponential

c. Constant - Wikipedia

d. Average

87. Quick sort is a recursive algorithm.

a. True Wikipedia ; Google

b. False

88. The asymptotic growth of n(n+1)/2 is:

O(n2) As the n^2 term has the largest contribution, the Big-O complexity is O(n^2)

a. O(n)

b. O(n+2)

c. O(n log n)

89. Approach of solving geometric problems by sweeping a line across the plane is called _____ sweep.

a. Line

b. Plane Page 18

c. Cube

d. Box

90. As per algorithm of Dynamic Programing, we need to store

a. First sub-problem only

b. Best solution only

c. Intermediate sub-problems Pg:75

d. Final solution only

91. In Sieve technique, we solve the problem

a. In recursive manner Pg:34

b. Non recursively

c. Using Merge Sort algorithm

d. Using Brute force technique

92. One of the limitation in 0/1 knapsack is that an item can either be _____ in the bag or not.

a. Use

b. Put Pg:91

c. Move

d. Store

93. Which one is not passed as parameter in Quick sort algorithm?

a. End of the array

b. Middle of the array

c. Array (containing input elements) Google

d. Start of the array

94. In the analysis of Selection algorithm, we get the convergent _____

a. Harmonic

b. Linear

c. Arithmetic

d. Geometric Pg:37

95. A Random Access Machine (RAM)is an idealized machine withrandom access memory.

a. Infinite large Pg:10

b. 512 MB

c. 256 MB

d. 2 GBs

96. While analyzing Selection algorithm, we make a number of passes, in fact it could be as many as

a. n(n+1)

b. log(n) Pg:37

c. n/3

d. n/4

97. In Random Access Machine (RAM), instructions are executed in

a. Parallel

b. Batch

c. One by One Pg:10

d. Multiple times

98. In selection problem, the rank of an element will be its _____ position

a. First

b. final Pg:34

c. Second last

d. Last

99. The worst-case running time of Merge sort is _____ in order to sort an array of n elements.

a. O(log n)

b. O(n)

c. O(n log n) page 40 and google

d. O(n)

100. f(n) and g(n) are asymptotically equivalent. This means that they have essentially the same _____.

a. Results

b. Variables

c. Size

d. Growth rates P:23

101. An algorithm is a mathematical entity. Which is independent of _____.

a. Programming language

b. Machine and Programming language

c. Compiler and Programming language

d. Programming language Compiler and Machine P: 07

102. In Quick sort algorithm, Pivots form ___

a. Stack

b. Queue

c. Binary Search Tree P:49

d. Graph

103. Counting sort is suitable for sorting the elements within range 1 to P. where

a. P is large

b. P is small P-57

c. P is very large

d. P is undetermined

104. In asymptotical analysis of n'(5 2)-3, as n becomes large, the dominant (fastest growing) term is some constant times

a. n_1

b. n

c. n+1

d. n*n P-23

105. ___ Items are not allowed in the 0/1 knapsack. a. Lighter

b. Fractional P-91

c. Whole

d. Weighty

106. Fibonacci Sequence was named on _____, a famous mathematician in 12th Century.

a. Fred Brooks

b. Grady Booch

c. Leonardo Pisano P-73

d. Edgar F. Codd

107. In Heap Sort algorithm, we build _____ for ascending sort.

a. Max heap P-41

b. Min heap

108. Bubble sort is not an in-place sorting algorithm.

a. True

b. False P-54

109. In partition algorithm, the subarray _____ has elements which are greater than pivot element x.

a. A[p...r]

b. A[p...q-1]

c. A[q]

d. A[q+1..r] P-46

110. In Heap Sort algorithm, if heap property is violated

a. We call Build heap procedure P-43

b. We call Heapify procedure

c. We ignore

d. Heap property can never be violated

111. _____ is not a characteristic of Random Access Machine.

a. Single-Processor P-10

b. Assigning a value to a variable

c. Locality of reference

d. Executing an arithmetic instruction

112. The only way to convert an empty string into a sting of j characters is by doing j insertions, represented as _____

a. E(i,j) = 1

b. E(I,0) = I

c. E(0,j) = j page 78

d. E(1,j)= j

113. In Selection problem, the Sieve technique works in _____.

a. Non-recursive manner

b. Constant time

c. Phases page 34

d. One complete go

114. Algorithm is a sequence of computational steps that ---- the input into output.

a. Merge

b. Assign

c. Transform page 7

d. Integrate

115. If pj dominates pi and pi dominates ph then pj also dominates ph, it means dominance relation is

a. Transitive page 18

b. Non Transitive

c. Equation

d. Symbolic

116. To find maximal points in brute-force algorithm each point of the space is compared against _____ of that space.

a. One other point

b. All other points page 11

c. Few other points

d. Most of the other points

117. In the following code the statement “cout<<j;”executes ---- ----- times. for (j=1; j<=5; j = j+2)

```
cout<<j;
```

a. 5 times

b. 2 times

c. 3 times

d. 0 times

118. In merge sort algorithm, we split the array around the _____ index q. a. Entring

e. b. Mid page 17

c. Exiting

d. Summing

119. In Selection problem, the Sieve technique _____.

a. Add some more input items each time

b. Do not work recursively

c. Do not uses Divide and Conquer approach

f. Eliminates undesired data items each time

120. Consider three matrices X, Y, Z of dimensions 1 x 2, 2 x 3, 3 x 4 respectively. The number of multiplications of X(YZ) is .

a. 16

b. 32

c. 26

g. d. 32 page 84

121. In Heap Sort algorithm, the total running time for Heapify procedure is _____

a. Theta (log n)

b. Order (log n)

c. Omega (log n)

h. O(1) i.e. Constant time

122. The sieve technique works where we have to find_____ items(s) from a large input.

a. Single page 34

Two

b. Three

c. Similar

123. In Dynamic Programming based solution of Knapsack Problem, if we decide to take an object i , then we gain_____

a. W(Total Weight of Knapsack)

b. V (Total Value of all items)

c. vi (Value of object i) page 93

d. Nome of the given option

124. While Sorting, the order domain means for any two input elements x and y _____ satisfies only.

a. x < y page 39

b. x > y

c. x = y

d. All of the above

125. For solving Selection problem, we introduced Sieve technique due to _____

a. Using Decrease and Conquer strategy page 34

b. Avoiding to sort all input data

c. Eliminating Rank of an element

d. Using Brute-force approach

126. _____ is one of the few problems, where provable lower bounds exist on how fast we can sort.

a. Searching

b. Sorting page 38

c. Both Searching & sorting

d. Growing

127. In plane sweep approach, a vertical line is swept across the 2d-plane from_____.

a. Right to Left

b. Left to Right page 18

c. Top to Bottom

d. Bottom to top

128. In generating Fibonacci sequence, we can avoid unnecessary repetitions by _____ process.

a. Tokenization

b. Memorization page 43

c. Randomization

d. Memorization

129. For _____ values of n, any algorithm is fast enough.

a. Small page 14

b. Medium

c. Large

d. Infinity

130. Dynamic programming comprises of _____.

a. Recursion only

b. Repetition only

c. Recursion with Repetition

d. No Repetition but Recursion page 75

131. The function f(n)=n(logn+1)/2 is asymptotically equalient t nlog n :Here Lower Bound means function f(n) grows asymptotically at __ as fast as nlog n.

a. Least page 23

b. Normal

c. Most

d. At

132. Counting sort has time complexity.

a. O(n+k)

b. O(n) page 58

c. O(k)

d. O(nlogn)

133. Due to left complete nature of binary tree, the heap can be stored in

a. Array page 40

b. Structures

c. Link List

d. Stack

134. Single item from a larger set of _____.

a. Constant

b. Pointers

c. Phases

d. n items page 34

135. In the clique cover problem, for two vertices to be in the same group, they must be _____ each other.

a. Apart from

b. Far from

c. Near to

d. Adjacent to page 76

136. How much time merge sort takes for an array of numbers?

a. T(n^2)

b. T(n)

c. T(log n)

d. T(n log n) page 40

137. In in-place sorting algorithm is one that uses arrays for storage.

a. No additional array page 54

b. An additional array

c. Both of above may be true according to algorithm

d. More than 3 arrays of one dimension

138. Brute-force algorithm for 2D-Maxima is operated by comparing _____ pairs of points.

a. Two

b. Some

c. Most

d. All page 18

139. While Sorting, the ordered domain means for any two input elements x and y ____ satisfies only.

a. x > y

b. x < y

c. x = y

d. All of the above page 38

140. Quick sort is.

a. Stable & in place

b. Not stable but in place page 54

c. Stable but not in place

d. Some time stable & some times in place

141. Which may be a stable sort?

a. Merger

b. Insertion

c. Both above page 54

d. None of the above

142. For the Sieve Technique we take time.

a. T(nk) page 34

b. IT(n / 3)

c. n^2

d. n/3

143. Continuation sort is suitable to sort the elements in range 1 to k.

a. K is Large

b. K is not known

c. K may be small or large

d. K is small page 54

144. Asymptotic growth rate of the function is taken over _____ case running time. .

a. Best

b. Worst page 14

c. Average

d. Normal

145. The sieve technique is a special case, where the number of sub problems is just.

a. 5

b. Many

c. 1 page 34

d. Few

146. In Quick sort, we don't have the control over the sizes of recursive calls.

a. True page 49

b. False

c. Less information to decide

d. Ether true or false

147. Before sweeping a vertical line in plane sweep approach, in start sorting of the points is done in increasing order of their _____ coordinates. .

a. X page 18

b. Y

c. Z

d. X , Y

148. Random access machine or RAM is a/an.

a. Machine build by Al-Khwarizmi

b. Mechanical machine

c. Mathematical model page 10

d. Electronics machine

149. The Huffman codes provide a method of encoding data inefficiently when coded using ASCII standard.

a. True

b. False page 99

150. A heap is a left-complete binary tree that confirms to the _____.

a. increasing order only

b. decreasing order only

c. heap order page 40

d. log n order

151. If we associate (x, y) integers pair to cars where x is the speed of the car and y is the negation of the price. High y value for a car means a _____ car.

a. Fast

b. Slow

c. Expensive

Cheap

152. Which one of the following sorting algorithms is the fastest?

a. Merge sort

Quick sort

b. Insertion sort

c. Heap sort

153. Quick sort algorithm divide the entire array into _____ sub arrays.

2

a. 3

b. 4

c. 5

154. In brute force algorithm, we measure running time T(n) based on _____.

a. Average-case time and best-case time

b. Worst-case time and average-case time page 46

c. Worst-case time and best-case time

d. Best-case time and staring-case time

155. For 2D Maxima problem. Plane Sweep algorithm first of all _____.

a. Sorts all points

b. Delete some points

c. Output the elements

d. Pushes all points on stack

156. There are _____ entries in the Edit Distance Matrix

a. ɵ (n)

b. b. Θ (n2) page 84

c. ɵ (n+2)

d. ɵ (n + 100)

157. Which symbol is used for Omega notation?

a. (O)

b. (ɵ)

c. (Ω)

c. (@)

158. Selection sort is a _____ sorting algorithm

d. a. In-place page 54

b. Not In-Place

c. Stable

d. in-partition

159. In Dynamic Programming based solution of knapsack problem, to compute entries of 'V', we will imply a(n) _____ approach.

a. Subjective

b. Inductive

c. Brute Force

d. Combination

160. We do not need to prove comparison-based sorting algorithms by mathematically. It always takes _____ time.

a. Big Oh nlog(n)

b. Omega nlog(n)

c. Omega n(n^2)

d. Theta nlog(n)

161. Merge sort is a/an _____ and _____ sorting algorithm

a. Not in-place, not stable one

b. In-place, not stable one

c. In-place, stable one

d. Not in-place, stable one page 54

162. Cubic function will _____ a quadratic function.

a. Prove

b. be equal to

c. overtake Page 25

d. find

163. Insertion sort is a _____ sorting algorithm

a. Unstable

b. In-place Page 54

c. Not In-Place

d. in-partition

164. To check whether a function grows faster or slower than the other function, we use some asymptotic notations, which is _____.

a. **Big-oh notation**

b. **Theta notation**

c. **Omega notation**

d. **All of the given**

165. Asymptotic growth of 8n^2 + 2n – 3 is:

a. **Θ(n^2 + n)**

b. **Θ (n^2)** page 14

c. **Θ(8n^2)**

d. **Θ(8n^2 + 2n)**

166. In the analysis of algorithms, _____ plays an important role.

a. **text analysis**

time

b. **growth rate**

c. **money**

167. In inductive approach of knapsack problem, we consider 2 cases, _____

Or _____.

a. Median, Mode

b. Recursive, Iterative

c. Leave object, Take object page 93

d. Sequentially. Parallel

168. Random Access Machine (RAM) can execute _____ instructions

a. only logical

b. parallel

c. only arithmetic

d. logical and arithmetic

169. Using _____ algorithm, efficiency is not given much importance

a. Greedy

b. Merge sort

c. Processing as there is no algorithm by this name

d. Brute Force

170. Bubble sort takes theta _____ in the worst case

a. (n2) page 39

b. (n)

c. log(n)

d. nlog(n)

171. If matrix A of dimension p × q is multiply with matrix B of dimension q × r, then dimension of resultant matrix is:

a. p × q

b. p × r page 84

c. q × r

d. r × p

172. Dynamic Programing algorithms often use some kind of _____ to store the results of intermediate sub-problems

a. table (Page 75)

b. variable

c. stack

d. loop

173. _____ is in-place sorting algorithm.

a. Bubble sort (Page 54)

b. Merge sort

c. Linear search

d. Binary Search

174. Which one of the following problems can be solved using dynamic problem?

a. Bubble sort problem

b. Matrix chain multiplication problem page 85

c. Greedy search problem

d. Fractional knapsack problem

175. In chain matrix multiplication, solutions of the sub-problems are stored in a _____.

a. Array

b. Table page 86

c. Tree

d. Link list

176. What is the average running time of a quick sort algorithm?

a. O(n^2)

b. O(n)

c. O(n log n) (Page 49)

d. O(log n)

177. Sorting Algorithms having O _____ running time are considered to be slow ones.

a. (n)

b. (n^2) (Page 39)

c. (nlog(n))

d. (log(n))

178. While solving Selection problem, in Sieve technique we partition input data _____

a. In increasing order

b. In decreasing order

c. According to Pivot

d. Randomly

179. _____ is the process of avoiding unnecessary repetitions by writing down the results of recursive calls and looking them up again if we need them later.

a. Loop

b. Memoization page 74

c. Recursion

d. Function

180. In average-case time the probability of seeing input is denoted by _____.

a. p{I}

b. p[I]

c. p<i>

d. p(i) page 13

181. While applying the Sieve technique to selection sort, how to choose a pivot element.

a. Through mean

b. Linear

c. Randomly page 35

d. Sequentially

182. Number of _____ of the pseudo code are counted to measure the running time.

a. Inputs

b. Outputs

c. Steps page 13

d. Pages

183. Developing a dynamic programming algorithm generally involves _____ separate steps.

a. One

b. Two page 75

c. Three

d. Four

184. 8n^2+2n+3 will exceed c28(n), no matter how large we make _____.

a. n

b. 2n

c. c2 page 25

d. this quadratic equation

185. The running time of quick sort algorithm_____.

a. Is impossible to compute

b. Has nothing to do with pivot selection

c. Is Random upon each execution

d. Greatly influenced by the selection of pivot page 49

186. _____ involves breaking up the problem into sub problems whose solutions can be combined to solve the global problem.

a. Complexity Theory

b. Dynamic programming solution

c. Divide and Conquer Strategy page 34

d. Greedy Algorithms

187. In _____ we have to find rank of an element from given input.

a. Merge sort algorithm

b. Selection problem page 34

c. Brute force technique

d. Plane Sweep algorithm

188. How many steps are involved to design the dynamic programming strategy?

a. 2

b. 3

c. 1

d. 4 page 92

189. In Bucket sort, if there are duplicates then each bin can be replaced by a a. Stack

b. Linked list page 69

c. Hash table

d. Heap

190. In merge sort algorithm, we split the array _____ to find index q.

a. from start

b. midway page 28

c. from end

d. both from start or end

191. Find the maximum value of the items which can carry using knapsack Knapsack weight capacity = 50.

Item Weight Value

11070,22020,33080,470 200

a. 280

b. 100

c. 90

200

192. In 2-d maxima problem a point p is said to be dominated by point q if _____.

a. p.x <= q.x

b. p.x <= q.x and p.y <= q.y page 17

c. p.y <= q.y

d. p.x >= q.x and p.y >=q.y

193. Sorting can be in _____.

a. Increasing order only

b. Decreasing order only

c. Both increasing and decreasing order

d. Random order

194. Recurrence can be described in terms of _____.

a. Array

b. Linear

c. Tree page 31

d. Graph

195. The brute-force algorithm for 2D-Maxima runs in order O(__) time.

a. n

b. n(log n)

c. n*n page 18

d. n3

196. In plane sweep approach of solving geometric problems, a _____ is swept across the plane.

a. Line page 18

b. Plane

c. Cube

d. Box

197. Which of the following is calculated with Big Omega notation?

a. Medium bounds

b. Upper bounds

c. Lower bounds Page - 25

d. Both upper and lower bounds

198. _____ is always based on divide and conquer strategy.

a. Bubble sort

b. Selection sort

c. Pigeon sort

d. Quick sort page 46

199. If a matrix has three rows and two columns, then dimensions of matrix will be:

a. 3x2

2x3

b. 3x3

c. 2x2

200. Asymptotic notations are used to describe _____ of an algorithm.

a. Length

b. running time google

c. size

d. compile time

201. Catalan numbers are related the number of different _____ on 'n' nodes.

a. Arrays

b. linked lists

c. binary trees page 85

d. functions

202. Applying the sieve technique to selection problem, _____ element is picked from array.

a. Output

b. Total

c. Input

d. Pivot page 35

203. Dynamic Programming approach is usually useful in solving _____ problems.

a. Normal

b. Optimization google

c. Array

d. Loop

204. In recursive formulation of knapsack Problem: V [0, j] = _____ for j>=0

a. -1

b. 0 page 93

c. 1

d. 2

205. _____ is a linear time sorting algorithm.

a. Merge sort

b. Radix sort page 71

c. Quick sort

d. Bubble sort

206. Quick sort is one of the _____ sorting algorithm.

a. Fastest page 19

b. Slowest

c. Major

d. Average

207. The time assumed for each basic operation to execute on RAM model of computation is _____.

a. Infinite

b. Continuous

c. Constant page 10

d. Variable

208. In Sieve Technique, we know the item of interest.

a. True

b. False page 34

209. While analyzing algorithms, _____ and _____ usually considered difficult to calculate.

a. Finite, Infinite

b. Floor, ceiling google

c. Row, Column

d. Graph, Tree

210. While analysis of the brute-force maxima algorithm, an array sorted in the reverse order is the type of _____ case input.

a. Best

b. Worst page 14

c. Somewhat bad

d. Average

211. _____ is not useful measure of central tendency of given input set especially when the distribution of values is highly skewed.

a. Mean

b. Mode

c. Average

d. Median page 34

212. In asymptotical analysis of n(n-3) and 4n*n, as n becomes large, the dominant (fastest growing) term is some constant times _____.

a. n+1

b. n-1

c. n

d. n*n page 23

213. In addition to passing in the array itself to Merge Sort algorithm, we will pass in other arguments which are indices.

a. Two P-38

b. Three

c. Four

d. Five

214. In 2d-maximal problem, a point is said to be if it is not dominated by any other point in that space.

a. Member

b. Minimal

c. Maximal P-11

d. Joint

215. Counting sort assumes that the numbers to be sorted are in the range _____.

a. K to n where n is large

b. 1 to k where k is small (P-57)

c. K to n where k is small

d. k to n where n is small

216. Insertion sort is an efficient algorithm for sorting a _____ number of elements

a. Large P-39

b. Small

c. Extra large

d. Medium

217. If the indices passed to merge sort algorithm are _____ then this means that there is only one element to sort.

a. Small page 28

b. Large

c. Equal

d. Not Equal

218. In Knapsack Problem, each item must be entirely accepted or rejected, is called _____ problem.

a. Fractional

b. 0-1 P-92

c. Linear

d. Optimal

219. If the time complexity of an algorithm is O(n). then it is called _____ time complexity.

a. Linear Wikipedia

b. Constant

c. Average

d. Exponential

220. In the case of _____ analysis does not depend upon on the distribution of input.

a. Merge sort

b. Quick sort P-50

c. Insertion sort

d. Heap sort

221. We can use the _____ Property to devise a recursive formulation of the edit distance problem.

a. Small substructure

b. Algorithmic

c. Real

d. Optimal substructure page 78

222. The following sequence is called _____

1,2,3,5,8,13,21,34,55,…..

a. Optimize sequence

b. Fibonacci sequence page 73

c. Optimal sequence

d. Overlapping sequence

223. Which one sorting algorithm is best suited to sort an array of 2 million elements?

a. Bubble sort

b. Insert sort

c. Merge sort

d. Quick sort

e. Ridx Sort page 71

224. We can improve the performance of quick sort if we could be able to _,_____.

a. Skip input elements somehow

b. Select two or more pivots page 34

c. Skip any sub-array completely

d. Eliminate recursive calls

225. The problem with the brute-force algorithm is that is uses _____ in pruning out de

a. Worst-case time

b. No intelligence page 18

c. Outside looping

d. Artificial intelligence

226. In chain matrix multiplication, the order of the matrices _____.

a. Can be changed

b. Can not be changed page 85

c. is equal

d. is reverse

227. In quick sort algorithm, we choose pivot_____.

a. Always the smallest element

b. Greater than 5

c. Randomly page 35

d. Less than 5

228. In Heap Sort algorithm. Heapify procedure is _____ in nature.

a. Recursive

b. Non-Recursive page 43

c. Fast

d. Slow

229. When matrix A of 5x 3 is multiplied with matrix B of 3 x 4 then the number of multiplications required will be _____.

a. 15

b. 12

c. 36

d.

60

230. An algorithm is said to be correct if for every _____ instance, it halts with the correct _____.

a. Input, Output page 13

b. Design, Analysis

c. Value, Key

d. Key, Analysis

231. In chain matrix multiplication, table is filled _____ to find the multiplication of matrix.

a. row wise

b. column wise

c. diagonally

d. bottom-to-up page 86

232. If we have an equation 8n2+7f*n + 5f + 6 then is large, _____ term will be muchxxxxxxxthe n term and will dominate the running time.

a. f g (n)

b. g (n) * 2

c. n * 2 page 23

d. f (n)

233. For quick sort algorithm. Partitioning takes theta _____.

a. (n)

b. log(n)

c. n log (n)

n2log (n)

234. In Heap Sort algorithm, the maximum levels an element can move upward is

_____

a. Theta (log n) page 43

b. Big-ch (log n)

c. Omega (log n)

d. 0 (1) i.e. Constant time

235. _____ programming is essentially recursion without repetition.

a. Fast

b. Dynamic page 75

c. Array

d. n (log n)

236. There are no hard formal rules to the syntax of the _____ code.

a. Basic

b. Programming

c. Pseudo

d. Assembly

237. In Heap Sort algorithm, to remove the maximum element every time.

a. We call Build-Heap procedure

b. Heap Sort algorithm terminates without result

c. We call heapify procedure

d. Nothing happens

238. Which process is used for avoiding unnecessary repetitions and looking them up again if we need them later.

a. Greedy Approach

b. Memoization page 74

c. Divide and conquer

d. Recursion

239. The worst-case running time of Quick sort is _____ in order to sort an array of n element.

a. O(n log n) page 49

b. O(n)

c. $O(n^2)$

d. O(log n)

240. Boolean operation is a _____ operation on an idealized RAM model of computation.

a. Starting

b. Basic page 10

c. Advance

d. Normal

241. In chain matrix multiplication, if there are n items, there are _____ ways in which outer most pair of parentheses can placed.

a. n^2

b. 2n

c. n+1

d. n-1 page 85

242. The number of nodes in a complete binary tree of height h is: a. 2 * (h+1) - 1

b. 2^(h+1) – 1 page 40

c. 2 * (h+1)

d. ((h+1)^2) – 1

Cs502 Solved MCQS for Final Term

July 2013

Including Current Quizzes

Question #

Bellman Ford Algorithm applies relaxations to every _____

Vertex of …

Edge of Graph and repeats this V-1 times (p159)

Question #

Floyd-Warshall Algorithm is _____

Based on dynamic programming (p161)

Question #

Running time of Floyd-Warshall Algorithm is _____

Θn^2

Θn^4

Θn^3 (p164)

3^n

Question #

Space used by Floyd-Warshall Algorithm is _____

Θn^2 (p164)

Θn^4

Θn^3

3^n

Question #

Huffman encoding Algorithm is a ____

Greedy Algorithm (p99)

Dynamic and Greedy

Question #

Graphs cannot be traversed by brute-force technique.

True (p117) False

Question #

Brute-force technique is always optimal for Graph traversal.

True **False (p117)**

Question #

For graph traversal, Breadth-first search strategy _____

is always recursive

Cannot be recursive

Cannot be non-recursive

Can be both recursive and non-recursive

Question #

Greedy algorithm can do very poorly for some problems.

True (p97) False

Question #

A greedy algorithm sometimes works well for optimization problems.

True (p97) False

Question #

A greedy algorithm does not work in phases.

True **False (p97)**

Question #

Greedy algorithm approach solves both 0/1 Knapsack and Fractional Knapsack problems.

True **False**

Question #

Dynamic Programming approach solves both 0/1 Knapsack and Fractional Knapsack problems.

True **False**

Question #

In _____ based solution of knapsack problem, we consider 2 cases. Leave object Or Take object.

Brute force

Dynamic programming (p93)

Question #

In brute force based solution of knapsack problem, we consider 2 cases. Leave object Or Take object.

True **False (p93)**

Question #

In DP based solution of knapsack problem, V[0, j] = _____ if j>=0

-1

0 (p93)

1
2

Question #

In DP based solution of knapsack problem, V[0, j] = 1 if
j>=0 True

False (p93)

Question #

In DP based solution of knapsack problem, to compute entries of V we will imply a/an _____ approach.

Subjective

Inductive (p93)

Brute force

Combination

Question #

In DP based solution of knapsack problem, if we decide to take an object i, then we gain a value of _____.

w i-1

vi-1
w i

vi (p93)

Question #

In Fractional Knapsack problem, one is allowed to take fraction of an item for _____

for fraction of the weight

for fraction of the value

Both, fraction of the weight and value (p109)

None

Question #

_____ approach is optimal for the fractional knapsack problem.

Divide and Conquer

Dynamic Programming

Greedy algorithm (p110)

Brute force

Question #

In ____ Knapsack Problem, limitation is that an item can either be put in the bag or not-fractional items are not allowed.

0 1 **0/1** Fractional

Question #

In Fractional Knapsack problem, the goal is to _____

minimize the value of items without exceeding the total weight limit of W

maximize the value of items without exceeding the total weight limit of W (p109)

minimize the value of items even if exceeding the total weight limit of W

maximize the value of items even if exceeding the total weight limit of W

Question #

In solving Fractional Knapsack problem, we do not need sorting.

True

False (p109)

Question #

Suppose that a graph G = (V,E) is implemented using adjacency lists. What is the complexity of a breadth-first traversal of G? Select correct option:

O(|V |^2)

O(|V | |E|)

O(|V |^2|E|)

O(|V | + |E|) (p116)

Question #

For traversing graphs, Breadth-first search can be visualized as a wave front propagating inwards towards root (or source) node.

True

False (p117)

Question #

Breadth-first search is not a traversal strategy for graphs.

True

False (p119)

Question #

Breadth-first search is not a popular algorithm technique used for traversing graphs.

True

False (p119)

Question #

Breadth-first search begins at a root node and inspects all the nodes except neighboring ones.
True

False (117)

Question #

Each time we traverse graph by Breath-first search algorithm, we count the distance from _____

Starting node (p117)

neighbors of the starting node

right most node

left most node

Question #

Suppose we are given a set S = {a1, a2, ... an} of n activities that are to be scheduled to use some resource. Each activity ai must be started at a given start time si and ends at a given finish time fi. We want to schedule the largest number of activities on the resource. It is an example of _____. Searching

Dynamic Programming

Activity selection (p105)

Divide and Conquer

Question #

In general, the Activity selection problem is to select a _____

minimum-size set of interfering activities

maximum-size set of mutually non-interfering activities (p105)

maximum-size set of interfering activities

minimum-size set of mutually non-interfering activities

Question #

In Activity selection (using Greedy approach), intuitively _____

Short activities are not preferable

There are always short activities as input

We do not like long activities (p105)

It does not matter about the length of activities

Question #

The activity schedule is a simple scheduling problem for which the ____ approach provides an optimal solution.

Dynamic Programming

Graph

Divide and Conquer

Greedy Algorithm (p105)

Question #

The activity scheduling is a simple scheduling problem for which the greedy algorithm approach provides a/an

_____solution.

Simple

Sub optimal

Optimal (p105)

Non optimal

Question #

A number of lectures are to be given in a single lecture hall. Optimum scheduling for this is an example of Activity selection.

True (p105)

False

Question #

In Activity scheduling algorithm, each activity is represented by a _____

Circle

Square

Triangle

Rectangle (p106)

Question #

In Activity scheduling algorithm, the width of a rectangle _____

Is always ignored

Directs towards recursion

Should be maximized

Indicates the duration of an activity (p106)

Question #

In Activity scheduling algorithm, the time is dominated by sorting of the activities by _____

start times

finish times (p106)

Question #

The complexity of Activity scheduling algorithm is _____

O (n)

O log n

O (n) log n (p106)

O n^2

Question #

In Activity scheduling algorithm, as base case if there are no activities then Greedy algorithm _____

cannot be optimized

is solved using Recursion

is transformed into Dynamic Programming

is trivially optimal (p109)

Question #

Bag is a _____

type of algorithm

data structure (p119)

program

compiler

Question #

For _____ problems, greedy approach always gets optimum.

Select correct option:

Two

Some (p97)

All

None of the

Question #

Fixed-length codes may not be efficient from the perspective of _____the total quantity of data.

Minimizing (p99)

Averaging

Maximizing

Summing

Question #

There are _____ nested loops in DP based algorithm for computing the minimum cost of chain matrix multiplication.

2

3 (p90)

4

5

Question #

Time complexity of DP based algorithm for computing the minimum cost of chain matrix Multiplication is ----.

log n n n^2 **n3 (p90)**

Question #

In greedy algorithm, at each phase, you take the_____ you can get right now, without regard for future consequences.

Worst Minimum Good **Best (p97)**

Question #

An optimization problem is one in which you want to find, not just a solution, but the _____ solution.

Simple Good **Best (p97)** Worst

Question #

In _____ algorithm, you hope that by choosing a local optimum at each step, you will end up at a global optimum.

Simple Non-Greedy. **Greedy (p97)** Brute force

Question #

The string |xyz|, if coded with ASCII code, the message length would be 24 bits.

True False

Question #

If the string |lmncde| is coded with ASCII code, the message length would be --- bits.

24 36 **48** 60

Question #

If there are n items, there are _____ possible combinations of the items.

---

2 n **2^n (p92)** 3^n

Question #

In Huffman encoding, for a given message string, the frequency of occurrence (relative probability) of each character in the message is determined last.

True **False (p100)**

Question #

In DP based solution of knapsack problem, to compute entries of V we will imply a/an _____ approach.

Subjective **Inductive (p93)** Brute force Combination

Question #

Inductive approach to compute entries of V is implied in _____ based solution of knapsack problem.

Brute force **Dynamic programming (p93)**

Question #

In brute force based solution of knapsack problem, we consider 2 cases, Leave object Or Take object.

True (p93) False

Question #

There are _____ nested loops in DP based algorithm for computing the minimum cost of chain matrix multiplication.

2

3

4

5 (not confirm page 90)

Question #

Those problems in which Greedy finds good, but not always best is called a greedy_____.

Algorithm

Solution

Heuristic (p97)

Result

Question #

The Huffman codes provide a method of _____ data efficiently.

Reading

Encoding (p99)

Decoding

Printing

Question #

Fixed-length codes are known for easy break up of a string into its individual characters.

True (p99) False

Question #

The Huffman codes provide a method of _____ data efficiently.

Reading **Encoding (p99)** Decoding Printing

Question #

In Huffman encoding, the _____is the number of occurrence of a character divided by the total characters in the message.

Counting

Parsing

Probability (p100)

Weight

Question #

In Huffman encoding, for a given message string, the frequency of occurrence (relative probability) of each _____in the message is done first.

Word Space Digit **Character (p100)**

Question #

The Huffman algorithm finds a (n) _____ solution.

Optimal (Reference) Non-optimal Exponential Polynomial

Question #

The Huffman algorithm finds an **exponential** solution True **False (Reference)**

Question #

The Huffman algorithm finds a **polynomial** solution True **False (Reference)**

Prefix Property:

The codewords assigned to characters by the Huffman algorithm have the property that no codeword is a prefix of any other:

Question #

Non-optimal or greedy algorithm for money change takes_____

O(k) (p99) O(kN) O(2k) O(N)

Question #

The Huffman codes provide a method of encoding data **inefficiently** when coded using ASCII standard.

True **False (p99)**

- The Huffman codes provide a method of encoding data **efficiently**.

Question #

The codeword assigned to characters by the Huffman algorithm have the property that no codeword is the **postfix** of any other. True **False**

Question #

The codeword assigned to characters by the Huffman algorithm have the property that no codeword is the **prefix** of any other. **True (p101)** False

Question #

Using ASCII standard the string “abacdaacacwe” will be encoded with _____ bits

Select correct option:

64

128

96 (p101 12x8=96)

120

Question #

Using ASCII standard the string abacdaacac will be encoded with _____ bits.

80 (p99) 160 320 100

- Consider the string " abacdaacac". if the string is coded with ASCII codes, the message length would **be**
- **10 × 8 = 80 bits.**

Question #

Using ASCII standard the string abacdaacac will be encoded with 160 bits. True **False (p99)**

Question #

Using ASCII standard the string abacdaacac will be encoded with 320 bits. True **False (p99)**

Question #

Using ASCII standard the string abacdaacac will be encoded with 100 bits. True **False (p99)**

Question #

Using ASCII standard the string abacdaacac will be encoded with 32 bytes. True **False (p99)**

Question #

The greedy part of the Huffman encoding algorithm is to first find two nodes with **smallest** frequency.

True (p100) False

Question #

The greedy part of the Huffman encoding algorithm is to first find two nodes with **character** frequency.

True **False (p100)**

Question #

The greedy part of the Huffman encoding algorithm is to first find two nodes with **larger** frequency.

True **False**

Question #

Huffman algorithm uses a greedy approach to generate an antefix code T that minimizes the expected length B (T) of the encoded string. True **False (p102)**

Question #

Huffman algorithm uses a greedy approach to generate a postfix code T that minimizes the expected length B (T) of the encoded string. True **False (p102)**

Question #

Huffman algorithm uses a greedy approach to generate a prefix code T that minimizes the expected length B (T) of the encoded string. **True (p102)** False

Question #

Huffman algorithm generates an optimum prefix code.

True (p102) False

Question #

A graph is said to acyclic if it contains _____

at least one cycle

exactly one cycle

---

always more than one cycle

no cycle (p116)

Question #

Graph is not a good way to model some sort of "connection" or "relationship" or "interaction" between pairs of objects taken from a set of objects.

True **False (p113)**

Question #

A graph is not connected if every vertex can reach every other vertex.

True **False (p116)**

Question #

Traversing a graph means visiting _____ in the graph.

no node

at least one

more than one node

every node (p119)

Question #

Graphs are important _____ model for many application problems.

Unsystematic

Mathematical (p113)

Haphazard

Unpredictable

Question #

In a digraph, the number of edges coming in of a vertex is **not** called the in-degree of that vertex.

True **False (p114)**

Question #

In a digraph, the number of edges coming out of a vertex is **not** called the out-degree of that vertex.

True **False** **(p114)**

Question #

For a digraph G = (V, E), Sum of in-degree(v) _____

Not equal to Sum of out-degree(v)

= Sum of out-degree(v) (p115)

< Sum of out-degree(v)
> Sum of out-degree(v)

Question #

Graphs cannot be represented by an adjacency matrix.

True **False** **(p116)**

Question #

A vertex a is not adjacent to vertex b if there is an edge from a to b.

True **False** **(p113)**

Question #

A **dense undirected** graph is:

Select correct option:

A graph in which E = O (V^2) (Reference)

A graph in which E = O (V)

A graph in which E = O (log V)

All items above may be used to characterize a dense undirected graph

Question #

What algorithm technique is used in the implementation of **Kruskal solution** for the MST?

Greedy Technique (p142)

Divide-and-Conquer Technique

Dynamic Programming Technique

The algorithm combines more than one of the above techniques

Question #

A digraph is strongly connected under what condition?

A digraph is strongly connected if for every pair of vertices u, v e V, u can reach v .

A digraph is strongly connected if for every pair of vertices u, v e V, u can reach v and vice versa.
(p135) A digraph is strongly connected if for at least one pair of vertex u, v e V, u can reach v and vice versa.

A digraph is strongly connected if at least one third pair of vertices u, v e V, u can reach v and vice versa.

Question #

The relationship between number of back edges and number of cycles in DFS is,

Both are equal

Back edges are half of cycles

Back edges are one quarter of cycles

There is no relationship between no. of edges and cycles (p131)

Question #

Back edge is?

(u, v) where v is an ancestor of u in the tree. (p128)

(u,v) where u is an ancestor of v in the tree.

(u, v) where v is an predecessor of u in the tree.

None of above

Question #

Forward edge is?

(u, v) where u is a proper descendent of v in the tree.

(u, v) where v is a proper descendent of u in the tree. (p129)

(u, v) where v is a proper ancestor of u in the tree.

(u, v) where u is a proper ancestor of v in the tree.

Question #

Edge (u, v) is a forward edge if

u is a proper descendant of v in the tree

v is a proper descendant of u in the tree

None of these

Question #

Cross edge is?

(u, v) where u and v are not ancestor of one another

(u, v) where u is ancestor of v and v is not descendent of u.

(u, v) where u and v are not ancestor or descendent of one another (p129)

(u, v) where u and v are either ancestor or descendent of one another.

Question #

If you find yourself in maze the better traversel approach will be?

Select correct option:

BFS

BFS and DFS both are valid (p119)

Level order

DFS

Question #

In digraph G=(V,E) ;G **has cycle** if and only if

Select correct option:

The DFS forest has forward edge.

The DFS forest has back edge (p131)

The DFS forest has both back and forward edge

BFS forest has forward edge

Question #

What is generally true of Adjacency List and Adjacency Matrix representations of graphs?

Select correct option:

Lists require less space than matrices but take longer to find the weight of an edge (v1,v2)

Lists require less space than matrices and they are faster to find the weight of an edge (v1, v2)

Lists require more space than matrices and they take longer to find the weight of an edge (v1, v2)

Lists require more space than matrices but are faster to find the weight of an edge (v1, v2)

Question #

There is relationship between number of **back edges** and **number of cycles** in DFS

Select correct option:

Both are equal.

Cycles are half of back edges.

Cycles are one fourth of back edges.

There is no relationship between back edges and number of cycle **(p131)**

Question #

You have an adjacency list for G, what is the time complexity to compute Graph transpose G^T.?

Select correct option:

(V+E) **(Reference)**

V.E

V

E

Question #

Although it requires more complicated data structures, **Prim's algorithm** for a minimum spanning tree **is better** than **Kruskal's** when the graph has a **large number** of vertices.

True False

Question #

Maximum number of vertices in a Directed Graph may be $|V^2|$

True False

Question #

Dijkstra's algorithm:

Has greedy approach to find all shortest paths

Has both greedy and dynamic approach to find all shortest paths

Has greedy approach to compute single source shortest paths to all other vertices (p154) Has both greedy and dynamic approach to compute single source shortest paths to all other vertices.

Question #

Dijkestra's single source shortest path algorithm works if all edges weights are non-negative and there are negative cost cycles. True **False**

Question #

Dijkestra s single source shortest path algorithm works if all edges weights are non-negative and there are no

negative cost cycles. **True (p159)** False

Question #

Dijkestra s single source shortest path algorithm works if all edges weights are negative and there are no negative cost cycles.

True **False**

Question #

Dijkstra's algorithm is operates by maintaining a subset of vertices.

True False

Question #

Floyd-Warshall algorithm is a dynamic programming algorithm; the genius of the algorithm is in the clever

recursive formulation of the shortest path problem. **True (p162)** False

Question #

Floyd-Warshall algorithm, as in the case with DP algorithms, we avoid recursive evaluation by generating a table for $d_{ij}^{(k)}$

True (p164) False

Question #

The term coloring came from the original application which was in **map drawing**. **True (p173)** False

Question #

The term "coloring" came from the original application which was in **architectural design**. True **False**

Question #

In the **clique cover** problem, for two vertices to be in the same group, they must be _____ each other.

Apart from Far from Near to **Adjacent to (p176)**

Question #

In the clique cover problem, for two vertices to be in the same group, they **must be adjacent** to each other.

True (p176) False

Question #

In the **clique cover** problem, for two vertices to be in the same group, they **must be apart** from each other.

True **False (p176)**

Question #

The difference between Prims algorithm and Dijkstra s algorithm is that Dijkstra s algorithm uses a different key.

True (p156) False

Question #

The difference between Prim s algorithm and Dijkstra s algorithm is that Dijkstra s algorithm uses a same key.

True **False (p156)**

Question #

You have an adjective list for G, what is the time complexity to computer graph transpose G^T.?

(V+E) (p138)

Question #

Given an adjacency list for G, it is possible to compute G^T in $\Theta(V + E)$ time.

It takes O(log V) to extract a vertex from the priority queue.

Question #

Overall time for Kruskal is $\Theta(E \log E) = \Theta(E \log V)$ if the graph is sparse. **True (p149)**

Question #

An optimization problem is one in which you want to find,

Not a solution

An algorithm

Good solution

The best solution (p97)

Question #

Although it requires more complicated data structures, Prim's algorithm for a minimum spanning tree is better than Kruskal's when the graph has a large number of vertices.

True False

Question #

If a graph has v vertices and e edges then to obtain a spanning tree we have to delete v edges.

v – e + 5 edges

v + e edges.

None of these

Question #

Bellman-Ford allows negative weights edges and negative cost cycles.

True **False (p159)**

- Bellman-Ford allows negative weights edges and no negative cost cycles.

Question #

We do sorting to,

keep elements in random positions

keep the algorithm run in linear order

keep the algorithm run in (log n) order

keep elements in increasing or decreasing order (p39)

Question #

After partitioning array in Quick sort, pivot is placed in a position such that

Values smaller than pivot are on left and larger than pivot are on right (Ref)

Values larger than pivot are on left and smaller than pivot are on right

Pivot is the first element of array

Pivot is the last element of array

Question #

Dynamic programming algorithms need to store the results of intermediate sub-problems.

True False

Question #

Which statement is true?

If a dynamic-programming problem satisfies the optimal-substructure property, then a locally optimal solution is globally optimal.

If a greedy choice property satisfies the optimal-substructure property, then a locally optimal solution is globally optimal.

Both of above None of above

Question #

What **general property** of the list indicates that the graph has an **isolated vertex**?

There is Null pointer at the end of list.

The Isolated vertex is not handled in list.

Only one value is entered in the list.

There is at least one null list.

Question #

Depth first search is **shortest path** algorithm that works on **un-weighted** graphs.

True **False (p153)**

- The **breadth-first-search** algorithm is a shortest-path algorithm that works on **un-weighted graphs.**

Question #

Which is true statement?

Select correct option:

Breadth first search is shortest path algorithm that works on un-weighted graphs (p153)
Depth first search is shortest path algorithm that works on un-weighted graphs.

Both of above are true.

None of above are true.

Question #

Shortest path problems can be solved efficiently by modeling the road map as a graph.

True False

Question #

Which is true statement in the following?

Kruskal algorithm is multiple source technique for finding MST.

Kruskal's algorithm is used to find minimum spanning tree of a graph, time complexity of this algorithm is O(EV) Both of above

Kruskal's algorithm (choose best non-cycle edge) is better than Prim's (choose best tree edge) when the graph has relatively few edges.

Question #

Kruskal's algorithm (choose best non-cycle edge) is better than Prim's (choose best tree edge) when the graph has relatively few edges.

True False

Question #

What is the time complexity to **extract a vertex** from the **priority queue** in **Prim's algorithm**?

Select correct option:

O (log E)

(V)

(V+E)

O (log V) (p152)

Question #

In Prim's algorithm, the additional information maintained by the algorithm is the **length of the shortest edge** from vertex v to points already in the tree.

a) True **b) False** c) unknown

Question #

Although it requires more complicated data structures, Prim's algorithm for a minimum spanning tree is better than Kruskal's when the graph has a **large number of vertices**.

a) True b) False c) unknown

Question #

Which statement is true?

► The running time of Bellman-Ford algorithm is T (VE)
► Both Dijkstra's algorithm and Bellman-Ford are based on performing **repeated relaxations**
► The 0-1 knapsack problem is hard to solve

Only i Only iii Both i and iii **All of these**

Question #

Which of the following arrays represent **descending (max) heaps**?

i. [10,7,7,2,4,6] ii. [10,7,6,2,4,7]

iii. [10,6,7,2,4,6] iv. [6,6,7,2,4,10]

Only ii Only iv Both ii and iv Both i and iii

Question #

Which of the following statement(s) is/are correct?

(a) O(n log n + n2) = O(n2). (b) O(n log n + n2) = O(n2 log 2n)

(c) O(c n2) = O(n2) where c is a constant. (d) O(c n2) = O(c) where c is a constant.

(e) O(c) = O(1) where c is a constant.

Only (a) & (e) Both (c) and (e)

Question #

Which of the shortest path algorithms would be most appropriate for finding paths in the graph with **negative edge weights** and **cycles**?

- Dijkstra's Algorithm
- Bellman-Ford Algorithm.
- Floyd Warshall Algorithm

Only ii Only iii

Both ii & iii

Question #

If, in a DFS forest of digraph G = (V, E), f[u] = f[v] for an edge (u, v) ? E then the edge is called

Back edge Forward edge Cross Edge Tree Edge None of these

Question #

Best and worst case times of an algorithm may be same. **True** False

Question #

Can an adjacency matrix for a directed graph ever not be square in shape? **Yes** No

Question #

Consider the following Huffman Tree. The binary code for the string **TEA** is:

10 00 010

011 00 010

10 00 110

11 10 110

String	Encoding
TEA	10 00 010
SEA	011 00 010
TEN	10 00 110

Question #

Can an adjacency matrix for a directed graph ever not be square in shape? Yes **No**

- No. since we want to describe the relationship between each node and each other node, we need precisely n^2 matrix entries.

Question #

Suppose we have two problems A and B .Problem A is polynomial-time reducible and problem B is NP-complete. If we reduce problem A into B then problem A becomes NP-complete **Yes** No

Question #

If a problem is NP-complete, it must also be in NP.

a) True b) False c) unknown

Question #

If a problem is in NP, it must also be in P.

True **False (p178)** unknown

Question #

If a problem is in NP-complete, it must also be in NP.

True False

Cs502 Solved MCQS for MidTerm

Including Current Quizes

26 May 2013

Question # 1

Word Algorithm comes from the name of the Muslim author:

Abu Ja'far Mohammad ibn Musaal-Khowarizmi. (p7)

Question # 2

Al-Khwarizmi's work was written in a book titled

al Kitab al-mukhatasar fi hisab al-jabr wa'l-muqabalah (p7)

Question # 3

Analysis of Selection algorithm ends up with,

Select correct option:

T(n) (p37) T(1 / 1 + n) T(n / 2) T((n / 2) + n)

Question # 4

The function f(n)= n(logn+1)/2 is asymptotically equivalent to n log n. Hre Upper Bound means the function f(n) grows asymptotically _____ faster than n log n.

Select correct option:

More Quiet **Not (p24)** At least

Question # 5

The running time of an algorithm would not depend upon the optimization by the compiler but that of an implementation of the algorithm would depend on it. Select correct option:

True **False (p13)**

Question # 6

After sorting in merge sort algorithm, merging process is invoked.

Select correct option:

True (p27) False

Question # 7

Asymptotic growth rate of the function is taken over_____ case running time.

Select correct option:

Best

Average

Worst (p14)

Normal

Question # 8

In analysis of f(n)=n(n/5)+n-10 log n, f(n) is asymptotically equivalent to _____.

n

2n

n+1

n^2 (p23)

Question # 9

f(n) and g(n) are asymptotically equivalent. This means that they have essentially the same _____ for large n.

Results Variables Size **Growth rates (p23)**

Question # 10

Algorithm is concerned with _____ issues.

Macro Micro 1 **Both Macro & Micro (p8)** Normal

Question # 11

We cannot make any significant improvement in the running time which is better than that of brute-force algorithm.

True **False (p18, p92)**

Question # 12

In addition to passing in the array itself to Merge Sort algorithm, we will pass in _____other arguments which are indices.

Two (p28) Three Four Five

Question # 13

For the worst-case running time analysis, the nested loop structure containing one “for” and one “while” loop, might be expressed as a pair of _____nested summations.

1 **2 (p14)** 3 4

Question # 14

The O-notation is used to state only the asymptotic _____bounds.

Two Lower **Upper (p25)** Both lower & upper

Question # 15

Which of the following is calculated with **big O notation**?

Lower bounds **Upper bounds (p25)**

Both upper and lower bound Medium bounds

Question # 16

The Ω-notation allows us to state only the asymptotic _____ bounds.

Lower (p25) Upper

Both upper and lower bound Medium bounds

Question # 17

The time assumed for each basic operation to execute on RAM model of computation is _____

Infinite Continuous **Constant (p10)** Variable

Question # 18A

If the indices passed to merge sort algorithm are not equal, the algorithm may return immediately.

True **False (p28)**

Question #18B

If the indices passed to merge sort algorithm are _____,then this means that there is only one element to sort.

Small Large **Equal (p28)** Not Equal

Question # 19

Brute-force algorithm uses no intelligence in pruning out decisions.

True (p18) False

Question # 20

In analysis, the **Lower Bound** means the function grows asymptotically **at least** as fast as its largest term.

True (p24) False

Question # 21

In analysis, the **Upper Bound** means the function grows asymptotically **no faster** than its largest term.

True (p24) False

Question # 22

For small values of n, any algorithm is fast enough. Running time does become an issue when n gets large.

True (p14) Fast

Question # 23

The array to be sorted is not passed as argument to the merge sort algorithm.

True **False (p28)**

Question # 24

In simple brute-force algorithm, we give no thought to efficiency.

True (p11) False

Question # 25

The ancient Roman politicians understood an important principle of good algorithm design that is plan-sweep algorithm.

True **False (p27) [Divide and Conquer]**

Question # 26

In 2d-space a point is said to be _____if it is not dominated by any other point in that space.

Select correct option:

Member Minimal **Maximal (p11)** Joint

Question # 27

An algorithm is a mathematical entity that is dependent on a specific programming language.

Select correct option:

True **False (p7)**

Question # 28

Fibonacci sequence was posted by _____

Leonardo Pisano (p73)

Question # 29

$8n^2 + 2n - 3$ will eventually exceed $c_2(n)$ no matter how large we make c2.

True (p25) False

Question # 30

If we associate (x, y) integers pair to cars where x is the speed of the car and y is the negation of the price. High y value for a car means a _____ car.

Fast Slow Expensive **Cheap (p11)**

Question # 31

The running time of quick sort depends heavily on the selection of:

No of inputs Arrangement of elements in array

Size of elements **Pivot element (p49)**

Question # 32

Which sorting algorithm is faster?

O(n^2) O(nlogn) **O(n+k) (p58)** O(n^3)

Question # 33

Q knapsack problem is called a "0-1" problem, because

Each item must be entirely accepted or rejected (p92)

Question # 34

Which statement is true?

Select correct option:

If a **dynamic-programming problem** satisfies the optimal-substructure property, then a locally optimal solution is globally optimal.

If a **greedy choice property** satisfies the optimal-substructure property, then a locally optimal solution is globally optimal.

Both of above (p77, p98) None of above

Question # 35

Merge sort is stable sort, but not an in-place algorithm

True (p54) False

Question # 36

In counting sort, once we know the ranks, we simply _____ numbers to their final positions in an output array.

Delete **copy (p57)** Mark arrange

Question # 37

Dynamic programming algorithms need to store the results of intermediate sub-problems. **True (p75)** False

Question # 38

A p × q matrix A can be multiplied with a q × r matrix B. The result will be a p × r matrix C. There are (p . r) total entries in C and each takes _____ to compute.

O (q) **O (1) (p84)** O (n^2) O (n^3)

Question # 39

Due to left-complete nature of binary tree, heaps can be stored in

Link list Structure **Array (p40)** None of above

Question # 40

Efficient algorithm requires less computational _____

Memory Running Time **Memory and Running Time (p9)** Energy

Question # 41

Merge sort makes two recursive calls. Which statement is true after these recursive calls finish, but before the merge step?

The array elements form a heap

Elements in each half of the array are sorted amongst themselves

Elements in the first half of the array are less than or equal to elements in the second half of the array

None of the above

Question # 42

Who invented Quick sort procedure?

Hoare Sedgewick Mellroy Coreman

Question # 43

Merge sort requires extra array storage. **True (p54)** False

- Mergesort is a stable algorithm but **not** an in-place algorithm. It requires extra array storage.

Question # 44

In which order we can sort?

Increasing order only decreasing order only

Increasing order or decreasing order both at the same time

Question # 45

In the analysis of **Selection algorithm**, we make a number of passes, in fact it could be as many as,

T(n) T(n / 2) **log n (p37)** n / 2 + n / 4

Question # 46

How much time **merge sort** takes for an array of numbers?

T(n^2) T(n) T(log n) **T(n log n) (p40)**

Question # 47A

Sieve Technique applies to problems where we are interested in finding a single item from a larger set of ___

n items (p34) phases pointers constant

Question # 47B

The sieve technique works in phases as follows. It applies to problems where we are interested in finding

a **single** item from a larger set of n items. **(p34)**

Question # 47C

The sieve technique works where we have to find _____ item(s) from a large input.

Single (p34) Two Three Similar

Question # 48

The sieve technique works in _____ as follows

Phases (p34) numbers integers routines

Question # 49

For the heap sort, access to nodes involves simple _____ operations.

Arithmetic (p41) binary algebraic logarithmic

Question # 50

The analysis of Selection algorithm shows the total running time is indeed _____in n,

Arithmetic geometric **linear (p39)** orthogonal

Question # 51

Slow sorting algorithms run in,

T(n^2) (p39) T(n) T(log n) T(n log n)

Question # 52

A heap is a left-complete binary tree that conforms to the

Increasing order only decreasing order only **heap order** (log n) order

Question # 53

For the heap sort we store the tree nodes in

Level-order traversal (p40) in-order traversal pre-order traversal post-order traversal

Question # 54A

In Sieve Technique we do not know which item is of interest **True (p34)** False

Question # 54B

In Sieve Technique, we know the item of interest.

True **False (p34)**

Question # 55

Divide-and-conquer as breaking the problem into a small number of

Pivot Sieve **smaller sub problems (p34)** Selection

Question # 56

For the sieve technique we solve the problem,

Recursively (p34) mathematically precisely accurately

Question # 57

The reason for introducing **Sieve Technique** algorithm is that it illustrates a very important special case of, **Divide-and-conquer (p34)** decrease and conquer greedy nature 2-dimension Maxima

Question # 58

Sorting is one of the few problems where provable _____ bonds exits on how fast we can sort,

Upper **Lower (p39)** Average log n

Question # 59

Sieve Technique can be applied to selection problem? **True (p35)** False

Question # 60

One of the clever aspects of heaps is that they can be stored in arrays without using any_____

Pointers (p40) constants variables functions

Question # 61

Heaps can be stored in arrays without using any pointers; this is due to the ___ nature of the binary tree,

Left-complete (p40) right-complete tree nodes tree leaves

Question # 62

How many elements do we eliminate in each time for the Analysis of Selection algorithm?

n / 2 elements (p36) (n / 2) + n elements n / 4 elements 2 n elements

Question # 63

The sieve technique is a special case, where the number of sub problems is just

5 Many **1 (p34)** few

Question # 64

For the Sieve Technique we take time

T(nk) (p34) T(n / 3) n^2 n/3

Question # 65

The number of nodes in a complete binary tree of height h is

Select correct option:

2^(h+1) – 1

2 * (h+1) – 1

2 * (h+1)

((h+1) ^ 2) – 1

Question # 66

Random access machine or RAM is a/an _____

Machine build by Al-Khwarizmi Mechanical machine

Electronics machine **Mathematical model (p10)**

Question # 67

Analysis of Selection algorithm ends up with,

T(n)

T(1 / 1 + n)

T(n / 2)

T((n / 2) + n)

Question # 68

Continuation sort is suitable to sort the elements in range 1 to k

1. K is Large 2. K is not known 3. K may be small or large **4. K is small (p57)**

Question # 69

Counting sort is suitable to sort the elements in range 1 to k:

K is large **K is small (p57)** K may be large or small None

Question # 70

Continuing sort has time complexity of?

1. O (n) (p58) 2. O(n+k) 3. O(nlogn)

Question # 71

Counting sort has time complexity:

O (n) (p58) O(n+k) O(k) O(nlogn)

Question # 72

Memoization is _____:

-To store previous results for further use.

-To avoid unnecessary repetitions by writing down the results of recursive calls and looking them again if needed later. (p74)

-To make the process accurate -None of the above

Question # 73

In Quick sort algorithm, constants hidden in T(n lg n) are:

Large Medium Not known **Small (Ref)**

Question # 74

Quick sort is based on divide and conquer paradigm; we divide the problem on base of pivot element and:

There is explicit combine process as well to conquer the solution.

No work is needed to combine the sub-arrays, the array is already sorted

Merging the subarrays None of above.

Question # 75

In RAM model instructions are executed _____

In Random Access Machine, instructions are executed_____

One after another (p10) Parallel Concurrent Random

Question # 76

In the analysis of Selection algorithm, we eliminate a constant fraction of the array with each phase; we get the

convergent _____ series in the analysis,

linear arithmetic **geometric (p37)** exponent

Question # 77

_____ is a graphical representation of an algorithm

Σ notation Θnotation **Flowchart (Ref)** Asymptotic notation

Question # 78

A RAM is an idealized machine with _____ random-access memory.

256MB 512MB **An infinitely large (p10)** 100GB

Question # 79

What type of instructions Random Access Machine (RAM) can execute? Choose best answer

Algebraic and logic Geometric and arithmetic

Arithmetic and logic (p10) Parallel and recursive

Question # 80

What will be the total number of **max comparisons** if we run **brute-force maxima** algorithm with n elements?

n^2 (p14) 2n/n n 8n

Question # 81

What is the solution to the recurrence T(n) = T(n/2)+n .

O(logn) **O(n)** O(nlogn) O(n2)

Question # 82

Consider the following code:

```
For(j=1; j<n;j++)
        For(k=1; k<15;k++)
                For(l=5; l<n; l++)
                {
                        Do_something_constant();
                }
```

What is the order of execution for this code.

O(n)

O(n3)

O(n2 log n)

O(n2)

Question # 83

Consider the following Algorithm: Fun(n){ if (n=1) return 1 else return (n * Fun(n-1)) } Recurrence for the above algorithm is:

nT(n-1)+1

2T(n-1)+1

T(n-1)+cn

T(n-1)+1

Question # 84

Consider the following Algorithm:

```
Factorial (n){
  if (n=1)
    return 1
  else
    return (n * Factorial(n-1))
{
```

Recurrence for the following algorithm is:

T(n) = T(n-1) +1

T(n) = nT(n-1) +1

T(n)= T(n-1) +n

T(n)=T(n(n-1)) +1

Question # 85

What is the total time to heapify?

O(log n) **(p43)** O(n log n) O(n2 log n) O(log2 n)

Question # 86

When we call heapify then at each level the comparison performed takes time

It will take Θ (1) (p43)

Time will vary according to the nature of input data

It cannot be predicted

It will take Θ (log n)

Question # 87

In Quick sort, we don't have the control over the sizes of recursive calls

-True (p49) -False -Less information to decide -Either true or false

Question # 88

Is it possible to sort without making comparisons? **Yes (p57)** No

Question # 89

If there are **Θ (n2) entries** in **edit distance matrix** then the total running time is

Θ (1) **Θ (n^2) (p84)** Θ (n) Θ (n log n)

Question # 90

For **Chain Matrix Multiplication** we cannot use divide and conquer approach because,

We do not know the optimum k (p86)

We use divide and conquer for sorting only

We can easily perform it in linear time

Size of data is not given

Question # 91

The Knapsack problem belongs to the domain of _____ problems.

Optimization (p91) NP Complete Linear Solution

Sorting

Question # 92

Suppose we have three items as shown in the following table, and suppose the capacity of the knapsack is 50 i.e.

W=50.

Item	Value	Weight
1	60	10
2	100	20
3	120	30

The optimal solution is to pick

Items 1 and 2

Items 1 and 3

Items 2 and 3

None of these

Question # 93

Theta asymptotic notation for T (n):

Select correct option:

Set of functions described by: c1g(n)<=f(n) for c1 some constant and n=n0

Set of functions described by c1g(n)>=f(n) for c1 some constant and n=n0

Theta for T(n)is actually upper and worst case complexity of the code

Set of functions described by: c1g(n)<=f(n)<=c2g(n) for c1 and c2 some constants and n=n0

Question # 94

A (an) _____ is a left-complete binary tree that conforms to the heap order.

Heap (page 40) Binary tree Binary search tree Array

Question # 95

A heap is a left-complete binary tree that conforms to the _____

increasing order only decreasing order only **heap order (p40)** (log n) order

Question # 96

Consider the following Algorithm: Fun(n){ if (n=1) return 1 else return (n * Fun(n-1)) } Recurrence for the above algorithm is: Select correct option:

nT(n-1)+1

2T(n-1)+1

T(n-1)+cn

T(n-1)+1

Question # 97

The recurrence relation of Tower of Hanoi is given below T(n)={1 if n=1 and 2T(n-1) if n >1 In order to move a tower of 5 rings from one peg to another, how many ring moves are required?

Select correct option: 16 10 **32 (p30)** 31

Question # 98

In Quick Sort Constants hidden in T(n log n) are

1. Large 2. Medium **3. Small (Reference)** 4. Not Known

Question # 99

In stable sorting algorithm:

One array is used

In which duplicating elements are not handled.

More than one arrays are required.

Duplicating elements remain in same relative position after sorting. (p54)

Question # 100

In in-place sorting algorithm is one that uses arrays for storage:

An additional array

No additional array (p54)

Both of above may be true according to algorithm

More than 3 arrays of one dimension

Question # 101

Which may be a stable sort?

1. Merger 2. Insertion **3. Both above (p54)** 4. None of the above

Question # 102

An in place sorting algorithm is one that uses _____ arrays for storage

1. Two dimensional arrays 2. More than one array

3. No Additional Array (p54) 4. None of the above

Question # 103

Quick sort is _____

Stable and In place **Not stable but in place (p54)**

Stable and not in place Sometime in place and sometime stable

Question # 104

One Example of **in place** but **not stable** sort is

Quick (p54) Heap Merge Bubble

Question # 105

Which may be stable sort?

Bubble sort Insertion sort **both of above (p54)**

Question # 106

Merge sort is stable sort, but not an in-place algorithm **True (p54)** False

A sorting algorithm is **stable** if duplicate elements remain in the same relative position after sorting. **(p54)**

Inplace (SBHIQ)	**Stable (MCBI)**
Selection Sort	Merge Sort
Bubble Sort	Counting Sort
Heap Sort	Bubble Sort
Insertion Sort	
Quick Sort	

Question # 107

$8n^2 + 2n - 3$ grows asymptotically at least as fast as $\mathbf{n^2}$.

Question # 108

Given a set of points P = {p1, p2, . . . , pn} in 2-space, output the set of maximal points of P, i.e., those points pi such that pi is not dominated by any other point of P.

True (p11) False

Question # 109

While Sorting, the ordered domain means for any two input elements x and y _____ satisfies only.

x < y

x > y

x = y

All of the above (p34)

Question # 110

Brute-force algorithm for 2D-Maxima is operated by comparing _____ pairs of points.

Two Some Most **All (p18)**

Question # 111

Merge sort is based on ____

Brute-force Plan-sweep Axis-sweep **Divide and Conqer (p27)**

Question # 112

In the statement "if (P[i].x < P[j].x) & (P[i].y < P[j].y)", the number of times elements of P are accessed is ____

1 2 3 **4 (p14)**

Question # 113

Al-Khawarizmi was a/an....

Artist **Mathematician** Astronomer Khalifah

Question # 114

In Sorting the key value or attribute ____ from an ordered domain.

Must be Not always

The key value need not be a number. It can be any object from a totally ordered domain. (p39)

Question # 115

In Selection problem, the Sieve technique works in ____

Non-recursive manner Constant time

Phases (p34) One complete go

Question # 116

In max heap (for Heap Sort algorithm), when every time maximum element is removed from top we replace it with ____ leaf in the tree.

Second last **Last (p41)** First Any

Question # 117

If input "n" is odd, the median will be ____

(n+1)/2 (p34) n/2

Question # 118

The function f(n)= n(logn+1)/2 is asymptotically equivalent to n log n. Hre Upper Bound means the function f(n) grows asymptotically _____ faster than n log n.

More Quiet **Not (p23)** At least

Question # 119

In asymptotical analysis of n(n - 3) and 4n*n, as n becomes large, the dominant (fastest growing) term is some constant times ____.

n+1

n-1

n

n*n (p23)

Question # 120

In asymptotical analysis of n*(5 + 2)-3 , as n becomes large, the dominant (fastest growing) term is some constant times ____.

n+1

n-1

n

n*n

Question # 121

A point p in 2-dimensional space is usually given by its integer coordinate(s)_____

Select correct option:

p.x only

p.y only

p.x & p.z

p.x & p.y (p10)

Question # 122

Algorithm analysts know for sure about efficient solutions for NP-complete problems.

True **False (p9)**

Question # 123

In _____ we have to find rank of an element from given input.

Select correct option:

Merge sort algorithm **Selection problem (p34)**

Brute force technique Plane Sweep algorithm

Question # 124

In Heap Sort algorithm, if heap property is violated _____

Select correct option:

We call Build heap procedure **We call Heapify procedure (p41)**

We ignore Heap property can never be violated

Question # 125

Upper bound requires that there exist positive constants c2 and n0 such that f(n) ____ c2n for all n <= n0

Select correct option:

Less than

Equal to or Less than (p25 => for all n >= n0)

Equal or Greater than

Greater than

Question # 126

A RAM is an idealized algorithm with takes an infinitely large random-access memory.

True **False (p10 => A RAM is an idealized machine)**

Question # 127

In simple brute-force algorithm, we give no thought to efficiency.

True (p11) False

Question # 128

_____ is one of the few problems, where provable lower bounds exist on how fast we can sort.

Select correct option:

Searching **Sorting (p39)**

Both Searching & Sorting Graphing

Question # 129

The total no of arguments passed to merge sort algorithm is _____

2

3 (p28 => Array itself + two indices)

4

5

Question # 130

While solving Selection problem, in Sieve technique we partition input data _____

In increasing order In decreasing order **According to Pivot (p35)** Randomly

Question # 131

The number of accesses to any element of space is not counted for the running time calculation of algorithm.

True **False (p14)**

Question # 132

In merge sort algorithm, to merge two lists of size n/2 to a list of size n, takes _____ time.

Theta (n) (p30) Theta log(n) Theta log2(n) Theta nlog(n)

Question # 133

In Heap Sort algorithm, Heapify procedure is _____ in nature.

Recursive (p43) Non-Recursive

Question # 134

In Heap Sort algorithm, we call Build-heap procedure _____

Only once Twice Thrice **As many times as we need**

Question # 135

In merge sort algorithm, we split the array around the _____ index q.

Entring **Mid (p28)** Exiting Summing

Question # 136

The Iteration method is used for _____

Comparing sorting algorithms only **Solving Recurrence relations (p31)**

Merging elements in Merge sort Dividing elements in Merge sort

Question # 137

Rank of an element can be defined as _____

One minus the number of elements that are smaller

Two plus the number of elements that are greater

One plus the number of elements that are smaller (p34)

Two minus the number of elements that are smaller

Question # 138

f(n) is a set of functions such that there exist positive constants c1, c2 and n0 such that 0 <= c1g(n) <= f(n) <= c2g(n) for all n >= n0 Then f(n) is asymptotically _____ g(n).

Select correct option:

Less than Greater than

Equivalent to (p23) Not equivalent to

Question # 139

Floor and ceiling are _____ to calculate while analyzing algorithms.

Select correct option:

Very easy **Usually considered difficult (p31)**

Question # 140 In Heap Sort algorithm, the maximum levels an element can move upward is _____

Select correct option:

Theta (log n) (p43) Order (log n) Omega (log n) O (1) i.e. Constant time

Question # 141

In Heap Sort algorithm, the total running time for Heapify procedure is _____

Select correct option:

Theta (log n) **Order (log n) (p43)** Omega (log n) O (1) i.e. Constant time

Question # 142

In pseudo code, the level of details depends on intended audience of the algorithm.

True (p12) False

Question # 143

Counting sort assumes that the numbers to be sorted are in the range ____

1 to k where k is small (p57)

CS502 Solved Grand Quiz 2020

Question No 1

In the analysis of algorithms, _____ plays an important role.

Select Correct Option

- Text analysis
- **Time**
- Growth rate
- Money

Question No 2

_____ overcomes the limitations of _____ by working as per positional notations of numbers.

Select Correct Option

- **Bubble sort, Radix sort**……(**For More Visit VUStudents.pk)**
- Counting sort, Radix sort
- Radix sort, Bubble sort
- Radix sort, Counting sort

Question No 3

In generating Fibonacci Sequence, we can avoid unnecessary repetitions by _____ process.

Select Correct Option

- Tokenization
- **Memoization**……(**For More Visit VUStudents.pk)**
- Randomization
- Memorization

Question No 4

In Dynamic Programming, our approach is to _____.

Select Correct Option

- **Develop the solution in a top-down fashion**
- Express the problem non-recursively
- Build the solution in a bottom-up fashion
- Input several sub-problems simultaneously

For More Visit VUStudents.pk

Question No 5

We can user the optimal substructure property to devise a _____ formulation of the edit distance problem.

Select Correct Option

- Selective
- Optimum
- Iterative
- **Recursive……(For More Visit VUStudents.pk)**

Question No 6

Chain matrix multiplication problem can be solved through _____ strategy.

Select Correct Option

- **Dynamic programming**……(**For More Visit VUStudents.pk)**
- Greedy
- Divide and conquer
- Implementing arrays

Question No 7

For 2D Maxima problem, Plane Sweep algorithm first of all _____-.

Select Correct Option

- **Sorts all points……(For More Visit VUStudents.pk)**
- Delete some points
- Output the elements
- Pushes all points on stack

Question No 8

_____ algorithm based on Dynamic Programming strategy.

Select Correct Option

- **Heap Sort……(For More Visit VUStudents.pk)**
- Binary Tree
- Quick Sort
- Edit distance

For More Visit VUStudents.pk

Question No 9

The only way to convert a string of i characters into the empty string is with i deletions, represented as _____--.

Select Correct Option

- E (0, j) = j
- E (i, j) = l
- E (0, i) = l
- **E (i, 0) = i……(For More Visit VUStudents.pk)**

Question No 10

Edit Distance algorithm based on _____ strategy.

Select Correct Option

- Greedy
- **Dynamic Programming……(For More Visit VUStudents.pk)**
- Divide and Conquer
- Brute force

Question No 11

In _____ we have to find rank of an element from given input.

Select Correct Option

5. Merge sort algorithm
6. **Selection problem**……(For More Visit VUStudents.pk)
7. Brute force technique
8. Plane Sweep algorithm

Question No 12

In the statement “output P[i].x, P[i].y”, the number of time elements of P are accessed is _____

Select Correct Option

P 1

Q 2……(For More Visit VUStudents.pk)

R 3

S 4

For More Visit VUStudents.pk

Question No 13

Rank of an element can be defined as _____-.

Select Correct Option

- ✓ One minus the number of elements that are smaller
- ✓ Two plus the number of elements that are greater
- ✓ **One plus the number of elements that are smaller**
- ✓ Two minus the number of elements that are smaller

Question No 14

8n2 + 3n – 3 will eventually exceed c2*(n) no matter how large we make _____-.

Select Correct Option

- ✓ n
- ✓ c2
- ✓ **2n……(For More Visit VUStudents.pk)**
- ✓ this equation

Question No 15

Matrix Multiplication is a(n) _____ operation.

Select Correct Option

- ✓ Commutative
- ✓ **Associative……(For More Visit VUStudents.pk)**
- ✓ Neither commutative nor associative
- ✓ Transitive

Question No 16

The number of nodes in a complete binary tree of height h is:

Select Correct Option

- ✓ 2 * (h+1) -1
- ✓ **2^(h+10 – 1**……(**For More Visit VUStudents.pk)**
- ✓ 2*(h+1)
- ✓ ((h+1) ^ 2) – 1

Question No 17

Applying the sieve technique to selection problem, _____ element is picked from array.

Select Correct Option

- ✓ Output
- ✓ Total
- ✓ Input
- ✓ **Pivot……(For More Visit VUStudents.pk)**

Question No 18

In Quick sort algorithm, we choose pivot _____--.

Select Correct Option

- ✓ **Always the smallest element** ……(**For More Visit VUStudents.pk)**
- ✓ Greater than 5
- ✓ Randomly
- ✓ Less than 5

Question No 19

Approach of solving geometric problems by sweeping a line across the plane is called

_____ sweep.

Select Correct Option

- ✓ Line
- ✓ **Plane……(For More Visit VUStudents.pk)**
- ✓ Cube
- ✓ Box

Question No 20

In Knapsack Problem, the goal is to put items in the knapsack such that the value of the items is _____ subject to weight limit of knapsack.

Select Correct Option

- ✓ Minimized
- ✓ Decrease
- ✓ **Maximized……(For More Visit VUStudents.pk)**
- ✓ None of the given options

For More Visit VUStudents.pk

For More Visit VUStudents.pk

Question No 21

Sorting is performed on the basis of _____.

Select Correct Option

- ✓ Computational resources
- ✓ Asymptotic notation
- ✓ Summation
- ✓ **Some key value or attribute**……(For More Visit VUStudents.pk)

Question No 22

In Asymptotical analysis of n(n-3) and 4n*n, as n becomes large, the dominant (fastest growing) term is some constant times_____.

Select Correct Option

- ✓ n+1
- ✓ n-1
- ✓ n
- ✓ **n*n……(**For More Visit VUStudents.pk)

Question No 23

In Heap Sort algorithm, we build _____ for ascending sort.

Select Correct Option

- ✓ Max heap
- ✓ **Min heap**……(For More Visit VUStudents.pk)
- ✓ Full heap
- ✓ Order heap

Question No 24

If we have an equation 8n2+7f*n+5f+6 then n is large, _____ term will be much larger than the n term and will dominate the running time.

Select Correct Option

- ✓ f(g(n)
- ✓ g(n)^2
- ✓ **n^2……(****For More Visit VUStudents.pk)**
- ✓ f(n)

Question No 25

_____ is the process of avoiding unnecessary repetitions by writing down the results of recursive calls and looking them up again if we need them later.

Select Correct Option

- ✓ Loop
- ✓ **Memoization……(For More Visit VUStudents.pk)**
- ✓ Recursion
- ✓ Function

Question No 26

The worst-case running time of merge sort is _____ in order to sort an array of n elements.

Select Correct Option

- ✓ O(log n)
- ✓ O(n)
- ✓ **O(nlogn)** ……(For More Visit VUStudents.pk)
- ✓ O(n^2)

Question No 27

If matrix A of dimension 2 x 4 is multiplied with matrix b of dimension 4 x 3, then the dimension of resultant matrix will be _____.

Select Correct Option

- ✓ **2 x 4**……(For More Visit VUStudents.pk)

- ✓ 4 x 3
- ✓ 3 x 4
- ✓ 2 x 3

Question No 28

Fibonacci Sequence was named on _____ , a famous mathematician in 12th Century.

Select Correct Option

- ✓ Fred Brooks
- ✓ Grady Booch
- ✓ **Leonardo Pisano**……(**For More Visit VUStudents.pk)**
- ✓ Edger F.Codd

For More Visit VUStudents.pk

For More Visit VUStudents.pk

Question No 29

Algorithm is a sequence of computational steps that _____ the input into output.

Select Correct Option

- ✓ Merge
- ✓ Assign
- ✓ **Transform**……(For More Visit VUStudents.pk)
- ✓ Integrate

Question No 30

If the indices passed to marge sort algorithm are _____, then this means that there is only one element to sort.

Select Correct Option

- ✓ Small
- ✓ Large
- ✓ **Equal**……
- ✓ Not Equal

CS502 Quiz 2 Fall 2020

Question No 1

The tricky part of _____ algorithm(s) is/are, how to detect whether the addition of an edge will create a cycle in viable set A.

Kruskal's (Page 147)

Question No 2

Overall Running time of Prim's algorithm is _____.

Θ(ElogV) (Page 153)

Question No 3

Finding the faster result of the shortest path from u to v for every pair of vertices u and v we use_____.

All-pairs shortest-paths problem (Page 154)

Question No 4

What is the time complexity to extract a vertex from the priority queue in Prim's algorithm?

O(log V) (Page 152)

Question No 5

Edge weights can be interpreted as distance _____.

in the shortest-paths (Page 153)

Question No 6

Dijkstra's Algorithm is used to solve_____problems.

Single-source shortest path (Page 159)

Question No 7

Problems such as the shortest route between cities can be solved efficiently by modeling the road map as a_____.

Graph (Page 153)

Question No 8

For _____ graphs, there is no distinction between forward and back edges.

Undirected (Page 130)

Question No 9

Bellman-Ford Algorithm does not allow G(graph) to have _____.

negative cost cycles (Page 159)

Question No 10

A topological sort of a DAG is a _____ ordering of the vertices of the DAG such that for each edge (u, v), u appears before v in the ordering.

Linear (Page 133)

Quiz File 2

Question No 1

Networks are _____ in the sense that it is possible from any location in the network to reach any other location in the digraph.

Complete (Page 135)

Question No 2

According to parenthesis lemma vertex u is a descendent of v vertex if and only if:

f [d[u], f[u]] ⊆ [d[v], f[v]] (Page 129)

Question No 3

For _____ graphs, there is no distinction between forward and back edges.

Undirected (Page 130)

Question No 4

There exist a unique path between any _____ vertices of a free tree.

Two (Page 142)

Question No 5

Keeping in mind the shortest-path, if given scenarios occur in computer networks like the internet where data packets have to be routed. The vertices are_____and Edges are

_____which may be wired or wireless.

Routers, communication links (Page 153)

Question No 6

In undirected graph, by convention all the edges are called _____ edges.

back (Page 130)

Question No 7

Bellman-Ford algorithm is slower than_____.

Dijkstra's (Page 159)

Question No 8

From the given option's which one is correct regarding the time complexity of Dijkstra's algorithm:

O(N^2)

Question No 9

In the shortest-paths problem, we are given a weighted of_____G = (V, E).

Un-directed graph

Question No 10

You have an adjacency list for G, what is the time complexity to compute Graph transpose G^T

?

(V+E)

CS502 – Midterm Spring 2013 (26 May-05 June)

How would you modify QUICKSORT to sort into non-increasing order?
Answer: - Click here for detail
To make QUICKSORT sort in non-increasing order we must modify PARTITION.

Q- We can avoid unnecessary repetitions for recursive calls?
Answer: - (Page 74)
We can avoid these unnecessary repetitions by writing down the results of recursive calls and looking them up again if we need them later. This process is called *memoization*.

Q-Write a pseudo code Fibonacci With memorization? -- (3)
Answer: - (Page 74)
MEMOFIB(n)
1 **if** (n < 2)
2 **then return** n
3 **if** (F[n] is undefined)
4 **then** F[n] MEMOFIB(n − 1) + MEMOFIB(n − 2)
5 **return** F[n]

Spelling correction in edit distance? 3 marks
Answer: - (Page 76)
If a text contains a word that is not in the dictionary, a 'close' word, i.e. one with a small edit distance, may be suggested as a correction. Most word processing applications, such as Microsoft Word, have spelling checking and correction facility. When Word, for example, finds an incorrectly spelled word, it makes suggestions of possible replacements.

Bubble sort?
Answer: - (Page 39)
Scan the array. Whenever two consecutive items are found that are out of order, swap them. Repeat until all consecutive items are in order.

What is the worst case running time for the Quick sort? What simple change is required in the algorithm to preserve its linear expected running time and makes it worst case time Θ(n log n)
Answer: - (Page 49)
Worst case running time is O (n).
The simple change which can change the running time of the edit distance algorithm is the number of entries n2.

CS502 – Midterm Spring 2013 (26 May-05 June)

QUICK, HEAP, COUNTING, MERG KA TABLE KA JIS ME STABLE OR IN-PLACE BTANEY THE 5 MARKS
Answer: Page 54

IN Place	Stable
Selection Sort	
Quick Sort	
Heap sort	

Bubble sort	Bubble sort
Insertion Sort	Insertion Sort
	Merge sort
	Counting sort

ADIT DISTANCE KI 3 APPLICATION K NAME BTANEY THE 3 MARKS
Answer: - (Page 76)
Spelling Correction
Plagiarism Detection
Speech Recognition

MEMOIZATION KI DIFINATION THI 2 MARKS
Answer: - Click here for detail
Memoization is an optimization technique used primarily to speed up computer programs by having function calls avoid repeating the calculation of results for previously processed inputs

WORS CASE OR AVERAGE CASE DEFINE KRNE THE 2 MARKS
Answer: - (Page 13)
Worst-case time is the maximum running time over all (legal) inputs of size n
Average-case time is the average running time over all inputs of size n.

CS502 – Midterm Spring 2013 (26 May-05 June)

Write Essential constraint of Counting sort.
Answer:
Essential constraint for the counting sort is that numbers to be sorted must be small integers i.e., 1…..k where k is small.

What are total numbers of entries in matrix for edit distance?
Answer: - (Page 84)
There are O(n2) entries in the matrix. Each entry E(i, j) takes O(1) time to compute. The total running time is O(n2).

WHAT IS the necessary assumption for average case analysis quick sort?
Answer: - (Page 50)
Average case depends on some assumption about the distribution of inputs. However, in the case of quicksort, the analysis does not depend on the distribution of input at all. It only depends upon the random choices of pivots that the algorithm makes

Suggest & describe modifications of the implementation of "Quick sort" that will improve its performance?
Answer: - (Page 76)

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Q1 : How we Heapify?
Answer:
If an element in the Heap is not at its proper place means it is violating the Heap Order, the Heapify procedure is used to fix it and place it at its proper position. In Heapify, we recursively swap the element with its larger one child and stop at a stage when this element is larger than both of its children or it becomes the leaf node.

Q2 : Describe Heap sort algorithm?
Answer: - (Page 43)

```
HEAPIFY( array A, int i, int m)
1 l   LEFT(i)
2 r   RIGHT(i)
3 max   i
4 if (l _ m)and(A[l] > A[max])
5 then max   l
6 if (r _ m)and(A[r] > A[max])
7 then max   r
8 if (max 6= i)
9 then SWAP(A[i],A[max])
10 HEAPIFY(A,max,m)
```

Q3 : How many elements are in matrix of edit distance?
Answer: Rep

Q6 : suggest and describe one modification of implementing quick sort? 5marks

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Edit Distance in Speech Recognition
Answer: - (Page 77)
Speech Recognition
Algorithms similar to those for the edit-distance problem are used in some speech recognition systems. Find a close match between a new utterance and one in a library of classified utterances.

What is sorting? Describe slow running sorting algorithms. 5 marks
Answer: - (Page 39)

Sorting is any process of arranging items in some sequence and/or in different sets, and accordingly,
There are a number of well-known slow O(n2) sorting algorithms. These include the following:
Bubble sort: Scan the array. Whenever two consecutive items are found that are out of order, swap them. Repeat until all consecutive items are in order.
Insertion sort: Assume that A[1..i − 1] have already been sorted. Insert A[i] into its proper position in this sub array. Create this position by shifting all larger elements to the right.
Selection sort: Assume that A[1..i − 1] contain the i − 1 smallest elements in sorted order. Find the smallest element in A[i..n] Swap it with A[i]. These algorithms are easy to implement. But they run in _(n2) time in the worst case

Catalan Numbers and their formula 3 marks
Answer: - (Page 85)
Catalan numbers are related the number of different binary trees on n nodes. Catalan number is given by the formula:

$$C(n) = \frac{1}{n+1}\binom{2n}{n}$$

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Q1.Heap Sort , Counting Sort , Quick Sort ,Merge Sort , in ,table ,And required to tell , which is inplace Algorithm and stable ? Marks 5
Answer: rep

Q2 . Suggest that how can make better improvement in Quick Sort algorithm .Marks 5
Answer: - (Page 54)
The best we have seen so far is O(n log n) algorithms for sorting. Is it possible to do better than O(n log n)? If a sorting algorithm is solely based on comparison of keys in the array then it is *impossible* to sort more efficiently than (n log n) time.

Q3. Consider three numbers with comparison based sortig algorithim and write possible combination in a1,a2,a3 .Marks 3
Answer: - (Page 85)
Consider sorting three numbers a1, a2, a3. There are 3! = 6 possible combinations:
(a1, a2, a3), (a1, a3, a2), (a3, a2, a1)
(a3, a1, a2), (a2, a1, a3), (a2, a3, a1)

Q4. What is better aproach of multiplication rather than straight form of Multiplication . Named that . Marks 3
Answer: - (Page 54)

Chain Matrix Multiplication-Dynamic Programming Formulation

Q6.worst case and average case algorithm of Quick Sort .Marks 2
Answer: - (Page 50)
Answer: Worst case = O(n2)
Average Case = O(n log n)

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1.worst case of bucket algorithm
Answer: - O(n)

Catalan Numbers and their formula 3 marks
Answer: rep

speech reorganization.2
Answer: rep

Worst and average case of quick sort algorithm.2 marks
Answer: rep

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Radix sort table ? 5 marks
Answer:
Radix sort:
In linear time sorting, counting sort is used for the numbers in the range 1 to k where k is small and it is based on determining the rank of each number in final sorted array. But it is useful only for small integers i.e., 1 ….k where k is small. But if k were very large, then the size of the rank array formed would also be very large which is not efficient. So solution for such cases is the Radix sort which works by sorting one digit at a time.

Example:
841 84[1] 8[4]1 [1]85

185 è 37[3] è 3[7]3 è [3]73

373 18[5] 1[8]5 [8]41

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1. **What are the worst and average run times of quick sort? (2 marks)**
   Answer: rep

2. **What are total numbers of entries in matrix for edit distance? (2 marks)**
   Answer: rep

Q.3 *What does dynamic programming have in common with divide and conquer ?*

Ans. : Both the divide and conquer and dynamic programming solve the problem by breaking it into number of subproblems. In both these methods solutions from subproblems are collected together to form a solution to given problem.

Q.4 *What is the principle difference between the two techniques ?*

Ans. : Both divide and conquer as well as dynamic programming solve the given problem by breaking it into subproblems. But in divide and conquer the subproblems are solved independently whereas in dynamic programming the subproblems share the solutions among themselves. The dynamic programming works on principle of optimality to solve the problem.

5. **Show that the running time of quick sort when the array A contains distinct elements and sorted in decreasing order ? (5 marks)**
   Answer:

3. (CLRS 7.2-3) Show that the running time of QUICKSORT is $\Theta(n^2)$ when the array A contains distinct elements and is sorted in decreasing order.

   Solution: On the first iteration of PARTITION the pivot element is chosen as the first element of A. Index i is incremented once and j is decremented until it reaches the pivot, i.e. the entire length of A. PARTITION returns to QUICKSORT the first element of A, which recursively sorts one subarray of size 1 and one of size $n-1$. This process is repeated for the subarray of size $n-1$. The running time of the entire computation is then given by the recurrence:

$$\begin{aligned} T(n) &= \begin{cases} \Theta(1) & n \leq 2 \\ T(n-1) + \Theta(n) & \text{otherwise} \end{cases} \\ &= \Theta(n^2). \end{aligned}$$

6. **Illustrate how Radix Sort works on the following words: Show the result of first 2 passes only.**
   COW, DOW, SEA, RUG, ROW, MOB, BOX, TAB, BAR, EAR, TAR, DIG, BIG, TEA, NOW, FOX
   Answer:
COW DOG SEA RUG ROW MOB BOX TAB BAR EAR TAR DIG BIG TEA NOW FOX
SEA TAB TEA BAR MOB EAR TAB TAR DOG SEA RUG TEA DIG DIG BIG BIG BAR
MOB EAR DOG TAR COW COW ROW ROW NOW NOW BOX BOX FOX FOX RUG BAR
BIG...

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Q: define heap and heap oerder
A *heap* is a left-complete binary tree that conforms to the *heap order*.

Q:Average-case Analysis of Quicksort
Average case analysis of Quick sort: The running time of Quick sort depends on the selection of pivot q which is done randomly. For average case analysis of quick sort, average is computed over all possible random choices of the pivot index q. The average case running time for quick sort is Θ (n log n).

Q:Worst case Analysis of Quick sort
Worst case analysis of Quick sort: For worst case we maximize over all possible values of q, means the selection of pivot q which gives the maximum (worst) time for sorting. The chances are that the pivot values q=1 (start) or n (end) or n/2 (middle) happen to give maximum time values. The worst case running time is O (n2).

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In Random Access Machine, instructions are executed _____ Instructions are executed
Answer:- one-by-one (there is no parallelism). (Page 10)

Dynamic Programming algorithm:
Answer: - The genius of the algorithm is in the clever recursive formulation of the shortest path problem. (Page 162)

For the heap sort, access to nodes involves simple __ **arithmetic** ___ operations. (Page 40)

Comparison based sorting algorithm cannot run faster than __ **Ω(n log n)** __ (Page 46)

Fibonacci sequence was posted by __ **Leonardo Pisano**, ___ (Page 73)

Mid

Quiz by Attiq Kundi

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CS-502

Fundamentals of Algorithms (CS

Multiple Choice Questions (MCQs)

Objective Questions

1. **An optimization problem is one in which you want to find,**
    1. Not a solution
    2. An algorithm
    3. Good solution
    4. The best solution

2. **In _____ algorithm, at any time, the subset of edges A forms a sin**
    1. Kruskal's
    2. Prim's
    3. Both Kruskal's and Prim's
    4. None of the given

3. **Counting sort assumes that the numbers to be sorted are in the range**
    1. k to n where n is large
    2. 1 to k where k is small
    3. k to n where k is small
    4. k to n where n is small

4. **In Dynamic Programming approach, we do not store the solution to ea problem in case if it reappears.**
    1. True

7. **Shortest path problems can be solved efficiently by modeling the road graph.**

    1. True
    2. False

8. **Adding an edge to a free tree creates.**

    1. A cycle
    2. A complete MST
    3. A partial MST
    4. A shortest path

9. **Although it requires more complicated data structures, Prim's algorith minimum spanning tree is better than Kruskal's when the graph has a of vertices.**

    1. True
    2. False

10. **In Prim's algorithm, we start with the root vertex r; it can be any verte**

    1. True
    2. False

11. **In the clique cover problem, for two vertices to be in the same group, t adjacent to each other.**

    1. True
    2. False

12. **According to parenthesis lemma. vertex u is a descendent of v vertex i**

    1. $[d[u], f[u]] \subseteq [d[v], f[v]]$

15. Dynamic Programming is a problem-solving approach in which _____

1. Problem is solved in Zero time
2. Solution is developed only at final stage
3. Both are correct
4. Both are incorrect

16. Networks are _____ in the sense that is possible from any location network to reach any other location in the digraph.

1. Complete
2. Incomplete
3. Not graphs
4. Transportation

17. We say that two vertices u and v are mutually not reachable if u can re vice versa.

1. True
2. False

18. We say that two vertices u and v are mutually _____ if u can reach versa.

1. Crossed
2. Forward
3. Reachable
4. Nor Reachable

19. Consider three matrices X, Y, Z of dimensions 1 × 2, 2 × 3, 3 × 4 respe number of multiplications of X(YZ) is:

1. 16
2. 23
3. 26

2. False

23. The Huffman algorithm finds a(n) _____ solution.

1. Optimal
2. Non-optimal
3. Exponential
4. Polynomial

24. Chain matrix multiplication problem can be solved through _____ s

1. Dynamic programming
2. Greedy
3. Divide and conquer
4. Sorting

25. Strongly connected components are not affected by reversal of all edg vertices reachability.

1. True
2. False

26. In kruskal's algorithm, the next edge is added to viable set A, if its add induce a cycle.

1. True
2. False

27. You have an adjacency list for G, what is the time compexity to comput transpose G^T.?

1. ? (V + E)
2. ? (V E)
3. ?(V)

2. False

31. The _____ given by DFS allow us to determine whether the graph c cycles.

1. Order
2. Time stamps
3. BFS traversing
4. Topological sort

32. If there are Θ (n^2) entries in edit distance matrix then each entry E (i, _____ time to compute.

1. Θ (n)
2. Θ (n^2)
3. Θ (1)
4. Θ (n log n)

33. We can use the optimal substructure property to devise a _____ fo the edit distance problem.

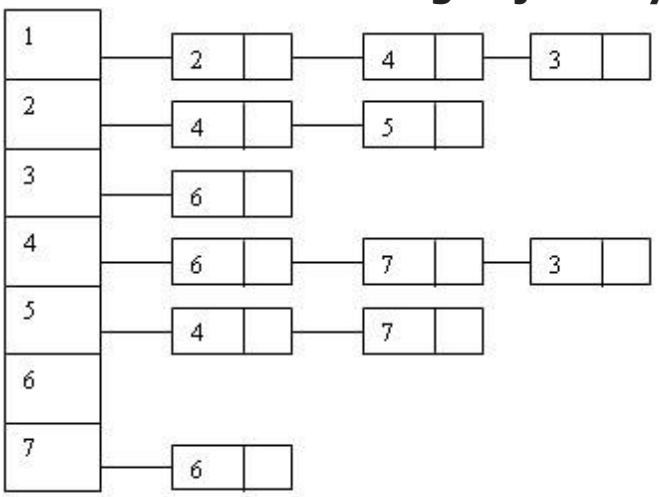
1. Selective
2. Optimum
3. Iterative
4. Recursive

34. The codeword assigned to characters by the Huffman algorithm have t that no codeword is the postfix of any other.

1. True
2. False

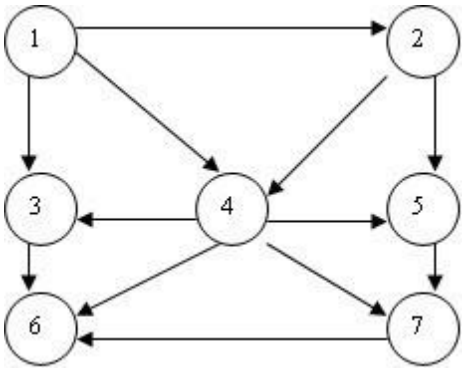
35. The term "coloring" came form the original application which was in ar design.

37. Consider the following adjacency list:

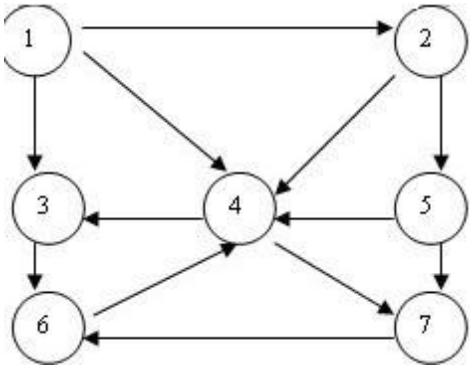


Which of the following graph(s) describe(s) the above adjacency list?

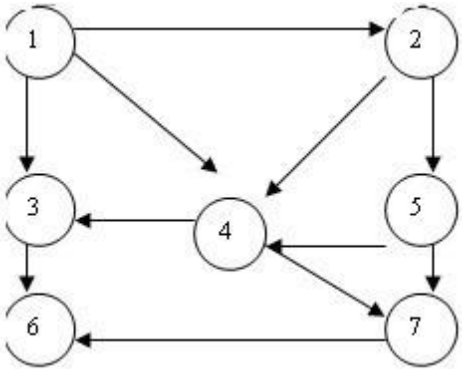
1.



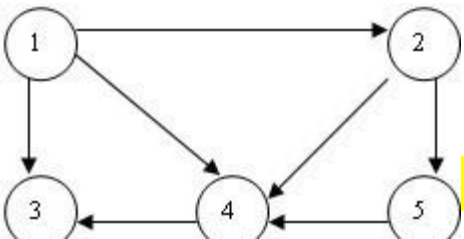
2.



3.



4.



4. Lists require more space than matrices but are faster to find the weight of (v1,v2)

40. By breaking any edge on cycle created in free tree, the free _____

1. Edge
2. Tree
3. Cycle
4. Vertex

41. The Huffman algorithm finds a polynomial solution.

1. True
2. False

42. Huffman algorithm uses a greedy approach to generate a postfix code minimizes the expected length B(T) of the encoded string.

1. True
2. False

43. You have an adjacency list for G, what is the time compexity to compu transpose G^T?

1. (V+E)
2. V.E
3. V
4. E

44. In strong components algorithm, first of all DFS is run for computing fi vertices.

1. True
2. False

48. The greedy part of the Huffman encoding algorithm is to first find two larger frequency.

1. True
2. False

49. In Dynamic Programming, our approach is to _____.

1. Develop the solution in a top-down fashion
2. Express the problem non-recursively
3. Build the solution in a bottom-up fashion
4. Input several sub-problems simultaneously

50. Heap sort is a/an _____ and _____ sorting algorithm.

1. Not in-place, not stable one
2. In-place, stable one
3. In-place, not stable one
4. Not in-place, stable one

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CS502 Grand Quiz

1. The sequence of merge sort algorithm is:
   a. Divide Combine-Conquer
   b. Conquer-Divide-Combine
   c. Divide-Conquer-Combine Page 27
   d. Combine-Divide-Conquer
2. In _____ Knapsack Problem, limitation is that an item can either be put in the bag or not. Fractional items are not allowed.
   a. 0
   b. 1
   c. 0/1 Page 91
   d. Fractional
3. In Selection algorithm, we assume pivot selection takes theta _____ running time.
   a. n Page - 36
   b. n2
   c. n3
   d. log (n)
4. In Heap Sort algorithm (using max heap), when every time maximum elements removed from top _____.
   a. We call merge Sort Algorithm
   b. it becomes Order n2 Algorithm
   c. Divide and Conquer strategy helps us
   d. We are left with a hole Page – 41
5. If matrix A of dimension p x q is multiply with matrix B of dimension q x r, then each entry in resultant matrix takes _____ time.
   a. O (q) Page - 84
   b. O (1)

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c. O (p x q)
d. O (q x r)

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6. _____ is a method of solving a problem in which we check all possible solutions to the problem to find the solution we need.
   a. Plane-Sweep Algorithm
   b. Sorting Algorithm
   c. Brute-Force Algorithm google
   d. Greedy approach
7. The worst case running time of Quick sort algorithm _____.
   a. Cannot be quadratic
   b. Is quadratic google QuickSort and its Analysis (codesdope.com) & http://personal.kent.edu/~rmuhamma/Algorithms/MyAlgorithms/Sorting/quickSort.htm
   c. Is always Exponential
   d. Is linear
8. In max heap (for Heap Sort algorithm), when every time maximum element is removed from top we replace it with _____ leaf in the tree.
   a. second last
   b. Last Page -41
   c. First
   d. Any
9. Quick sort algorithm was developed by -
   a. AlferdAho
   b. Sedgewick
   c. John Vincent Atanasoff
   d. Tony Hoare – Google wikipedia
10. If Matrix-A has dimensions "3x2" and Matrix-B has dimensions "2x3", then multiplication of Matrix-A and Matrix-B will result a new Matrix-C having dimensions.
    a. 3x2
    b. 2x3
    c. 2x2
    d. 3x3 http://www.calcul.com/show/calculator/matrix-multiplication

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11. For comparison-based sorting algorithms, it is possible to sort more efficiently than Omega n log(n) time.
    a. Always
    b. Not P-54
    c. Sometimes

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d. Sometimes not

12. Dynamic Programming approach is usually useful in solving optimization problems.
    a. True
    b. False

13. In Sorting the key value or attribute_____ from an ordered domain.
    a. Must be page 39
    b. Not always
    c. May be
    d. Occasionally

14. Result of asymptotical analysis of n(n -3) and 4n*n is that _____
    a. n(n-1) is asymptotically Less
    b. n(n-1) is asymptotically Greater
    c. Both are asymptotically Not equivalent
    d. Both are asymptotically Equivalent page 23 ($4n*n= 4n^2$)

15. Floor and ceiling are _____ to calculate while analyzing algorithms a. Very easy
    b. Usually considered difficult P-31
    c. 3rd Option is missing
    d. 4th Option is missing

16. _____ of reference is an important fact of current processor technology.
    a. Defining
    b. Assigning
    c. Formality
    d. Locality P-8

17. In max-heap, largest element is stored at root node. Where is the smallest element stored?
    a. Right Node
    b. Leaf Node
    c. Middle Node

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d. Left Node

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18. In average-case time analysis of Quick sort algorithm, the most balanced case for partition is when we divide the list of elements into _.
    a. Equal no. of pieces as of input elements
    b. Single piece exactly
    c. Two nearly equal pieces
    d. Three nearly equal pieces
19. Which of the following is calculated with Big O notation?
    a. Medium bounds
    b. Upper bounds Page - 25
    c. Lower bounds
    d. Both upper and lower bounds
20. Edit distance algorithm based on _____ strategy
    a. Greedy
    b. Dynamic Programming Page - 81
    c. Divide and Conquer
    d. Searching
21. In Heapsort Algorithm, total time taken by heapify procedure is _____
    a. O (log n) Page-43
    b. O (log2 n)
    c. O (n log n)
    d. O (n2 log n)
22. Al-Khwarizmi was a/an _____
    a. Artist
    b. Mathematician P-7
    c. Astronomer
    d. Khalifah
23. When matrix A of 5x3is multiply with metric B of 3x4 then the number of multiplication required is: Not found exactly
    a. 15
    b. 12

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c. 36

d. 60 **Not Found exactly but as per formula at page 84,**

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24. Pseudo code of algorithms are to be read by _____.
    a. People Page -12
    b. RAM
    c. Computer
    d. Compiler
25. The sieve technique is a special case, where the number of sub-problems is Just _____
    a. 1 P-34
    b. 2
    c. 3
    d. 4
26. When a recursive algorithm revisits the same problem over and over again, we say that the optimization problem has _____ sub-problems.
    a. Overlapping – Google Search
    b. Over costing
    c. Optimized
    d. Three
27. Sieve technique is very important special case of Divide-and-Conquer strategy.
    a. True P-34
    b. False
28. In order to say anything meaningful about our algorithms, it will be important for us to settle on a _____.
    a. Java Program
    b. C++ Program
    c. Pseudo program
    d. Mathematical model of computation P-10
29. Merge sort is based on _____.
    a. Brute-force
    b. Plan-sweep
    c. Axis-sweep

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d. Divide and Conquer **P-27**

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30. What time does Merge Sort algorithm take in order to sort an array of 'n' numbers?
    a. (n)
    b. (log n)
    c. (n^2)

    d. (n log n) Google Search 31. In Heap Sort
    31. algorithm, the first step is to _____.
    a. Call Build-Heap procedure Page - 46
    b. Sort the array in descending order
    c. Call Heapify procedure
    d. Find the number of input elements
32. The definition of theta-notation relies on proving _____ asymptotic bound.
    a. One
    b. Lower
    c. Upper
    d. Both lower & upper Page - 25
33. In merge sort algorithm, to merge two lists of size n/2 to a list of size n, takes _____ time.
    a. Theta (n) Page - 32
    b. Theta log(n)
    c. Theta log2(n)
    d. Theta n log(n)
34. We can make _____ recursive calls in Fibonacci Sequence.
    a. Infinite
    b. Finite google
    c. Only one
    d. Zero
35. Following is NOT the application of Edit Distance problem.
    a. Speech recognition

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b. Spelling Correction

c. Ascending Sort

d. Computational Molecular Biology

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36. In plane sweep approach, a vertical line is swept across the 2d-plane and structure is used for holding the maximal points lying to the left of the sweep line.
    a. Array
    b. Queue
    c. Stack Page - 18
    d. Tree
37. When a heapify procedure is applied to the root node to restore the heap, then at each level, the comparison performed takes time:
    a. It will take (log n).
    b. It can not be predicted
    c. It will take O (1). Page - 43
    d. Time will vary according to the nature of input data.
38. _____ time is the maximum running time over all legal inputs.
    a. Worst-case Page - 13
    b. Average-case
    c. Best-case
    d. Good-case
39. Efficient algorithm requires less computational...
    a. Memory
    b. Running Time
    c. Memory and Running Time Page - 9
    d. Energy
40. For average-case time analysis of Quick sort algorithm, Pivot selection is on average basis from _____
    a. half of the input values
    b. all possible random values Page - 50
    c. Pivot is input separately
    d. values greater than 5
41. Selection algorithm takes theta _____

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a. (n2)

b. (n)

c. log(n)

d. n log(n)

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42. Recurrence can be described in terms of a tree.
    a. Yes Page - 31
    b. No
43. Time complexity of Dynamic Programming based algorithm for computing the minimum cost of Chain Matrix Multiplication is _____
    a. Log n
    b. n
    c. n^2 (n square)
    d. n^3 (n cube) Page -90
44. The Iteration method is used for _____
    a. Comparing sorting algorithms only
    b. Solving Recurrence relations Page 31
    c. Merging elements in Merge sort
    d. Dividing elements in Merge sort
45. In 3-Dimensional space, a point P has _____ coordinate(s).
    a. (X, Y)
    b. (X, 0)
    c. (0, Y)
    d. (X,Y, Z)
46. Chain matrix multiplication problem can be solved through _____ strategy.
    a. Dynamic programming Page - 85
    b. Greedy
    c. Divide and conquer
    d. Sorting
47. Merge sort have running time....running time of Heap sort. Not found exactly
    a. Greater than
    b. Less than Google
    c. Equal to
    d. Different than

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48. Median is not useful measure of central tendency of given input set especially when the distribution of values is highly skewed.
    a. True
    b. False Page – 34

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49. We do not need to mathematically prove that for comparison-based sorting algorithms always takes Omega nlog (n) time.
    a. True Google & VU Tech (pg 46 not very clear)
    b. False
50. The Omega-notation allows us to state only the asymptotic _____ bounds.
    a. Middle
    b. Lower Page 25
    c. Upper
    d. Both lower & upper
51. Both lower & upperSorting can be in _____
    a. Increasing order only
    b. Decreasing order only
    c. Both Increasing and Decreasing order GOOGLR Search
    d. Random order
52. Radix sort performs sorting the numbers _____ digit (s) at a time.
    a. One Page - 71
    b. Two
    c. Three
    d. All
53. Quicksort is a/an _____ and _____ sorting algorithm.
    a. Not in place, not stable one
    b. In place , not stable one Page - 54
    c. In place , stable one
    d. Not in place , stable one
54. Consider three matrices X,Y,Z of dimensions 1x2, 2x3,3x4 respectively. The number of multiplications of (XY) Z is:
    a. 18 As per lecture slides
    b. 32
    c. 24
    d. 30

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55. In Fibonacci Sequence, unnecessary repetitions do not exist at all.

a. True

b. False **Page – 74**

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56. It is not a Fibonacci sequence . 1,1,1,2,3,5,8,13,21,34,55,…..
    a. True Page - 73
    b. False
57. Heap sort is a/ an _____ and _____ sorting algorithem.
    a. Not in place, not stable one
    b. In place , not stable one Page - 54
    c. In place , stable one
    d. Not in place , stable one
58. Identify the True Statement
    a. The knapsack problem does not belong to the domain of optimization problems.
    b. The knapsack problem belongs to the domain of optimization problems. Page - 91
    c. The Knapsack problem cannot be solved by using dynamic programming
    d. The knapsack problem is optimally solved by using brute force algorithm.
59. In Dynamic Programming, our approach is to _____
    a. Develop the solution in a top-down fashion
    b. Express the problem non-recursively
    c. Build the solution in a bottom-up fashion Page - 75
    d. Input several sub-problems simultaneously
60. Counting sort is suitable to sort the elements in range 1 to K;
    a. K is large
    b. K is small Page - 57
    c. K may be large or small
    d. None
61. We can multiply two matrices A and B only when they are compatible which means
    a. Number of columns in A must be equal to number of rows in B. it seems Correct as per page 84

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b. Number of rows and columns do not matter

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c. Number of columns in A must be equal to number of columns in B
d. Number of rows in A must be equal to number of rows in B

62. Matrix multiplication is a (n) _____ operation.
a. Commutative
b. Associative Page 85
c. Neither commutative nor associative
d. Commutative but not associative

63. In Dynamic Programming approach, solution is modified / changed
a. Always once
b. At each stage google and wikipedia
c. Only for specific problems
d. At 4th stage only

64. In Knapsack problem, the goal is to put items in the Knapsack such that the value of the items is _____ subject to weight limit of knapsack.
a. Minimized
b. Decreased
c. Maximized Page - 91
d. None of the given options

65. An in-place sorting algorithm is one that _____ uses additional array for storage.
a. Always
b. Permanently
c. Does not Page - 54
d. Sometime

66. Memoization is a part of Dynamic Programming Strategy.
a. True Page - 74
b. False

67. If matrix A of dimension 2x4 is multiply with matrix B of dimension 4x3, then the dimension of resultant matrix is Not found exactly
a. 2x4

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b. 4x3

c. 3x4

d. 2x3 It seems correct as per second last Para of page 84

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68. In Dynamic Programming approach, we do not store the solution to each sub-problem in case if it reappears.
    a. True
    b. False Page - 75
69. Dynamic Programming is a problem-solving approach in which___
    a. Problem is solved in Zero time
    b. Solution is developed only at final stage
    c. Both are correct
    d. Both are incorrect google
70. In Fibonacci sequence, each term is calculated by____ previous__ terms.
    a. Subtracting, Two
    b. Adding, Three
    c. Adding, Two Page - 73
    d. Multiplying, Two
71. Selection sort is not an in-place sorting algorithm.
    a. True Page - 54
    b. False
72. If there are θ (n^2) entries in edit distance matrix then the total running time is:
    a. θ (n)
    b. θ (1)
    c. θ (n^2) Page – 84
    d. θ (n logn)
73. The only way to convert a string of i characters into the empty string is with i deletions, represented as
    a. E(0.j) =j
    b. E(i.j) = 1
    c. E(0.i) = j
    d. E (i.0)=I Page - 78
74. Dynamic programming formulation of the matrix chain multiplication problem will store the solutions of each sub problem in an

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a. Array
b. Table Page - 86
c. Variable
d. class

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75. We can use the optimal substructure property to devise a formulation of the edit distance problem.
    a. Selective
    b. Optimum
    c. Iterative
    d. Recursive Page - 78
76. Sorting is performed on the basis of _____.
    a. Computational resources
    b. Asymptotic notation
    c. Summation
    d. Some key value of attribute page- 39
77. In Heap Sort algorithm, we call Build-heap procedure _____.
    a. Only once page 46
    b. Twice
    c. Thrice
    d. As many times as we need
78. Radix sort is not a non-comparative integer sorting algorithm.
    a. True Google Search
    b. False
79. In the statement "output P[1].x, P[1].y", the number of times elements of P are accessed is _____.
    a. 1
    b. 2 page 14
    c. 3
    d. 4
80. The main purpose of mathematical analysis is measuring the _____ required by the algorithm.
    a. Space
    b. Execution time P-13
    c. Inputs & outputs

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d. Execution time and memory

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81. _____ provides us more accurate result when input values are not closer with each other
    a. Average
    b. Median P-34
    c. Mode
    d. Mean
82. The process of _____ ends when you are left with such tiny pieces remaining that it is trivial to solve them.
    a. Brute-force
    b. Plan-sweep
    c. Divide and Conquer P-27
    d. Axis-sweep
83. _____ overcomes the limitations of _____ by working as per positional notations of numbers.
    a. Counting sort, Radix sort
    b. Radix sort, Counting sort P-71
84. Memorization is a part of Dynamic Programming strategy.
    a. True P-74
    b. False
85. Rank of an element can be defined as _____.
    a. One minus the number of elements that are smaller
    b. Two plus the number of elements that are greater
    c. One plus the number of elements that are smaller P-34
    d. Two minus the number of elements that are smaller
86. If the time complexity of an algorithm is given by O (1), then its time complexity would be
    a. Polynomial
    b. Exponential
    c. Constant - Wikipedia
    d. Average

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87. Quick sort is a recursive algorithm.

a. True **Wikipedia ; Google**
b. False

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88. The asymptotic growth of n(n+1)/2 is:
    a. $O(n^2)$ As the n^2 term has the largest contribution, the Big-O complexity is O(n^2)
    b. O(n)
    c. O(n+2)
    d. O(n log n)
89. Approach of solving geometric problems by sweeping a line across the plane is called _____ sweep.
    a. Line
    b. Plane Page 18
    c. Cube
    d. Box
90. As per algorithm of Dynamic Programing, we need to store
    a. First sub-problem only
    b. Best solution only
    c. Intermediate sub-problems Pg:75
    d. Final solution only
91. In Sieve technique, we solve the problem
    a. In recursive manner Pg:34
    b. Non recursively
    c. Using Merge Sort algorithm
    d. Using Brute force technique
92. One of the limitation in 0/1 knapsack is that an item can either be _____ in the bag or not.
    a. Use
    b. Put Pg:91
    c. Move
    d. Store
93. Which one is not passed as parameter in Quick sort algorithm?
    a. End of the array
    b. Middle of the array

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c. Array (containing input elements) **Google**

d. Start of the array

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94. In the analysis of Selection algorithm, we get the convergent _____
    a. Harmonic
    b. Linear
    c. Arithmetic
    d. Geometric Pg:37
95. A Random Access Machine (RAM)is an idealized machine withrandom access memory.
    a. Infinite large Pg:10
    b. 512 MB
    c. 256 MB
    d. 2 GBs
96. While analyzing Selection algorithm, we make a number of passes, in fact it could be as many as
    a. n(n+1)
    b. log(n) Pg:37
    c. n/3
    d. n/4
97. In Random Access Machine (RAM), instructions are executed in
    a. Parallel
    b. Batch
    c. One by One Pg:10
    d. Multiple times
98. In selection problem, the rank of an element will be its _____ position
    a. First
    b. final Pg:34
    c. Second last
    d. Last
99. The worst-case running time of Merge sort is _____ in order to sort an array of n elements.
    a. O(log n)

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b. O(n)

c. O(n log n) page 40 and google

d. O(n)

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100. f(n) and g(n) are asymptotically equivalent. This means that they have essentially the same _____.
   a. Results
   b. Variables
   c. Size
   d. Growth rates P:23

101. An algorithm is a mathematical entity. Which is independent of _____.
   a. Programming language
   b. Machine and Programming language
   c. Compiler and Programming language
   d. Programming language Compiler and Machine P:07

102. In Quick sort algorithm, Pivots form ___
   a. Stack
   b. Queue
   c. Binary Search Tree P:49
   d. Graph

103. Counting sort is suitable for sorting the elements within range 1 to P. where
   a. P is large
   b. P is small P-57
   c. P is very large
   d. P is undetermined

104. In asymptotical analysis of n'(5 2)-3, as n becomes large, the dominant (fastest growing) term is some constant times
   a. n_1
   b. n
   c. n+1
   d. n*n P-23

105. ___ Items are not allowed in the 0/1 knapsack. a. Lighter
   b. Fractional P-91

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c. Whole
d. Weighty

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106. Fibonacci Sequence was named on _____, a famous mathematician in 12th Century.
    a. Fred Brooks
    b. Grady Booch
    c. Leonardo Pisano P-73
    d. Edgar F. Codd
107. In Heap Sort algorithm, we build _____ for ascending sort.
    a. Max heap P-41
    b. Min heap
108. Bubble sort is not an in-place sorting algorithm.
    a. True
    b. False P-54
109. In partition algorithm, the subarray _____ has elements which are greater than pivot element x.
    a. A[p...r]
    b. A[p...q-1]
    c. A[q]
    d. A[q+1..r] P-46
110. In Heap Sort algorithm, if heap property is violated
    a. We call Build heap procedure P-43
    b. We call Heapify procedure
    c. We ignore
    d. Heap property can never be violated
111. _____ is not a characteristic of Random Access Machine.
    a. Single-Processor P-10
    b. Assigning a value to a variable
    c. Locality of reference
    d. Executing an arithmetic instruction
112. The only way to convert an empty string into a sting of j characters is by doing j insertions, represented as _____

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a. E(i,j) = 1
b. E(I,0) = I
c. E(0,j) = j page 78
d. E(1,j)= j

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113. In Selection problem, the Sieve technique works in _____.
    a. Non-recursive manner
    b. Constant time
    c. Phases page 34
    d. One complete go
114. Algorithm is a sequence of computational steps that ---- the input into output.
    a. Merge
    b. Assign
    c. Transform page 7
    d. Integrate
115. If pj dominates pi and pi dominates ph then pj also dominates ph, it means dominance relation is
    a. Transitive page 18
    b. Non Transitive
    c. Equation
    d. Symbolic
116. To find maximal points in brute-force algorithm each point of the space is compared against _____ of that space.
    a. One other point
    b. All other points page 11
    c. Few other points
    d. Most of the other points
117. In the following code the statement "cout<<j;"executes --------- times. for (j=1; j<=5; j = j+2)
        cout<<j;
    a. 5 times
    b. 2 times
    c. 3 times

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d. 0 times

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118. In merge sort algorithm, we split the array around the _____ index q. a. Entring
    b. Mid page 17
    c. Exiting
    d. Summing
119. In Selection problem, the Sieve technique _____.
    a. Add some more input items each time
    b. Do not work recursively
    c. Do not uses Divide and Conquer approach
    d. Eliminates undesired data items each time
120. Consider three matrices X, Y, Z of dimensions 1 x 2, 2 x 3, 3 x 4 respectively. The number of multiplications of X(YZ) is .
    a. 16
    b. 32
    c. 26
    d. 32 page 84
121. In Heap Sort algorithm, the total running time for Heapify procedure is _____
    a. Theta (log n)
    b. Order (log n)
    c. Omega (log n)
    d. O(1) i.e. Constant time
122. The sieve technique works where we have to find_____ items(s) from a large input.
    a. Single page 34
    b. Two
    c. Three
    d. Similar
123. In Dynamic Programming based solution of Knapsack Problem, if we decide to take an object i , then we gain_____

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a. W(Total Weight of Knapsack)
b. V (Total Value of all items)
c. vi (Value of object i) page 93
d. Nome of the given option

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124. While Sorting, the order domain means for any two input elements x and y _____ satisfies only.
   a. x < y page 39
   b. x > y
   c. x = y
   d. All of the above

125. For solving Selection problem, we introduced Sieve technique due to _____
   a. Using Decrease and Conquer strategy page 34
   b. Avoiding to sort all input data
   c. Eliminating Rank of an element
   d. Using Brute-force approach

126. _____ is one of the few problems, where provable lower bounds exist on how fast we can sort.
   a. Searching
   b. Sorting page 38
   c. Both Searching & sorting
   d. Growing

127. In plane sweep approach, a vertical line is swept across the 2d-plane from_____.
   a. Right to Left
   b. Left to Right page 18
   c. Top to Bottom
   d. Bottom to top

128. In generating Fibonacci sequence, we can avoid unnecessary repetitions by _____ process.
   a. Tokenization
   b. Memorization page 43
   c. Randomization
   d. Memorization

129. For _____ values of n, any algorithm is fast enough.

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a. Small page 14
b. Medium
c. Large
d. Infinity

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130. Dynamic programming comprises of _____.
    a. Recursion only
    b. Repetition only
    c. Recursion with Repetition
    d. No Repetition but Recursion page 75

131. The function f(n)=n(logn+1)/2 is asymptotically equalient t nlog n :Here Lower Bound means function f(n) grows asymptotically at __ as fast as nlog n.
    a. Least page 23
    b. Normal
    c. Most
    d. At

132. Counting sort has time complexity.
    a. O(n+k)
    b. O(n) page 58
    c. O(k)
    d. O(nlogn)

133. Due to left complete nature of binary tree, the heap can be stored in
    a. Array page 40
    b. Structures
    c. Link List
    d. Stack

134. Single item from a larger set of _____.
    a. Constant
    b. Pointers
    c. Phases
    d. n items page 34

135. In the clique cover problem, for two vertices to be in the same group, they must be _____ each other.
    a. Apart from
    b. Far from

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c. Near to

d. Adjacent to page 76

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136. How much time merge sort takes for an array of numbers?
    a. T(n^2)
    b. T(n)
    c. T(log n)
    d. T(n log n) page 40

137. In in-place sorting algorithm is one that uses arrays for storage.
    a. No additional array page 54
    b. An additional array
    c. Both of above may be true according to algorithm
    d. More than 3 arrays of one dimension

138. Brute-force algorithm for 2D-Maxima is operated by comparing _____ pairs of points.
    a. Two
    b. Some
    c. Most
    d. All page 18

139. While Sorting, the ordered domain means for any two input elements x and y ____ satisfies only.
    a. x > y
    b. x < y
    c. x = y
    d. All of the above page 38

140. Quick sort is.
    a. Stable & in place
    b. Not stable but in place page 54
    c. Stable but not in place
    d. Some time stable & some times in place

141. Which may be a stable sort?
    a. Merger
    b. Insertion

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c. Both above **page 54**

d. None of the above

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142. For the Sieve Technique we take time.
   a. T(nk) page 34
   b. IT(n / 3)
   c. n^2
   d. n/3

143. Continuation sort is suitable to sort the elements in range 1 to k.
   a. K is Large
   b. K is not known
   c. K may be small or large
   d. K is small page 54

144. Asymptotic growth rate of the function is taken over _____ case running time. .
   a. Best
   b. Worst page 14
   c. Average
   d. Normal

145. The sieve technique is a special case, where the number of sub problems is just.
   a. 5
   b. Many
   c. 1 page 34
   d. Few

146. In Quick sort, we don't have the control over the sizes of recursive calls.
   a. True page 49
   b. False
   c. Less information to decide
   d. Ether true or false

147. Before sweeping a vertical line in plane sweep approach, in start sorting of the points is done in increasing order of their _____ coordinates. .
   a. X page 18

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b. Y
c. Z
d. X , Y

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148. Random access machine or RAM is a/an.
    a. Machine build by Al-Khwarizmi
    b. Mechanical machine
    c. Mathematical model page 10
    d. Electronics machine
149. The Huffman codes provide a method of encoding data inefficiently when coded using ASCII standard.
    a. True
    b. False page 99
150. A heap is a left-complete binary tree that confirms to the _____.
    a. increasing order only
    b. decreasing order only
    c. heap order page 40
    d. log n order
151. If we associate (x, y) integers pair to cars where x is the speed of the car and y is the negation of the price. High y value for a car means a _____ car.
    a. Fast
    b. Slow
    c. Expensive
    d. Cheap
152. Which one of the following sorting algorithms is the fastest?
    a. Merge sort
    b. Quick sort
    c. Insertion sort
    d. Heap sort
153. Quick sort algorithm divide the entire array into _____ sub arrays.
    a. 2
    b. 3
    c. 4
    d. 5
154. In brute force algorithm, we measure running time T(n) based on _____.

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a. Average-case time and best-case time
b. Worst-case time and average-case time page 46
c. Worst-case time and best-case time
d. Best-case time and staring-case time

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155. For 2D Maxima problem. Plane Sweep algorithm first of all _____.
   a. Sorts all points
   b. Delete some points
   c. Output the elements
   d. Pushes all points on stack

156. There are _____ entries in the Edit Distance Matrix
   a. ɵ (n)
   b. ɵ (n_2) page 84
   c. ɵ (n+2)
   d. ɵ (n + 100)

157. Which symbol is used for Omega notation?
   a. (O)
   b. (ɵ)
   c. (Ω)
   d. (@)

158. Selection sort is a _____ sorting algorithm
   a. In-place page 54
   b. Not In-Place
   c. Stable
   d. in-partition

159. In Dynamic Programming based solution of knapsack problem, to compute entries of 'V', we will imply a(n) _____ approach.
   a. Subjective
   b. Inductive
   c. Brute Force
   d. Combination

160. We do not need to prove comparison-based sorting algorithms by mathematically. It always takes _____ time.
   a. Big Oh nlog(n)
   b. Omega nlog(n) NOT SURE

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c. Omega n(n^2)
d. Theta nlog(n)

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161. Merge sort is a/an _____ and _____ sorting algorithm
    a. Not in-place, not stable one
    b. In-place, not stable one
    c. In-place, stable one
    d. Not in-place, stable one page 54
162. Cubic function will _____ a quadratic function.
    a. Prove
    b. be equal to
    c. overtake Page 25
    d. find
163. Insertion sort is a _____ sorting algorithm
    a. Unstable
    b. In-place Page 54
    c. Not In-Place
    d. in-partition
164. To check whether a function grows faster or slower than the other function, we use some asymptotic notations, which is _____.
    a. Big-oh notation
    b. Theta notation
    c. Omega notation
    d. All of the given
165. Asymptotic growth of 8n^2 + 2n – 3 is:
    a. Θ(n^2 + n)
    b. Θ (n^2) page 14
    c. Θ(8n^2)
    d. Θ(8n^2 + 2n)
166. In the analysis of algorithms, _____ plays an important role.
    a. text analysis
    b. time
    c. growth rate

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d. money

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167. In inductive approach of knapsack problem, we consider 2 cases, _____ Or _____.
   a. Median, Mode
   b. Recursive, Iterative
   c. Leave object, Take object page 93
   d. Sequentially. Parallel

168. Random Access Machine (RAM) can execute _____ instructions
   a. only logical
   b. parallel
   c. only arithmetic
   d. logical and arithmetic

169. Using _____ algorithm, efficiency is not given much importance
   a. Greedy
   b. Merge sort
   c. Processing as there is no algorithm by this name
   d. Brute Force

170. Bubble sort takes theta _____ in the worst case
   a. (n2) page 39
   b. (n)
   c. log(n)
   d. nlog(n)

171. If matrix A of dimension p × q is multiply with matrix B of dimension q × r, then dimension of resultant matrix is:
   a. p × q
   b. p × r page 84
   c. q × r
   d. r × p

172. Dynamic Programing algorithms often use some kind of _____ to store the results of intermediate sub-problems
   a. table (Page 75)

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b. variable
c. stack
d. loop

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173. _____ is in-place sorting algorithm.
   a. Bubble sort (Page 54)
   b. Merge sort
   c. Linear search
   d. Binary Search

174. Which one of the following problems can be solved using dynamic problem?
   a. Bubble sort problem
   b. Matrix chain multiplication problem page 85
   c. Greedy search problem
   d. Fractional knapsack problem

175. In chain matrix multiplication, solutions of the sub-problems are stored in a _____.
   a. Array
   b. Table page 86
   c. Tree
   d. Link list

176. What is the average running time of a quick sort algorithm?
   a. O(n^2)
   b. O(n)
   c. O(n log n) (Page 49)
   d. O(log n)

177. Sorting Algorithms having O _____ running time are considered to be slow ones.
   a. (n)
   b. (n^2) (Page 39)
   c. (nlog(n))
   d. (log(n))

178. While solving Selection problem, in Sieve technique we partition input data _____
   a. In increasing order

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b. In decreasing order
c. According to Pivot
d. Randomly

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179. _____ is the process of avoiding unnecessary repetitions by writing down the results of recursive calls and looking them up again if we need them later.
    a. Loop
    b. Memoization page 74
    c. Recursion
    d. Function

180. In average-case time the probability of seeing input is denoted by _____.
    a. p{I}
    b. p[I]
    c. p<i>
    d. p(i) page 13

181. While applying the Sieve technique to selection sort, how to choose a pivot element.
    a. Through mean
    b. Linear
    c. Randomly page 35
    d. Sequentially

182. Number of _____ of the pseudo code are counted to measure the running time.
    a. Inputs
    b. Outputs
    c. Steps page 13
    d. Pages

183. Developing a dynamic programming algorithm generally involves _____ separate steps.
    a. One
    b. Two page 75
    c. Three
    d. Four

184. 8n^2+2n+3 will exceed c28(n), no matter how large we make _____.
    a. n

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b. 2n

c. c2 page 25

d. this quadratic equation

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185. The running time of quick sort algorithm_____.
   a. Is impossible to compute
   b. Has nothing to do with pivot selection
   c. Is Random upon each execution
   d. Greatly influenced by the selection of pivot page 49

186. _____ involves breaking up the problem into sub problems whose solutions can be combined to solve the global problem.
   a. Complexity Theory
   b. Dynamic programming solution
   c. Divide and Conquer Strategy page 34
   d. Greedy Algorithms

187. In _____ we have to find rank of an element from given input.
   a. Merge sort algorithm
   b. Selection problem page 34
   c. Brute force technique
   d. Plane Sweep algorithm

188. How many steps are involved to design the dynamic programming strategy?
   a. 2
   b. 3
   c. 1
   d. 4 page 92

189. In Bucket sort, if there are duplicates then each bin can be replaced by a a. Stack
   b. Linked list page 69
   c. Hash table
   d. Heap

190. In merge sort algorithm, we split the array _____ to find index q.
   a. from start
   b. midway page 28
   c. from end

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d. both from start or end

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191. Find the maximum value of the items which can carry using knapsack Knapsack weight capacity = 50.

Item Weight Value
11070
22020
33080
470 200

a. 280
b. 100
c. 90
d. 200

192. In 2-d maxima problem a point p is said to be dominated by point q if _____.

a. p.x <= q.x
b. p.x <= q.x and p.y <= q.y page 17
c. p.y <= q.y
d. p.x >= q.x and p.y >=q.y

193. Sorting can be in _____.

a. Increasing order only
b. Decreasing order only
c. Both increasing and decreasing order
d. Random order

194. Recurrence can be described in terms of _____.

a. Array
b. Linear
c. Tree page 31
d. Graph

195. The brute-force algorithm for 2D-Maxima runs in order O(__) time.

a. n
b. n(log n)

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c. n*n page 18

d. n3

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196. In plane sweep approach of solving geometric problems, a _____ is swept across the plane.
   a. Line page 18
   b. Plane
   c. Cube
   d. Box

197. Which of the following is calculated with Big Omega notation?
   a. Medium bounds
   b. Upper bounds
   c. Lower bounds Page - 25
   d. Both upper and lower bounds

198. _____ is always based on divide and conquer strategy.
   a. Bubble sort
   b. Selection sort
   c. Pigeon sort
   d. Quick sort page 46

199. If a matrix has three rows and two columns, then dimensions of matrix will be:
   a. 3x2
   b. 2x3
   c. 3x3
   d. 2x2

200. Asymptotic notations are used to describe _____ of an algorithm.
   a. Length
   b. running time google
   c. size
   d. compile time

201. Catalan numbers are related the number of different _____ on 'n' nodes.
   a. Arrays
   b. linked lists

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c. binary trees **page 85**

d. functions

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202. Applying the sieve technique to selection problem, _____ element is picked from array.
   a. Output
   b. Total
   c. Input
   d. Pivot page 35

203. Dynamic Programming approach is usually useful in solving _____ problems.
   a. Normal
   b. Optimization google
   c. Array
   d. Loop

204. In recursive formulation of knapsack Problem: V [0, j] = _____ for j>=0
   a. -1
   b. 0 page 93
   c. 1
   d. 2

205. _____ is a linear time sorting algorithm.
   a. Merge sort
   b. Radix sort page 71
   c. Quick sort
   d. Bubble sort

206. Quick sort is one of the _____ sorting algorithm.
   a. Fastest page 19
   b. Slowest
   c. Major
   d. Average

207. The time assumed for each basic operation to execute on RAM model of computation is _____.

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a. Infinite
b. Continuous
c. Constant page 10
d. Variable

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208. In Sieve Technique, we know the item of interest. a. True
b. False page 34

209. While analyzing algorithms, _____ and _____ usually considered difficult to calculate.
a. Finite, Infinite
b. Floor, ceiling google
c. Row, Column
d. Graph, Tree

210. While analysis of the brute-force maxima algorithm, an array sorted in the reverse order is the type of _____ case input.
a. Best
b. Worst page 14
c. Somewhat bad
d. Average

211. _____ is not useful measure of central tendency of given input set especially when the distribution of values is highly skewed.
a. Mean
b. Mode
c. Average
d. Median page 34

212. In asymptotical analysis of n(n-3) and 4n*n, as n becomes large, the dominant (fastest growing) term is some constant times _____.
a. n+1
b. n-1
c. n
d. n*n page 23

213. In addition to passing in the array itself to Merge Sort algorithm, we will pass in other arguments which are indices.

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a. Two P-38
b. Three
c. Four
d. Five

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214. In 2d-maximal problem, a point is said to be if it is not dominated by any other point in that space.
    a. Member
    b. Minimal
    c. Maximal P-11
    d. Joint

215. Counting sort assumes that the numbers to be sorted are in the range _____.
    a. K to n where n is large
    b. 1 to k where k is small (P-57)
    c. K to n where k is small
    d. k to n where n is small

216. Insertion sort is an efficient algorithm for sorting a _____ number of elements
    a. Large P-39
    b. Small
    c. Extra large
    d. Medium

217. If the indices passed to merge sort algorithm are _____ then this means that there is only one element to sort.
    a. Small page 28
    b. Large
    c. Equal
    d. Not Equal

218. In Knapsack Problem, each item must be entirely accepted or rejected, is called _____ problem.
    a. Fractional
    b. 0-1 P-92
    c. Linear
    d. Optimal

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219. If the time complexity of an algorithm is O(n). then it is called _____ time complexity.
   a. Linear Wikipedia
   b. Constant
   c. Average
   d. Exponential

220. In the case of _____ analysis does not depend upon on the distribution of input.
   a. Merge sort
   b. Quick sort P-50
   c. Insertion sort
   d. Heap sort

221. We can use the _____ Property to devise a recursive formulation of the edit distance problem.
   a. Small substructure
   b. Algorithmic
   c. Real
   d. Optimal substructure page 78

222. The following sequence is called _____
   1,2,3,5,8,13,21,34,55,.....
   a. Optimize sequence
   b. Fibonacci sequence page 73
   c. Optimal sequence
   d. Overlapping sequence

223. Which one sorting algorithm is best suited to sort an array of 2 million elements?
   a. Bubble sort
   b. Insert sort
   c. Merge sort
   d. Quick sort

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e. Ridx Sort page 71

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224. We can improve the performance of quick sort if we could be able to _,_____.
    a. Skip input elements somehow
    b. Select two or more pivots page 34
    c. Skip any sub-array completely
    d. Eliminate recursive calls
225. The problem with the brute-force algorithm is that is uses _____ in pruning out de
    a. Worst-case time
    b. No intelligence page 18
    c. Outside looping
    d. Artificial intelligence
226. In chain matrix multiplication, the order of the matrices _____.
    a. Can be changed
    b. Can not be changed page 85
    c. is equal
    d. is reverse
227. In quick sort algorithm, we choose pivot_____.
    a. Always the smallest element
    b. Greater than 5
    c. Randomly page 35
    d. Less than 5
228. In Heap Sort algorithm. Heapify procedure is _____ in nature.
    a. Recursive
    b. Non-Recursive page 43
    c. Fast
    d. Slow
229. When matrix A of 5x 3 is multiplied with matrix B of 3 x 4 then the number of multiplications required will be _____.
    a. 15

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b. 12
c. 36
d. 60

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230. An algorithm is said to be correct if for every _____ instance, it halts with the correct _____.
   a. Input, Output page 13
   b. Design, Analysis
   c. Value, Key
   d. Key, Analysis

231. In chain matrix multiplication, table is filled _____ to find the multiplication of matrix.
   a. row wise
   b. column wise
   c. diagonally
   d. bottom-to-up page 86

232. If we have an equation 8n2+7f*n + 5f + 6 then is large, _____ term will be muchxxxxxxxthe n term and will dominate the running time.
   a. f g (n)
   b. g (n) * 2
   c. n * 2 page 23
   d. f (n)

233. For quick sort algorithm. Partitioning takes theta _____.
   a. (n)
   b. log(n)
   c. n log (n)
   d. n2log (n)

234. In Heap Sort algorithm, the maximum levels an element can move upward is _____
   a. Theta (log n) page 43
   b. Big-ch (log n)
   c. Omega (log n)
   d. 0 (1) i.e. Constant time

235. _____ programming is essentially recursion without repetition.

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a. Fast
b. Dynamic page 75
c. Array
d. n (log n)

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236. There are no hard formal rules to the syntax of the _____ code.
   a. Basic
   b. Programming
   c. Pseudo
   d. Assembly

237. In Heap Sort algorithm, to remove the maximum element every time.
   a. We call Build-Heap procedure
   b. Heap Sort algorithm terminates without result
   c. We call heapify procedure
   d. Nothing happens

238. Which process is used for avoiding unnecessary repetitions and looking them up again if we need them later.
   a. Greedy Approach
   b. Memoization page 74
   c. Divide and conquer
   d. Recursion

239. The worst-case running time of Quick sort is _____ in order to sort an array of n element.
   a. O(n log n) page 49
   b. O(n)
   c. $O(n^2)$
   d. O(log n)

240. Boolean operation is a _____ operation on an idealized RAM model of computation.
   a. Starting
   b. Basic page 10
   c. Advance
   d. Normal

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241. In chain matrix multiplication, if there are n items, there are _____ ways in which outer most pair of parentheses can placed.
    a. n^2
    b. 2n
    c. n+1
    d. n-1 page 85

242. The number of nodes in a complete binary tree of height h is: a. 2 * (h+1) - 1
    b. 2^(h+1) – 1 page 40
    c. 2 * (h+1)
    d. ((h+1)^2) – 1

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CS502 GRAND QUIZ SOLVED

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1. The sequence of merge sort algorithm is:
   a. Divide Combine-Conquer
   b. Conquer-Divide-Combine
   c. Divide-Conquer-Combine Page 27
   d. Combine-Divide-Conquer

2. In_____Knapsack Problem, limitation is that an item can either be put in thebag or not. Fractional items are not allowed.
   a. 0
   b. 1
   c. 0/1 Page 91
   d. Fractional

3. In Selection algorithm, we assume pivot selection takes theta _____runnin gtime.
   a. n Page - 36
   b. n2
   c. n3
   d. log (n)

4. In Heap Sort algorithm (using max heap), when every time maximum

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elementsremoved from top___.

a. We call merge Sort Algorithm
b. it becomes Order n2 Algorithm
c. Divide and Conquer strategy helps us
d. We are left with a hole Page – 41

5. If matrix A of dimension p x q is multiply with matrix B of dimensionq x r, then each entry in resultant matrix takes _____time.

a. O (q) Page - 84
b. O (1)
c. O (p x q)
d. O (q x r)

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6. _____is a method of solving a problem in which we check all possiblesolutions to the problem to find the solution we need.
   a. Plane-Sweep Algorithm
   b. Sorting Algorithm
   c. Brute-Force Algorithm google
   d. Greedy approach
7. The worst case running time of Quick sort algorithm__.
   a. Cannot be quadratic
   b. Is quadratic google QuickSort and its Analysis (codesdope.com) & http://personal.kent.edu/~rmuhamma/Algorithms/MyAlgorithms/Sorting/quickSort.htm
   c. Is always Exponential
   d. Is linear
8. In max heap (for Heap Sort algorithm), when every time maximum element isremoved from top we replace it with__leaf in the tree.
   a. second last
   b. Last Page -41
   c. First
   d. Any
9. Quick sort algorithm was developed by -
   a. AlferdAho
   b. Sedgewick
   c. John Vincent Atanasoff
   d. Tony Hoare – Google wikipedia
10. If Matrix-A has dimensions "3x2" and Matrix-B has dimensions "2x3", thenmultiplication of Matrix-A and Matrix-B will result a new Matrix-C having dimensions.

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a. 3x2
b. 2x3
c. 2x2
d. 3x3 **http://www.calcul.com/show/calculator/matrix-multiplication**

11. For comparison-based sorting algorithms, it is possible to sort more efficientlythan Omega n log(n) time.
a. Always
b. Not **P-54**
c. Sometimes

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d. Sometimes not

12. Dynamic Programming approach is usually useful in solving optimizationproblems.

a. True https://en.wikipedia.org/wiki/Dynamic_programming & http://web.mit.edu/15.053/www/AMP-Chapter-11.pdf

b. False

13. In Sorting the key value or attribute___from an ordered domain.

a. Must be page 39

b. Not always

c. May be

d. Occasionally

14. Result of asymptotical analysis of n(n -3) and 4n*n is that___

a. n(n-1) is asymptotically Less

b. n(n-1) is asymptotically Greater

c. Both are asymptotically Not equivalent

d. Both are asymptotically Equivalent page 23 ($4n*n= 4n^2$)

15. Floor and ceiling are_____to calculate while analyzing algorithms

a. Very easy

b. Usually considered difficult P-31

c. 3rd Option is missing

d. 4th Option is missing

16. _____of reference is an important fact of current processor technology.

a. Defining

b. Assigning

c. Formality

d. Locality P-8

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17. In max-heap, largest element is stored at root node. Where is the smallestelement stored?
    a. Right Node
    b. Leaf Node
    c. Middle Node
    d. Left Node

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18. In average-case time analysis of Quick sort algorithm, the most balanced casefor partition is when we divide the list of elements into _.
    - **a. Equal no. of pieces as of input elements**
    - b. Single piece exactly
    - c. Two nearly equal pieces
    - d. Three nearly equal pieces
19. Which of the following is calculated with Big O notation?
    - a. Medium bounds
    - **b. Upper bounds Page - 25**
    - c. Lower bounds
    - d. Both upper and lower bounds
20. Edit distance algorithm based on_____strategy
    - a. Greedy
    - **b. Dynamic Programming Page - 81**
    - c. Divide and Conquer
    - d. Searching
21. In Heapsort Algorithm, total time taken by heapify procedure is____
    - **a. O (log n) Page-43**
    - b. O (log2 n)
    - c. O (n log n)
    - d. O (n2 log n)
22. Al-Khwarizmi was a/an _____
    - a. Artist
    - **b. Mathematician P-7**
    - c. Astronomer

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d. Khalifah

23. When matrix A of 5x3is multiply with metric B of 3x4 then the number ofmultiplication required is: Not found exactly

a. 15
b. 12
c. 36
d. 60 Not Found exactly but as per formula at page 84,

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24. Pseudo code of algorithms are to be read by ___.
    a. **People** **Page -12**
    b. RAM
    c. Computer
    d. Compiler
25. The sieve technique is a special case, where the number of sub-problems is Just _____
    a. **1** **P-34**
    b. 2
    c. 3
    d. 4
26. When a recursive algorithm revisits the same problem over and over again, wesay that the optimization problem has_sub-problems.
    a. **Overlapping** **– Google Search**
    b. Over costing
    c. Optimized
    d. Three
27. Sieve technique is very important special case of Divide-and-Conquer strategy.
    a. **True** **P-34**
    b. False
28. In order to say anything meaningful about our algorithms, it will be importantfor us to settle on a__.
    a. Java Program
    b. C++ Program
    c. Pseudo program

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d. Mathematical model of computation P-10

29. Merge sort is based on_____.

a. Brute-force
b. Plan-sweep
c. Axis-sweep
d. Divide and Conquer P-27

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30. What time does Merge Sort algorithm take in order to sort an array of 'n'numbers?
    a. Θ(n)
    b. Θ (log n)
    c. Θ (n^2)
    d. Θ (n log n) Google Search
31. In Heap Sort algorithm, the first step is to _____.
    a. Call Build-Heap procedure Page - 46
    b. Sort the array in descending order
    c. Call Heapify procedure
    d. Find the number of input elements
32. The definition of theta-notation relies on proving_____asymptotic bound.
    a. One
    b. Lower
    c. Upper
    d. Both lower & upper Page - 25
33. In merge sort algorithm, to merge two lists of size n/2 to a list of size n, takes _____time.
    a. Theta (n) Page - 32
    b. Theta log(n)
    c. Theta log2(n)
    d. Theta n log(n)
34. We can make _____recursive calls in Fibonacci Sequence.
    a. Infinite
    b. Finite google

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c. Only one
d. Zero

35. Following is NOT the application of Edit Distance problem.

a. Speech recognition
b. Spelling Correction
c. Ascending Sort **Page - 76**
d. Computational Molecular Biology

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36. In plane sweep approach, a vertical line is swept across the 2d-plane and structure is used for holding the maximal points lying to the left of the sweepline.
    a. Array
    b. Queue
    c. Stack Page - 18
    d. Tree
37. When a heapify procedure is applied to the root node to restore the heap, thenat each level, the comparison performed takes time:
    a. It will take (log n).
    b. It can not be predicted
    c. It will take O (1). Page - 43
    d. Time will vary according to the nature of input data.
38. _____time is the maximum running time over all legal inputs.
    a. Worst-case Page - 13
    b. Average-case
    c. Best-case
    d. Good-case
39. Efficient algorithm requires less computational...
    a. Memory
    b. Running Time
    c. Memory and Running Time Page - 9
    d. Energy
40. For average-case time analysis of Quick sort algorithm, Pivot selection is onaverage basis from _____
    a. half of the input values
    b. all possible random values Page - 50

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c. Pivot is input separately

d. values greater than 5

41. Selection algorithm takes theta ____

a. (n2)

b. (n) **Page - 37**

c. log(n)

d. n log(n)

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42. Recurrence can be described in terms of a tree.
    a. Yes Page - 31
    b. No
43. Time complexity of Dynamic Programming based algorithm for computing theminimum cost of Chain Matrix Multiplication is ____
    a. Log n
    b. n
    c. n^2 (n square)
    d. n^3 (n cube) Page -90
44. The Iteration method is used for ___
    a. Comparing sorting algorithms only
    b. Solving Recurrence relations Page 31
    c. Merging elements in Merge sort
    d. Dividing elements in Merge sort
45. In 3-Dimensional space, a point P has___coordinate(s).
    a. (X, Y)
    b. (X, 0)
    c. (0, Y)
    d. (X,Y, Z)
46. Chain matrix multiplication problem can be solved through___strategy.
    a. Dynamic programming Page - 85
    b. Greedy
    c. Divide and conquer
    d. Sorting
47. Merge sort have running time running time of Heap sort. Not found exactly

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a. Greater than

b. Less than Google

c. Equal to

d. Different than

48. Median is not useful measure of central tendency of given input set especiallywhen the distribution of values is highly skewed.

a. True

b. False Page – 34

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49. We do not need to mathematically prove that for comparison-based sortingalgorithms always takes Omega nlog (n) time.
    a. True Google & VU Tech (pg 46 not very clear)
    b. False
50. The Omega-notation allows us to state only the asymptotic__bounds.
    a. Middle
    b. Lower Page 25
    c. Upper
    d. Both lower & upper
51. Both lower & upperSorting can be in _____
    a. Increasing order only
    b. Decreasing order only
    c. Both Increasing and Decreasing order – (My opinion)
    d. Random order
52. Radix sort performs sorting the numbers____digit (s) at a time.
    a. One Page - 71
    b. Two
    c. Three
    d. All
53. Quicksort is a/an_____and_____sorting algorithm.
    a. Not in place, not stable one
    b. In place , not stable one Page - 54
    c. In place , stable one
    d. Not in place , stable one
54. Consider three matrices X,Y,Z of dimensions 1x2, 2x3,3x4 respectively. Thenumber of multiplications of (XY) Z is:

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a. **18 As per lecture slides**
b. 32
c. 24
d. 30

55. In Fibonacci Sequence, unnecessary repetitions do not exist at all.

a. True
b. **False Page – 74**

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56. It is not a Fibonacci sequence . 1,1,1,2,3,5,8,13,21,34,55,…..

a. True Page - 73

b. False

57. Heap sort is a/ an_____and_____ sorting algorithem.

a. Not in place, not stable one

b. In place , not stable one Page - 54

c. In place , stable one

d. Not in place , stable one

58. Identify the True Statement

a. The knapsack problem does not belong to the domain of optimizationproblems.

b. The knapsack problem belongs to the domain of optimization problems. Page - 91

c. The Knapsack problem cannot be solved by using dynamicprogramming

d. The knapsack problem is optimally solved by using brute forcealgorithm.

59. In Dynamic Programming, our approach is to _____

a. Develop the solution in a top-down fashion

b. Express the problem non-recursively

c. Build the solution in a bottom-up fashion Page - 75

d. Input several sub-problems simultaneously

60. Counting sort is suitable to sort the elements in range 1 to K;

a. K is large

b. K is small Page - 57

c. K may be large or small

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d. None

61. We can multiply two matrices A and B only when they are compatible whichmeans

a. Number of columns in A must be equal to number of rows in B.it seems Correct as per page 84 (https://www.varsitytutors.com/hotmath/hotmath_help/topics/matrix-multiplication#:~:text=You%20can%20only%20multiply%20two,an%20m%C3%97p%20mat rix.)

b. Number of rows and columns do not matter

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c. Number of columns in A must be equal to number of columns in B
d. Number of rows in A must be equal to number of rows in B

62. Matrix multiplication is a (n)_____operation.
a. Commutative
b. Associative Page 85
c. Neither commutative nor associative
d. Commutative but not associative

63. In Dynamic Programming approach, solution is modified / changed
a. Always once
b. At each stage google and wikipedia
c. Only for specific problems
d. At 4th stage only

64. In Knapsack problem, the goal is to put items in the Knapsack such that thevalue of the items is __subject to weight limit of knapsack.
a. Minimized
b. Decreased
c. Maximized Page - 91
d. None of the given options

65. An in-place sorting algorithm is one that_____uses additional array forstorage.
a. Always
b. Permanently
c. Does not Page - 54
d. Sometime

66. Memoization is a part of Dynamic Programming Strategy.
a. True Page - 74

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b. False

67. If matrix A of dimension 2x4 is multiply with matrix B of dimension 4x3, thenthe dimension of resultant matrix is Not found exactly

a. 2x4
b. 4x3
c. 3x4
d. 2x3 It seems correct as per second last Para of page 84

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68. In Dynamic Programming approach, we do not store the solution to each sub-problem in case if it reappears.
    a. True
    b. False Page - 75
69. Dynamic Programming is a problem-solving approach in which___
    a. Problem is solved in Zero time
    b. Solution is developed only at final stage
    c. Both are correct
    d. Both are incorrect google
70. In Fibonacci sequence, each term is calculated by_____ previous _____terms.
    a. Subtracting, Two
    b. Adding, Three
    c. Adding, Two Page - 73
    d. Multiplying, Two
71. Selection sort is not an in-place sorting algorithm.
    a. True Page - 54
    b. False
72. If there are θ (n^2) entries in edit distance matrix then the total running time is:
    a. θ (n)
    b. θ (1)
    c. **θ (n^2) Page – 84**
    d. θ (n logn)
73. The only way to convert a string of i characters into the empty string is with ideletions, represented as
    a. E(0.j) =j

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b. E(i.j) = 1
c. E(0.i) = j
d. E (i.0)=I Page - 78

74. Dynamic programming formulation of the matrix chain multiplication problemwill store the solutions of each sub problem in an
a. Array
b. Table Page - 86
c. Variable
d. class

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75. We can use the optimal substructure property to devise a formulation of theedit distance problem.
    a. Selective
    b. Optimum
    c. Iterative
    d. Recursive Page - 78
76. Sorting is performed on the basis of_____.
    a. Computational resources
    b. Asymptotic notation
    c. Summation
    d. Some key value of attribute page- 39
77. In Heap Sort algorithm, we call Build-heap procedure_____.
    a. Only once page 46
    b. Twice
    c. Thrice
    d. As many times as we need
78. Radix sort is not a non-comparative integer sorting algorithm.
    a. True Google Search
    b. False
79. In the statement “output P[1].x, P[1].y”, the number of times elements of P areaccessed is_____.
    a. 1
    b. 2 page 14
    c. 3
    d. 4
80. The main purpose of mathematical analysis is measuring the

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_____require dby the algorithm.

a. Space

b. Execution time P-13

c. Inputs & outputs

d. Execution time and memory

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81. _____provides us more accurate result when input values are not closer witheach other
    a. Average
    b. Median P-34
    c. Mode
    d. Mean
82. The process of_____ends when you are left with such tiny pieces remainingthat it is trivial to solve them.
    a. Brute-force
    b. Plan-sweep
    c. Divide and Conquer P-27
    d. Axis-sweep
83. _____overcomes the limitations of_____by working as perpositional notations of numbers.
    a. Counting sort, Radix sort
    b. Radix sort, Counting sort P-71
84. Memorization is a part of Dynamic Programming strategy.
    a. True P-74
    b. False
85. Rank of an element can be defined as _____.
    a. One minus the number of elements that are smaller
    b. Two plus the number of elements that are greater
    c. One plus the number of elements that are smaller P-34
    d. Two minus the number of elements that are smaller
86. If the time complexity of an algorithm is given by O (1), then its timecomplexity would be

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a. Polynomial
b. Exponential
c. Constant - Wikipedia
d. Average

87. Quick sort is a recursive algorithm.

a. True Wikipedia ; Google
b. False

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88. The asymptotic growth of n(n+1)/2 is:
    - **a. $O(n^2)$** **As the n^2 term has the largest contribution, the Big-O complexity is O(n^2)**
    - b. O(n)
    - c. O(n+2)
    - d. O(n log n)

89. Approach of solving geometric problems by sweeping a line across the plane iscalled____sweep.
    - a. Line
    - **b. Plane Page 18**
    - c. Cube
    - d. Box

90. As per algorithm of Dynamic Programing, we need to store
    - a. First sub-problem only
    - b. Best solution only
    - **c. Intermediate sub-problems Pg:75**
    - d. Final solution only

91. In Sieve technique, we solve the problem
    - **a. In recursive manner Pg:34**
    - b. Non recursively
    - c. Using Merge Sort algorithm
    - d. Using Brute force technique

92. One of the limitation in 0/1 knapsack is that an item can either be___ inthe bag or not.
    - a. Use
    - **b. Put Pg:91**
    - c. Move

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d. Store

93. Which one is not passed as parameter in Quick sort algorithm?

a. End of the array
b. Middle of the array
c. Array (containing input elements) Google
d. Start of the array

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94. In the analysis of Selection algorithm, we get the convergent ____
    a. Harmonic
    b. Linear
    c. Arithmetic
    d. Geometric Pg:37

95. A Random Access Machine (RAM)is an idealized machine withrandom accessmemory.
    a. Infinite large Pg:10
    b. 512 MB
    c. 256 MB
    d. 2 GBs

96. While analyzing Selection algorithm, we make a number of passes, in fact itcould be as many as
    a. n(n+1)
    b. log(n) Pg:37
    c. n/3
    d. n/4

97. In Random Access Machine (RAM), instructions are executed in
    a. Parallel
    b. Batch
    c. One by One Pg:10
    d. Multiple times

98. In selection problem, the rank of an element will be its____ position
    a. First
    b. final Pg:34
    c. Second last

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d. Last

99. The worst-case running time of Merge sort is __in order to sort an array ofn elements.

a. O(log n)

b. O(n)

c. O(n log n) page 40 and google

d. O(n)

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100. f(n) and g(n) are asymptotically equivalent. This means that they haveessentially the same_.
   a. Results
   b. Variables
   c. Size
   d. Growth rates P:23

101. An algorithm is a mathematical entity. Which is independent of___.
   a. Programming language
   b. Machine and Programming language
   c. Compiler and Programming language
   d. Programming language Compiler and Machine P:07

102. In Quick sort algorithm, Pivots form__
   a. Stack
   b. Queue
   c. Binary Search Tree P:49
   d. Graph

103. Counting sort is suitable for sorting the elements within range 1 to P. where
   a. P is large
   b. P is small P-57
   c. P is very large
   d. P is undetermined

104. In asymptotical analysis of n'(5 2)-3, as n becomes large, the dominant(fastest growing) term is some constant times
   a. n_1
   b. n

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c. n+1

d. n*n P-23

105. ___Items are not allowed in the 0/1 knapsack.

a. Lighter

b. Fractional P-91

c. Whole

d. Weighty

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106. Fibonacci Sequence was named on___, a famous mathematician in 12thCentury.
   a. Fred Brooks
   b. Grady Booch
   c. Leonardo Pisano P-73
   d. Edgar F. Codd

107. In Heap Sort algorithm, we build____for ascending sort.
   a. Max heap P-41
   b. Min heap

108. Bubble sort is not an in-place sorting algorithm.
   a. True
   b. False P-54

109. In partition algorithm, the subarray ____has elements which are greaterthan pivot element x.
   a. A[p...r]
   b. A[p...q-1]
   c. A[q]
   d. A[q+1..r] P-46

110. In Heap Sort algorithm, if heap property is violated
   a. We call Build heap procedure P-43
   b. We call Heapify procedure
   c. We ignore
   d. Heap property can never be violated

111. _____is not a characteristic of Random Access Machine.
   a. Single-Processor P-10
   b. Assigning a value to a variable

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c. Locality of reference
d. Executing an arithmetic instruction

112. The only way to convert an empty string into a sting of j characters is bydoing j insertions, represented as ___

a. E(i,j) = 1
b. E(I,0) = I
c. E(0,j) = j page 78
d. E(1,j)= j

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113. In Selection problem, the Sieve technique works in_____.
   a. Non-recursive manner
   b. Constant time
   c. Phases page 34
   d. One complete go

114. Algorithm is a sequence of computational steps that the input into output.
   a. Merge
   b. Assign
   c. Transform page 7
   d. Integrate

115. If pj dominates pi and pi dominates ph then pj also dominates ph, it meansdominance relation is
   a. Transitive page 18
   b. Non Transitive
   c. Equation
   d. Symbolic

116. To find maximal points in brute-force algorithm each point of the space iscompared against___of that space.
   a. One other point
   b. All other points page 11
   c. Few other points
   d. Most of the other points

117. In the following code the statement “cout<<j;”executes ----times.
   for (j=1; j<=5; j =

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j+2)cout<<j;

a. 5 times
b. 2 times
c. 3 times
d. 0 times

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118. In merge sort algorithm, we split the array around the___index q.
    a. Entrin
    gb.**Mid page 17**
    c. Exiting
    d. Summing
119. In Selection problem, the Sieve technique_____.
    a. Add some more input items each time
    b. Do not work recursively
    c. Do not uses Divide and Conquer approach
    d. Eliminates undesired data items each time
120. Consider three matrices X, Y, Z of dimensions 1 x 2, 2 x 3, 3 x 4respectively. The number of multiplications of X(YZ) is .
    a. 16
    b. 32
    c. 26
    d. 32 page 84
121. In Heap Sort algorithm, the total running time for Heapify procedure is _____
    a. Theta (log n)
    b. Order (log n)
    c. Omega (log n)
    d. O(1) i.e. Constant time
122. The sieve technique works where we have to find___items(s) from alarge input.
    a. Single page 34
    b. Two

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c. Three
d. Similar

123. In Dynamic Programming based solution of Knapsack Problem, if we decideto take an object i , then we gain___

a. W(Total Weight of Knapsack)
b. V (Total Value of all items)
c. vi (Value of object i) page 93
d. Nome of the given option

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124. While Sorting, the order domain means for any two input elements x and y _____satisfies only.
   a. x < y page 39
   b. x > y
   c. x = y
   d. All of the above

125. For solving Selection problem, we introduced Sieve technique due to _____
   a. Using Decrease and Conquer strategy page 34
   b. Avoiding to sort all input data
   c. Eliminating Rank of an element
   d. Using Brute-force approach

126. _____ is one of the few problems, where provable lower bounds exist onhow fast we can sort.
   a. Searching
   b. Sorting page 38
   c. Both Searching & sorting
   d. Growing

127. In plane sweep approach, a vertical line is swept across the 2d-planefrom_____.
   a. Right to Left
   b. Left to Right page 18
   c. Top to Bottom
   d. Bottom to top

128. In generating Fibonacci sequence, we can avoid unnecessary repetitions by

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_____process.

a. Tokenization
b. Memorization page 43
c. Randomization
d. Memorization

129. For_____values of n, any algorithm is fast enough.

a. Small page 14
b. Medium
c. Large
d. Infinity

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130. Dynamic programming comprises of_____.
   a. Recursion only
   b. Repetition only
   c. Recursion with Repetition
   d. No Repetition but Recursion page 75

131. The function f(n)=n(logn+1)/2 is asymptotically equalient t nlog n :Here Lower Bound means function f(n) grows asymptotically at__as fast as nlog n.
   a. Least page 23
   b. Normal
   c. Most
   d. At

132. Counting sort has time complexity.
   a. O(n+k)
   b. O(n) page 58
   c. O(k)
   d. O(nlogn)

133. Due to left complete nature of binary tree, the heap can be stored in
   a. Array page 40
   b. Structures
   c. Link List
   d. Stack

134. Single item from a larger set of_____.
   a. Constant
   b. Pointers
   c. Phases
   d. n items page 34

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135. In the clique cover problem, for two vertices to be in the same group, theymust be_____each other.

a. Apart from
b. Far from
c. Near to
d. Adjacent to page 76

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136. How much time merge sort takes for an array of numbers?
   a. T(n^2)
   b. T(n)
   c. T(log n)
   d. T(n log n) page 40

137. In in-place sorting algorithm is one that uses arrays for storage.
   a. No additional array page 54
   b. An additional array
   c. Both of above may be true according to algorithm
   d. More than 3 arrays of one dimension

138. Brute-force algorithm for 2D-Maxima is operated by comparing __ pairs of points.
   a. Two
   b. Some
   c. Most
   d. All page 18

139. While Sorting, the ordered domain means for any two input elements x andy _satisfies only.
   a. x > y
   b. x < y
   c. x = y
   d. All of the above page 38

140. Quick sort is.
   a. Stable & in place
   b. Not stable but in place page 54
   c. Stable but not in place

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d. Some time stable & some times in place

141. Which may be a stable sort?

a. Merger

b. Insertion

c. Both above page 54

d. None of the above

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142. For the Sieve Technique we take time.
   a. T(nk) page 34
   b. IT(n / 3)
   c. n^2
   d. n/3
143. Continuation sort is suitable to sort the elements in range 1 to k.
   a. K is Large
   b. K is not known
   c. K may be small or large
   d. K is small page 54
144. Asymptotic growth rate of the function is taken over___case runningtime. .
   a. Best
   b. Worst page 14
   c. Average
   d. Normal
145. The sieve technique is a special case, where the number of sub problems isjust.
   a. 5
   b. Many
   c. 1 page 34
   d. Few
146. In Quick sort, we don't have the control over the sizes of recursive calls.
   a. True page 49
   b. False

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c. Less information to decide
d. Ether true or false

147. Before sweeping a vertical line in plane sweep approach, in start sorting ofthe points is done in increasing order of their coordinates. .

a. X page 18
b. Y
c. Z
d. X , Y

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148. Random access machine or RAM is a/an.
   a. Machine build by Al-Khwarizmi
   b. Mechanical machine
   c. Mathematical model page 10
   d. Electronics machine

149. The Huffman codes provide a method of encoding data inefficiently whencoded using ASCII standard.
   a. True
   b. False page 99

150. A heap is a left-complete binary tree that confirms to the____.
   a. increasing order only
   b. decreasing order only
   c. heap order page 40
   d. log n order

151. If we associate (x, y) integers pair to cars where x is the speed of the car andy is the negation of the price. High y value for a car means a ___ car.
   a. Fast
   b. Slow
   c. Expensive
   d. Cheap

152. Which one of the following sorting algorithms is the fastest?
   a. Merge sort
   b. Quick sort
   c. Insertion sort
   d. Heap sort

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153. Quick sort algorithm divide the entire array into_____sub arrays.
   a. **2**
   b. 3
   c. 4
   d. 5

154. In brute force algorithm, we measure running time T(n) based on___.
   a. Average-case time and best-case time
   b. Worst-case time and average-case time page 46
   c. Worst-case time and best-case time
   d. Best-case time and staring-case time

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155. For 2D Maxima problem. Plane Sweep algorithm first of all_____.
   a. Sorts all points
   b. Delete some points
   c. Output the elements
   d. Pushes all points on stack

156. There are_____ entries in the Edit Distance Matrix
   a. ɵ (n)
   b. ɵ (n^2) page 84
   c. ɵ (n+2)
   d. ɵ (n + 100)

157. Which symbol is used for Omega notation?
   a. (O)
   b. (ɵ)
   c. (Ω)
   d. (@)

158. Selection sort is a_____sorting algorithm
   a. In-place page 54
   b. Not In-Place
   c. Stable
   d. in-partition

159. In Dynamic Programming based solution of knapsack problem, to computeentries of 'V', we will imply a(n)__approach.
   a. Subjective
   b. Inductive
   c. Brute Force
   d. Combination

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160. We do not need to prove comparison-based sorting algorithms bymathematically. It always takes __time.

a. Big Oh nlog(n)
b. Omega nlog(n)
c. Omega n(n^2)
d. Theta nlog(n)

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161. Merge sort is a/an_____and_____sorting algorithm
   a. Not in-place, not stable one
   b. In-place, not stable one
   c. In-place, stable one
   d. Not in-place, stable one page 54

162. Cubic function will_____a quadratic function.
   a. Prove
   b. be equal to
   c. overtake Page 25
   d. find

163. Insertion sort is a_____sorting algorithm
   a. Unstable
   b. In-place Page 54
   c. Not In-Place
   d. in-partition

164. To check whether a function grows faster or slower than the other function,we use some asymptotic notations, which is_.
   a. Big-oh notation
   b. Theta notation
   c. Omega notation
   d. All of the given

165. Asymptotic growth of 8n^2 + 2n – 3 is:
   a. Θ(n^2 + n)
   b. Θ (n^2) page 14
   c. Θ(8n^2)
   d. Θ(8n^2 + 2n)

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166. In the analysis of algorithms, _____plays an important role.

a. text analysis

b. time

c. growth rate

d. money

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167. In inductive approach of knapsack problem, we consider 2 cases, ___Or _____.
   a. Median, Mode
   b. Recursive, Iterative
   c. Leave object, Take object page 93
   d. Sequentially. Parallel

168. Random Access Machine (RAM) can execute_____instructions
   a. only logical
   b. parallel
   c. only arithmetic
   d. logical and arithmetic

169. Using _____algorithm, efficiency is not given much importance
   a. Greedy
   b. Merge sort
   c. Processing as there is no algorithm by this name
   d. Brute Force

170. Bubble sort takes theta _____in the worst case
   a. (n2) page 39
   b. (n)
   c. log(n)
   d. nlog(n)

171. If matrix A of dimension p × q is multiply with matrix B of dimension q × r,then dimension of resultant matrix is:
   a. p × q
   b. p × r page 84
   c. q × r

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d. r × p

172. Dynamic Programing algorithms often use some kind of____ to storethe results of intermediate sub-problems

a. table (Page 75)
b. variable
c. stack
d. loop

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173. _____is in-place sorting algorithm.
   a. Bubble sort (Page 54)
   b. Merge sort
   c. Linear search
   d. Binary Search

174. Which one of the following problems can be solved using dynamic problem?
   a. Bubble sort problem
   b. Matrix chain multiplication problem page 85
   c. Greedy search problem
   d. Fractional knapsack problem

175. In chain matrix multiplication, solutions of the sub-problems are stored in a _____.
   a. Array
   b. Table page 86
   c. Tree
   d. Link list

176. What is the average running time of a quick sort algorithm?
   a. O(n^2)
   b. O(n)
   c. O(n log n) (Page 49)
   d. O(log n)

177. Sorting Algorithms having O_____running time are considered to beslow ones.
   a. (n)

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b. (n^2) (Page 39)

c. (nlog(n))

d. (log(n))

178. While solving Selection problem, in Sieve technique we partition input data

_____

a. In increasing order

b. In decreasing order

c. According to Pivot

d. Randomly

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179. _____ is the process of avoiding unnecessary repetitions by writing down the results of recursive calls and looking them up again if we need themlater.
   a. Loop
   b. Memoization page 74
   c. Recursion
   d. Function

180. In average-case time the probability of seeing input is denoted by _.
   a. p{I}
   b. p[I]
   c. p<i>
   d. p(i) page 13

181. While applying the Sieve technique to selection sort, how to choose a pivotelement.
   a. Through mean
   b. Linear
   c. Randomly page 35
   d. Sequentially

182. Number of_____of the pseudo code are counted to measure the runningtime.
   a. Inputs
   b. Outputs
   c. Steps page 13
   d. Pages

183. Developing a dynamic programming algorithm generally involves __ separate steps.

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a. One
b. Two page 75
c. Three
d. Four

184. 8n^2+2n+3 will exceed c28(n), no matter how large we make_.
a. n
b. 2n
c. c2 page 25
d. this quadratic equation

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185. The running time of quick sort algorithm_____.
   a. Is impossible to compute
   b. Has nothing to do with pivot selection
   c. Is Random upon each execution
   d. Greatly influenced by the selection of pivot page 49

186. _____involves breaking up the problem into sub problems whosesolutions can be combined to solve the global problem.
   a. Complexity Theory
   b. Dynamic programming solution
   c. Divide and Conquer Strategy page 34
   d. Greedy Algorithms

187. In_____we have to find rank of an element from given input.
   a. Merge sort algorithm
   b. Selection problem page 34
   c. Brute force technique
   d. Plane Sweep algorithm

188. How many steps are involved to design the dynamic programming strategy?
   a. 2
   b. 3
   c. 1
   d. 4 page 92

189. In Bucket sort, if there are duplicates then each bin can be replaced by a
   a. Stack
   b. Linked list page 69

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c. Hash table
d. Heap

190. In merge sort algorithm, we split the array ___to find index q.
a. from start
b. midway page 28
c. from end
d. both from start or end

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191. Find the maximum value of the items which can carry using knapsackKnapsack weight capacity = 50.

Item	Weight	Value
1	10	70
2	20	20
3	30	80
4	70	200

a. 280
b. 100
c. 90
d. 200

192. In 2-d maxima problem a point p is said to be dominated by point q if _____.

a. p.x <= q.x
b. p.x <= q.x and p.y <= q.y page 17
c. p.y <= q.y
d. p.x >= q.x and p.y >=q.y

193. Sorting can be in _____.

a. Increasing order only
b. Decreasing order only
c. Both increasing and decreasing order
d. Random order

194. Recurrence can be described in terms of_____.

a. Array
b. Linear

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c. Tree page 31

d. Graph

195. The brute-force algorithm for 2D-Maxima runs in order O() time.

a. n

b. n(log n)

c. n*n page 18

d. n3

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196. In plane sweep approach of solving geometric problems, a____ isswept across the plane.

a. Line page 18

b. Plane

c. Cube

d. Box

197. Which of the following is calculated with Big Omega notation?

a. Medium bounds

b. Upper bounds

c. Lower bounds Page - 25

d. Both upper and lower bounds

198. _____is always based on divide and conquer strategy.

a. Bubble sort

b. Selection sort

c. Pigeon sort

d. Quick sort page 46

199. If a matrix has three rows and two columns, then dimensions of matrix willbe:

a. 3x2

b. 2x3

c. 3x3

d. 2x2

200. Asymptotic notations are used to describe ___of an algorithm.

a. Length

b. running time google

c. size

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d. compile time

201. Catalan numbers are related the number of different __on ‘n’ nodes.

a. Arrays
b. linked lists
c. binary trees page 85
d. functions

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202. Applying the sieve technique to selection problem,_____element ispicked from array.
   a. Output
   b. Total
   c. Input
   d. Pivot page 35

203. Dynamic Programming approach is usually useful in solving ___problems.
   a. Normal
   b. Optimization google
   c. Array
   d. Loop

204. In recursive formulation of knapsack Problem:
V [0, j] =_____for j>=0
   a. -1
   b. 0 page 93
   c. 1
   d. 2

205. _____ is a linear time sorting algorithm.
   a. Merge sort
   b. Radix sort page 71
   c. Quick sort
   d. Bubble sort

206. Quick sort is one of the____sorting algorithm.
   a. Fastest page 19
   b. Slowest

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c. Major
d. Average

207. The time assumed for each basic operation to execute on RAM model ofcomputation is_.

a. Infinite
b. Continuous
c. Constant page 10
d. Variable

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208. In Sieve Technique, we know the item of interest.
    a. True
    b. False page 34

209. While analyzing algorithms, _____and _____usually considereddifficult to calculate.
    a. Finite, Infinite
    b. Floor, ceiling google
    c. Row, Column
    d. Graph, Tree

210. While analysis of the brute-force maxima algorithm, an array sorted in thereverse order is the type of__case input.
    a. Best
    b. Worst page 14
    c. Somewhat bad
    d. Average

211. _____is not useful measure of central tendency of given input setespecially when the distribution of values is highly skewed.
    a. Mean
    b. Mode
    c. Average
    d. Median page 34

212. In asymptotical analysis of n(n-3) and 4n*n, as n becomes large, thedominant (fastest growing) term is some constant times _____.

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a. n+1
b. n-1
c. n
d. n*n page 23

213. In addition to passing in the array itself to Merge Sort algorithm, we willpass in other arguments which are indices.

a. **Two P-38**
b. Three
c. Four
d. Five

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214. In 2d-maximal problem, a point is said to be if it is not dominated by anyother point in that space.
   a. Member
   b. Minimal
   c. Maximal P-11
   d. Joint

215. Counting sort assumes that the numbers to be sorted are in the range _____.
   a. K to n where n is large
   b. 1 to k where k is small (P-57)
   c. K to n where k is small
   d. k to n where n is small

216. Insertion sort is an efficient algorithm for sorting a_____number ofelements
   a. Large P-39
   b. Small
   c. Extra large
   d. Medium

217. If the indices passed to merge sort algorithm are_____then this meansthat there is only one element to sort.
   a. Small page 28
   b. Large
   c. Equal
   d. Not Equal

218. In Knapsack Problem, each item must be entirely accepted or rejected, iscalled__problem.

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a. Fractional
b. 0-1 P-92
c. Linear
d. Optimal

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219. If the time complexity of an algorithm is O(n). then it is called _ __timecomplexity.

a. **Linear Wikipedia**
b. Constant
c. Average
d. Exponential

220. In the case of_____analysis does not depend upon on the distributionof input.

a. Merge sort
b. Quick sort P-50
c. Insertion sort
d. Heap sort

221. We can use the_____Property to devise a recursive formulation ofthe edit distance problem.

a. Small substructure
b. Algorithmic
c. Real
d. Optimal substructure page 78

222. The following sequence is called _____ 1,2,3,5,8,13,21,34,55,…..

a. Optimize sequence
b. Fibonacci sequence page 73
c. Optimal sequence
d. Overlapping sequence

223. Which one sorting algorithm is best suited to sort an array of 2 millionelements?

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a. Bubble sort
b. Insert sort
c. Merge sort
d. Quick sort
e. Ridx Sort page 71

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224. We can improve the performance of quick sort if we could be able to _'_____.
   a. Skip input elements somehow
   b. Select two or more pivots page 34
   c. Skip any sub-array completely
   d. Eliminate recursive calls

225. The problem with the brute-force algorithm is that is uses____in pruning out de
   a. Worst-case time
   b. No intelligence page 18
   c. Outside looping
   d. Artificial intelligence

226. In chain matrix multiplication, the order of the matrices_____.
   a. Can be changed
   b. Can not be changed page 85
   c. is equal
   d. is reverse

227. In quick sort algorithm, we choose pivot_____.
   a. Always the smallest element
   b. Greater than 5
   c. Randomly page 35
   d. Less than 5

228. In Heap Sort algorithm. Heapify procedure is_____in nature.
   a. Recursive
   b. Non-Recursive page 43
   c. Fast

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d. Slow

229. When matrix A of 5x 3 is multiplied with matrix B of 3 x 4 then the numberof multiplications required will be___.

a. 15
b. 12
c. 3
6d.
60

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230. n algorithm is said to be correct if for every _____instance, it halts withthe correct__.

a. Input, Output page 13

b. Design, Analysis

c. Value, Key

d. Key, Analysis

231. In chain matrix multiplication, table is filled_____ to find themultiplication of matrix.

a. row wise

b. column wise

c. diagonally

d. bottom-to-up page 86

232. If we have an equation 8n2+7f*n + 5f + 6 then is large,_____term willbe muchxxxxxxxthe n term and will dominate the running time.

a. f g (n)

b. g (n) * 2

c. n * 2 page 23

d. f (n)

233. For quick sort algorithm. Partitioning takes theta_____.

a. (n)

b. log(n)

c. n log (n)

d. n2log (n)

234. In Heap Sort algorithm, the maximum levels an element can move upward is

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_____

a. Theta (log n) page 43

b. Big-ch (log n)

c. Omega (log n)

d. 0 (1) i.e. Constant time

235. _____programming is essentially recursion without repetition.

a. Fast

b. Dynamic page 75

c. Array

d. n (log n)

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236. There are no hard formal rules to the syntax of the_____code.
   a. Basic
   b. Programmin
   gc. **Pseudo**
   d. Assembly

237. In Heap Sort algorithm, to remove the maximum element every time.
   a. We call Build-Heap procedure
   b. Heap Sort algorithm terminates without
   resultc. We call heapify procedure
   d. Nothing happens

238. Which process is used for avoiding unnecessary repetitions and lookingthem up again if we need them later.
   a. Greedy Approach
   b. Memoization page 74
   c. Divide and conquer
   d. Recursion

239. The worst-case running time of Quick sort is_____in order to sort anarray of n element.
   a. O(n log n) page 49
   b. O(n)
   c. $O(n^2)$
   d. O(log n)

240. Boolean operation is a _____operation on an idealized RAM model ofcomputation.
   a. Starting

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b. Basic page 10

c. Advance

d. Normal

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241. In chain matrix multiplication, if there are n items, there are____ waysin which outer most pair of parentheses can placed.

a. n^2
b. 2n
c. n+1
d. n-1 page 85

242. The number of nodes in a complete binary tree of height h is:a. 2 * (h+1) - 1
b. 2^(h+1) – 1 page 40
c. 2 * (h+1)
d. ((h+1)^2) – 1

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MIDTERM EXAMINATION
Fall 2011
CS502- Fundamentals of Algorithms

Question No: 1 (Marks: 1) - Please choose one
Due to left complete nature of binary tree, the heap can be stored in

► **Arrays (Page 40)**
► Structures
► Link Lis
►Stack

Question No: 1 (Marks: 1) - Please choose one
What type of instructions Random Access Machine (RAM) can execute?

►Algebraic and logic
►Geometric and arithmetic
►**Arithmetic and logic (Page 10)**
►Parallel and recursive

Question No: 1 (Marks: 1) - Please choose one
For Chain Matrix Multiplication we can not use divide and conquer approach because,

►**We do not know the optimum k (Page 86)**
►We use divide and conquer for sorting only
►We can easily perform it in linear time
►Size of data is not given

Question No: 1 (Marks: 1) - Please choose one
What is the total time to heapify?

► **O(log n) (Page 43)**
► O(n log n)
► O(n^2 log n)
► O($\log^2$ n)

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Question No: 1 (Marks: 1) - Please choose one
word Algorithm comes from the name of the muslim author _____

►**Abu Ja'far Mohammad ibn Musa al-Khowarizmi**.

Question No: 1 (Marks: 1) - Please choose one
al-Khwarizmi's work was written in a book titled _____

►**al Kitab al-mukhatasar fi hisab al-jabr wa'l-muqabalah**

MIDTERM EXAMINATION
Spring 2010
CS502- Fundamentals of Algorithms

Question No: 1 (Marks: 1) - Please choose one
Random access machine or RAM is a/an

► Machine build by Al-Khwarizmi
► Mechanical machine
► Electronics machine
► **Mathematical model (Page 10)**

Question No: 2 (Marks: 1) - Please choose one
_____ is a graphical representation of an algorithm

► Σ notation
► Θ notation
► **Flowchart** **Click here for detail**
► Asymptotic notation

Question No: 3 (Marks: 1) - Please choose one
A RAM is an idealized machine with _____ random-access memory.

► 256MB
► 512MB
► **an infinitely large (Page 10)**
► 100GB

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Question No: 4 (Marks: 1) - Please choose one
What type of instructions Random Access Machine (RAM) can execute? Choose best answer

- ► Algebraic and logic
- ► Geometric and arithmetic
- ► **Arithmetic and logic (Rep)**
- ► Parallel and recursive

Question No: 5 (Marks: 1) - Please choose one
What will be the total number of max comparisons if we run brute-force maxima algorithm with n elements?

- ► n^2
- ► $n^{\frac{n}{2}}$
- ► n **(Page 14)**
- ► n^8

Question No: 6 (Marks: 1) - Please choose one
What is the solution to the recurrence $T(n) = T(n/2)+n$.

- ► $O(\log n)$
- ► **$O(n)$ (Page 37)**
- ► $O(n\log n)$
- ► $O(n^2)$

Question No: 7 (Marks: 1) - Please choose one
Consider the following code:

```
For(j=1; j<n;j++)
        For(k=1; k<15;k++)
                For(l=5; l<n; l++)
                {
                        Do_something_constant();
                }
```

What is the order of execution for this code.

- ► **O(n)**
- ► O(n^3)
- ► O($n^2 \log n$)
- ► O(n^2)

Question No: 8 (Marks: 1) - Please choose one
What is the total time to heapify?

- ► **O(log n) rep**
- ► O(n log n)
- ► O(n^2 log n)
- ► O($\log^2$ n)

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Question No: 9 (Marks: 1) - Please choose one
Consider the following Algorithm:

```
Factorial (n){
  if (n=1)
    return 1
  else
    return (n * Factorial(n-1))
```

Recurrence for the following algorithm is:

- ► **T(n) = T(n-1) +1**
- ► **T(n) = nT(n-1) +1**
- ► **T(n)= T(n-1) +n**
- ► **T(n)=T(n(n-1)) +1**

Question No: 10 (Marks: 1) - Please choose one
When we call heapify then at each level the comparison performed takes time

- ► **It will take Θ (1) (Page 43)**
- ► Time will vary according to the nature of input data
- ► It can not be predicted
- ► It will take Θ (log n)

Question No: 11 (Marks: 1) - Please choose one
In Quick sort, we don't have the control over the sizes of recursive calls

- ► **True (Page 40)**
- ► False
- ► Less information to decide
- ► Either true or false

Question No: 12 (Marks: 1) - Please choose one
Is it possible to sort without making comparisons?

- ► **Yes (Page 57)**
- ► No

Question No: 13 (Marks: 1) - Please choose one
If there are Θ (n^2) entries in edit distance matrix then the total running time is

- ► Θ (1)
- ► **Θ (n^2)** **Click here for detail**
- ► Θ (n)
- ► Θ (n log n)

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Question No: 14 (Marks: 1) - Please choose one
For Chain Matrix Multiplication we can not use divide and conquer approach because,

- ► **We do not know the optimum k (Page 86)**
- ► We use divide and conquer for sorting only
- ► We can easily perform it in linear time
- ► Size of data is not given

Question No: 15 (Marks: 1) - Please choose one
The Knapsack problem belongs to the domain of _____ problems.

- ► **Optimization (Page 91)**
- ► NP Complete
- ► Linear Solution
- ► Sorting

Question No: 16 (Marks: 1) - Please choose one
Suppose we have three items as shown in the following table, and suppose the capacity of the knapsack is 50 i.e. W = 50.

Item	Value	Weight
1	60	10
2	100	20
3	120	30

The optimal solution is to pick

- ► Items 1 and 2
- ► Items 1 and 3
- ► **Items 2 and 3 (correct)**
- ► None of these

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MIDTERM EXAMINATION
Spring 2010
CS502- Fundamentals of Algorithms

Question No: 1 (Marks: 1) - Please choose one
For the Sieve Technique we take time
► **T(nk) (Page 34)**
►T(n / 3)
►n^2
►n/3

Question No: 1 (Marks: 1) - Please choose one
Sieve Technique applies to problems where we are interested in finding a single item from a larger set of _____
Select correct option:
►**n items (Page 34)**
►phases
►pointers
►constant

Question No: 1 (Marks: 1) - Please choose one
_____ graphical representation of algorithm.
►asymptotic
►**Flowchart (rep)**

Question No: 1 (Marks: 1) - Please choose one
who invented the quick sort

►**C.A.R. Hoare** Click here for detail

Question No: 1 (Marks: 1) - Please choose one
main elements to a divide-and-conquer

►**Divide, conquer, combine (Page 27)**

Question No: 1 (Marks: 1) - Please choose one
Mergesort is a stable algorithm but not an in-place algorithm.

►**True (Page 54)**
►false

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Question No: 1 (Marks: 1) - Please choose one
Counting sort the numbers to be sorted are in the range 1 to k where k is small.
►**True (Page 57)**
►False

MIDTERM EXAMINATION
Spring 2007
CS502- Fundamentals of Algorithms

Question No: 1 (Marks: 1) - Please choose one
Total time for heapify is:
►O ($\log^2 n$)
►O (n log n)
►O (n^2 log n)
►**O (log n) Rep**

Question No: 1 (Marks: 1) - Please choose one
If an algorithm has a complexity of $\log_2 n + n\log_2 n + n$. we could say that it has complexity

►O(n)
►O(n $\log_2$ n)
►O(3)
►O($\log_2$ ($\log_2$ n))
►O ($\log_2$ n)

Question No: 1 (Marks: 1) - Please choose one
In RAM model instructions are executed

►**One after another (Page 10)**
►Parallel
►Concurrent
►Random

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Question No: 1 (Marks: 1) - Please choose one
In selection algorithm, because we eliminate a constant fraction of the array with each phase, we get the

►**Convergent geometric series (Page 37)**
►Divergent geometric series
►None of these

Question No: 1 (Marks: 1) - Please choose one
Due to left-complete nature of binary tree, heaps can be stored in

►Link list
►Structure
►**Array (Page 40)**
►None of above

CS609- System Programming
Midterm Quizzes (Quiz No.1 & 2)

Quiz No.1 (04 – MAY - 2013)

Question No: 1 (Marks: 1) - Please choose one
The time assumed for each basic operation to execute on RAM model of computation is-----

Infinite
Continuous
Constant (Page 10)
Variable

Question No: 1 (Marks: 1) - Please choose one
If the indices passed to merge sort algorithm are not equal, the algorithm may return immediately.

True
False (Page 28)

Question No: 1 (Marks: 1) - Please choose one
Brute-force algorithm uses no intelligence in pruning out decisions.

True (Page 18)
False

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Question No: 1 (Marks: 1) - Please choose one
In analysis, the Upper Bound means the function grows asymptotically no faster than its largest term.

True (Page 24)
False

Question No: 1 (Marks: 1) - Please choose one
For small values of n, any algorithm is fast enough. Running time does become an issue when n gets large.

True (Page 14)
Fast

Question No: 1 (Marks: 1) - Please choose one
The array to be sorted is not passed as argument to the merge sort algorithm.

True
False

Question No: 1 (Marks: 1) - Please choose one
In simple brute-force algorithm, we give no thought to efficiency.

True (Page 11)
False

Question No: 1 (Marks: 1) - Please choose one
The ancient Roman politicians understood an important principle of good algorithm design that is plan-sweep algorithm.

True
False (Page 27) [Divide and Conquer]

Question No: 1 (Marks: 1) - Please choose one
In 2d-space a point is said to be _____if it is not dominated by any other point in that space.

Member
Minimal
Maximal (Page 11)
Joint

Question No: 1 (Marks: 1) - Please choose one
An algorithm is a mathematical entity that is dependent on a specific programming language.
True
False (Page 7)

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Question No: 1 (Marks: 1) - Please choose one
The running time of an algorithm would not depend upon the optimization by the compiler but that of an implementation of the algorithm would depend on it.

True (Page 13)
False

Question No: 1 (Marks: 1) - Please choose one
F (n) and g (n) are asymptotically equivalent. This means that they have essentially the same _____ for large n.

Results
Variables
Size
Growth rates (Page 23)

Question No: 1 (Marks: 1) - Please choose one
8n2 + 2n - 3 will eventually exceed c2*(n) no matter how large we make c2.

True (Page 25)
False

Question No: 1 (Marks: 1) - Please choose one
If we associate (x, y) integers pair to cars where x is the speed of the car and y is the negation of the price. High y value for a car means a _____ car.
Fast
Slow
Expensive
Cheap (Page 11)

Question No: 1 (Marks: 1) - Please choose one
The function f(n)= n(logn+1)/2 is asymptotically equivalent to n log n. Here Upper Bound means the function f(n) grows asymptotically _____ faster than n log n.
More
Quiet
Not (Page 24)
At least

Question No: 1 (Marks: 1) - Please choose one
After sorting in merge sort algorithm, merging process is invoked.
Select correct option:
True (Page 28)
False

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Question No: 1 (Marks: 1) - Please choose one
Asymptotic growth rate of the function is taken over_____ case running time.
Select correct option:

Best
Average
Worst (Page 14)
Normal

Question No: 1 (Marks: 1) - Please choose one
In analysis of f (n) =n (n/5) +n-10 log n, f (n) is asymptotically equivalent to _____.

n
2n
n+1
n2 (Page 23)

Question No: 1 (Marks: 1) - Please choose one
Algorithm is concerned with.....issues.

Macro
Micro
Both Macro & Micro (Page 8)
Normal

Question No: 1 (Marks: 1) - Please choose one
We cannot make any significant improvement in the running time which is better than that of brute-force algorithm.

True
False (Page 18)

Question No: 1 (Marks: 1) - Please choose one
In addition to passing in the array itself to Merge Sort algorithm, we will pass in _____other arguments which are indices.

Two (Page 28)
Three
Four
Five

Question No: 1 (Marks: 1) - Please choose one
Consider the following Algorithm: Fun(n){ if (n=1) return 1 else return (n * Fun(n-1)) } Recurrence for the above algorithm is:

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nT(n-1)+1
2T(n-1)+1
T(n-1)+cn
T(n-1)+1

Question No: 1 (Marks: 1) - Please choose one
In analysis, the Lower Bound means the function grows asymptotically at least as fast as its largest term.

True (Page 24)
False

Question No: 1 (Marks: 1) - Please choose one
Efficient algorithm requires less computational…….

Memory
Running Time
Memory and Running Time (Page 9)
Energy

Question No: 1 (Marks: 1) - Please choose one
The O-notation is used to state only the asymptotic _____bounds.

Two
Lower
Upper (Page 25)
Both lower & upper

Question No: 1 (Marks: 1) - Please choose one
For the worst-case running time analysis, the nested loop structure containing one "for" and one "while" loop, might be expressed as a pair of _____nested summations.

1
2 (Page 16)
3
4

Question No: 1 (Marks: 1) - Please choose one
Before sweeping a vertical line in plane sweep approach, in start sorting of the points is done in increasing order of their _____coordinates.

X (Page 18)
Y
Z
X & Y

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Question No: 1 (Marks: 1) - Please choose one
Brute-force algorithm for 2D-Maxima is operated by comparing _____ pairs of points.

Two
Some
Most
All (Page 18)

Question No: 1 (Marks: 1) - Please choose one
The function f(n)=n(logn+1)/2 is asymptotically equivalent to nlog n. Here Lower Bound means function f(n) grows asymptotically at _____ as fast as nlog n.

Normal
Least (Page 23)
Most
All

Question No: 1 (Marks: 1) - Please choose one
The definition of Theta-notation relies on proving _____asymptotic bound.

One
Lower
Upper
Both lower & upper (Page 25) rep

Question No: 1 (Marks: 1) - Please choose one
In plane sweep approach, a vertical line is swept across the 2d-plane and _____structure is used for holding the maximal points lying to the left of the sweep line.

Array
Queue
Stack (Page 18)
Tree

Question No: 1 (Marks: 1) - Please choose one
Algorithm analysts know for sure about efficient solutions for NP-complete problems.
Select correct option:
True
False (Page 9)

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Quiz No.1 (2012)

Question No: 1 of 10 (Marks: 1) - Please choose one
The number of nodes in a complete binary tree of height h is

2^(h+1) – 1 (Page 40)
2 * (h+1) – 1
2 * (h+1)
((h+1) ^ 2) – 1

Question No: 1 of 10 (Marks: 1) - Please choose one
The analysis of Selection algorithm shows the total running time is indeed _____in n,

arithmetic
geometric
linear (Page 37)
orthogonal

Question No: 1 of 10 (Marks: 1) - Please choose one
A (an) _____ is a left-complete binary tree that conforms to the heap order

heap (Page 40)
binary tree
binary search tree
array

Question No: 1 of 10 (Marks: 1) - Please choose one
Analysis of Selection algorithm ends up with,

T(n) (Page 37)
T(1 / 1 + n)
T(n / 2)
T((n / 2) + n)

Question No: 1 of 10 (Marks: 1) - Please choose one
For the sieve technique we solve the problem,

recursively (Page 34)
mathematically
precisely
accurately

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Question No: 1 of 10 (Marks: 1) - Please choose one
A heap is a left-complete binary tree that conforms to the _____

increasing order only
decreasing order only
heap order (Page 40)
(log n) order

Question No: 1 of 10 (Marks: 1) - Please choose one
In which order we can sort?

increasing order only
decreasing order only
increasing order or decreasing order (Page 39)
both at the same time

Question No: 1 of 10 (Marks: 1) - Please choose one
Divide-and-conquer as breaking the problem into a small number of

pivot
Sieve
smaller sub problems (Page 34)
Selection

Question No: 1 of 10 (Marks: 1) - Please choose one
For the heap sort we store the tree nodes in

level-order traversal (Page 40)
in-order traversal
pre-order traversal
post-order traversal

Question No: 1 of 10 (Marks: 1) - Please choose one
The sieve technique works in _____ as follows

Phases (Page 34)
numbers
integers
routines

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CS502 - Fundamentals of Algorithms
Quiz No.1 12-11-2012

Question No: 1 of 10 (Marks: 1) - Please choose one
We do sorting to,

keep elements in random positions
keep the algorithm run in linear order
keep the algorithm run in (log n) order
keep elements in increasing or decreasing order

Question No: 1 of 10 (Marks: 1) - Please choose one
Heaps can be stored in arrays without using any pointers; this is due to the _____ nature of the binary tree,

left-complete (Page 40)
right-complete
tree nodes
tree leaves

Question No: 1 of 10 (Marks: 1) - Please choose one
Sieve Technique can be applied to selection problem?

True (Page 35)
False

Question No: 1 of 10 (Marks: 1) - Please choose one
In Sieve Technique we do not know which item is of interest

True (Page 34)
False

Question No: 1 of 10 (Marks: 1) - Please choose one
In the analysis of Selection algorithm, we eliminate a constant fraction of the array with each phase; we get the convergent _____ series in the analysis,

linear
arithmetic
geometric (Page 37)
exponent

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Question No: 1 of 10 (Marks: 1) - Please choose one
For the heap sort, access to nodes involves simple _____ operations.

arithmetic (Page 41)
binary
algebraic
logarithmic

Question No: 1 of 10 (Marks: 1) - Please choose one
Slow sorting algorithms run in,

T(n^2) (Page 39)
T(n)
T(log n)

Question No: 1 of 10 (Marks: 1) - Please choose one
In the analysis of Selection algorithm, we make a number of passes, in fact it could be as many as,
T(n)
T(n / 2)
log n (Page 37)
n / 2 + n / 4

Question No: 1 of 10 (Marks: 1) - Please choose one
The sieve technique is a special case, where the number of sub problems is just
5
many
1 (Page 34)
few

Question No: 1 of 10 (Marks: 1) - Please choose one
How many elements do we eliminate in each time for the Analysis of Selection algorithm?

(n / 2)+n elements
(n / 2) elements (Page 37)
n / 4 elements
2 n elements

Question No: 1 of 10 (Marks: 1) - Please choose one
One of the clever aspects of heaps is that they can be stored in arrays without using any _____.
pointers (Page 40)
constants
variables
functions

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Question No: 1 of 10 (Marks: 1) - Please choose one
How much time merge sort takes for an array of numbers?

T(n^2)
T(n)
T(log n)
T(n log n) (Page 40)

Question No: 1 of 10 (Marks: 1) - Please choose one
The reason for introducing Sieve Technique algorithm is that it illustrates a very important special case of,
divide-and-conquer (Page 34)
decrease and conquer
greedy nature
2-dimension Maxima

Question No: 1 of 10 (Marks: 1) - Please choose one
In Sieve Technique we do not know which item is of interest
True (Page 34) rep
False

Question No: 1 of 10 (Marks: 1) - Please choose one
Theta asymptotic notation for T (n) :

Set of functions described by: c1g(n)Set of functions described by c1g(n)>=f(n) for c1 s
Theta for T(n)is actually upper and worst case comp (Not sure)
Set of functions described by:
c1g(n)

Question No: 1 of 10 (Marks: 1) - Please choose one
Memoization is?

To store previous results for future use
To avoid this unnecessary repetitions by writing down the results of recursive calls and looking them up again if we need them later (page 74)
To make the process accurate
None of the above

Question No: 1 of 10 (Marks: 1) - Please choose one
Which sorting algorithm is faster
O (n log n) Page 26
O n^2
O (n+k)
O n^3

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Question No: 1 of 10 (Marks: 1) - Please choose one
Quick sort is

Stable & in place
Not stable but in place (Page 54)
Stable but not in place
Some time stable & some times in place

Question No: 1 of 10 (Marks: 1) - Please choose one
One example of in place but not stable algorithm is

Merger Sort
Quick Sort (Page 54)
Continuation Sort
Bubble Sort

Question No: 1 of 10 (Marks: 1) - Please choose one
Cont sort is suitable to sort the elements in range 1 to k
K is Large
K is not known
K may be small or large
K is small (Page 57)

Question No: 1 of 10 (Marks: 1) - Please choose one
In place stable sorting algorithm.

If duplicate elements remain in the same relative position after sorting (Page 54)
One array is used
More than one arrays are required
Duplicating elements not handled

Question No: 1 of 10 (Marks: 1) - Please choose one
Which may be a stable sort?
Merger
Insertion (Page 54)
Both above
None of the above

Question No: 1 of 10 (Marks: 1) - Please choose one
An in place sorting algorithm is one that uses ___ arrays for storage

Two dimensional arrays
More than one array
No Additional Array (Page 54)
None of the above

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Question No: 1 of 10 (Marks: 1) - Please choose one
Sieve Technique applies to problems where we are interested in finding a single item from a larger set of _____

n items (Page 34)
phases
pointers
constant

Question No: 1 of 10 (Marks: 1) - Please choose one
Sorting is one of the few problems where provable _____ bonds exits on how fast we can sort,

upper
lower (Page 39)
average
log n

Question No: 1 of 10 (Marks: 1) - Please choose one
Counting sort has time complexity:

O(n) (Page 58)
O(n+k)
O(k)
O(nlogn)

Question No: 1 of 10 (Marks: 1) - Please choose one
The running time of quick sort depends heavily on the selection of

No of inputs
Arrangement of elements in array
Size o elements
Pivot elements (Page 49)

Question No: 1 of 10 (Marks: 1) - Please choose one
Which may be stable sort:

Bubble sort
Insertion sort
Both of above (Page 54)

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Question No: 1 of 10 (Marks: 1) - Please choose one
One Example of in place but not stable sort is

Quick (Page 54)
Heap
Merge
Bubble

Question No: 1 of 10 (Marks: 1) - Please choose one
In Quick Sort Constants hidden in T(n log n) are

Large
Medium
Small Click here for detail
Not Known

Question No: 1 of 10 (Marks: 1) - Please choose one
Quick sort is based on divide and conquer paradigm; we divide the problem on base of pivot element and:

There is explicit combine process as well to conquer the solution.
No work is needed to combine the sub-arrays, the array is already sorted
Merging the sub arrays
None of above. (Page 51)
Ref: - random choices for the pivot element and each choice have an equal probability of 1/n of occurring. So we can modify the above recurrence to compute an average rather than a max

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CS501 - Quiz No.2 (Spring 2013)

Question No: 1 of 10 (Marks: 1) - Please choose one
A point p in 2-dimensional space is usually given by its integer coordinate(s)_____

p.x only
p.y only
p.x & p.z
p.x & p.y (Page 10)

Question No: 1 of 10 (Marks: 1) - Please choose one
In _____ we have to find rank of an element from given input.

Merge sort algorithm
Selection problem (Page 34)
Brute force technique
Plane Sweep algorithm

Question No: 1 of 10 (Marks: 1) - Please choose one
In Heap Sort algorithm, if heap property is violated _____

We call Build heap procedure
We call Heapify procedure
We ignore
Heap property can never be violated

Question No: 1 of 10 (Marks: 1) - Please choose one
Upper bound requires that there exist positive constants c2 and n0 such that f(n) ____ c2n for all n <= n0(ye question ghalat lag raha hai mujhae

Less than
Equal to or Less than (Page 25)
Equal or Greater than
Greater than

Question No: 1 of 10 (Marks: 1) - Please choose one
A RAM is an idealized algorithm with takes an infinitely large random-access memory.

True
False (Page 10)

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Question No: 1 of 10 (Marks: 1) - Please choose one
_____ is one of the few problems, where provable lower bounds exist on how fast we can sort.

Searching
Sorting (Page)
Both Searching & Sorting
Graphing

Question No: 1 of 10 (Marks: 1) - Please choose one
Floor and ceiling are _____ to calculate while analyzing algorithms.

Very easy
Usually considered difficult (Page 31)

Question No: 1 of 10 (Marks: 1) - Please choose one
In Heap Sort algorithm, the maximum levels an element can move upward is _____

Theta (log n) (Page 43)
Order (log n)
Omega (log n)
O (1) i.e. Constant time

Question No: 1 of 10 (Marks: 1) - Please choose one
A point p in 2-dimensional space is usually given by its integer coordinate(s)_____

p.x only p.y
only p.x & p.z
p.x & p.y (Page 17)

Question No: 1 of 10 (Marks: 1) - Please choose one
In Heap Sort algorithm, the total running time for Heapify procedure is _____

Theta (log n) (Page 43)
Order (log n)
Omega (log n)
O (1) i.e. Constant time

Question No: 1 of 10 (Marks: 1) - Please choose one
Algorithm is a mathematical entity, which is independent of a specific machine and operating system.

True
False (Page 7)

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Question No: 1 of 10 (Marks: 1) - Please choose one
While Sorting, the ordered domain means for any two input elements x and y _____ satisfies only.

x < y
x > y
x = y
All of the above (Page 39)

Question No: 1 of 10 (Marks: 1) - Please choose one
Quick sort is best from the perspective of Locality of reference.

True (Page 9)
False

Question No: 1 of 10 (Marks: 1) - Please choose one
Sorting can be in _____

Increasing order only
Decreasing order only
Both Increasing and Decreasing order (Page 39)
Random order

Question No: 1 of 10 (Marks: 1) - Please choose one
In Heap Sort algorithm, we build _____ for ascending sort.

Max heap (Page 41)
Min heap

Question No: 1 of 10 (Marks: 1) - Please choose one
In Sieve Technique, we know the item of interest.

True
False (Page 34)

Question No: 1 of 10 (Marks: 1) - Please choose one
While solving Selection problem, in Sieve technique we partition input data _____

In increasing order
In decreasing order
According to Pivot (Page 35)
Randomly

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Question No: 1 of 10 (Marks: 1) - Please choose one
In pseudo code, the level of details depends on intended audience of the algorithm.

True (Page 12)
False

Question No: 1 of 10 (Marks: 1) - Please choose one
The sieve technique works where we have to find _____ item(s) from a large input.

Single (Page 34)
Two
Three
Similar

Question No: 1 of 10 (Marks: 1) - Please choose one
If the indices passed to merge sort algorithm are _____,then this means that there is only one element to sort.

Small
Large
Equal (Page 28)
Not Equal

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Design and Analysis of Algorithms

Sohail Aslam

January 2004

Contents

1 Introduction **7**

1.1 Origin of word: *Algorithm* 7

1.2 Algorithm: Informal Definition 7

1.3 Algorithms, Programming 7

1.4 Implementation Issues 8

1.5 Course in Review 9

1.6 Analyzing Algorithms 9

1.7 Model of Computation 10

1.8 Example: 2-dimension maxima 10

1.9 Brute-Force Algorithm 11

1.10 Running Time Analysis 13

1.10.1 Analysis of the brute-force maxima algorithm. 14

1.11 Analysis: A Harder Example 16

1.11.1 2-dimension Maxima Revisited 17

1.11.2 Plane-sweep Algorithm 18

1.11.3 Analysis of Plane-sweep Algorithm 21

1.11.4 Comparison of Brute-force and Plane sweep algorithms 21

2 Asymptotic Notation **23**

3 Divide and Conquer Strategy **27**

3.1 Merge Sort 27

3.1.1 Analysis of Merge Sort 29

3.1.2 The Iteration Method for Solving Recurrence Relations 31

3.1.3 Visualizing Recurrences Using the Recursion Tree 31

3.1.4 A Messier Example . . . 32
3.2 Selection Problem . . . 34
3.2.1 Sieve Technique . . . 34
3.2.2 Applying the Sieve to Selection . . . 35
3.2.3 Selection Algorithm . . . 35
3.2.4 Analysis of Selection . . . 36

4 Sorting **39**
4.1 Slow Sorting Algorithms . . . 39
4.2 Sorting in $O(n \log n)$ time . . . 40
4.2.1 Heaps . . . 40
4.2.2 Heapsort Algorithm . . . 41
4.2.3 Heapify Procedure . . . 43
4.2.4 Analysis of Heapify . . . 43
4.2.5 BuildHeap . . . 43
4.2.6 Analysis of BuildHeap . . . 44
4.2.7 Analysis of Heapsort . . . 46
4.3 Quicksort . . . 46
4.3.1 Partition Algorithm . . . 46
4.3.2 Quick Sort Example . . . 47
4.3.3 Analysis of Quicksort . . . 49
4.3.4 Worst Case Analysis of Quick Sort . . . 49
4.3.5 Average-case Analysis of Quicksort . . . 50
4.4 In-place, Stable Sorting . . . 54
4.5 Lower Bounds for Sorting . . . 54

5 Linear Time Sorting **57**
5.1 Counting Sort . . . 57
5.2 Bucket or Bin Sort . . . 69
5.3 Radix Sort . . . 71

6 Dynamic Programming **73**
6.1 Fibonacci Sequence . . . 73

6.2 Dynamic Programming . . . 75
6.3 Edit Distance . . . 75
6.3.1 Edit Distance: Applications . . . 76
6.3.2 Edit Distance Algorithm . . . 77
6.3.3 Edit Distance: Dynamic Programming Algorithm . . . 77
6.3.4 Analysis of DP Edit Distance . . . 84
6.4 Chain Matrix Multiply . . . 84
6.4.1 Chain Matrix Multiplication-Dynamic Programming Formulation . . . 85
6.5 0/1 Knapsack Problem . . . 91
6.5.1 0/1 Knapsack Problem: Dynamic Programming Approach . . . 93

7 Greedy Algorithms **97**
7.1 Example: Counting Money . . . 97
7.1.1 Making Change: Dynamic Programming Solution . . . 98
7.1.2 Complexity of Coin Change Algorithm . . . 99
7.2 Greedy Algorithm: Huffman Encoding . . . 99
7.2.1 Huffman Encoding Algorithm . . . 100
7.2.2 Huffman Encoding: Correctness . . . 102
7.3 Activity Selection . . . 105
7.3.1 Correctness of Greedy Activity Selection . . . 107
7.4 Fractional Knapsack Problem . . . 109

8 Graphs **113**
8.1 Graph Traversal . . . 116
8.1.1 Breadth-first Search . . . 117
8.1.2 Depth-first Search . . . 119
8.1.3 Generic Graph Traversal Algorithm . . . 119
8.1.4 DFS - Timestamp Structure . . . 125
8.1.5 DFS - Cycles . . . 130
8.2 Precedence Constraint Graph . . . 131
8.3 Topological Sort . . . 133
8.4 Strong Components . . . 135
8.4.1 Strong Components and DFS . . . 137

8.5 Minimum Spanning Trees . . . 142
8.5.1 Computing MST: Generic Approach . . . 143
8.5.2 Greedy MST . . . 144
8.5.3 Kruskal's Algorithm . . . 147
8.5.4 Prim's Algorithm . . . 149
8.6 Shortest Paths . . . 153
8.6.1 Dijkstra's Algorithm . . . 154
8.6.2 Correctness of Dijkstra's Algorithm . . . 158
8.6.3 Bellman-Ford Algorithm . . . 159
8.6.4 Correctness of Bellman-Ford . . . 160
8.6.5 Floyd-Warshall Algorithm . . . 161

9 Complexity Theory **169**
9.1 Decision Problems . . . 170
9.2 Complexity Classes . . . 170
9.3 Polynomial Time Verification . . . 171
9.4 The Class NP . . . 172
9.5 Reductions . . . 173
9.6 Polynomial Time Reduction . . . 177
9.7 NP-Completeness . . . 177
9.8 Boolean Satisfiability Problem: Cook's Theorem . . . 178
9.9 Coping with NP-Completeness . . . 184

Chapter 1

Introduction

1.1 Origin of word: *Algorithm*

The word *Algorithm* comes from the name of the muslim author *Abu Ja'far Mohammad ibn Musa al-Khowarizmi*. He was born in the eighth century at Khwarizm (Kheva), a town south of river Oxus in present Uzbekistan. Uzbekistan, a Muslim country for over a thousand years, was taken over by the Russians in 1873.

His year of birth is not known exactly. Al-Khwarizmi parents migrated to a place south of Baghdad when he was a child. It has been established from his contributions that he flourished under Khalifah Al-Mamun at Baghdad during 813 to 833 C.E. Al-Khwarizmi died around 840 C.E.

Much of al-Khwarizmi's work was written in a book titled *al Kitab al-mukhatasar fi hisab al-jabr wa'l-muqabalah* (The Compendious Book on Calculation by Completion and Balancing). It is from the titles of these writings and his name that the words *algebra* and *algorithm* are derived. As a result of his work, al-Khwarizmi is regarded as the most outstanding mathematician of his time

1.2 Algorithm: Informal Definition

An algorithm is any well-defined computational procedure that takes some values, or set of values, as input and produces some value, or set of values, as output. An algorithm is thus a sequence of computational steps that transform the input into output.

1.3 Algorithms, Programming

A good understanding of algorithms is essential for a good understanding of the most basic element of computer science: programming. Unlike a program, an algorithm is a mathematical entity, which is independent of a specific programming language, machine, or compiler. Thus, in some sense, algorithm

design is all about the mathematical theory behind the design of good programs.

Why study algorithm design? There are many facets to good program design. Good algorithm design is one of them (and an important one). To be really complete algorithm designer, it is important to be aware of programming and machine issues as well. In any important programming project there are two major types of issues, macro issues and micro issues.

Macro issues involve elements such as how does one coordinate the efforts of many programmers working on a single piece of software, and how does one establish that a complex programming system satisfies its various requirements. These macro issues are the primary subject of courses on software engineering.

A great deal of the programming effort on most complex software systems consists of elements whose programming is fairly mundane (input and output, data conversion, error checking, report generation). However, there is often a small critical portion of the software, which may involve only tens to hundreds of lines of code, but where the great majority of computational time is spent. (Or as the old adage goes: 80% of the execution time takes place in 20% of the code.) The micro issues in programming involve how best to deal with these small critical sections.

It may be very important for the success of the overall project that these sections of code be written in the most efficient manner possible. An unfortunately common approach to this problem is to first design an inefficient algorithm and data structure to solve the problem, and then take this poor design and attempt to fine-tune its performance by applying clever coding tricks or by implementing it on the most expensive and fastest machines around to boost performance as much as possible. The problem is that if the underlying design is bad, then often no amount of fine-tuning is going to make a substantial difference.

Before you implement, first be sure you have a good design. This course is all about how to design good algorithms. Because the lesson cannot be taught in just one course, there are a number of companion courses that are important as well. CS301 deals with how to design good data structures. This is not really an independent issue, because most of the fastest algorithms are fast because they use fast data structures, and vice versa. In fact, many of the courses in the computer science program deal with efficient algorithms and data structures, but just as they apply to various applications: compilers, operating systems, databases, artificial intelligence, computer graphics and vision, etc. Thus, a good understanding of algorithm design is a central element to a good understanding of computer science and good programming.

1.4 Implementation Issues

One of the elements that we will focus on in this course is to try to study algorithms as pure mathematical objects, and so ignore issues such as programming language, machine, and operating system. This has the advantage of clearing away the messy details that affect implementation. But these details may be very important.

For example, an important fact of current processor technology is that of locality of reference. Frequently accessed data can be stored in registers or cache memory. Our mathematical analysis will usually ignore these issues. But a good algorithm designer can work within the realm of mathematics, but still keep an

open eye to implementation issues down the line that will be important for final implementation. For example, we will study three fast sorting algorithms this semester, heap-sort, merge-sort, and quick-sort. From our mathematical analysis, all have equal running times. However, among the three (barring any extra considerations) quick sort is the fastest on virtually all modern machines. Why? It is the best from the perspective of locality of reference. However, the difference is typically small (perhaps 10-20% difference in running time).

Thus this course is not the last word in good program design, and in fact it is perhaps more accurately just the first word in good program design. The overall strategy that I would suggest to any programming would be to first come up with a few good designs from a mathematical and algorithmic perspective. Next prune this selection by consideration of practical matters (like locality of reference). Finally prototype (that is, do test implementations) a few of the best designs and run them on data sets that will arise in your application for the final fine-tuning. Also, be sure to use whatever development tools that you have, such as profilers (programs which pin-point the sections of the code that are responsible for most of the running time).

1.5 Course in Review

This course will consist of four major sections. The first is on the mathematical tools necessary for the analysis of algorithms. This will focus on asymptotics, summations, recurrences. The second element will deal with one particularly important algorithmic problem: sorting a list of numbers. We will show a number of different strategies for sorting, and use this problem as a case-study in different techniques for designing and analyzing algorithms.

The final third of the course will deal with a collection of various algorithmic problems and solution techniques. Finally we will close this last third with a very brief introduction to the theory of NP-completeness. NP-complete problem are those for which no efficient algorithms are known, but no one knows for sure whether efficient solutions might exist.

1.6 Analyzing Algorithms

In order to design good algorithms, we must first agree the criteria for measuring algorithms. The emphasis in this course will be on the design of efficient algorithm, and hence we will measure algorithms in terms of the amount of computational resources that the algorithm requires. These resources include mostly running time and memory. Depending on the application, there may be other elements that are taken into account, such as the number disk accesses in a database program or the communication bandwidth in a networking application.

In practice there are many issues that need to be considered in the design algorithms. These include issues such as the ease of debugging and maintaining the final software through its life-cycle. Also, one of the luxuries we will have in this course is to be able to assume that we are given a clean, fully-specified mathematical description of the computational problem. In practice, this is often not the case, and the algorithm must be designed subject to only partial knowledge of the final specifications. Thus, in practice

it is often necessary to design algorithms that are simple, and easily modified if problem parameters and specifications are slightly modified. Fortunately, most of the algorithms that we will discuss in this class are quite simple, and are easy to modify subject to small problem variations.

1.7 Model of Computation

Another goal that we will have in this course is that our analysis be as independent as possible of the variations in machine, operating system, compiler, or programming language. Unlike programs, algorithms to be understood primarily by people (i.e. programmers) and not machines. Thus gives us quite a bit of flexibility in how we present our algorithms, and many low-level details may be omitted (since it will be the job of the programmer who implements the algorithm to fill them in).

But, in order to say anything meaningful about our algorithms, it will be important for us to settle on a mathematical model of computation. Ideally this model should be a reasonable abstraction of a standard generic single-processor machine. We call this model a *random access machine* or RAM.

A RAM is an idealized machine with an infinitely large random-access memory. Instructions are executed one-by-one (there is no parallelism). Each instruction involves performing some basic operation on two values in the machines memory (which might be characters or integers; let's avoid floating point for now). Basic operations include things like assigning a value to a variable, computing any basic arithmetic operation (+, - , $\times$, integer division) on integer values of any size, performing any comparison (e.g. $x \leq 5$) or boolean operations, accessing an element of an array (e.g. $A[10]$). We assume that each basic operation takes the same constant time to execute.

This model seems to go a good job of describing the computational power of most modern (nonparallel) machines. It does not model some elements, such as efficiency due to locality of reference, as described in the previous lecture. There are some "loop-holes" (or hidden ways of subverting the rules) to beware of. For example, the model would allow you to add two numbers that contain a billion digits in constant time. Thus, it is theoretically possible to derive nonsensical results in the form of efficient RAM programs that cannot be implemented efficiently on any machine. Nonetheless, the RAM model seems to be fairly sound, and has done a good job of modelling typical machine technology since the early 60's.

1.8 Example: 2-dimension maxima

Let us do an example that illustrates how we analyze algorithms. Suppose you want to buy a car. You want the pick the fastest car. But fast cars are expensive; you want the cheapest. You cannot decide which is more important: speed or price. Definitely do not want a car if there is another that is both faster and cheaper. We say that the fast, cheap car *dominates* the slow, expensive car relative to your selection criteria. So, given a collection of cars, we want to list those cars that are not dominated by any other.

Here is how we might model this as a formal problem.

- Let a point p in 2-dimensional space be given by its integer coordinates, $p = (p.x, p.y)$.

- A point p is said to be dominated by point q if $p.x \leq q.x$ and $p.y \leq q.y$.
- Given a set of n points, $P = \{p_1, p_2, \ldots, p_n\}$ in 2-space a point is said to be maximal if it is not dominated by any other point in P.

The car selection problem can be modelled this way: For each car we associate (x, y) pair where x is the speed of the car and y is the negation of the price. High y value means a cheap car and low y means expensive car. Think of y as the money left in your pocket after you have paid for the car. Maximal points correspond to the fastest and cheapest cars.

The *2-dimensional Maxima* is thus defined as

- Given a set of points $P = \{p_1, p_2, \ldots, p_n\}$ in 2-space, output the set of maximal points of P, i.e., those points p_i such that p_i is not dominated by any other point of P.

Here is set of maximal points for a given set of points in 2-d.

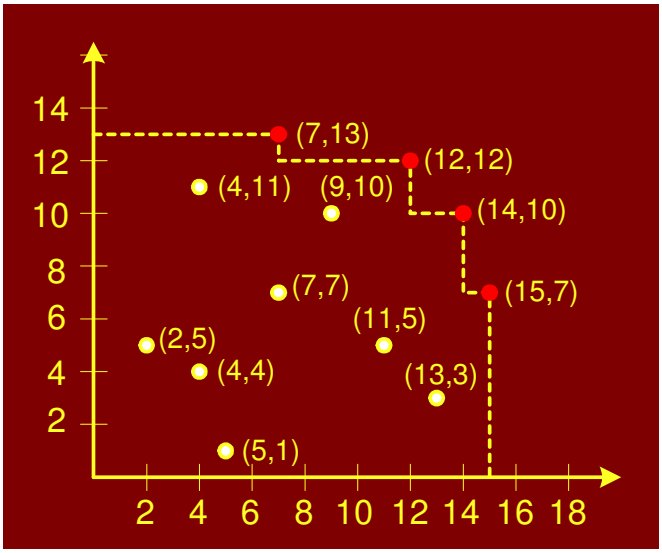


Figure 1.1: Maximal points in 2-d

Our description of the problem is at a fairly mathematical level. We have intentionally not discussed how the points are represented. We have not discussed any input or output formats. We have avoided programming and other software issues.

1.9 Brute-Force Algorithm

To get the ball rolling, let's just consider a simple brute-force algorithm, with no thought to efficiency. Let $P = \{p_1, p_2, \ldots, p_n\}$ be the initial set of points. For each point p_i, test it against all other points p_j. If p_i is not dominated by any other point, then output it.

This English description is clear enough that any (competent) programmer should be able to implement it. However, if you want to be a bit more formal, it could be written in pseudocode as follows:

```
MAXIMA(int n, Point P[1...n])
1  for i ← 1 to n
2  do maximal ← true
3     for j ← 1 to n
4     do
5        if (i ≠ j) and (P[i].x ≤ P[j].x) and (P[i].y ≤ P[j].y)
6           then maximal ← false;  break
7     if (maximal = true)
8        then output P[i]
```

There are no formal rules to the syntax of this pseudo code. In particular, do not assume that more detail is better. For example, I omitted type specifications for the procedure Maxima and the variable maximal, and I never defined what a Point data type is, since I felt that these are pretty clear from context or just unimportant details. Of course, the appropriate level of detail is a judgement call. Remember, algorithms are to be read by people, and so the level of detail depends on your intended audience. When writing pseudo code, you should omit details that detract from the main ideas of the algorithm, and just go with the essentials.

You might also notice that I did not insert any checking for consistency. For example, I assumed that the points in P are all distinct. If there is a duplicate point then the algorithm may fail to output even a single point. (Can you see why?) Again, these are important considerations for implementation, but we will often omit error checking because we want to see the algorithm in its simplest form.

Here are a series of figures that illustrate point domination.

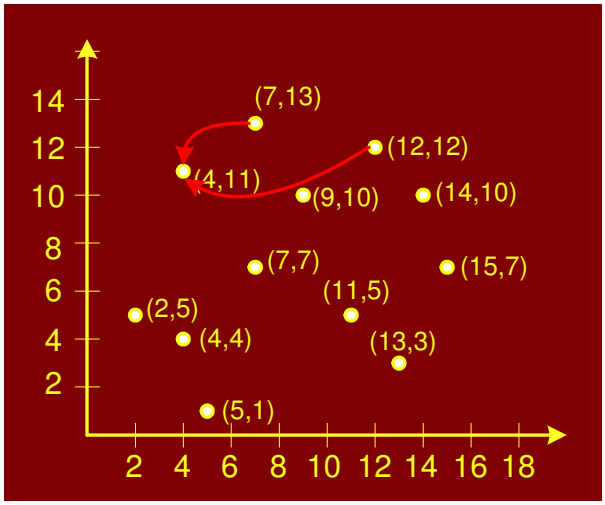


Figure 1.2: Points that dominate $(4, 11)$

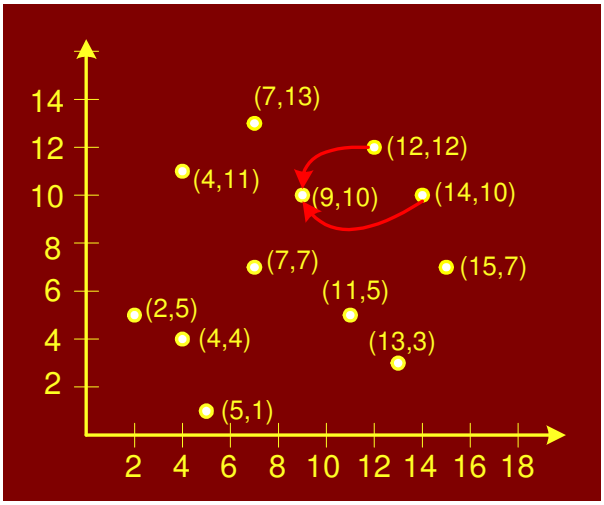


Figure 1.3: Points that dominate $(9, 10)$

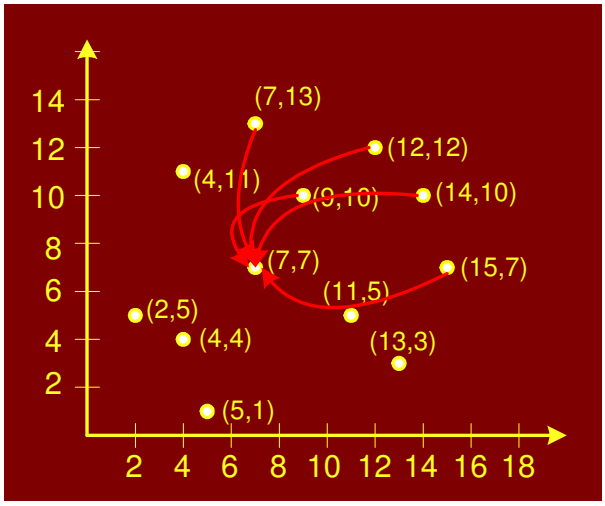


Figure 1.4: Points that dominate $(7, 7)$

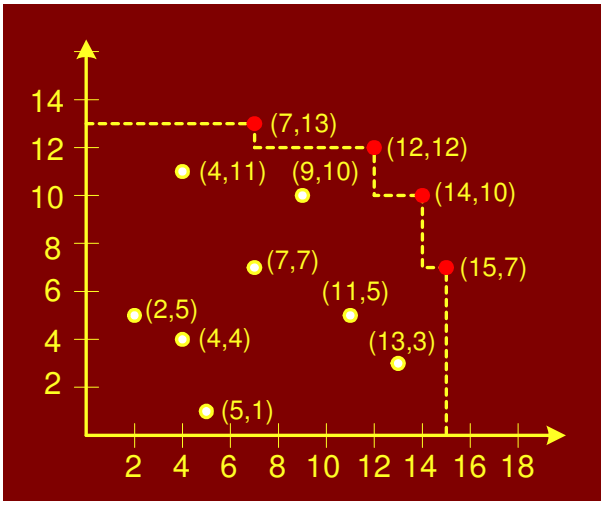


Figure 1.5: The maximal points

1.10 Running Time Analysis

The main purpose of our mathematical analysis will be measuring the execution time. We will also be concerned about the space (memory) required by the algorithm.

The running time of an implementation of the algorithm would depend upon the speed of the computer, programming language, optimization by the compiler etc. Although important, we will ignore these technological issues in our analysis.

To measure the running time of the brute-force 2-d maxima algorithm, we could count the number of steps of the pseudo code that are executed or, count the number of times an element of P is accessed or, the number of comparisons that are performed.

The running time depends upon the input size, e.g. n Different inputs of the same size may result in different running time. For example, breaking out of the inner loop in the brute-force algorithm depends not only on the input size of P but also the structure of the input.

Two criteria for measuring running time are *worst-case time* and *average-case time*.

Worst-case time is the maximum running time over all (legal) inputs of size n. Let I denote an input instance, let $|I|$ denote its length, and let $T(I)$ denote the running time of the algorithm on input I. Then

$$T_{worst}(n) = \max_{|I|=n} T(I)$$

Average-case time is the average running time over all inputs of size n. Let $p(I)$ denote the probability of seeing this input. The average-case time is the weighted sum of running times with weights

being the probabilities:

$$T_{avg}(n) = \sum_{|I|=n} p(I)T(I)$$

We will almost always work with worst-case time. Average-case time is more difficult to compute; it is difficult to specify probability distribution on inputs. Worst-case time will specify an upper limit on the running time.

1.10.1 Analysis of the brute-force maxima algorithm.

Assume that the input size is n, and for the running time we will count the number of time that any element of P is accessed. Clearly we go through the outer loop n times, and for each time through this loop, we go through the inner loop n times as well. The condition in the if-statement makes four accesses to P. The output statement makes two accesses for each point that is output. In the worst case every point is maximal (can you see how to generate such an example?) so these two access are made for each time through the outer loop.

```
MAXIMA(int n, Point P[1...n])
1  for i ← 1 to n                    n times
2  do maximal ← true
3     for j ← 1 to n                 n times
4     do
5        if (i ≠ j)&(P[i].x ≤ P[j].x)&(P[i].y ≤ P[j].y)   4 accesses
6           then maximal ← false break
7     if maximal
8       then output P[i].x, P[i].y    2 accesses
```

Thus we might express the worst-case running time as a pair of nested summations, one for the i-loop and the other for the j-loop:

$$\begin{aligned} T(n) &= \sum_{i=1}^{n} \left(2 + \sum_{j=1}^{n} 4\right) \\ &= \sum_{i=1}^{n} (4n + 2) \\ &= (4n + 2)n = 4n^2 + 2n \end{aligned}$$

For small values of n, any algorithm is fast enough. What happens when n gets large? Running time does become an issue. When n is large, n^2 term will be much larger than the n term and will *dominate* the running time.

We will say that the worst-case running time is $\Theta(n^2)$. This is called the asymptotic growth rate of the function. We will discuss this Θ-notation more formally later.

The analysis involved computing a summation. Summation should be familiar but let us review a bit here. Given a finite sequence of values $a_1, a_2, \ldots, a_n$, their sum $a_1 + a_2 + \ldots + a_n$ is expressed in summation notation as

$$\sum_{i=1}^{n} a_i$$

If $n = 0$, then the sum is additive identity, 0.

Some facts about summation: If c is a constant

$$\sum_{i=1}^{n} ca_i = c \sum_{i=1}^{n} a_i$$

and

$$\sum_{i=1}^{n} (a_i + b_i) = \sum_{i=1}^{n} a_i + \sum_{i=1}^{n} b_i$$

Some important summations that should be committed to memory.

Arithmetic series

$$\begin{aligned} \sum_{i=1}^{n} i &= 1 + 2 + \ldots + n \\ &= \frac{n(n+1)}{2} = \Theta(n^2) \end{aligned}$$

Quadratic series

$$\begin{aligned} \sum_{i=1}^{n} i^2 &= 1 + 4 + 9 + \ldots + n^2 \\ &= \frac{2n^3 + 3n^2 + n}{6} = \Theta(n^3) \end{aligned}$$

Geometric series

$$\begin{aligned} \sum_{i=1}^{n} x^i &= 1 + x + x^2 + \ldots + x^n \\ &= \frac{x^{(n+1)} - 1}{x - 1} = \Theta(n^2) \end{aligned}$$

If $0 < x < 1$ then this is $\Theta(1)$, and if $x > 1$, then this is $\Theta(x^n)$.

Harmonic series For $n \geq 0$

$$\begin{aligned} H_n &= \sum_{i=1}^{n} \frac{1}{i} \\ &= 1 + \frac{1}{2} + \frac{1}{3} + \ldots + \frac{1}{n} \approx \ln n \\ &= \Theta(\ln n) \end{aligned}$$

1.11 Analysis: A Harder Example

Let us consider a harder example.

```
NESTED-LOOPS()
1  for i ← 1 to n
2  do
3     for j ← 1 to 2i
4     do k = j ...
5        while (k ≥ 0)
6        do k = k − 1 ...
```

How do we analyze the running time of an algorithm that has complex nested loop? The answer is we write out the loops as summations and then solve the summations. To convert loops into summations, we work from inside-out.

Consider the *inner most while* loop.

```
NESTED-LOOPS()
1  for i ← 1 to n
2  do for j ← 1 to 2i
3     do k = j
4        while (k ≥ 0)
5        do k = k − 1
```

It is executed for $k = j, j-1, j-2, \ldots, 0$. Time spent inside the while loop is constant. Let $I()$ be the time spent in the while loop. Thus

$$I(j) = \sum_{k=0}^{j} 1 = j + 1$$

Consider the *middle for* loop.

```
NESTED-LOOPS()
1  for i ← 1 to n
2  do for j ← 1 to 2i
3     do k = j
4        while (k ≥ 0)
5        do k = k − 1
```

Its running time is determined by i. Let $M()$ be the time spent in the for loop:

$$\begin{aligned} M(i) &= \sum_{j=1}^{2i} I(j) \\ &= \sum_{j=1}^{2i} (j+1) \\ &= \sum_{j=1}^{2i} j + \sum_{j=1}^{2i} 1 \\ &= \frac{2i(2i+1)}{2} + 2i \\ &= 2i^2 + 3i \end{aligned}$$

Finally the *outer-most for* loop.

```
NESTED-LOOPS()
1  for i ← 1 to n
2  do for j ← 1 to 2i
3       do k = j
4          while (k ≥ 0)
5          do k = k − 1
```

Let $T()$ be running time of the entire algorithm:

$$\begin{aligned} T(n) &= \sum_{i=1}^{n} M(i) \\ &= \sum_{i=1}^{n} (2i^2 + 3i) \\ &= \sum_{i=1}^{n} 2i^2 + \sum_{i=1}^{n} 3i \\ &= 2\frac{2n^3 + 3n^2 + n}{6} + 3\frac{n(n+1)}{2} \\ &= \frac{4n^3 + 15n^2 + 11n}{6} \\ &= \Theta(n^3) \end{aligned}$$

1.11.1 2-dimension Maxima Revisited

Recall the 2-d maxima problem: Let a point p in 2-dimensional space be given by its integer coordinates, $p = (p.x, p.y)$. A point p is said to *dominated* by point q if $p.x \leq q.x$ and $p.y \leq q.y$. Given a set of n

points, $P = \{p_1, p_2, \ldots, p_n\}$ in 2-space a point is said to be *maximal* if it is not dominated by any other point in P. The problem is to output all the maximal points of P. We introduced a brute-force algorithm that ran in $\Theta(n^2)$ time. It operated by comparing *all pairs* of points. Is there an approach that is significantly better?

The problem with the brute-force algorithm is that it uses no intelligence in *pruning* out decisions. For example, once we know that a point p_i is dominated by another point p_j, we do not need to use p_i for eliminating other points. This follows from the fact that dominance relation is *transitive*. If p_j dominates p_i and p_i dominates p_h then p_j also dominates p_h; p_i is not needed.

1.11.2 Plane-sweep Algorithm

The question is whether we can make an significant improvement in the running time? Here is an idea for how we might do it. We will sweep a vertical line across the plane from left to right. As we sweep this line, we will build a structure holding the maximal points lying to the left of the sweep line. When the sweep line reaches the rightmost point of P , then we will have constructed the complete set of maxima. This approach of solving geometric problems by sweeping a line across the plane is called *plane sweep*.

Although we would like to think of this as a continuous process, we need some way to perform the plane sweep in discrete steps. To do this, we will begin by sorting the points in increasing order of their x-coordinates. For simplicity, let us assume that no two points have the same y-coordinate. (This limiting assumption is actually easy to overcome, but it is good to work with the simpler version, and save the messy details for the actual implementation.) Then we will advance the sweep-line from point to point in n discrete steps. As we encounter each new point, we will update the current list of maximal points.

We will sweep a vertical line across the 2-d plane from left to right. As we sweep, we will build a structure holding the maximal points lying to the left of the sweep line. When the sweep line reaches the rightmost point of P, we will have the complete set of maximal points. We will store the existing maximal points in a list The points that p_i dominates will appear at the end of the list because points are sorted by x-coordinate. We will scan the list left to right. Every maximal point with y-coordinate less than p_i will be eliminated from computation. We will add maximal points onto the end of a list and delete from the end of the list. We can thus use a stack to store the maximal points. The point at the top of the stack will have the highest x-coordinate.

Here are a series of figures that illustrate the plane sweep. The figure also show the content of the stack.

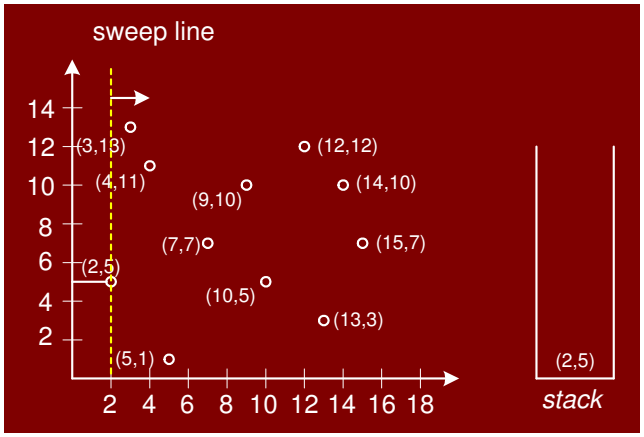


Figure 1.6: Sweep line at $(2, 5)$

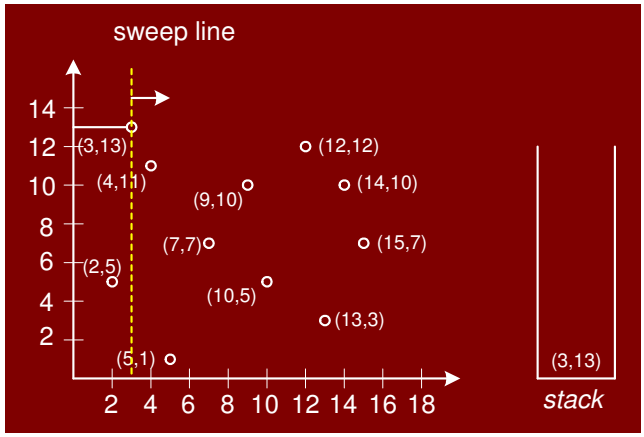


Figure 1.7: Sweep line at $(3, 13)$

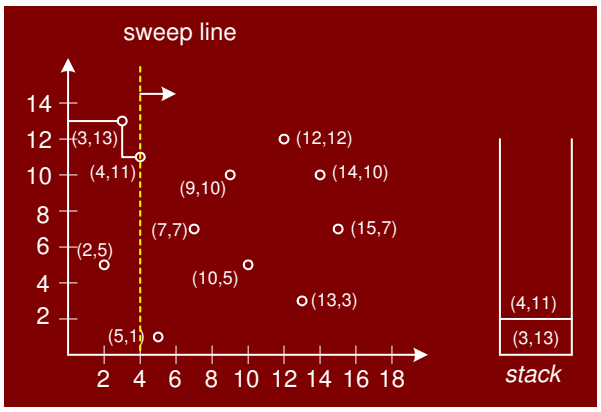


Figure 1.8: Sweep line at $(4, 11)$

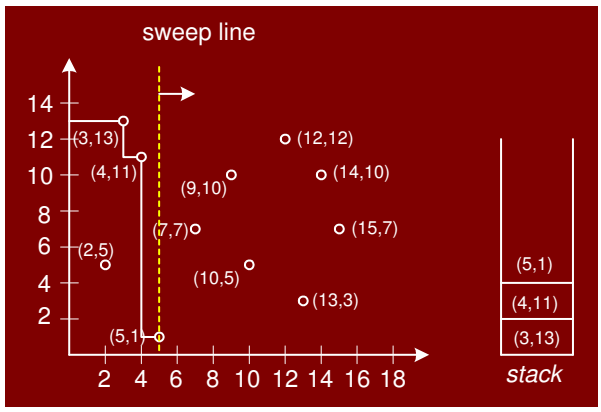


Figure 1.9: Sweep line at $(5, 1)$

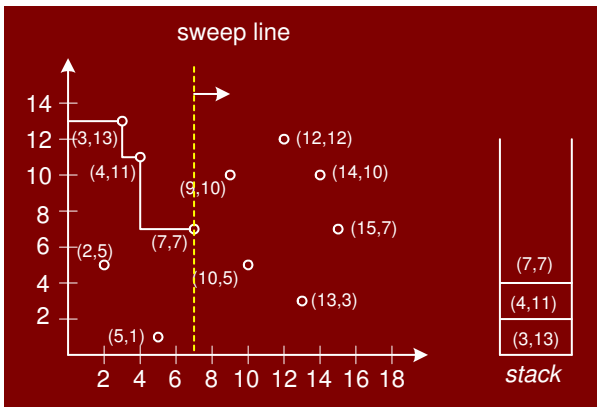


Figure 1.10: Sweep line at $(7, 7)$

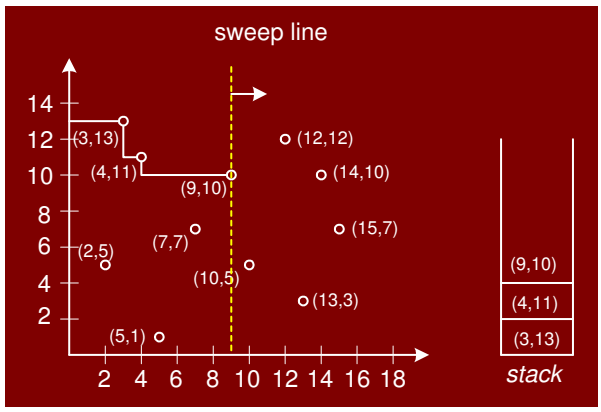


Figure 1.11: Sweep line at $(9, 10)$

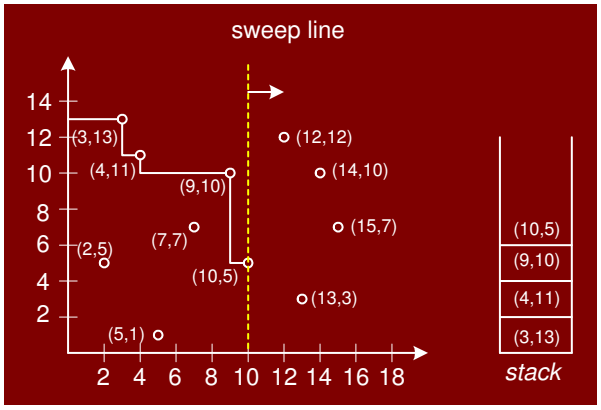


Figure 1.12: Sweep line at $(10, 5)$

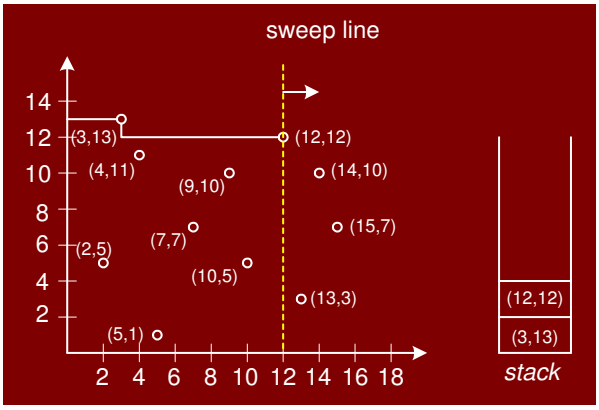


Figure 1.13: Sweep line at $(12, 12)$

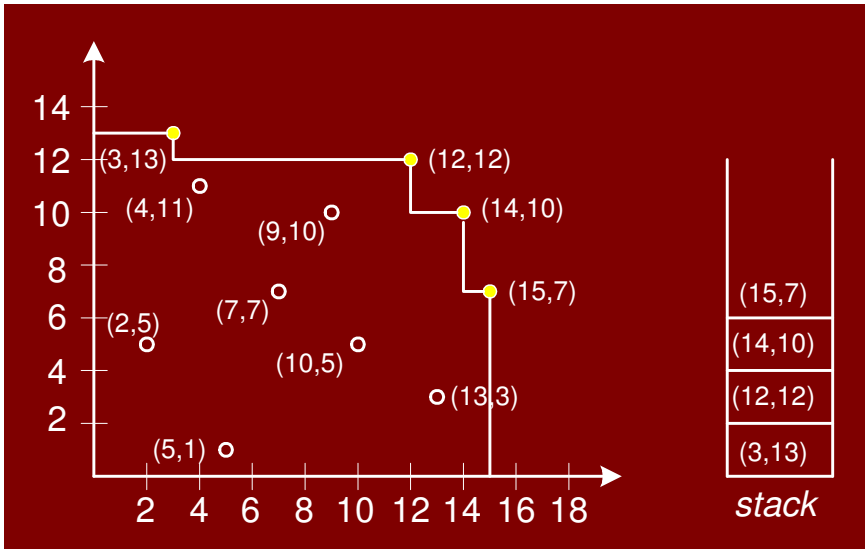


Figure 1.14: The final maximal set

Here is the algorithm.

PLANE-SWEEP-MAXIMA$(n,\ P[1..n])$

```
1  sort P in increasing order by x;
2  stack s;
3  for i ← 1 to n
4  do
5      while (s.notEmpty() & s.top().y ≤ P[i].y)
6      do s.pop();
7      s.push(P[i]);
8  output the contents of stack s;
```

1.11.3 Analysis of Plane-sweep Algorithm

Sorting takes $\Theta(n \log n)$; we will show this later when we discuss sorting. The for loop executes n times. The inner loop (seemingly) could be iterated $(n-1)$ times. It seems we still have an $n(n-1)$ or $\Theta(n^2)$ algorithm. Got fooled by simple minded loop-counting. The while loop will not execute more n times over the entire course of the algorithm. Why is this? Observe that the total number of elements that can be pushed on the stack is n since we execute exactly one push each time during the outer for-loop.

We pop an element off the stack each time we go through the inner while-loop. It is impossible to pop more elements than are ever pushed on the stack. Therefore, the inner while-loop cannot execute more than n times over the entire course of the algorithm. (*Make sure that you understand this*).

The for-loop iterates n times and the inner while-loop also iterates n time for a total of $\Theta(n)$. Combined with the sorting, the runtime of entire plane-sweep algorithm is $\Theta(n \log n)$.

1.11.4 Comparison of Brute-force and Plane sweep algorithms

How much of an improvement is plane-sweep over brute-force? Consider the ratio of running times:

$$\frac{n^2}{n \log n} = \frac{n}{\log n}$$

n	$\log n$	$\frac{n}{\log n}$
100	7	15
1000	10	100
10000	13	752
100000	17	6021
1000000	20	50171

For $n = 1,000,000$, if plane-sweep takes 1 second, the brute-force will take about 14 hours!. From this we get an idea about the importance of asymptotic analysis. It tells us which algorithm is better for large values of n. As we mentioned before, if n is not very large, then almost any algorithm will be fast. But efficient algorithm design is most important for large inputs, and the general rule of computing is that input sizes continue to grow until people can no longer tolerate the running times. Thus, by designing algorithms efficiently, you make it possible for the user to run large inputs in a reasonable amount of time.

Chapter 2

Asymptotic Notation

You may be asking that we continue to use the notation $\Theta()$ but have never defined it. Let's remedy this now. Given any function $g(n)$, we define $\Theta(g(n))$ to be a set of functions that *asymptotically equivalent* to $g(n)$. Formally:

$$\begin{aligned}\Theta(g(n)) = \{f(n) \mid &\text{there exist positive}\\ &\text{constants } c_1, c_2 \text{ and } n_0 \text{ such that}\\ &0 \leq c_1 g(n) \leq f(n) \leq c_2 g(n)\\ &\text{for all } n \geq n_0\}\end{aligned}$$

This is written as "$f(n) \in \Theta(g(n))$" That is, $f(n)$ and $g(n)$ are *asymptotically equivalent.* This means that they have essentially the same growth rates for large n. For example, functions like

- $4n^2$,
- $(8n^2 + 2n - 3)$,
- $(n^2/5 + \sqrt{n} - 10\log n)$
- $n(n-3)$

are all asymptotically equivalent. As n becomes large, the *dominant* (fastest growing) term is some constant times n^2.

Consider the function

$$f(n) = 8n^2 + 2n - 3$$

Our informal rule of keeping the largest term and ignoring the constant suggests that $f(n) \in \Theta(n^2)$. Let's see why this bears out formally. We need to show two things for

$$f(n) = 8n^2 + 2n - 3$$

Lower bound $f(n) = 8n^2 + 2n - 3$ grows asymptotically at least as fast as n^2,

Upper bound $f(n)$ grows no faster asymptotically than n^2,

Lower bound: $f(n)$ grows asymptotically at least as fast as n^2. For this, need to show that there exist positive constants c_1 and n_0, such that $f(n) \geq c_1 n^2$ for all $n \geq n_0$. Consider the reasoning

$$f(n) = 8n^2 + 2n - 3 \geq 8n^2 - 3 = 7n^2 + (n^2 - 3) \geq 7n^2$$

Thus $c_1 = 7$. We implicitly assumed that $2n \geq 0$ and $n^2 - 3 \geq 0$ These are not true for all n but if $n \geq \sqrt{3}$, then both are true. So select $n_0 \geq \sqrt{3}$. We then have $f(n) \geq c_1 n^2$ for all $n \geq n_0$.

Upper bound: $f(n)$ grows asymptotically no faster than n^2. For this, we need to show that there exist positive constants c_2 and n_0, such that $f(n) \leq c_2 n^2$ for all $n \geq n_0$. Consider the reasoning

$$f(n) = 8n^2 + 2n - 3 \leq 8n^2 + 2n \leq 8n^2 + 2n^2 = 10n^2$$

Thus $c_2 = 10$. We implicitly made the assumption that $2n \leq 2n^2$. This is not true for all n but it is true for all $n \geq 1$ So select $n_0 \geq 1$. We thus have $f(n) \leq c_2 n^2$ for all $n \geq n_0$.

From lower bound we have $n_0 \geq \sqrt{3}$ From upper bound we have $n_0 \geq 1$. Combining the two, we let n_0 be the larger of the two: $n_0 \geq \sqrt{3}$. In conclusion, if we let $c_1 = 7$, $c_2 = 10$ and $n_0 \geq \sqrt{3}$, we have

$$7n^2 \leq 8n^2 + 2n - 3 \leq 10n^2 \quad \text{for all } n \geq \sqrt{3}$$

We have thus established

$$0 \leq c_1 g(n) \leq f(n) \leq c_2 g(n) \qquad \text{for all } n \geq n_0$$

Here are plots of the three functions. Notice the bounds.

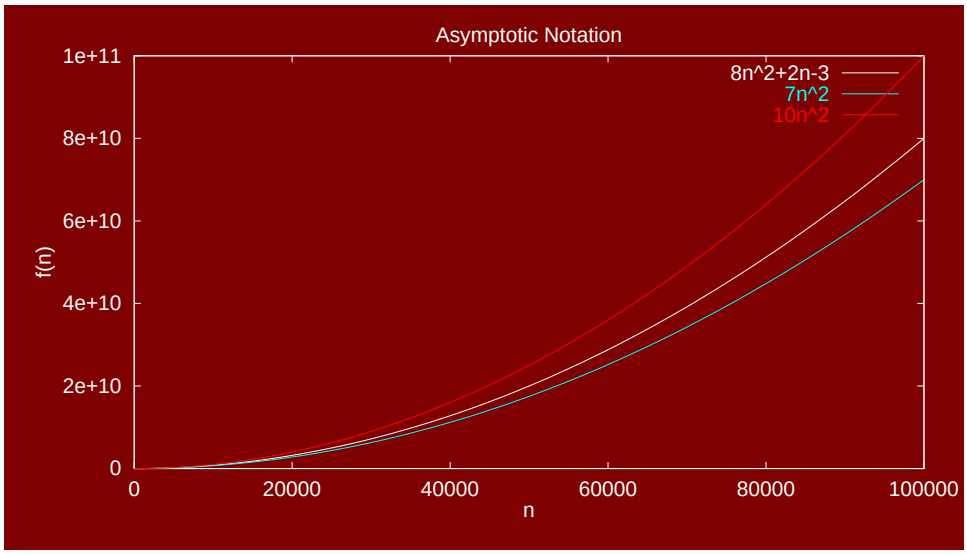


Figure 2.1: Asymptotic Notation Example

We have established that $f(n) \in n^2$. Let's show why $f(n)$ is not in some other asymptotic class. First, let's show that $f(n) \notin \Theta(n)$. Show that $f(n) \notin \Theta(n)$. If this were true, we would have had to satisfy

both the upper and lower bounds. The lower bound is satisfied because $f(n) = 8n^2 + 2n - 3$ does grow at least as fast asymptotically as n. But the upper bound is false. Upper bounds requires that there exist positive constants c_2 and n_0 such that $f(n) \leq c_2 n$ for all $n \geq n_0$.

Informally we know that $f(n) = 8n^2 + 2n - 3$ will eventually exceed $c_2 n$ no matter how large we make c_2. To see this, suppose we assume that constants c_2 and n_0 did exist such that $8n^2 + 2n - 3 \leq c_2 n$ for all $n \geq n_0$ Since this is true for all sufficiently large n then it must be true in the limit as n tends to infinity. If we divide both sides by n, we have

$$\lim_{n \to \infty} \left(8n + 2 - \frac{3}{n} \right) \leq c_2.$$

It is easy to see that in the limit, the left side tends to ∞. So, no matter how large c_2 is, the statement is violated. Thus $f(n) \notin \Theta(n)$.

Let's show that $f(n) \notin \Theta(n^3)$. The idea would be to show that the lower bound $f(n) \geq c_1 n^3$ for all $n \geq n_0$ is violated. (c_1 and n_0 are positive constants). Informally we know this to be true because any cubic function will overtake a quadratic.

If we divide both sides by n^3:

$$\lim_{n \to \infty} \left(\frac{8}{n} + \frac{2}{n^2} - \frac{3}{n^3} \right) \geq c_1$$

The left side tends to 0. The only way to satisfy this is to set $c_1 = 0$. But by hypothesis, c_1 is positive. This means that $f(n) \notin \Theta(n^3)$.

The definition of Θ-notation relies on proving both a lower and upper asymptotic bound. Sometimes we only interested in proving one bound or the other. The O-notation is used to state only the asymptotic *upper* bounds.

$$\begin{aligned} O(g(n)) = \{ f(n) \mid & \text{there exist positive} \\ & \text{constants } c \text{ and } n_0 \text{ such that} \\ & 0 \leq f(n) \leq cg(n) \\ & \text{for all } n \geq n_0 \} \end{aligned}$$

The Ω-notation allows us to state only the asymptotic *lower* bounds.

$$\begin{aligned} \Omega(g(n)) = \{ f(n) \mid & \text{there exist positive} \\ & \text{constants } c \text{ and } n_0 \text{ such that} \\ & 0 \leq cg(n) \leq f(n) \\ & \text{for all } n \geq n_0 \} \end{aligned}$$

The three notations:

$$\begin{aligned}\Theta(g(n)):&\ 0 \le c_1 g(n) \le f(n) \le c_2 g(n)\\ O(g(n)):&\ 0 \le f(n) \le cg(n)\\ \Omega(g(n)):&\ 0 \le cg(n) \le f(n)\\ &\text{for all } n \ge n_0\end{aligned}$$

These definitions suggest an alternative way of showing the asymptotic behavior. We can use limits for define the asymptotic behavior. Limit rule for Θ-notation:

$$\lim_{n\to\infty} \frac{f(n)}{g(n)} = c,$$

for some constant $c > 0$ (strictly positive but not infinity) then $f(n) \in \Theta(g(n))$. Similarly, the limit rule for O-notation is

$$\lim_{n\to\infty} \frac{f(n)}{g(n)} = c,$$

for some constant $c \ge 0$ (nonnegative but not infinite) then $f(n) \in O(g(n))$ and limit rule for Ω-notation:

$$\lim_{n\to\infty} \frac{f(n)}{g(n)} \neq 0,$$

(either a strictly positive constant or infinity) then $f(n) \in \Omega(g(n))$.

Here is a list of common asymptotic running times:

- $\Theta(1)$: Constant time; can't beat it!
- $\Theta(\log n)$: Inserting into a balanced binary tree; time to find an item in a sorted array of length n using binary search.
- $\Theta(n)$: About the fastest that an algorithm can run.
- $\Theta(n \log n)$: Best sorting algorithms.
- $\Theta(n^2)$, $\Theta(n^3)$: Polynomial time. These running times are acceptable when the exponent of n is small or n is not to large, e.g., $n \le 1000$.
- $\Theta(2^n)$, $\Theta(3^n)$: Exponential time. Acceptable only if n is small, e.g., $n \le 50$.
- $\Theta(n!)$, $\Theta(n^n)$: Acceptable only for really small n, e.g. $n \le 20$.

Chapter 3

Divide and Conquer Strategy

The ancient Roman politicians understood an important principle of good algorithm design (although they were probably not thinking about algorithms at the time). You divide your enemies (by getting them to distrust each other) and then conquer them piece by piece. This is called *divide-and-conquer*. In algorithm design, the idea is to take a problem on a large input, break the input into smaller pieces, solve the problem on each of the small pieces, and then combine the piecewise solutions into a global solution. But once you have broken the problem into pieces, how do you solve these pieces? The answer is to apply divide-and-conquer to them, thus further breaking them down. The process ends when you are left with such tiny pieces remaining (e.g. one or two items) that it is trivial to solve them. Summarizing, the main elements to a divide-and-conquer solution are

Divide: the problem into a small number of pieces

Conquer: solve each piece by applying divide and conquer to it *recursively*

Combine: the pieces together into a global solution.

3.1 Merge Sort

Divide and conquer strategy is applicable in a huge number of computational problems. The first example of divide and conquer algorithm we will discuss is a simple and efficient sorting procedure called We are given a sequence of n numbers A, which we will assume are stored in an array $A[1..n]$. The objective is to output a permutation of this sequence sorted in increasing order. This is normally done by permuting the elements within the array A. Here is how the merge sort algorithm works:

- (Divide:) split A down the middle into two subsequences, each of size roughly $n/2$
- (Conquer:) sort each subsequence by calling merge sort recursively on each.
- (Combine:) merge the two sorted subsequences into a single sorted list.

Let's design the algorithm top-down. We'll assume that the procedure that merges two sorted list is available to us. We'll implement it later. Because the algorithm is called recursively on sublists, in addition to passing in the array itself, we will pass in two indices, which indicate the first and last indices of the sub-array that we are to sort. The call $MergeSort(A, p, r)$ will sort the sub-array $A[p : r]$ and return the sorted result in the same sub-array.Here is the overview. If $r = p$, then this means that there is only one element to sort, and we may return immediately. Otherwise if $(p \neq r)$ there are at least two elements, and we will invoke the divide-and-conquer. We find the index q, midway between p and r, namely $q = (p + r)/2$ (rounded down to the nearest integer). Then we split the array into sub-arrays $A[p : q]$ and $A[q + 1 : r]$. Call MergeSort recursively to sort each sub-array. Finally, we invoke a procedure (which we have yet to write) which merges these two sub-arrays into a single sorted array.

Here is the MergeSort algorithm.

```
MERGE-SORT( array A, int p, int r)
1  if (p < r)
2    then
3          q ← (p + r)/2
4          MERGE-SORT(A, p, q)       // sort A[p..q]
5          MERGE-SORT(A, q + 1, r) // sort A[q + 1..r]
6          MERGE(A, p, q, r)    // merge the two pieces
```

The following figure illustrates the dividing (splitting) procedure.

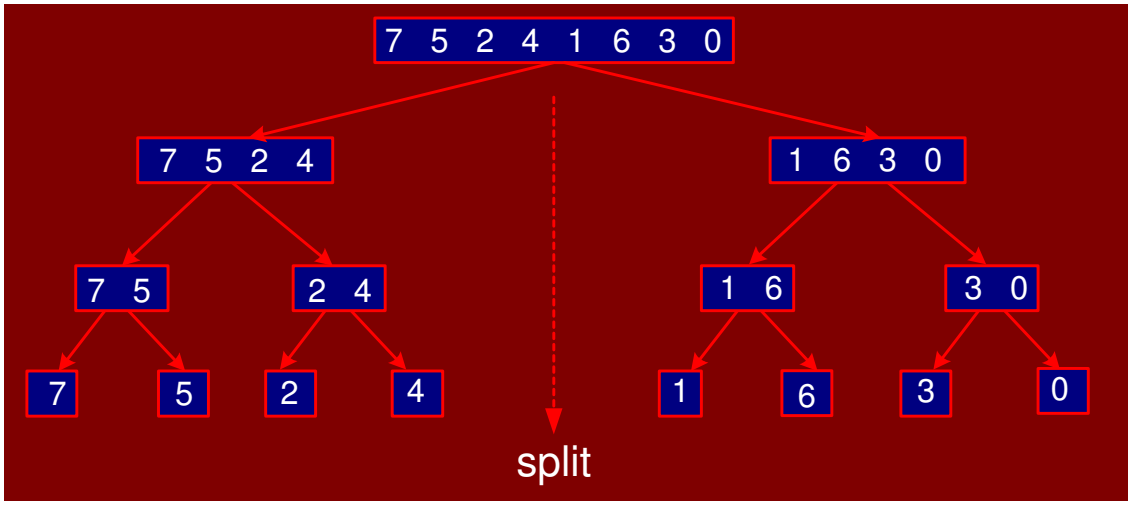


Figure 3.1: Merge sort divide phase

Merging: All that is left is to describe the procedure that merges two sorted lists. $Merge(A, p, q, r)$ assumes that the left sub-array, $A[p : q]$, and the right sub-array, $A[q + 1 : r]$, have already been sorted. We merge these two sub-arrays by copying the elements to a temporary working array called B. For convenience, we will assume that the array B has the same index range A, that is, $B[p : r]$. (One nice thing about pseudo-code, is that we can make these assumptions, and leave them up to the programmer to figure out how to implement it.) We have to indices i and j, that point to the current elements of each

sub-array. We move the smaller element into the next position of B (indicated by index k) and then increment the corresponding index (either i or j). When we run out of elements in one array, then we just copy the rest of the other array into B. Finally, we copy the entire contents of B back into A. (The use of the temporary array is a bit unpleasant, but this is impossible to overcome entirely. It is one of the shortcomings of MergeSort, compared to some of the other efficient sorting algorithms.)

Here is the merge algorithm:

```
MERGE( array A, int p, int q int r)
 1  int B[p..r]; int i ← k ← p; int j ← q + 1
 2  while (i ≤ q) and (j ≤ r)
 3  do if (A[i] ≤ A[j])
 4        then B[k++] ← A[i++]
 5        else B[k++] ← A[j++]
 6  while (i ≤ q)
 7  do B[k++] ← A[i++]
 8  while (j ≤ r)
 9  do B[k++] ← A[j++]
10  for i ← p to r
11  do A[i] ← B[i]
```

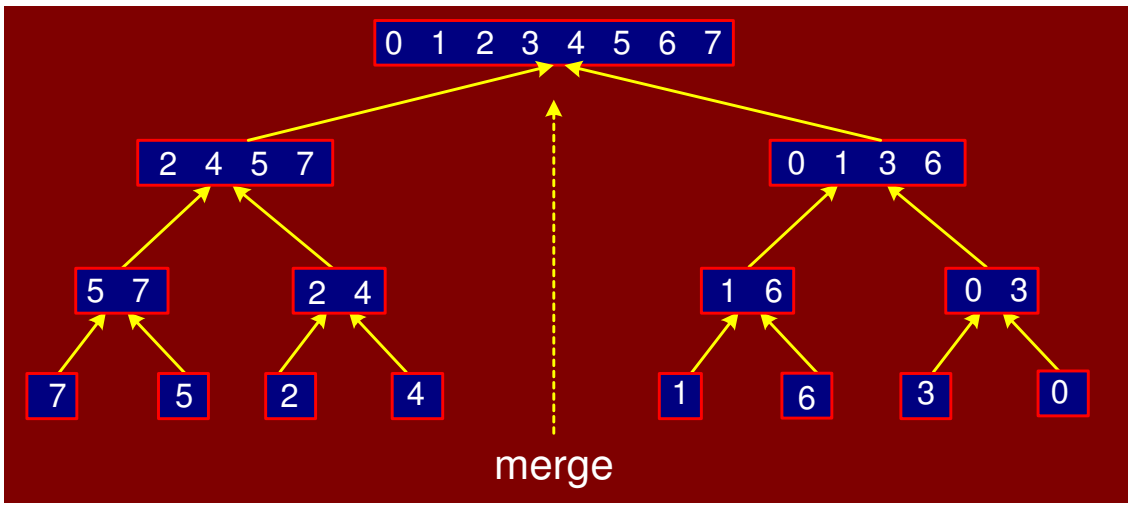


Figure 3.2: Merge sort: combine phase

3.1.1 Analysis of Merge Sort

First let us consider the running time of the procedure $Merge(A, p, q, r)$. Let $n = r - p + 1$ denote the total length of both the left and right sub-arrays. What is the running time of Merge as a function of n? The algorithm contains four loops (none nested in the other). It is easy to see that each loop can be executed at most n times. (If you are a bit more careful you can actually see that all the while-loops

together can only be executed n times in total, because each execution copies one new element to the array B, and B only has space for n elements.) Thus the running time to Merge n items is $\Theta(n)$. Let us write this without the asymptotic notation, simply as n. (We'll see later why we do this.)

Now, how do we describe the running time of the entire MergeSort algorithm? We will do this through the use of a recurrence, that is, a function that is defined recursively in terms of itself. To avoid circularity, the recurrence for a given value of n is defined in terms of values that are strictly smaller than n. Finally, a recurrence has some basis values (e.g. for $n = 1$), which are defined explicitly.

Let $T(n)$ denote the worst case running time of MergeSort on an array of length n. If we call MergeSort with an array containing a single item ($n = 1$) then the running time is constant. We can just write $T(n) = 1$, ignoring all constants. For $n > 1$, MergeSort splits into two halves, sorts the two and then merges them together. The left half is of size $\lceil n/2 \rceil$ and the right half is $\lfloor n/2 \rfloor$. How long does it take to sort elements in sub array of size $\lceil n/2 \rceil$? We do not know this but because $\lceil n/2 \rceil < n$ for $n > 1$, we can express this as $T(\lceil n/2 \rceil)$. Similarly the time taken to sort right sub array is expressed as $T(\lfloor n/2 \rfloor)$. In conclusion we have

$$T(n) = \begin{cases} 1 & \text{if } n = 1, \\ T(\lceil n/2 \rceil) + T(\lfloor n/2 \rfloor) + n & \text{otherwise} \end{cases}$$

This is called *recurrence relation*, i.e., a recursively defined function. Divide-and-conquer is an important design technique, and it naturally gives rise to recursive algorithms. It is thus important to develop mathematical techniques for solving recurrences, either exactly or asymptotically.

Let's expand the terms.

$$\begin{aligned} T(1) &= 1 \\ T(2) &= T(1) + T(1) + 2 = 1 + 1 + 2 = 4 \\ T(3) &= T(2) + T(1) + 3 = 4 + 1 + 3 = 8 \\ T(4) &= T(2) + T(2) + 4 = 4 + 4 + 4 = 12 \\ T(5) &= T(3) + T(2) + 5 = 8 + 4 + 5 = 17 \\ &\cdots \\ T(8) &= T(4) + T(4) + 8 = 12 + 12 + 8 = 32 \\ &\cdots \\ T(16) &= T(8) + T(8) + 16 = 32 + 32 + 16 = 80 \\ &\cdots \\ T(32) &= T(16) + T(16) + 32 = 80 + 80 + 32 = 192 \end{aligned}$$

What is the pattern here? Let's consider the ratios $T(n)/n$ for powers of 2:

$$\begin{aligned} T(1)/1 &= 1 & \qquad T(8)/8 &= 4 \\ T(2)/2 &= 2 & \qquad T(16)/16 &= 5 \\ T(4)/4 &= 3 & \qquad T(32)/32 &= 6 \end{aligned}$$

This suggests $T(n)/n = \log n + 1$ Or, $T(n) = n \log n + n$ which is $\Theta(n \log n)$ (using the limit rule).

3.1.2 The Iteration Method for Solving Recurrence Relations

Floor and ceilings are a pain to deal with. If n is assumed to be a power of 2 ($2^k = n$), this will simplify the recurrence to

$$T(n) = \begin{cases} 1 & \text{if } n = 1, \\ 2T(n/2) + n & \text{otherwise} \end{cases}$$

The iteration method turns the recurrence into a summation. Let's see how it works. Let's expand the recurrence:

$$\begin{aligned} T(n) &= 2T(n/2) + n \\ &= 2(2T(n/4) + n/2) + n \\ &= 4T(n/4) + n + n \\ &= 4(2T(n/8) + n/4) + n + n \\ &= 8T(n/8) + n + n + n \\ &= 8(2T(n/16) + n/8) + n + n + n \\ &= 16T(n/16) + n + n + n + n \end{aligned}$$

If n is a power of 2 then let $n = 2^k$ or $k = \log n$.

$$\begin{aligned} T(n) &= 2^k T(n/(2^k)) + \underbrace{(n + n + n + \cdots + n)}_{k \text{ times}} \\ &= 2^k T(n/(2^k)) + kn \\ &= 2^{(\log n)} T(n/(2^{(\log n)})) + (\log n)n \\ &= 2^{(\log n)} T(n/n) + (\log n)n \\ &= nT(1) + n \log n = n + n \log n \end{aligned}$$

3.1.3 Visualizing Recurrences Using the Recursion Tree

Iteration is a very powerful technique for solving recurrences. But, it is easy to get lost in all the symbolic manipulations and lose sight of what is going on. Here is a nice way to visualize what is going on in iteration. We can describe any recurrence in terms of a tree, where each expansion of the recurrence takes us one level deeper in the tree.

Recall that the recurrence for MergeSort (which we simplified by assuming that n is a power of 2, and hence could drop the floors and ceilings)

$$T(n) = \begin{cases} 1 & \text{if } n = 1, \\ 2T(n/2) + n & \text{otherwise} \end{cases}$$

Suppose that we draw the recursion tree for MergeSort, but each time we merge two lists, we label that node of the tree with the time it takes to perform the associated (nonrecursive) merge. Recall that to

merge two lists of size $m/2$ to a list of size m takes $\Theta(m)$ time, which we will just write as m. Below is an illustration of the resulting recursion tree.

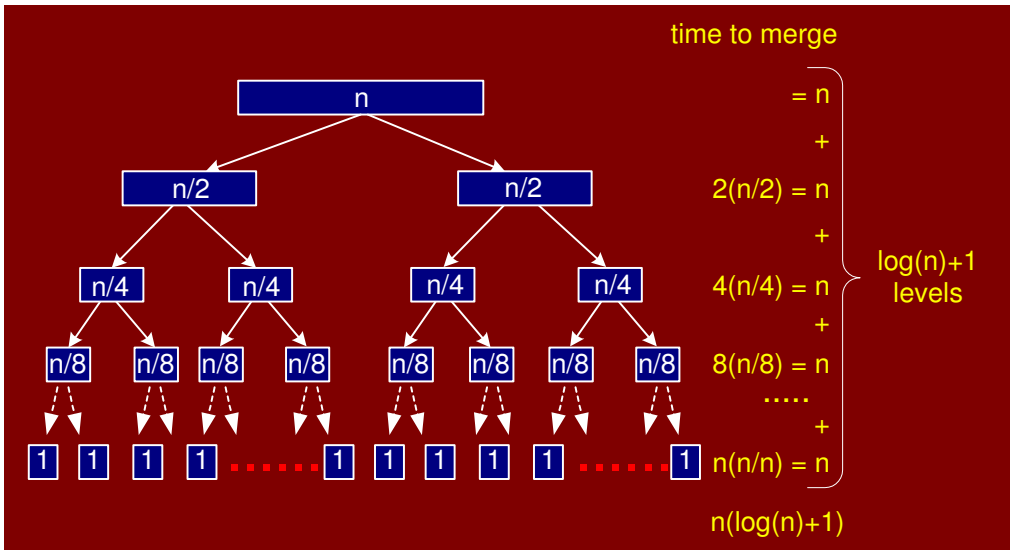


Figure 3.3: Merge sort Recurrence Tree

3.1.4 A Messier Example

The merge sort recurrence was too easy. Let's try a messier recurrence.

$$T(n) = \begin{cases} 1 & \text{if } n = 1, \\ 3T(n/4) + n & \text{otherwise} \end{cases}$$

Assume n to be a power of 4, i.e., $n = 4^k$ and $k = \log_4 n$

$$\begin{aligned} T(n) &= 3T(n/4) + n \\ &= 3(3T(n/16) + (n/4) + n \\ &= 9T(n/16) + 3(n/4) + n \\ &= 27T(n/64) + 9(n/16) + 3(n/4) + n \\ &= \dots \\ &= 3^k T(\frac{n}{4^k}) + 3^{k-1}(n/4^{k-1}) \\ &\qquad + \cdots + 9(n/16) + 3(n/4) + n \\ &= 3^k T(\frac{n}{4^k}) + \sum_{i=0}^{k-1} \frac{3^i}{4^i} n \end{aligned}$$

With $n = 4^k$ and $T(1) = 1$

$$\begin{aligned} T(n) &= 3^k T(\frac{n}{4^k}) + \sum_{i=0}^{k-1} \frac{3^i}{4^i} n \\ &= 3^{\log_4 n} T(1) + \sum_{i=0}^{(\log_4 n)-1} \frac{3^i}{4^i} n \\ &= n^{\log_4 3} + \sum_{i=0}^{(\log_4 n)-1} \frac{3^i}{4^i} n \end{aligned}$$

We used the formula $a^{\log_b n} = n^{\log_b a}$. n remains constant throughout the sum and $3^i/4^i = (3/4)^i$; we thus have

$$T(n) = n^{\log_4 3} + n \sum_{i=0}^{(\log_4 n)-1} \left(\frac{3}{4}\right)^i$$

The sum is a geometric series; recall that for $x \neq 1$

$$\sum_{i=0}^{m} x_i = \frac{x^{m+1} - 1}{x - 1}$$

In this case $x = 3/4$ and $m = \log_4 n - 1$. We get

$$T(n) = n^{\log_4 3} + n \frac{(3/4)^{\log_4 n + 1} - 1}{(3/4) - 1}$$

Applying the log identity once more

$$\begin{aligned} (3/4)^{\log_4 n} &= n^{\log_4 (3/4)} = n^{\log_4 3 - \log_4 4} \\ &= n^{\log_4 3 - 1} = \frac{n^{\log_4 3}}{n} \end{aligned}$$

If we plug this back, we get

$$\begin{aligned} T(n) &= n^{\log_4 3} + n \frac{\frac{n^{\log_4 3}}{n} - 1}{(3/4) - 1} \\ &= n^{\log_4 3} + \frac{n^{\log_4 3} - n}{-1/4} \\ &= n^{\log_4 3} + 4(n - n^{\log_4 3}) \\ &= 4n - 3n^{\log_4 3} \end{aligned}$$

With $\log_4 3 \approx 0.79$, we finally have the result!

$$T(n) = 4n - 3n^{\log_4 3} \approx 4n - 3n^{0.79} \in \Theta(n)$$

3.2 Selection Problem

Suppose we are given a set of n numbers. Define the *rank* of an element to be one plus the number of elements that are smaller. Thus, the rank of an element is its final position if the set is sorted. The minimum is of rank 1 and the maximum is of rank n.

Consider the set: $\{5, 7, 2, 10, 8, 15, 21, 37, 41\}$. The rank of each number is its position in the sorted order.

position	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*
Number	2	5	7	8	10	15	21	37	41

For example, rank of 8 is 4, one plus the number of elements smaller than 8 which is 3.

The selection problem is stated as follows:

> Given a set A of n distinct numbers and an integer k, $1 \leq k \leq n$, output the element of A of rank k.

Of particular interest in statistics is the *median*. If n is odd then the median is defined to be element of rank $(n+1)/2$. When n is even, there are two choices: $n/2$ and $(n+1)/2$. In statistics, it is common to return the average of the two elements.

Medians are useful as a measures of the *central tendency* of a set especially when the distribution of values is highly skewed. For example, the median income in a community is a more meaningful measure than average. Suppose 7 households have monthly incomes 5000, 7000, 2000, 10000, 8000, 15000 and 16000. In sorted order, the incomes are 2000, 5000, 7000, 8000, 10000, 15000, 16000. The median income is 8000; median is element with rank 4: $(7+1)/2 = 4$. The average income is 9000. Suppose the income 16000 goes up to 450,000. The median is still 8000 but the average goes up to 71,000. Clearly, the average is not a good representative of the majority income levels.

The selection problem can be easily solved by simply sorting the numbers of A and returning $A[k]$. Sorting, however, requires $\Theta(n \log n)$ time. The question is: can we do better than that? In particular, is it possible to solve the selections problem in $\Theta(n)$ time? The answer is yes. However, the solution is far from obvious.

3.2.1 Sieve Technique

The reason for introducing this algorithm is that it illustrates a very important special case of divide-and-conquer, which I call the *sieve technique*. We think of divide-and-conquer as breaking the problem into a small number of smaller subproblems, which are then solved recursively. The sieve technique is a special case, where the number of subproblems is just 1.

The sieve technique works in phases as follows. It applies to problems where we are interested in finding a single item from a larger set of n items. We do not know which item is of interest, however after doing some amount of analysis of the data, taking say $\Theta(nk)$ time, for some constant k, we find that we do not

know what the desired the item is, but we can identify a large enough number of elements that cannot be the desired value, and can be eliminated from further consideration. In particular "large enough" means that the number of items is at least some fixed constant fraction of n (e.g. $n/2$, $n/3$). Then we solve the problem recursively on whatever items remain. Each of the resulting recursive solutions then do the same thing, eliminating a constant fraction of the remaining set.

3.2.2 Applying the Sieve to Selection

To see more concretely how the sieve technique works, let us apply it to the selection problem. We will begin with the given array $A[1..n]$. We will pick an item from the array, called the *pivot element* which we will denote by x. We will talk about how an item is chosen as the pivot later; for now just think of it as a random element of A.

We then *partition* A into three parts.

1. $A[q]$ contains the pivot element x,
2. $A[1..q-1]$ will contain all the elements that are less than x and
3. $A[q+1..n]$ will contains all elements that are greater than x.

Within each sub array, the items may appear in any order. The following figure shows a partitioned array:

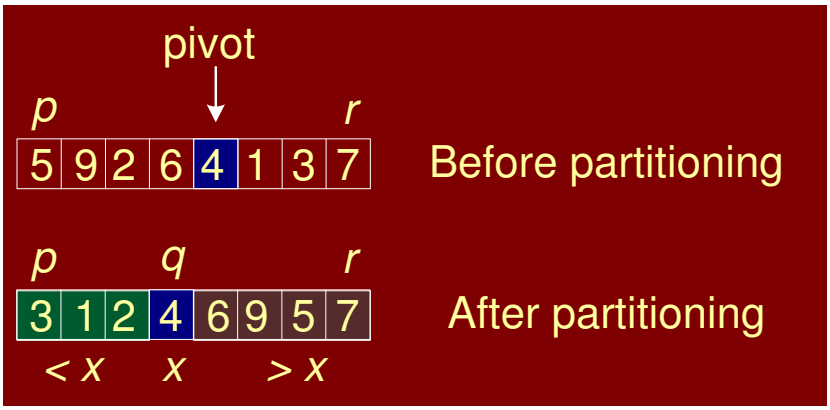


Figure 3.4: $A[p..r]$ partitioned about the pivot x

3.2.3 Selection Algorithm

It is easy to see that the rank of the The rank of the pivot x is $q-p+1$ in $A[p..r]$. Let $rank_x = q-p+1$. If $k = rank_x$ then the pivot is k^{th} smallest. If $k < rank_x$ then search $A[p..q-1]$ recursively. If $k > rank_x$ then search $A[q+1..r]$ recursively. Find element of rank $(k-q)$ because we eliminated q smaller elements in A.

SELECT(array A, int p, int r, int k)

```
1  if (p = r)
2    then return A[p]
3    else  x ← CHOOSE_PIVOT(A, p, r)
4          q ← PARTITION(A, p, r, x)
5          rank_x ← q − p + 1
6          if k = rank_x
7            then return x
8          if k < rank_x
9            then return SELECT(A, p, q − 1, k)
10           else  return SELECT(A, q + 1, r, k − q)
```

Example: select the 6^{th} smallest element of the set $\{5, 9, 2, 6, 4, 1, 3, 7\}$

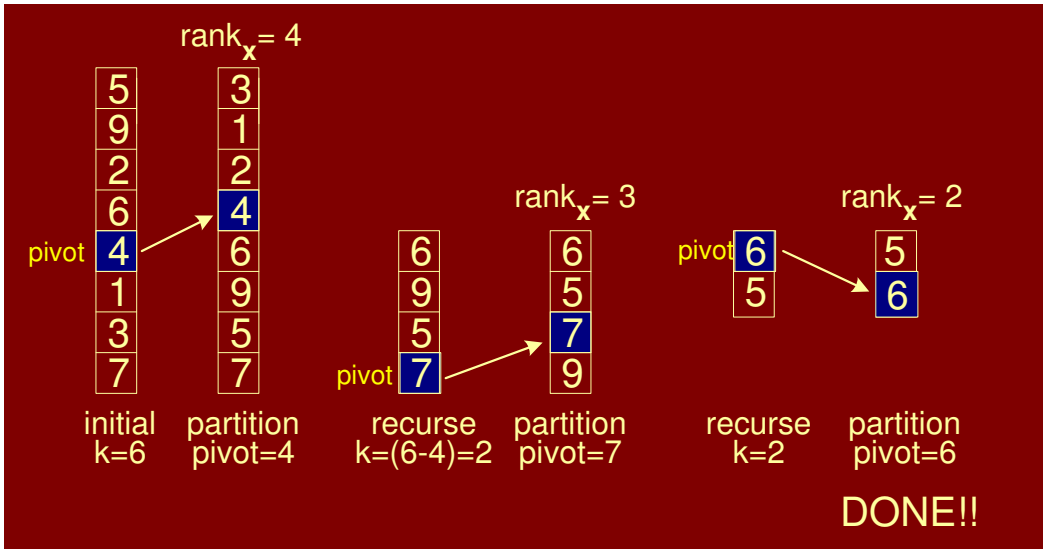


Figure 3.5: Sieve example: select 6^{th} smallest element

3.2.4 Analysis of Selection

We will discuss how to choose a pivot and the partitioning later. For the moment, we will assume that they both take $\Theta(n)$ time. How many elements do we eliminate in each time? If x is the largest or the smallest then we may only succeed in eliminating one element.

$$5, 9, 2, 6, 4, \boxed{1}, 3, 7 \qquad \text{pivot is } 1$$
$$\boxed{1}, 5, 9, 2, 6, 4, 3, 7 \qquad \text{after partition}$$

Ideally, x should have a rank that is neither too large or too small.

Suppose we are able to choose a pivot that causes exactly half of the array to be eliminated in each phase.

This means that we recurse on the remaining $n/2$ elements. This leads to the following recurrence:

$$T(n) = \begin{cases} 1 & \text{if } n = 1, \\ T(n/2) + n & \text{otherwise} \end{cases}$$

If we expand this recurrence, we get

$$\begin{aligned} T(n) &= n + \frac{n}{2} + \frac{n}{4} + \dots \\ &\leq \sum_{i=0}^{\infty} \frac{n}{2^i} \\ &= n \sum_{i=0}^{\infty} \frac{1}{2^i} \end{aligned}$$

Recall the formula for infinite geometric series; for any $|c| < 1$,

$$\sum_{i=0}^{\infty} c^i = \frac{1}{1-c}$$

Using this we have

$$T(n) \leq 2n \in \Theta(n)$$

Let's think about how we ended up with a $\Theta(n)$ algorithm for selection. Normally, a $\Theta(n)$ time algorithm would make a single or perhaps a constant number of passes of the data set. In this algorithm, we make a number of passes. In fact it could be as many as $\log n$.

However, because we eliminate a constant fraction of the array with each phase, we get the convergent geometric series in the analysis. This shows that the total running time is indeed *linear* in n. This lesson is well worth remembering. It is often possible to achieve linear running times in ways that you would not expect.

Chapter 4

Sorting

For the next series of lectures, we will focus on sorting. There a number of reasons for sorting. Here are a few important ones. Procedures for sorting are parts of many large software systems. Design of efficient sorting algorithms is necessary to achieve overall efficiency of these systems.

Sorting is well studied problem from the analysis point of view. Sorting is one of the few problems where provable lower bounds exist on how fast we can sort. In sorting, we are given an array $A[1..n]$ of n numbers We are to reorder these elements into increasing (or decreasing) order.

More generally, A is an array of objects and we sort them based on one of the attributes - the *key value*. The key value need not be a number. It can be any object from a *totally ordered domain*. Totally ordered domain means that for any two elements of the domain, x and y, either $x < y$, $x = y$ or $x > y$.

4.1 Slow Sorting Algorithms

There are a number of well-known slow $O(n^2)$ sorting algorithms. These include the following:

Bubble sort: Scan the array. Whenever two consecutive items are found that are out of order, swap them. Repeat until all consecutive items are in order.

Insertion sort: Assume that $A[1..i-1]$ have already been sorted. Insert $A[i]$ into its proper position in this sub array. Create this position by shifting all larger elements to the right.

Selection sort: Assume that $A[1..i-1]$ contain the $i-1$ smallest elements in sorted order. Find the smallest element in $A[i..n]$ Swap it with $A[i]$.

These algorithms are easy to implement. But they run in $\Theta(n^2)$ time in the worst case.

4.2 Sorting in $O(n \log n)$ time

We have already seen that Mergesort sorts an array of numbers in $\Theta(n \log n)$ time. We will study two others: *Heapsort* and *Quicksort*.

4.2.1 Heaps

A *heap* is a left-complete binary tree that conforms to the *heap order*. The heap order property: in a (min) heap, for every node X, the key in the parent is smaller than or equal to the key in X. In other words, the parent node has key smaller than or equal to both of its children nodes. Similarly, in a max heap, the parent has a key larger than or equal both of its children Thus the smallest key is in the root in a min heap; in the max heap, the largest is in the root.

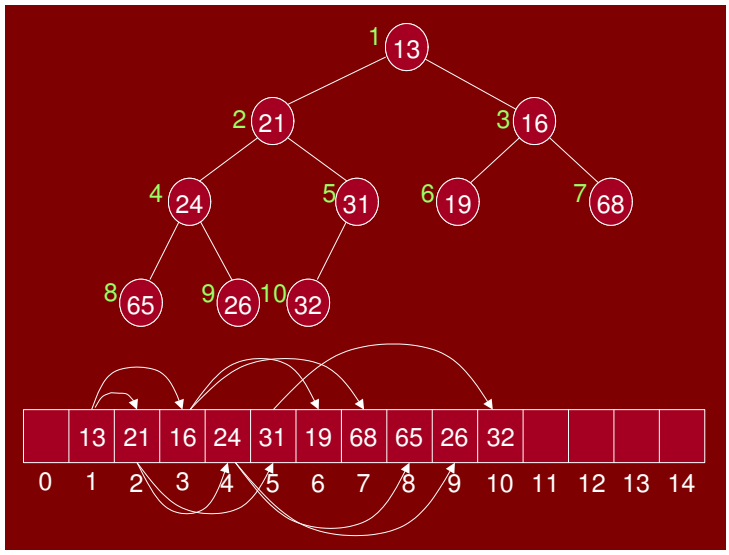


Figure 4.1: A min-heap

The number of nodes in a complete binary tree of height h is

$$n = 2^0 + 2^1 + 2^2 + \cdots + 2^h = \sum_{i=0}^{h} 2^i = 2^{h+1} - 1$$

h in terms of n is

$$h = (\log(n+1)) - 1 \approx \log n \in \Theta(\log n)$$

One of the clever aspects of heaps is that they can be stored in arrays without using any pointers. This is due to the left-complete nature of the binary tree. We store the tree nodes in level-order traversal. Access

to nodes involves simple arithmetic operations:

$$\begin{aligned} \texttt{left}(i) &: \text{returns } 2i, \text{ index of left child of node } i. \\ \texttt{right}(i) &: \text{returns } 2i+1, \text{ the right child.} \\ \texttt{parent}(i) &: \text{returns } \lfloor i/2 \rfloor, \text{ the parent of } i. \end{aligned}$$

The root is at position 1 of the array.

4.2.2 Heapsort Algorithm

We build a max heap out of the given array of numbers $A[1..n]$. We repeatedly extract the the maximum item from the heap. Once the max item is removed, we are left with a hole at the root. To fix this, we will replace it with the last leaf in tree. But now the heap order will very likely be destroyed. We will apply a heapify procedure to the root to restore the heap. Figure 4.2 shows an array being sorted.

```
HEAPSORT( array A, int n)
1  BUILD-HEAP(A, n)
2  m ← n
3  while (m ≥ 2)
4  do SWAP(A[1], A[m])
5     m ← m − 1
6     HEAPIFY(A, 1, m)
```

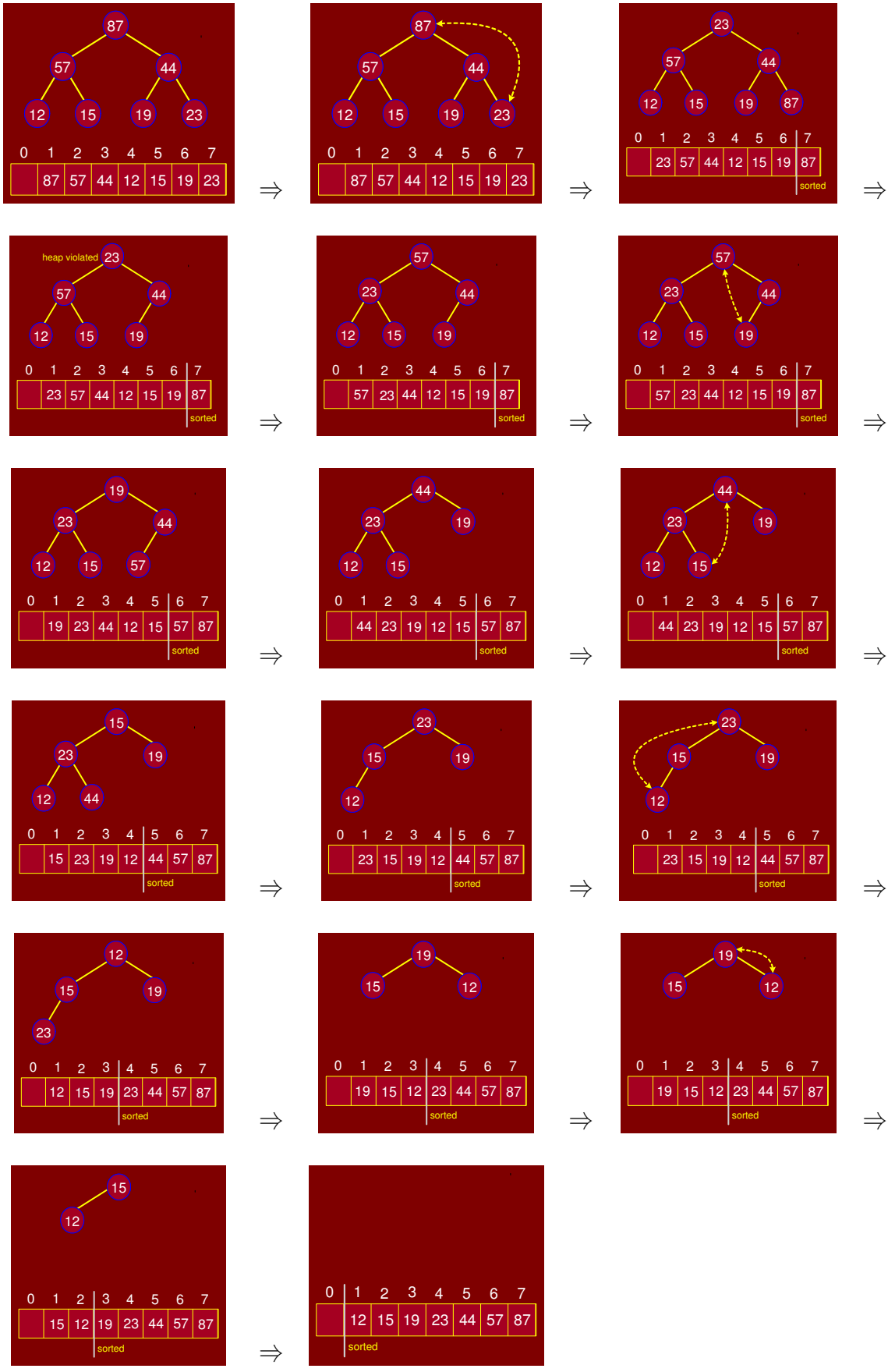


Figure 4.2: Example of heap sort

4.2.3 Heapify Procedure

There is one principal operation for maintaining the heap property. It is called Heapify. (In other books it is sometimes called sifting down.) The idea is that we are given an element of the heap which we suspect may not be in valid heap order, but we assume that all of other the elements in the subtree rooted at this element are in heap order. In particular this root element may be too small. To fix this we "sift" it down the tree by swapping it with one of its children. Which child? We should take the larger of the two children to satisfy the heap ordering property. This continues recursively until the element is either larger than both its children or until its falls all the way to the leaf level. Here is the algorithm. It is given the heap in the array A, and the index i of the suspected element, and m the current active size of the heap. The element $A[max]$ is set to the maximum of $A[i]$ and it two children. If $max \neq i$ then we swap $A[i]$ and $A[max]$ and then recurse on $A[max]$.

```
HEAPIFY( array A, int i, int m)
 1  l ← LEFT(i)
 2  r ← RIGHT(i)
 3  max ← i
 4  if (l ≤ m)and(A[l] > A[max])
 5     then max ← l
 6  if (r ≤ m)and(A[r] > A[max])
 7     then max ← r
 8  if (max ≠ i)
 9     then SWAP(A[i], A[max])
10          HEAPIFY(A, max, m)
```

4.2.4 Analysis of Heapify

We call heapify on the root of the tree. The maximum levels an element could move up is $\Theta(\log n)$ levels. At each level, we do simple comparison which $O(1)$. The total time for heapify is thus $O(\log n)$. Notice that it is not $\Theta(\log n)$ since, for example, if we call heapify on a leaf, it will terminate in $\Theta(1)$ time.

4.2.5 BuildHeap

We can use Heapify to build a heap as follows. First we start with a heap in which the elements are not in heap order. They are just in the same order that they were given to us in the array A. We build the heap by starting at the leaf level and then invoke Heapify on each node. (Note: We cannot start at the top of the tree. Why not? Because the precondition which Heapify assumes is that the entire tree rooted at node i is already in heap order, except for i.) Actually, we can be a bit more efficient. Since we know that each leaf is already in heap order, we may as well skip the leaves and start with the first non-leaf node. This will be in position $n/2$. (Can you see why?)

Here is the code. Since we will work with the entire array, the parameter m for Heapify, which indicates the current heap size will be equal to n, the size of array A, in all the calls.

```
BUILDHEAP( array A, int n)
1  for i ← n/2 downto 1
2  do
3      HEAPIFY(A, i, n)
```

4.2.6 Analysis of BuildHeap

For convenience, we will assume $n = 2^{h+1} - 1$ where h is the height of tree. The heap is a left-complete binary tree. Thus at each level j, $j < h$, there are 2^j nodes in the tree. At level h, there will be 2^h or less nodes. How much work does buildHeap carry out? Consider the heap in Figure 4.3:

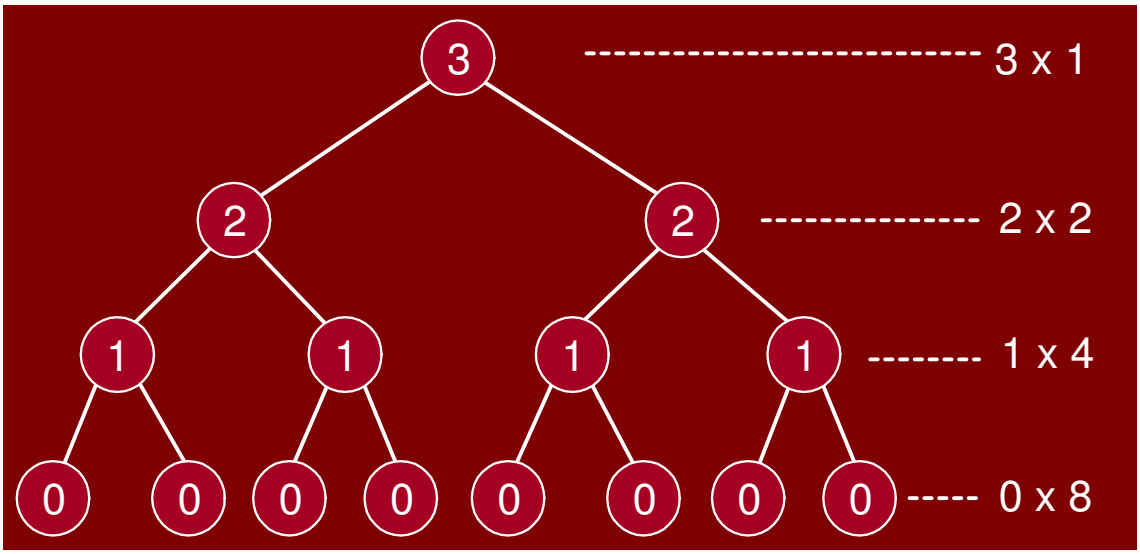


Figure 4.3: Total work performed in buildheap

At the bottom most level, there are 2^h nodes but we do not heapify these. At the next level up, there are 2^{h-1} nodes and each might shift down 1. In general, at level j, there are 2^{h-j} nodes and each may shift down j levels.

So, if count from bottom to top, level-by-level, the total time is

$$T(n) = \sum_{j=0}^{h} j2^{h-j} = \sum_{j=0}^{h} j\frac{2^h}{2^j}$$

We can factor out the 2^h term:

$$T(n) = 2^h \sum_{j=0}^{h} \frac{j}{2^j}$$

How do we solve this sum? Recall the geometric series, for any constant $x < 1$

$$\sum_{j=0}^{\infty} x^j = \frac{1}{1-x}$$

Take the derivative with respect to x and multiply by x

$$\sum_{j=0}^{\infty} jx^{j-1} = \frac{1}{(1-x)^2} \qquad \sum_{j=0}^{\infty} jx^j = \frac{x}{(1-x)^2}$$

We plug $x = 1/2$ and we have the desired formula:

$$\sum_{j=0}^{\infty} \frac{j}{2^j} = \frac{1/2}{(1-(1/2))^2} = \frac{1/2}{1/4} = 2$$

In our case, we have a bounded sum, but since we the infinite series is bounded, we can use it as an easy approximation:

$$\begin{aligned} T(n) &= 2^h \sum_{j=0}^{h} \frac{j}{2^j} \\ &\leq 2^h \sum_{j=0}^{\infty} \frac{j}{2^j} \\ &\leq 2^h \cdot 2 = 2^{h+1} \end{aligned}$$

Recall that $n = 2^{h+1} - 1$. Therefore

$$T(n) \leq n + 1 \in O(n)$$

The algorithm takes at least $\Omega(n)$ time since it must access every element at once. So the total time for BuildHeap is $\Theta(n)$.

BuildHeap is a relatively complex algorithm. Yet, the analysis yield that it takes $\Theta(n)$ time. An intuitive way to describe why it is so is to observe an important fact about binary trees The fact is that the vast majority of the nodes are at the lowest level of the tree. For example, in a complete binary tree of height h, there is a total of $n \approx 2^{h+1}$ nodes.

The number of nodes at the bottom three levels alone is

$$2^h + 2^{h-1} + 2^{h-2} = \frac{n}{2} + \frac{n}{4} + \frac{n}{8} = \frac{7n}{8} = 0.875n$$

Almost 90% of the nodes of a complete binary tree reside in the 3 lowest levels. Thus, algorithms that operate on trees should be efficient (as BuildHeap is) on the bottom-most levels since that is where most of the weight of the tree resides.

4.2.7 Analysis of Heapsort

Heapsort calls BuildHeap once. This takes $\Theta(n)$. Heapsort then extracts roughly n maximum elements from the heap. Each extract requires a constant amount of work (swap) and $O(\log n)$ heapify. Heapsort is thus $O(n \log n)$.

Is HeapSort $\Theta(n \log n)$? The answer is yes. In fact, later we will show that comparison based sorting algorithms can not run faster than $\Omega(n \log n)$. Heapsort is such an algorithm and so is Mergesort that we saw ealier.

4.3 Quicksort

Our next sorting algorithm is Quicksort. It is one of the fastest sorting algorithms known and is the method of choice in most sorting libraries. Quicksort is based on the divide and conquer strategy. Here is the algorithm:

```
QUICKSORT( array A, int p, int r)
1  if (r > p)
2    then
3         i ← a random index from [p..r]
4          swap A[i] with A[p]
5         q ← PARTITION(A, p, r)
6         QUICKSORT(A, p, q − 1)
7         QUICKSORT(A, q + 1, r)
```

4.3.1 Partition Algorithm

Recall that the partition algorithm partitions the array $A[p..r]$ into three sub arrays about a pivot element x. $A[p..q-1]$ whose elements are less than or equal to x, $A[q] = x$, $A[q+1..r]$ whose elements are greater than x

We will choose the first element of the array as the pivot, i.e. $x = A[p]$. If a different rule is used for selecting the pivot, we can swap the chosen element with the first element. We will choose the pivot randomly.

The algorithm works by maintaining the following *invariant condition*. $A[p] = x$ is the pivot value. $A[p..q-1]$ contains elements that are less than x, $A[q+1..s-1]$ contains elements that are greater than

or equal to x $A[s..r]$ contains elements whose values are currently unknown.

```
PARTITION( array A, int p, int r)
1  x ← A[p]
2  q ← p
3  for s ← p + 1 to r
4  do if (A[s] < x)
5        then q ← q + 1
6             swap A[q] with A[s]
7   swap A[p] with A[q]
8  return q
```

Figure 4.4 shows the execution trace partition algorithm.

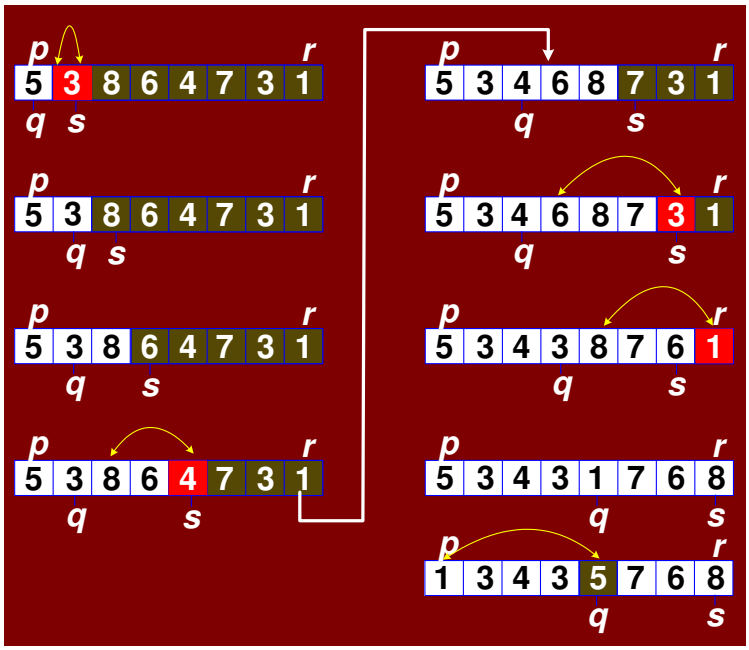


Figure 4.4: Trace of partitioning algorithm

4.3.2 Quick Sort Example

The Figure 4.5 trace out the quick sort algorithm. The first partition is done using the last element, 10, of the array. The left portion are then partitioned about 5 while the right portion is partitioned about 13. Notice that 10 is now at its final position in the eventual sorted order. The process repeats as the algorithm recursively partitions the array eventually sorting it.

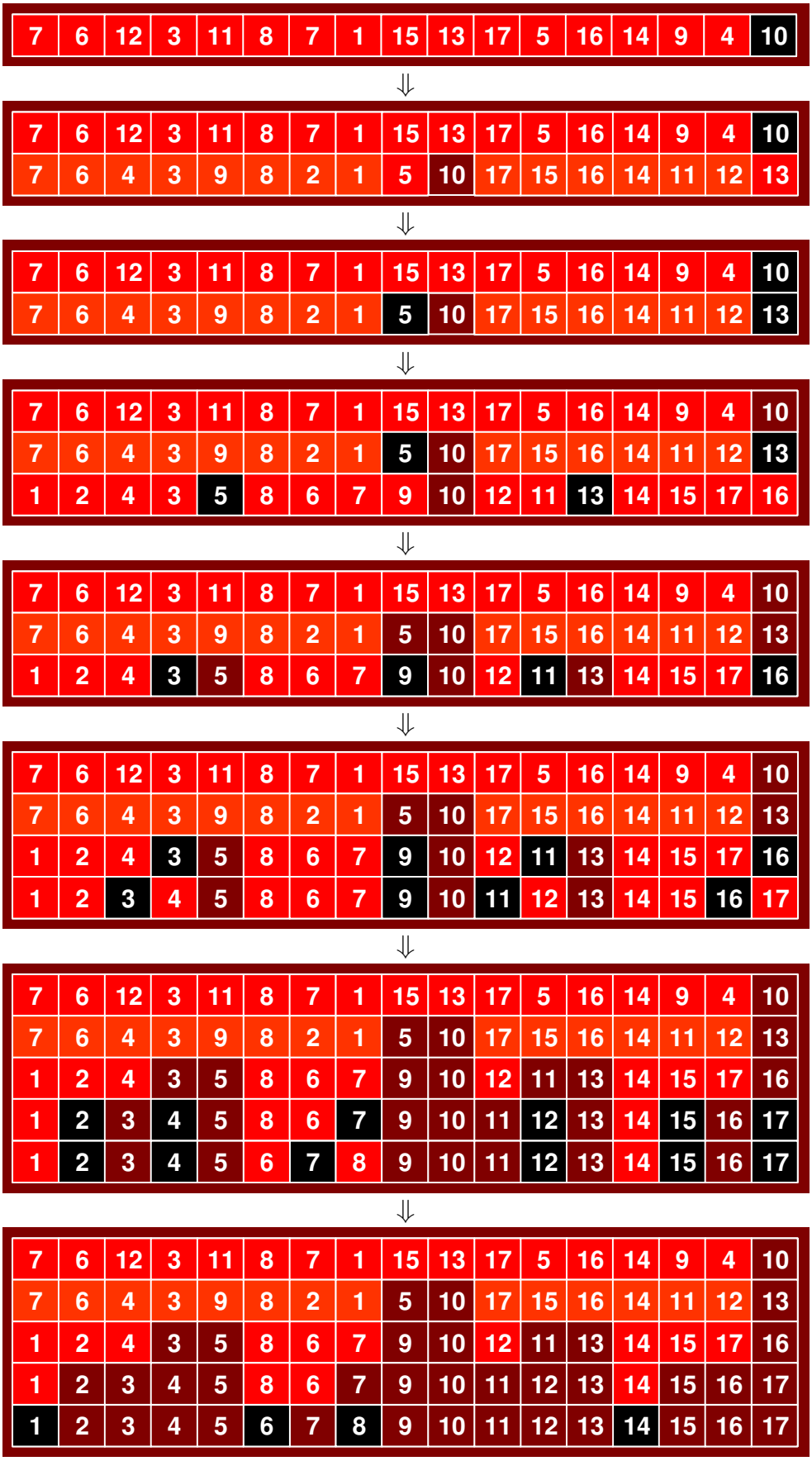


Figure 4.5: Example of quick sort

It is interesting to note (but not surprising) that the pivots form a binary search tree. This is illustrated in Figure 4.6.

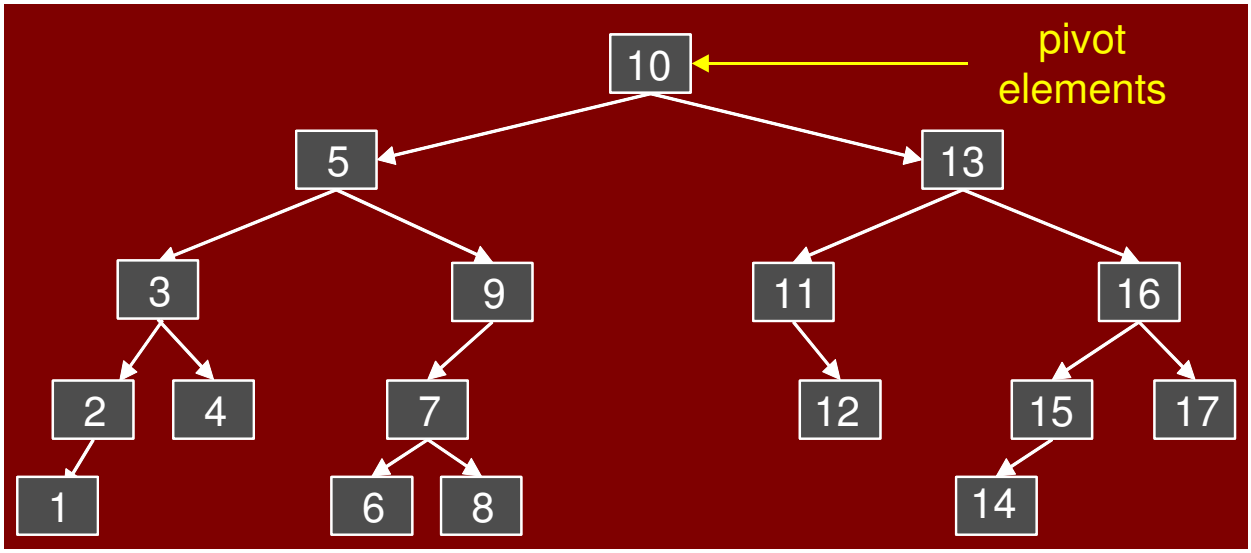


Figure 4.6: Quicksort BST

4.3.3 Analysis of Quicksort

The running time of quicksort depends heavily on the selection of the pivot. If the rank of the pivot is very large or very small then the partition (BST) will be unbalanced. Since the pivot is chosen randomly in our algorithm, the expected running time is $O(n \log n)$. The worst case time, however, is $O(n^2)$. Luckily, this happens rarely.

4.3.4 Worst Case Analysis of Quick Sort

Let's begin by considering the worst-case performance, because it is easier than the average case. Since this is a recursive program, it is natural to use a recurrence to describe its running time. But unlike MergeSort, where we had control over the sizes of the recursive calls, here we do not. It depends on how the pivot is chosen. Suppose that we are sorting an array of size n, $A[1 : n]$, and further suppose that the pivot that we select is of rank q, for some q in the range 1 to n. It takes $\Theta(n)$ time to do the partitioning and other overhead, and we make two recursive calls. The first is to the subarray $A[1 : q - 1]$ which has $q - 1$ elements, and the other is to the subarray $A[q + 1 : n]$ which has $n - q$ elements. So if we ignore the $\Theta(n)$ (as usual) we get the recurrence:

$$T(n) = T(q - 1) + T(n - q) + n$$

This depends on the value of q. To get the worst case, we maximize over all possible values of q. Putting is together, we get the recurrence

$$T(n) = \begin{cases} 1 & \text{if } n \leq 1 \\ \max\limits_{1 \leq q \leq n} (T(q-1) + T(n-q) + n) & \text{otherwise} \end{cases}$$

Recurrences that have max's and min's embedded in them are very messy to solve. The key is determining which value of q gives the maximum. (A rule of thumb of algorithm analysis is that the worst cases tends to happen either at the extremes or in the middle. So I would plug in the value $q = 1$, $q = n$, and $q = n/2$ and work each out.) In this case, the worst case happens at either of the extremes. If we expand the recurrence for $q = 1$, we get:

$$\begin{aligned} T(n) &\leq T(0) + T(n-1) + n \\ &= 1 + T(n-1) + n \\ &= T(n-1) + (n+1) \\ &= T(n-2) + n + (n+1) \\ &= T(n-3) + (n-1) + n + (n+1) \\ &= T(n-4) + (n-2) + (n-1) + n + (n+1) \\ &= T(n-k) + \sum_{i=-1}^{k-2} (n-i) \end{aligned}$$

For the basis $T(1) = 1$ we set $k = n - 1$ and get

$$\begin{aligned} T(n) &\leq T(1) + \sum_{i=-1}^{n-3} (n-i) \\ &= 1 + (3 + 4 + 5 + \cdots + (n-1) + n + (n+1)) \\ &\leq \sum_{i=1}^{n+1} i = \frac{(n+1)(n+2)}{2} \in \Theta(n^2) \end{aligned}$$

4.3.5 Average-case Analysis of Quicksort

We will now show that in the average case, quicksort runs in $\Theta(n \log n)$ time. Recall that when we talked about average case at the beginning of the semester, we said that it depends on some assumption about the distribution of inputs. However, in the case of quicksort, the analysis does not depend on the distribution of input at all. It only depends upon the random choices of pivots that the algorithm makes. This is good, because it means that the analysis of the algorithm's performance is the same for all inputs. In this case the average is computed over all possible random choices that the algorithm might make for the choice of the pivot index in the second step of the QuickSort procedure above.

To analyze the average running time, we let $T(n)$ denote the average running time of QuickSort on a list of size n. It will simplify the analysis to assume that all of the elements are distinct. The algorithm has n

random choices for the pivot element, and each choice has an equal probability of $1/n$ of occuring. So we can modify the above recurrence to compute an average rather than a max, giving:

$$T(n) = \begin{cases} 1 & \text{if } n \leq 1 \\ \frac{1}{n}\sum_{q=1}^{n}(T(q-1)+T(n-q)+n) & \text{otherwise} \end{cases}$$

The time $T(n)$ is the weighted sum of the times taken for various choices of q. I.e.,

$$\begin{aligned} T(n) = \Big[&\frac{1}{n}(T(0)+T(n-1)+n) \\ &+\frac{1}{n}(T(1)+T(n-2)+n) \\ &+\frac{1}{n}(T(2)+T(n-3)+n) \\ &+\cdots+\frac{1}{n}(T(n-1)+T(0)+n)\Big] \end{aligned}$$

We have not seen such a recurrence before. To solve it, expansion is possible but it is rather tricky. We will attempt a constructive induction to solve it. We know that we want a $\Theta(n \log n)$. Let us assume that $T(n) \leq cn \log n$ for $n \geq 2$ where c is a constant.

For the base case $n = 2$ we have

$$\begin{aligned} T(n) &= \frac{1}{2}\sum_{q=1}^{2}(T(q-1)+T(2-q)+2) \\ &= \frac{1}{2}\Big[(T(0)+T(1)+2)+(T(1)+T(0)+2)\Big] \\ &= \frac{8}{2} = 4. \end{aligned}$$

We want this to be at most $c2\log 2$, i.e.,

$$T(2) \leq c2\log 2$$

or

$$4 \leq c2\log 2$$

therefore

$$c \geq 4/(2\log 2) \approx 2.88.$$

For the induction step, we assume that $n \geq 3$ and The induction hypothesis is that for any $n' < n$, we have $T(n') \leq cn' \log n'$. We want to prove that it is true for $T(n)$. By expanding $T(n)$ and moving the

factor of n outside the sum we have

$$\begin{aligned} T(n) &= \frac{1}{n}\sum_{q=1}^{n}(T(q-1)+T(n-q)+n) \\ &= \frac{1}{n}\sum_{q=1}^{n}(T(q-1)+T(n-q))+n \\ &= \frac{1}{n}\sum_{q=1}^{n}T(q-1)+\frac{1}{n}\sum_{q=1}^{n}T(n-q)+n \end{aligned}$$

$$T(n) = \frac{1}{n}\sum_{q=1}^{n}T(q-1)+\frac{1}{n}\sum_{q=1}^{n}T(n-q)+n$$

Observe that the two sums add up the same values $T(0)+T(1)+\cdots+T(n-1)$. One counts up and other counts down. Thus we can replace them with $2\sum_{q=0}^{n-1}T(q)$. We will extract $T(0)$ and $T(1)$ and treat them specially. These two do not follow the formula.

$$\begin{aligned} T(n) &= \frac{2}{n}\Big(\sum_{q=0}^{n-1}T(q)\Big)+n \\ &= \frac{2}{n}\Big(T(0)+T(1)+\sum_{q=2}^{n-1}T(q)\Big)+n \end{aligned}$$

We will apply the induction hypothesis for $q < n$ we have

$$\begin{aligned} T(n) &= \frac{2}{n}\Big(T(0)+T(1)+\sum_{q=2}^{n-1}T(q)\Big)+n \\ &\leq \frac{2}{n}\Big(1+1+\sum_{q=2}^{n-1}(cq\log q)\Big)+n \\ &= \frac{2c}{n}\Big(\sum_{q=2}^{n-1}(cq\ln q)\Big)+n+\frac{4}{n} \end{aligned}$$

We have never seen this sum before:

$$S(n) = \sum_{q=2}^{n-1}(cq\ln q)$$

Recall from calculus that for any monotonically increasing function $f(x)$

$$\sum_{i=a}^{b-1}f(i) \leq \int_{a}^{b}f(x)dx$$

The function $f(x) = x\ln x$ is monotonically increasing, and so

$$S(n) = \sum_{q=2}^{n-1} (cq\ln q) \leq \int_2^n x\ln x\, dx \tag{4.1}$$

We can integrate this by parts:

$$\int_2^n x\ln x\, dx = \frac{x^2}{2}\ln x - \frac{x^2}{4}\Big|_{x=2}^{n}$$

$$\begin{aligned}\int_2^n x\ln x\, dx &= \frac{x^2}{2}\ln x - \frac{x^2}{4}\Big|_{x=2}^{n} \\ &= \Big(\frac{n^2}{2}\ln n - \frac{n^2}{4}\Big) - (2\ln 2 - 1) \\ &\leq \frac{n^2}{2}\ln n - \frac{n^2}{4}\end{aligned}$$

We thus have

$$S(n) = \sum_{q=2}^{n-1} (cq\ln q) \leq \frac{n^2}{2}\ln n - \frac{n^2}{4}$$

Plug this back into the expression for $T(n)$ to get

$$T(n) = \frac{2c}{n}\Big(\frac{n^2}{2}\ln n - \frac{n^2}{4}\Big) + n + \frac{4}{n}$$

$$\begin{aligned}T(n) &= \frac{2c}{n}\Big(\frac{n^2}{2}\ln n - \frac{n^2}{4}\Big) + n + \frac{4}{n} \\ &= cn\ln n - \frac{cn}{2} + n + \frac{4}{n} \\ &= cn\ln n + n\big(1 - \frac{c}{2}\big) + \frac{4}{n}\end{aligned}$$

$$T(n) = cn\ln n + n\big(1 - \frac{c}{2}\big) + \frac{4}{n}$$

To finish the proof,we want all of this to be at most $cn\ln n$. For this to happen, we will need to select c such that

$$n\big(1 - \frac{c}{2}\big) + \frac{4}{n} \leq 0$$

If we select $c = 3$, and use the fact that $n \geq 3$ we get

$$\begin{aligned}n\big(1 - \frac{c}{2}\big) + \frac{4}{n} &= \frac{3}{n} - \frac{n}{2} \\ &\leq 1 - \frac{3}{2} = -\frac{1}{2} \leq 0.\end{aligned}$$

From the basis case we had $c \geq 2.88$. Choosing $c = 3$ satisfies all the constraints. Thus $T(n) = 3n\ln n \in \Theta(n\log n)$.

4.4 In-place, Stable Sorting

An *in-place* sorting algorithm is one that uses no additional array for storage. A sorting algorithm is *stable* if duplicate elements remain in the same relative position after sorting.

$9 \mid 3 \mid 3' \mid 5 \mid 6 \mid 5' \mid 2 \mid 1 \mid 3''$	unsorted
$1 \mid 2 \mid 3 \mid 3' \mid 3'' \mid 5 \mid 5' \mid 6 \mid 9$	stable sort
$1 \mid 2 \mid 3' \mid 3 \mid 3'' \mid 5' \mid 5 \mid 6 \mid 9$	unstable

Bubble sort, insertion sort and selection sort are in-place sorting algorithms. Bubble sort and insertion sort can be implemented as stable algorithms but selection sort cannot (without significant modifications). Mergesort is a stable algorithm but not an in-place algorithm. It requires extra array storage. Quicksort is not stable but is an in-place algorithm. Heapsort is an in-place algorithm but is not stable.

4.5 Lower Bounds for Sorting

The best we have seen so far is $O(n \log n)$ algorithms for sorting. Is it possible to do better than $O(n \log n)$? If a sorting algorithm is solely based on comparison of keys in the array then it is *impossible* to sort more efficiently than $\Omega(n \log n)$ time. All algorithms we have seen so far are comparison-based sorting algorithms.

Consider sorting three numbers a_1, a_2, a_3. There are $3! = 6$ possible combinations:

$$(a_1, a_2, a_3), (a_1, a_3, a_2), (a_3, a_2, a_1)$$
$$(a_3, a_1, a_2), (a_2, a_1, a_3), (a_2, a_3, a_1)$$

One of these permutations leads to the numbers in sorted order.

The comparison based algorithm defines a *decision tree*. Here is the tree for the three numbers.

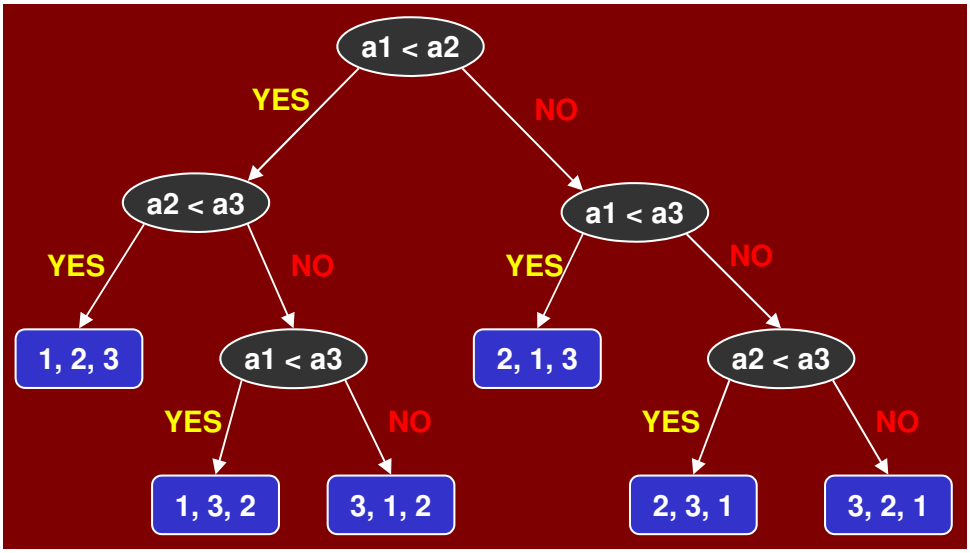


Figure 4.7: Decision Tree

For n elements, there will be $n!$ possible permutations. The height of the tree is exactly equal to $T(n)$, the running time of the algorithm. The height is $T(n)$ because any path from the root to a leaf corresponds to a sequence of comparisons made by the algorithm.

Any binary tree of height $T(n)$ has at most $2^{T(n)}$ leaves. Thus a comparison based sorting algorithm can distinguish between at most $2^{T(n)}$ different final outcomes. So we have

$$\begin{aligned} 2^{T(n)} &\geq n! \qquad \text{and therefore} \\ T(n) &\geq \log(n!) \end{aligned}$$

We can use *Stirling's approximation* for $n!$:

$$n! \geq \sqrt{2\pi n}\left(\frac{n}{e}\right)^n$$

Thereofore

$$\begin{aligned} T(n) &\geq \log\left(\sqrt{2\pi n}\left(\frac{n}{e}\right)^n\right) \\ &= \log(\sqrt{2\pi n}) + n\log n - n\log e \\ &\in \Omega(n\log n) \end{aligned}$$

We thus have the following theorem.

Theorem 1
Any comparison-based sorting algorithm has worst-case running time $\Omega(n\log n)$.

Chapter 5

Linear Time Sorting

The lower bound implies that if we hope to sort numbers faster than $O(n \log n)$, we cannot do it by making comparisons alone. Is it possible to sort without making comparisons? The answer is yes, but only under very restrictive circumstances. Many applications involve sorting small integers (e.g. sorting characters, exam scores, etc.). We present three algorithms based on the theme of speeding up sorting in special cases, by not making comparisons.

5.1 Counting Sort

We will consider three algorithms that are faster and work by not making comparisons. Counting sort assumes that the numbers to be sorted are in the range 1 to k where k is small. The basic idea is to determine the rank of each number in final sorted array.

Recall that the rank of an item is the number of elements that are less than or equal to it. Once we know the ranks, we simply copy numbers to their final position in an output array.

The question is how to find the rank of an element without comparing it to the other elements of the array?. The algorithm uses three arrays. As usual, $A[1..n]$ holds the initial input, $B[1..n]$ holds the sorted output and $C[1..k]$ is an array of integers. $C[x]$ is the rank of x in A, where $x \in [1..k]$. The algorithm is remarkably simple, but deceptively clever. The algorithm operates by first constructing C. This is done in two steps. First we set $C[x]$ to be the number of elements of $A[j]$ that are equal to x. We can do this initializing C to zero, and then for each j, from 1 to n, we increment $C[A[j]]$ by 1. Thus, if $A[j] = 5$, then the 5th element of C is incremented, indicating that we have seen one more 5. To determine the number of elements that are less than or equal to x, we replace $C[x]$ with the sum of elements in the sub array $R[1 : x]$. This is done by just keeping a running total of the elements of C.

$C[x]$ now contains the rank of x. This means that if $x = A[j]$ then the final position of $A[j]$ should be at position $C[x]$ in the final sorted array. Thus, we set $B[C[x]] = A[j]$. Notice We need to be careful if there are duplicates, since we do not want them to overwrite the same location of B. To do this, we decrement

C[i] after copying.

```
COUNTING-SORT( array A, array B, int k)
 1  for i ← 1 to k
 2  do C[i] ← 0          [k times]
 3  for j ← 1 to length[A]
 4  do C[A[j]] ← C[A[j]] + 1          [n times]
 5  // C[i] now contains the number of elements = i
 6  for i ← 2 to k
 7  do C[i] ← C[i] + C[i − 1]          [k times]
 8  // C[i] now contains the number of elements ≤ i
 9  for j ← length[A] downto 1
10  do B[C[A[j]]] ← A[j]
11     C[A[j]] ← C[A[j]] − 1          [n times]
```

There are four (unnested) loops, executed k times, n times, $k-1$ times, and n times, respectively, so the total running time is $\Theta(n+k)$ time. If $k = O(n)$, then the total running time is $\Theta(n)$.

Figure 5.1 through 5.19 shows an example of the algorithm. You should trace through the example to convince yourself how it works.

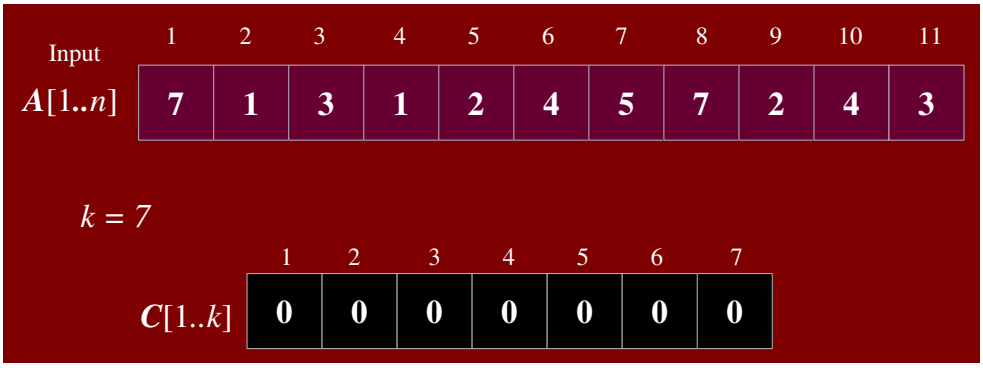


Figure 5.1: Initial A and C arrays.

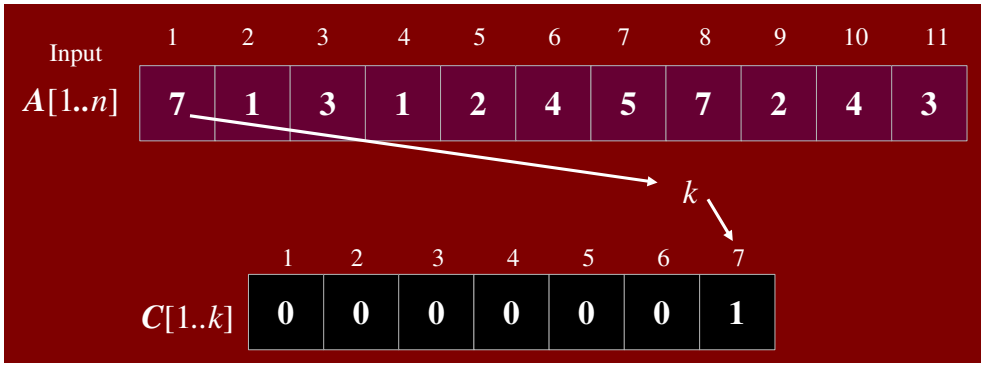


Figure 5.2: $A[1] = 7$ processed

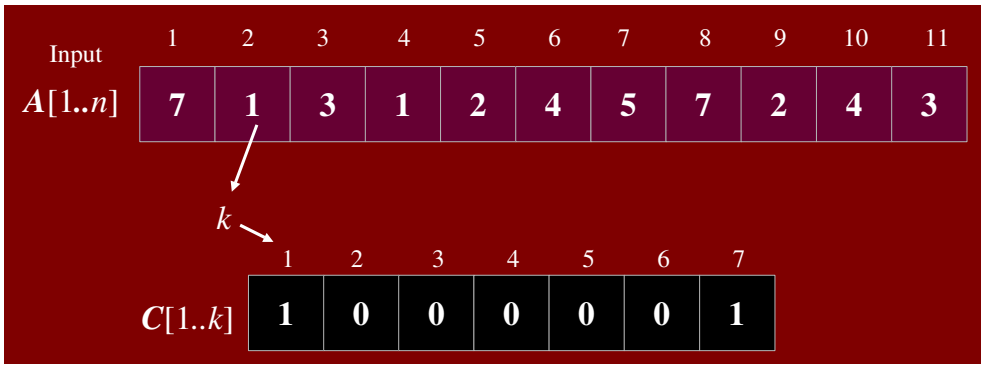


Figure 5.3: $A[2] = 1$ processed

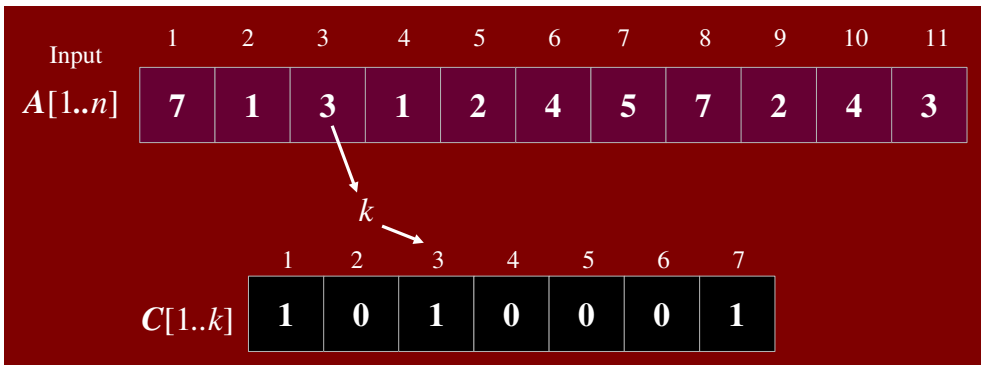


Figure 5.4: $A[3] = 3$ processed

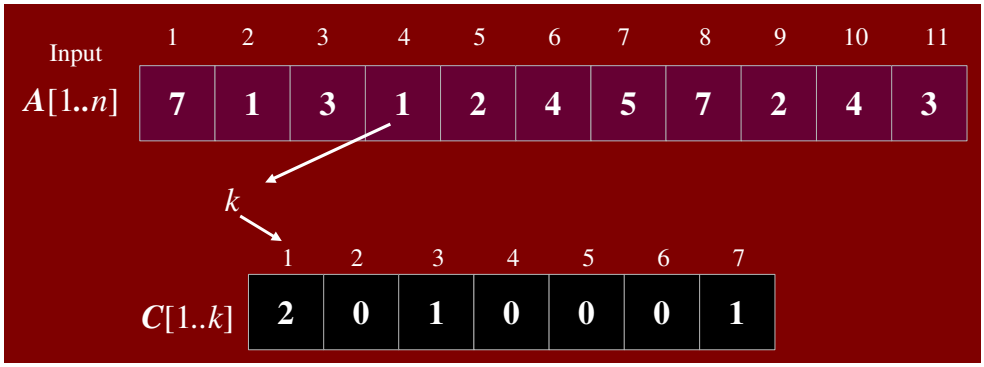


Figure 5.5: $A[4] = 1$ processed

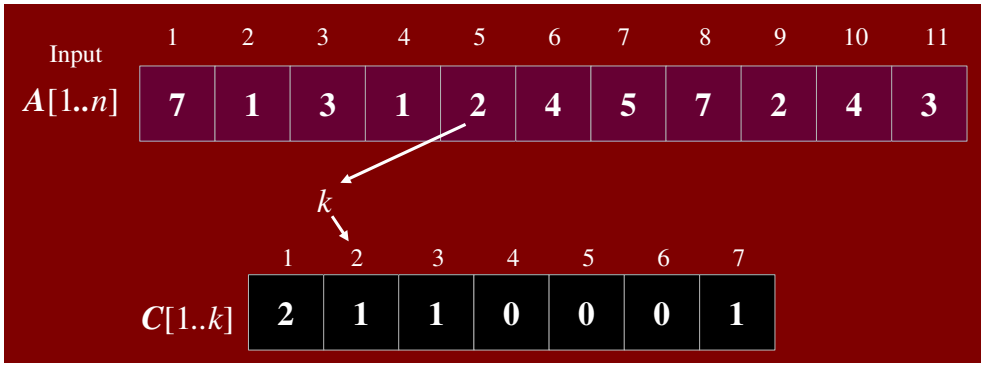


Figure 5.6: $A[5] = 2$ processed

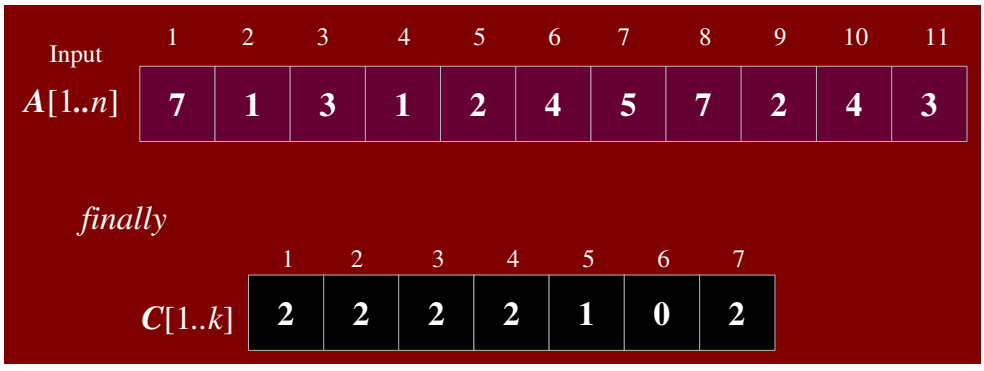


Figure 5.7: C now contains count of elements of A

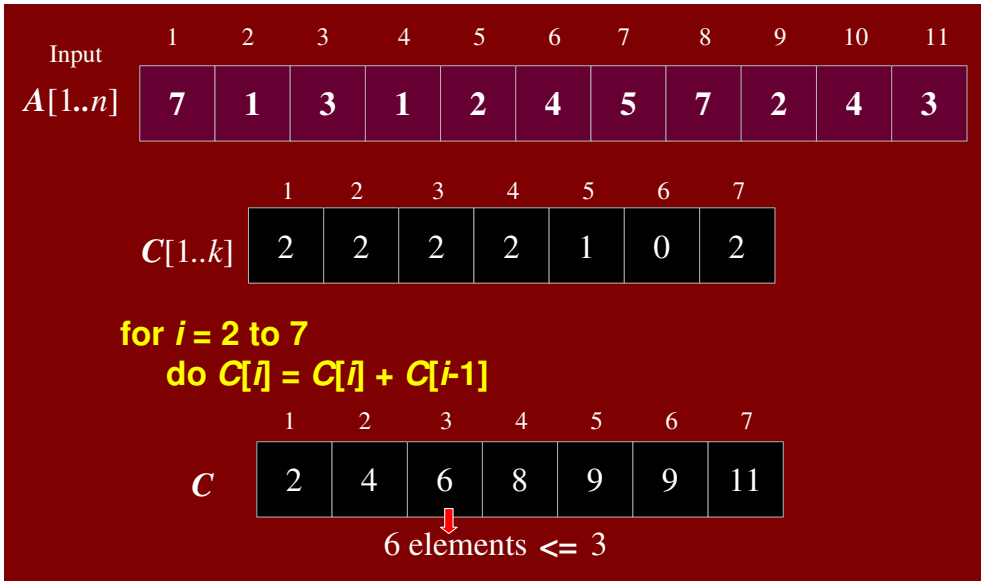


Figure 5.8: C set to rank each number of A

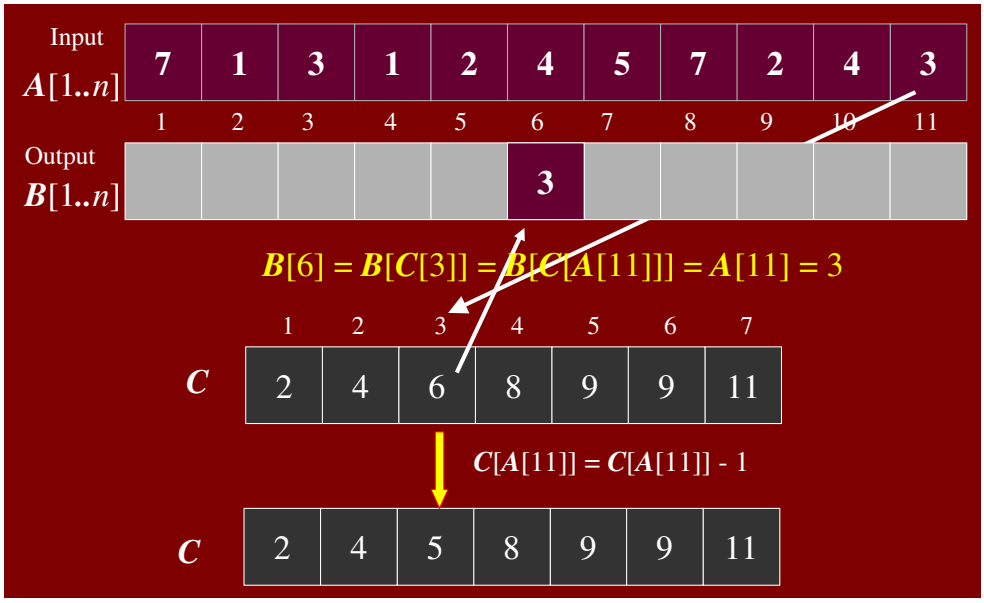


Figure 5.9: A[11] placed in output array B

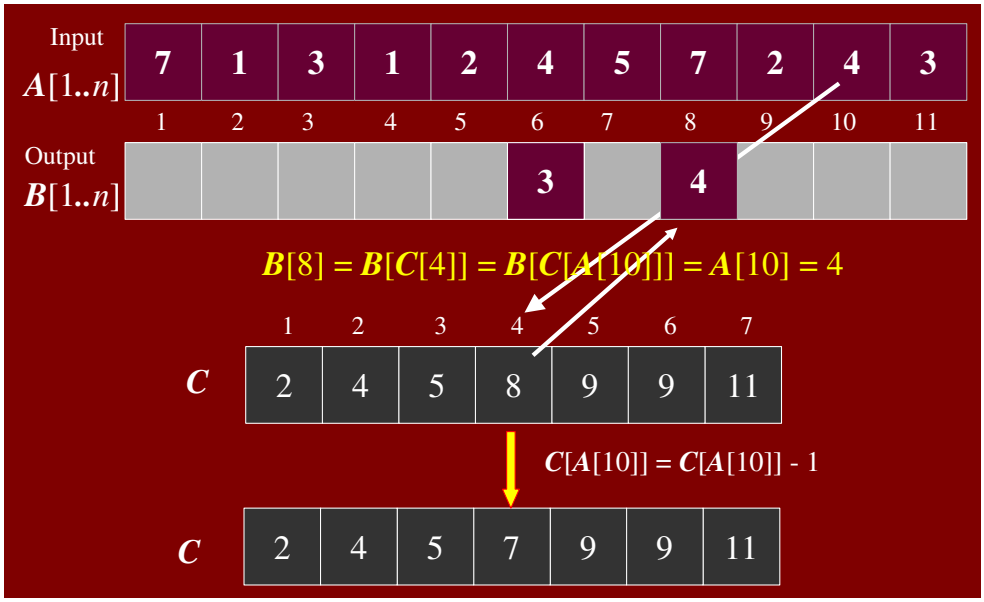


Figure 5.10: $A[10]$ placed in output array B

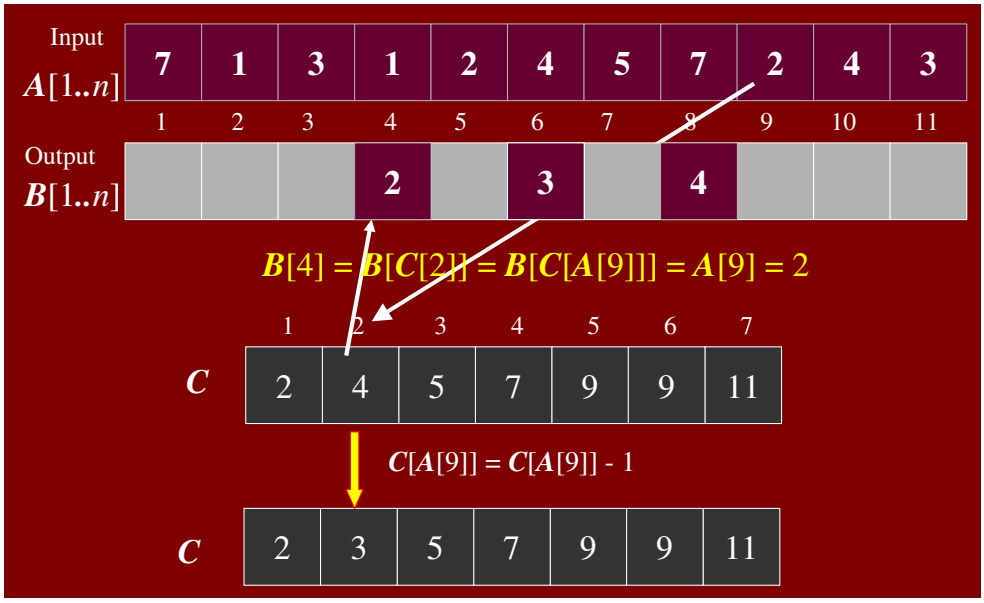


Figure 5.11: $A[9]$ placed in output array B

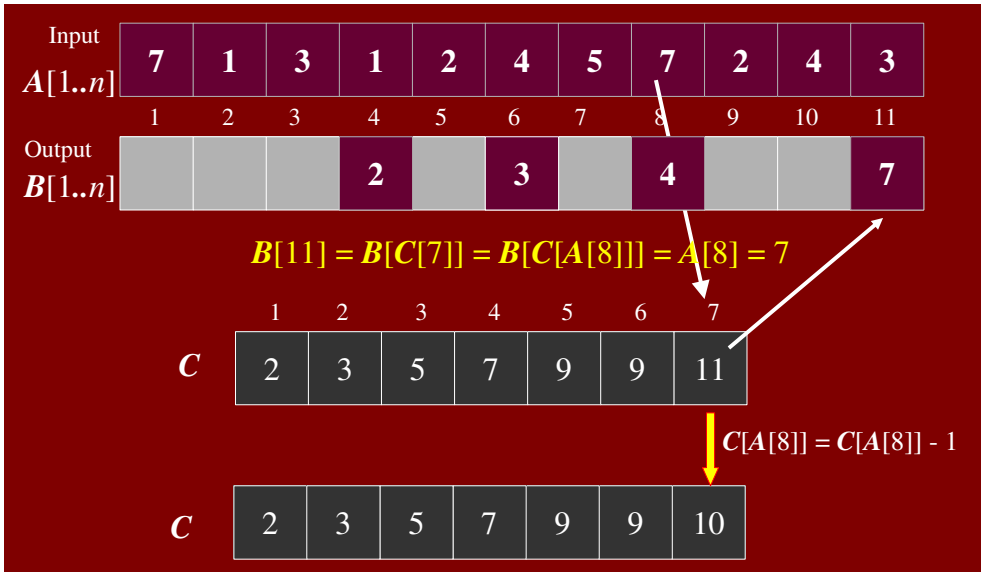


Figure 5.12: A[8] placed in output array B

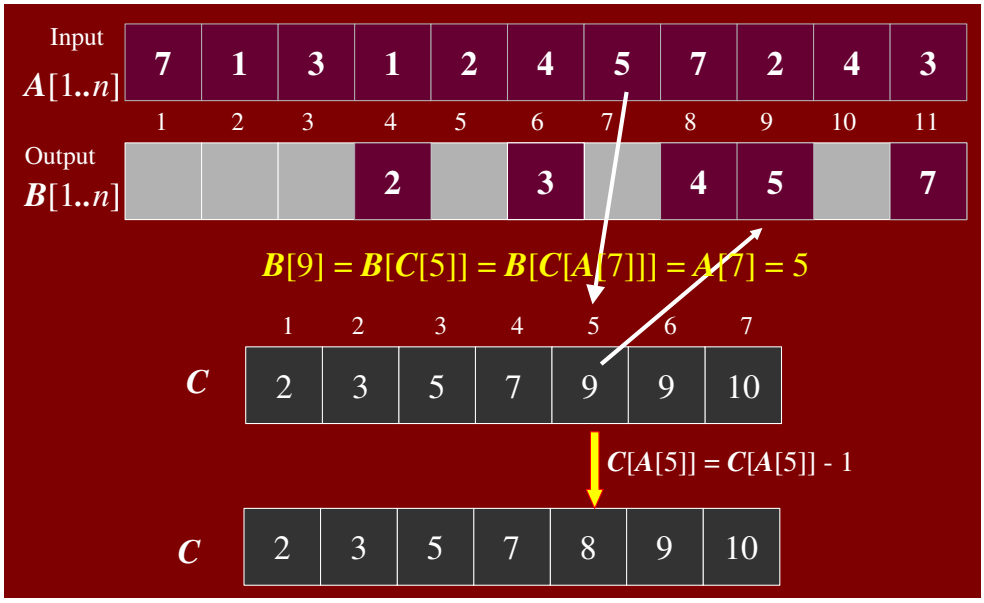


Figure 5.13: A[7] placed in output array B

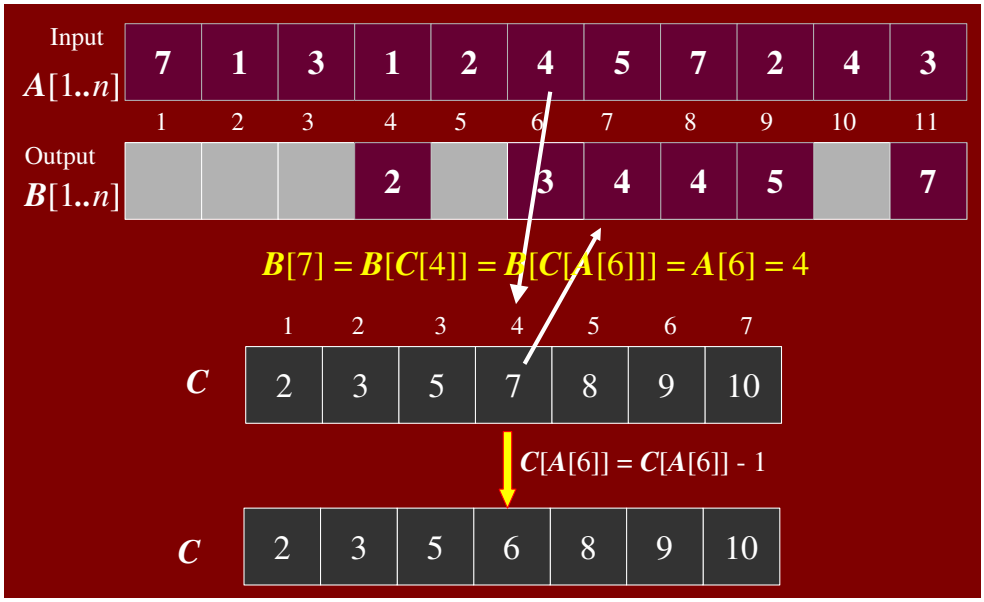


Figure 5.14: A[6] placed in output array B

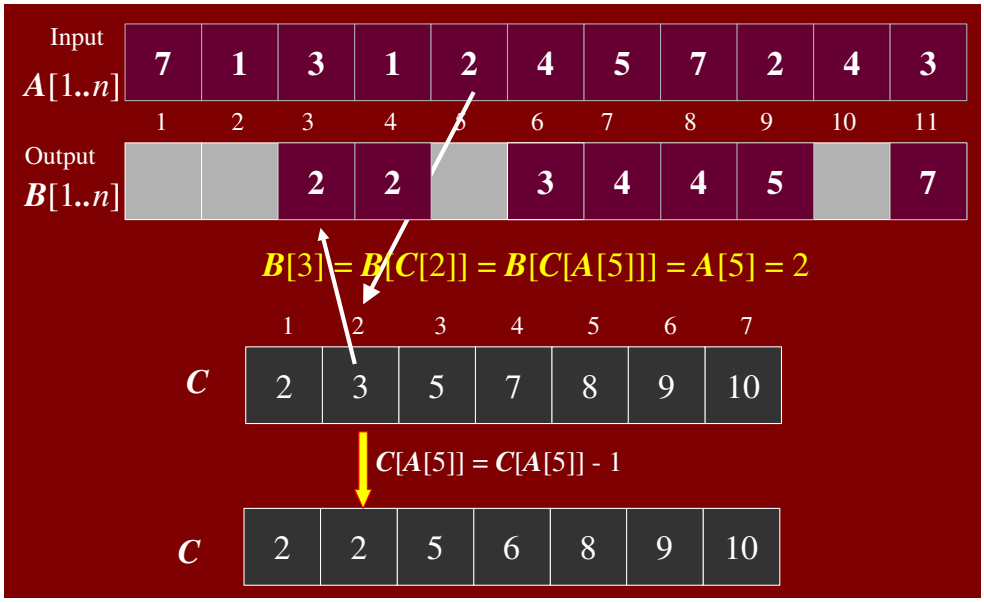


Figure 5.15: A[5] placed in output array B

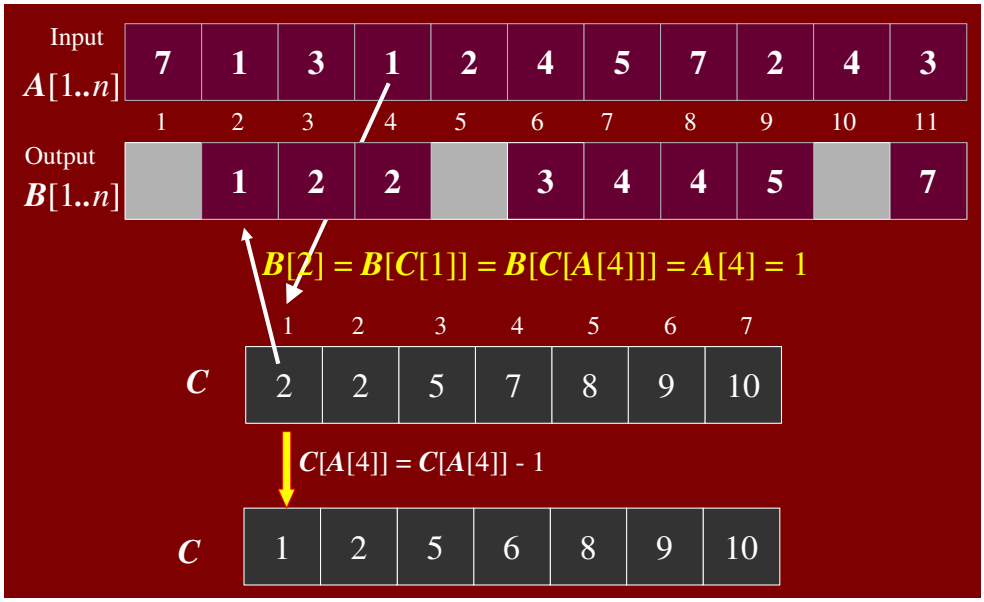


Figure 5.16: A[4] placed in output array B

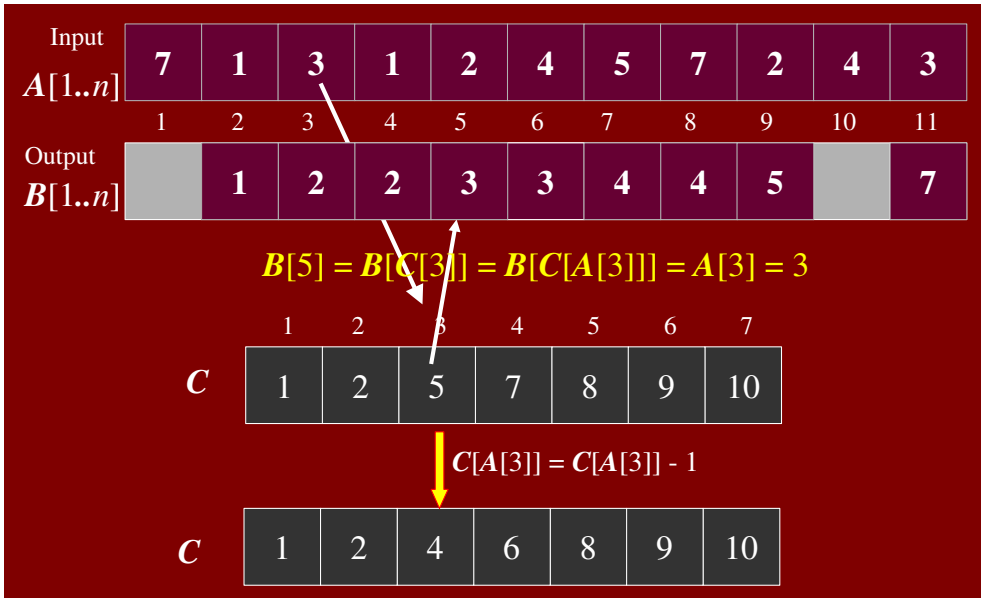


Figure 5.17: A[3] placed in output array B

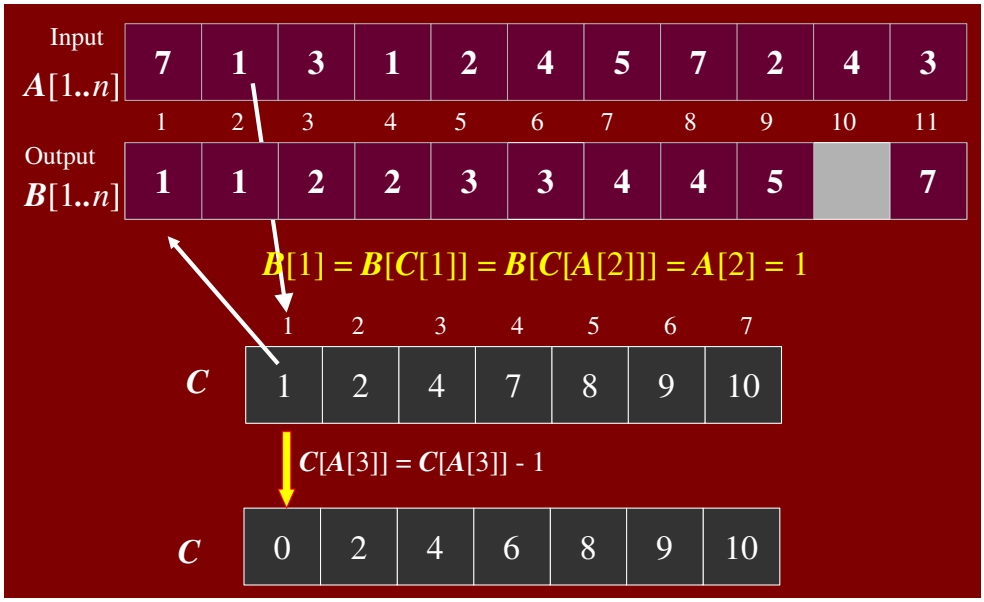


Figure 5.18: A[2] placed in output array B

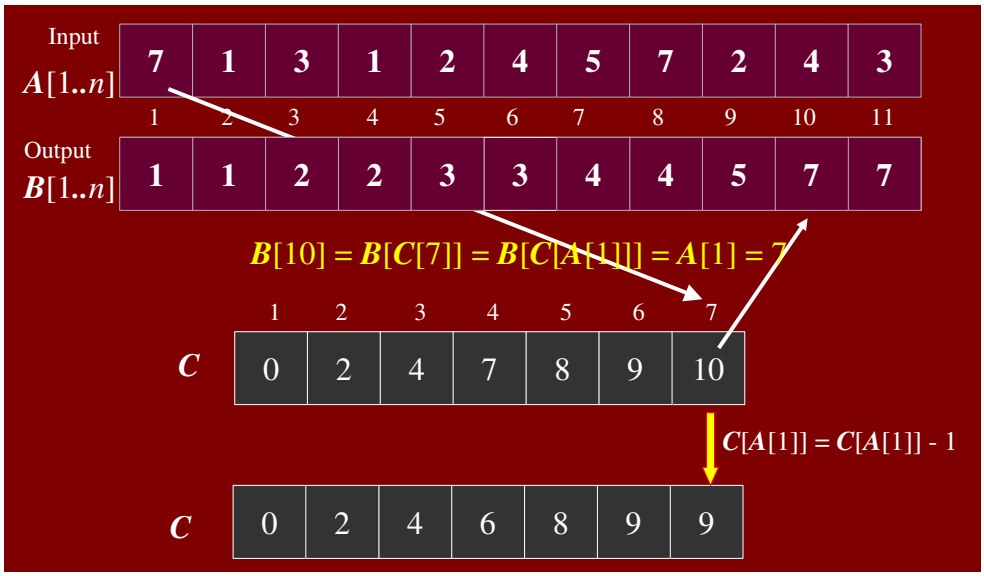


Figure 5.19: B now contains the final sorted data.

Counting sort is not an in-place sorting algorithm but it is stable. Stability is important because data are often carried with the keys being sorted. radix sort (which uses counting sort as a subroutine) relies on it to work correctly. Stability achieved by running the loop down from n to 1 and not the other way around:

```
COUNTING-SORT( array A, array B, int k)
1  ⋮
2  for j ← length[A] downto 1
3  do B[C[A[j]]] ← A[j]
4     C[A[j]] ← C[A[j]] − 1
```

Figure 5.20 illustrates the stability. The numbers 1, 2, 3, 4, and 7, each appear twice. The two 4's have been given the superscript "*****". Numbers are placed in the output B array starting from the right. The two 4's maintain their relative position in the B array. If the sorting algorithm had caused 4^{**} to end up on the left of 4^{*}, the algorithm would be termed unstable.

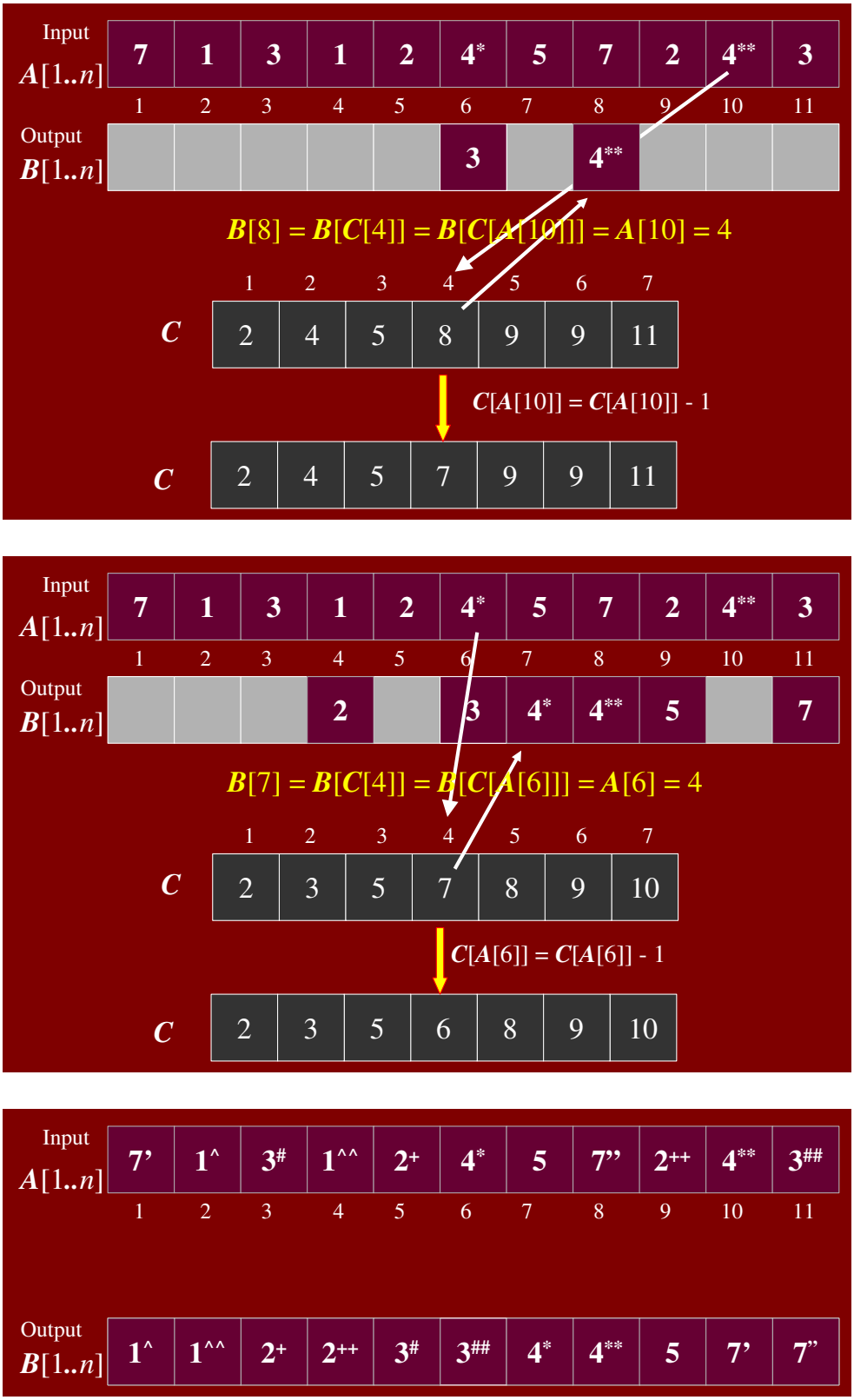


Figure 5.20: Stability of counting sort

5.2 Bucket or Bin Sort

Assume that the keys of the items that we wish to sort lie in a small fixed range and that there is only one item with each value of the key. Then we can sort with the following procedure:

1. Set up an array of "bins" - one for each value of the key - in order,

2. Examine each item and use the value of the key to place it in the appropriate bin.

Now our collection is sorted and it only took n operations, so this is an $O(n)$ operation. However, note that it will only work under very restricted conditions. To understand these restrictions, let's be a little more precise about the specification of the problem and assume that there are m values of the key. To recover our sorted collection, we need to examine each bin. This adds a third step to the algorithm above,

3. Examine each bin to see whether there's an item in it.

which requires m operations. So the algorithm's time becomes:

$$T(n) = c_1 n + c_2 m$$

and it is strictly $O(n + m)$. If $m \leq n$, this is clearly $O(n)$. However if $m >> n$, then it is $O(m)$. An implementation of bin sort might look like:

```
BUCEKTSORT( array A, int n, int M)
1  // Pre-condition: for 1 ≤ i ≤ n, 0 ≤ a[i] < M
2  // Mark all the bins empty
3  for i ← 1 to M
4  do bin[i] ← Empty
5  for i ← 1 to n
6  do bin[A[i]] ← A[i]
```

If there are *duplicates*, then each bin can be replaced by a *linked list*. The third step then becomes:

3. Link all the lists into one list.

We can add an item to a linked list in $O(1)$ time. There are n items requiring $O(n)$ time. Linking a list to another list simply involves making the tail of one list point to the other, so it is $O(1)$. Linking m such lists obviously takes $O(m)$ time, so the algorithm is still $O(n + m)$. Figures 5.21 through 5.23 show the algorithm in action using linked lists.

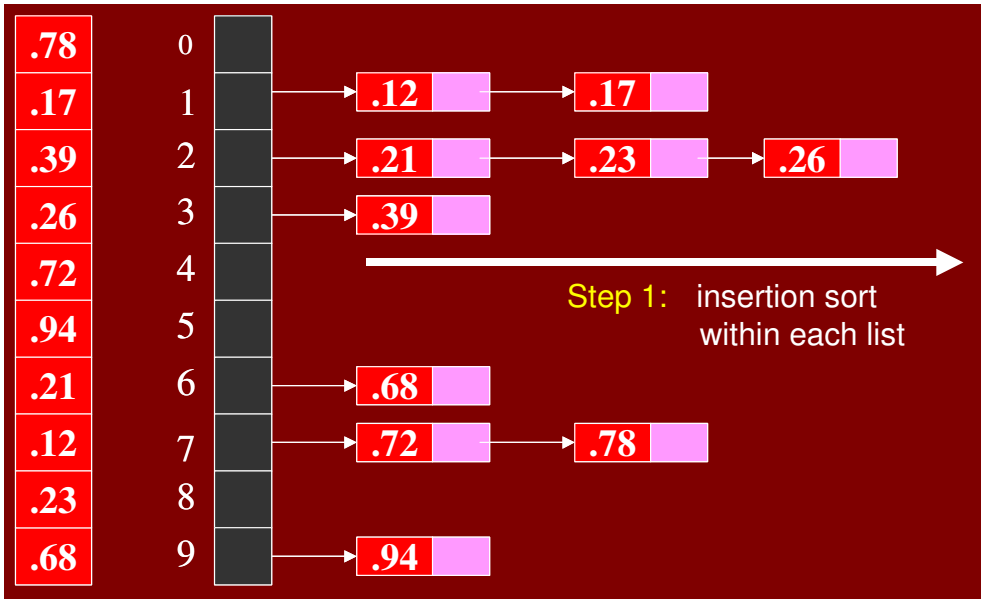


Figure 5.21: Bucket sort: step 1, placing keys in bins in sorted order

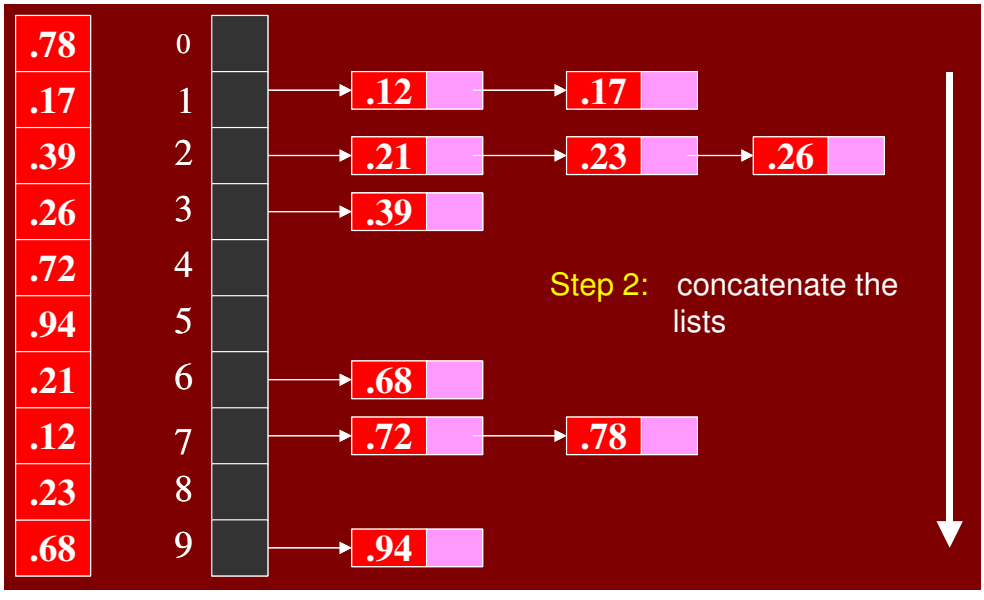


Figure 5.22: Bucket sort: step 2, concatenate the lists

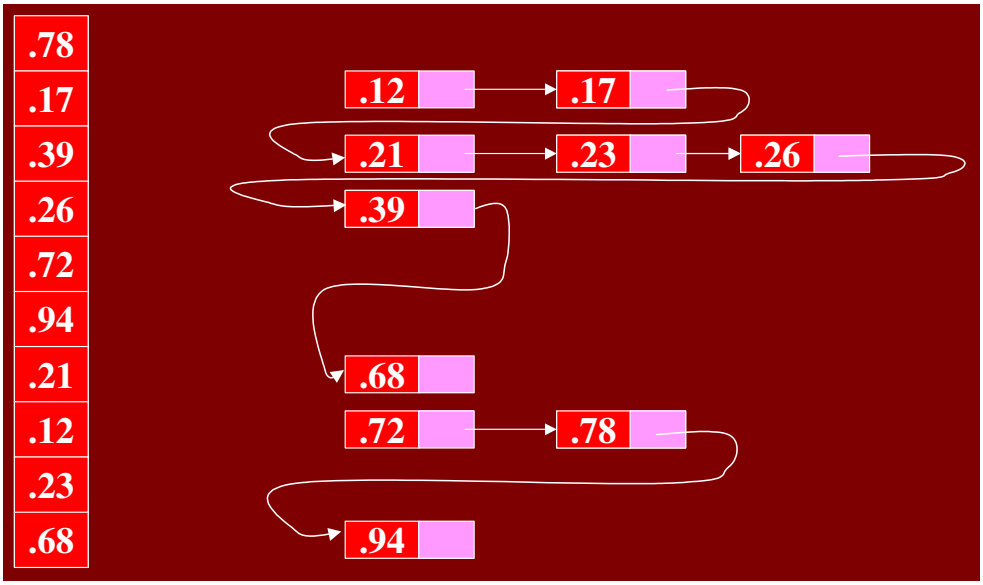


Figure 5.23: Bucket sort: the final sorted sequence

5.3 Radix Sort

The main shortcoming of counting sort is that it is useful for small integers, i.e., $1..k$ where k is small. If k were a million or more, the size of the rank array would also be a million. Radix sort provides a nice work around this limitation by sorting numbers one digit at a time.

576		49[4]		9[5]4		[1]76		176
494		19[4]		5[7]6		[1]94		194
194		95[4]		1[7]6		[2]78		278
296	$\Rightarrow$	57[6]	$\Rightarrow$	2[7]8	$\Rightarrow$	[2]96	$\Rightarrow$	296
278		29[6]		4[9]4		[4]94		494
176		17[6]		1[9]4		[5]76		576
954		27[8]		2[9]6		[9]54		954

Here is the algorithm that sorts $A[1..n]$ where each number is d digits long.

```
RADIX-SORT( array A, int n, int d)
1  for i ← 1 to d
2  do stably sort A w.r.t i^th lowest order digit
```

Chapter 6

Dynamic Programming

6.1 Fibonacci Sequence

Suppose we put a pair of rabbits in a place surrounded on all sides by a wall. How many pairs of rabbits can be produced from that pair in a year if it is supposed that every month each pair begets a new pair which from the second month on becomes productive? Resulting sequence is $1, 1, 2, 3, 5, 8, 13, 21, 34, 55, \ldots$ where each number is the sum of the two preceding numbers.

This problem was posed by Leonardo Pisano, better known by his nickname Fibonacci (son of Bonacci, born 1170, died 1250). This problem and many others were in posed in his book *Liber abaci*, published in 1202. The book was based on the arithmetic and algebra that Fibonacci had accumulated during his travels. The book, which went on to be widely copied and imitated, introduced the Hindu-Arabic place-valued decimal system and the use of Arabic numerals into Europe. The rabbits problem in the third section of Liber abaci led to the introduction of the Fibonacci numbers and the Fibonacci sequence for which Fibonacci is best remembered today.

This sequence, in which each number is the sum of the two preceding numbers, has proved extremely fruitful and appears in many different areas of mathematics and science. The *Fibonacci Quarterly* is a modern journal devoted to studying mathematics related to this sequence. The Fibonacci numbers F_n are defined as follows:

$$F_0 = 0$$
$$F_1 = 1$$
$$F_n = F_{n-1} + F_{n-2}$$

The recursive definition of Fibonacci numbers gives us a recursive algorithm for computing them:

```
FIB(n)
1  if (n < 2)
2     then return n
3     else  return FIB(n − 1) + FIB(n − 2)
```

Figure **??** shows four levels of recursion for the call $\mathtt{fib}(8)$:

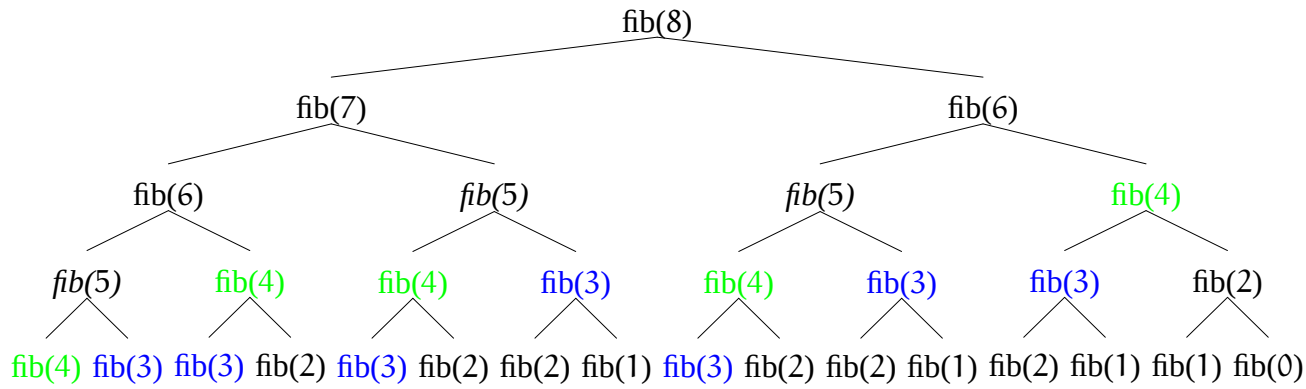


Figure 6.1: Recursive calls during computation of Fibonacci number

A single recursive call to fib(n) results in one recursive call to fib(n − 1), two recursive calls to fib(n − 2), three recursive calls to fib(n − 3), five recursive calls to fib(n − 4) and, in general, F_{k-1} recursive calls to fib(n − k) For each call, we're recomputing the same fibonacci number from scratch.

We can avoid this unnecessary repetitions by writing down the results of recursive calls and looking them up again if we need them later. This process is called *memoization*. Here is the algorithm with memoization.

```
MEMOFIB(n)
1  if (n < 2)
2     then return n
3  if (F[n] is undefined)
4     then F[n] ← MEMOFIB(n − 1) + MEMOFIB(n − 2)
5  return F[n]
```

If we trace through the recursive calls to MEMOFIB, we find that array F[] gets filled from bottom up. I.e., first F[2], then F[3], and so on, up to F[n]. We can replace recursion with a simple for-loop that just fills up the array F[] in that order.

This gives us our first explicit *dynamic programming* algorithm.

```
ITERFIB(n)
1  F[0] ← 0
2  F[1] ← 1
3  for i ← 2 to n
4  do
5      F[i] ← F[i − 1] + F[i − 2]
6  return F[n]
```

This algorithm clearly takes only $O(n)$ time to compute F_n. By contrast, the original recursive algorithm takes $\Theta(\phi^n)$, $\phi = \frac{1+\sqrt{5}}{2} \approx 1.618$. ITERFIB achieves an exponential speedup over the original recursive algorithm.

6.2 Dynamic Programming

Dynamic programming is essentially recursion without repetition. Developing a dynamic programming algorithm generally involves two separate steps:

- **Formulate problem recursively**. Write down a formula for the whole problem as a simple combination of answers to smaller subproblems.
- **Build solution to recurrence from bottom up**. Write an algorithm that starts with base cases and works its way up to the final solution.

Dynamic programming algorithms need to store the results of intermediate subproblems. This is often *but not always* done with some kind of table. We will now cover a number of examples of problems in which the solution is based on dynamic programming strategy.

6.3 Edit Distance

The words “computer” and “commuter” are very similar, and a change of just one letter, p-¿m, will change the first word into the second. The word “sport” can be changed into “sort” by the deletion of the ‘p’, or equivalently, ‘sort’ can be changed into ‘sport’ by the insertion of ‘p’. The edit distance of two strings, s1 and s2, is defined as the minimum number of point mutations required to change s1 into s2, where a point mutation is one of:

- change a letter,
- insert a letter or

- delete a letter

For example, the edit distance between *FOOD* and *MONEY* is at most four:

$$\underline{\mathrm{F}}\mathrm{OOD} \longrightarrow \mathrm{MO}\underline{\mathrm{O}}\mathrm{D} \longrightarrow \mathrm{MON}\underline{\ }\mathrm{D}$$
$$\longrightarrow \mathrm{MONE}\underline{\mathrm{D}} \longrightarrow \mathrm{MONEY}$$

6.3.1 Edit Distance: Applications

There are numerous applications of the Edit Distance algorithm. Here are some examples:

Spelling Correction

If a text contains a word that is not in the dictionary, a 'close' word, i.e. one with a small edit distance, may be suggested as a correction. Most word processing applications, such as Microsoft Word, have spelling checking and correction facility. When Word, for example, finds an incorrectly spelled word, it makes suggestions of possible replacements.

Plagiarism Detection

If someone copies, say, a C program and makes a few changes here and there, for example, change variable names, add a comment of two, the edit distance between the source and copy may be small. The edit distance provides an indication of similarity that might be too close in some situations.

Computational Molecular Biology DNA is a polymer. The monomer units of DNA are nucleotides, and the polymer is known as a "polynucleotide." Each nucleotide consists of a 5-carbon sugar (deoxyribose), a nitrogen containing base attached to the sugar, and a phosphate group. There are four different types of nucleotides found in DNA, differing only in the nitrogenous base. The four nucleotides are given one letter abbreviations as shorthand for the four bases.

- A-adenine
- G-guanine
- C-cytosine
- T-thymine

Double-helix of DNA molecule with nucleotides Figure of Double-helix of DNA molecule with nucleotides goes here

The edit distance like algorithms are used to compute a distance between DNA sequences (strings over A,C,G,T, or protein sequences (over an alphabet of 20 amino acids), for various purposes, e.g.:

- to find genes or proteins that may have shared functions or properties
- to infer family relationships and evolutionary trees over different organisms.

Speech Recognition

Algorithms similar to those for the edit-distance problem are used in some speech recognition systems. Find a close match between a new utterance and one in a library of classified utterances.

6.3.2 Edit Distance Algorithm

A better way to display this editing process is to place the words above the other:

S	*D*	*I*	*M*	*D*	*M*
M	A	_	T	H	S
A	_	R	T	_	S

The first word has a gap for every insertion (I) and the second word has a gap for every deletion (D). Columns with two different characters correspond to substitutions (S). Matches (M) do not count. The *Edit transcript* is defined as a string over the alphabet M, S, I, D that describes a transformation of one string into another. For example

S	*D*	*I*	*M*	*D*	*M*	
1+	1+	1+	0+	1+	0+	= 4

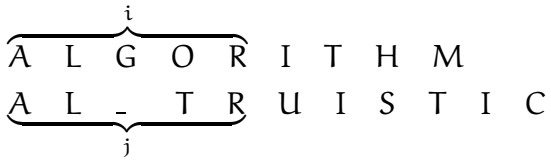
In general, it is not easy to determine the optimal edit distance. For example, the distance between *ALGORITHM* and *ALTRUISTIC* is at most 6.

A	L	G	O	R	_	I	_	T	H	M
A	L	_	T	R	U	I	S	T	I	C

Is this optimal?

6.3.3 Edit Distance: Dynamic Programming Algorithm

Suppose we have an m-character string A and an n-character string B. Define $E(i,j)$ to be the edit distance between the first i characters of A and the first j characters of B. For example,



The edit distance between entire strings A and B is $E(m,n)$. The gap representation for the edit sequences has a crucial "*optimal substructure*" property. If we remove the last column, the remaining columns must represent the shortest edit sequence for the remaining substrings. The edit distance is 6 for the following two words.

A L G O R _ I _ T H M
A L _ T R U I S T I C

If we remove the last column, the edit distance reduces to 5.

A L G O R _ I _ T H
A L _ T R U I S T I

We can use the optimal substructure property to devise a recursive formulation of the edit distance problem. There are a couple of obvious base cases:

- The only way to convert an empty string into a string of j characters is by doing j insertions. Thus

$$E(0, j) = j$$

- The only way to convert a string of i characters into the empty string is with i deletions:

$$E(i, 0) = i$$

There are four possibilities for the last column in the shortest possible edit sequence:

Deletion: Last entry in bottom row is empty.

$$\overbrace{\begin{matrix} A & L & G \end{matrix}}^{i=3} \begin{matrix} O & R & I & T & H & M \\ \underbrace{A \quad L \quad _}_{j=2} & T & R & U & I & S & T & I & C \end{matrix}$$

In this case

$$E(i, j) = E(i-1, j) + 1$$

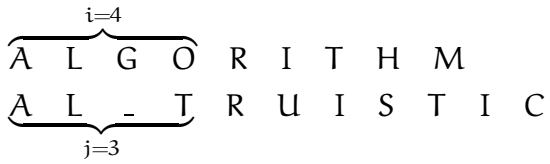
Insertion: The last entry in the top row is empty.

$$\overbrace{\begin{matrix} A & L & G & O & R & _ \end{matrix}}^{i=5} \begin{matrix} I & T & H & M \\ \underbrace{A \quad L \quad _ \quad T \quad R \quad U}_{j=5} & I & S & T & I & C \end{matrix}$$

In this case

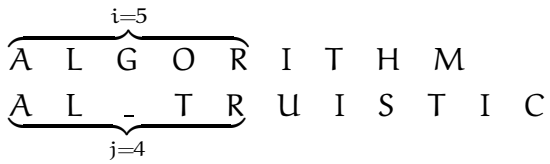
$$E(i, j) = E(i, j-1) + 1$$

Substitution: Both rows have characters in the last column.



If the characters are different, then

$$E(i,j) = E(i-1,j-1) + 1$$



If characters are same, no substitution is needed:

$$E(i,j) = E(i-1,j-1)$$

Thus the edit distance $E(i,j)$ is the smallest of the four possibilities:

$$E(i,j) = \min \begin{pmatrix} E(i-1,j)+1 & \\ E(i,j-1)+1 & \\ E(i-1,j-1)+1 & \text{if } A[i] \neq B[j] \\ E(i-1,j-1) & \text{if } A[i] = B[j] \end{pmatrix}$$

Consider the example of edit between the words “ARTS” and “MATHS”:

A	R	T	S	
M	A	T	H	S

The edit distance would be in $E(4,5)$. If we recursion to compute, we will have

$$E(4,5) = \min \begin{pmatrix} E(3,5)+1 & \\ E(4,4)+1 & \\ E(3,4)+1 & \text{if } A[4] \neq B[5] \\ E(3,4) & \text{if } A[4] = B[5] \end{pmatrix}$$

Recursion clearly leads to the same repetitive call pattern that we saw in Fibonnaci sequence. To avoid this, we will use the DP approach. We will build the solution bottom-up. We will use the base case $E(0,j)$ to fill first row and the base case $E(i,0)$ to fill first column. We will fill the remaining E matrix row by row.

		A	R	T	S
	0	→1	→2	→3	→4
M					
A					
T					
H					
S					

		A	R	T	S
	0	→1	→2	→3	→4
M	↓ 1				
A	↓ 2				
T	↓ 3				
H	↓ 4				
S	↓ 5				

Table 6.1: First row and first column entries using the base cases

We can now fill the second row. The table not only shows the values of the cells $E[i, j]$ but also arrows that indicate how it was computed using values in $E[i-1, j]$, $E[i, j-1]$ and $E[i-1, j-1]$. Thus, if a cell $E[i, j]$ has a down arrow from $E[i-1, j]$ then the minimum was found using $E[i-1, j]$. For a right arrow, the minimum was found using $E[i, j-1]$. For a diagonal down right arrow, the minimum was found using $E[i-1, j-1]$. There are certain cells that have two arrows pointed to it. In such a case, the minimum could be obtained from the diagonal $E[i-1, j-1]$ and either of $E[i-1, j]$ and $E[i, j-1]$. We will use these arrows later to determine the edit script.

		A	R	T	S
	0	→1	→2	→3	→4
M	↓ 1	↘ 1			
A	↓ 2				
T	↓ 3				
H	↓ 4				
S	↓ 5				

		A	R	T	S
	0	→1	→2	→3	→4
M	↓ 1	↘ 1	↘ →2		
A	↓ 2				
T	↓ 3				
H	↓ 4				
S	↓ 5				

Table 6.2: Computing $E[1,1]$ and $E[1,2]$

		A	R	T	S
	0	→1	→2	→3	→4
M	↓ 1	↘ 1	↘ →2	↘ →3	
A	↓ 2				
T	↓ 3				
H	↓ 4				
S	↓ 5				

		A	R	T	S
	0	→1	→2	→3	→4
M	↓ 1	↘ 1	↘ →2	↘ →3	↘ →4
A	↓ 2				
T	↓ 3				
H	↓ 4				
S	↓ 5				

Table 6.3: Computing $E[1,3]$ and $E[1,4]$

An edit script can be extracted by following a unique path from $E[0,0]$ to $E[4,5]$. There are three possible paths in the current example. Let us follow these paths and compute the edit script. In an actual implementation of the dynamic programming version of the edit distance algorithm, the arrows would be recorded using an appropriate data structure. For example, each cell in the matrix could be a record with fields for the value (numeric) and flags for the three incoming arrows.

		A	R	T	S
	0	→1	→2	→3	→4
M	↓ 1	↘ 1	↘ →2	↘ →3	↘ →4
A	↓ 2	↘ 1	↘ →2	↘ →3	↘ →4
T	↓ 3	↓ 2	↘ 2	↘ 2	→3
H	↓ 4	↓ 3	↘↓ 3	↘↓ 3	↘ 3
S	↓ 5	↓ 4	↘↓ 4	↘↓ 4	↘ 3

Table 6.4: The final table with all $E[i, j]$ entries computed

		A	R	T	S
	0	→1	→2	→3	→4
M	↓ 1	↘ 1	↘ →2	↘ →3	↘ →4
A	↓ 2	↘ 1	↘ →2	↘ →3	↘ →4
T	↓ 3	↓ 2	↘ 2	↘ 2	→3
H	↓ 4	↓ 3	↘↓ 3	↘↓ 3	↘ 3
S	↓ 5	↓ 4	↘↓ 4	↘↓ 4	↘ 3

Solution path 1:

1+	0+	1+	1+	0	= 3
D	**M**	**S**	**S**	**M**	
M	A	T	H	S	
_	A	R	T	S	

		A	R	T	S
	0	→1	→2	→3	→4
M	↓ 1	↘ 1	↘ →2	↘ →3	↘ →4
A	↓ 2	↘ 1	↘ →2	↘ →3	↘ →4
T	↓ 3	↓ 2	↘ 2	↘ 2	→3
H	↓ 4	↓ 3	↘↓ 3	↘↓ 3	↘ 3
S	↓ 5	↓ 4	↘↓ 4	↘↓ 4	↘ 3

Table 6.5: Possible edit scripts. The red arrows from $E[0,0]$ to $E[4,5]$ show the paths that can be followed to extract edit scripts.

		A	R	T	S
	0	→1	→2	→3	→4
M	↓ 1	↘ 1	↘ →2	↘ →3	↘ →4
A	↓ 2	↘ 1	↘ →2	↘ →3	↘ →4
T	↓ 3	↓ 2	↘ 2	↘ 2	→3
H	↓ 4	↓ 3	↘↓ 3	↘↓ 3	↘ 3
S	↓ 5	↓ 4	↘↓ 4	↘↓ 4	↘ 3

Solution path 2:

1+	1+	0+	1+	0	= 3
S	**S**	**M**	**D**	**M**	
M	A	T	H	S	
A	R	T	_	S	

		A	R	T	S
	0	→1	→2	→3	→4
M	↓ 1	↘ 1	↘ →2	↘ →3	↘ →4
A	↓ 2	↘ 1	↘ →2	↘ →3	↘ →4
T	↓ 3	↓ 2	↘ 2	↘ 2	→3
H	↓ 4	↓ 3	↘↓ 3	↘↓ 3	↘ 3
S	↓ 5	↓ 4	↘↓ 4	↘↓ 4	↘ 3

Solution path 3:

1+	0+	1+	0+	1+	0	= 3
D	**M**	**I**	**M**	**D**	**M**	
M	A	_	T	H	S	
_	A	R	T	_	S	

6.3.4 Analysis of DP Edit Distance

There are $\Theta(n^2)$ entries in the matrix. Each entry $E(i,j)$ takes $\Theta(1)$ time to compute. The total running time is $\Theta(n^2)$.

6.4 Chain Matrix Multiply

Suppose we wish to multiply a series of matrices:

$$A_1 A_2 \ldots A_n$$

In what order should the multiplication be done? A $p \times q$ matrix A can be multiplied with a $q \times r$ matrix B. The result will be a $p \times r$ matrix C. In particular, for $1 \leq i \leq p$ and $1 \leq j \leq r$,

$$C[i,j] = \sum_{k=1}^{q} A[i,k]B[k,j]$$

There are $(p \cdot r)$ total entries in C and each takes $O(q)$ to compute.

$$C[i,j] = \sum_{k=1}^{q} A[i,k]B[k,j]$$

Thus the total number of multiplications is $p \cdot q \cdot r$. Consider the case of 3 matrices: A_1 is 5×4, A_2 is 4×6 and A_3 is 6×2 The multiplication can be carried out either as $((A_1A_2)A_3)$ or $(A_1(A_2A_3))$. The cost of the two is

$$((A_1A_2)A_3) = (5 \cdot 4 \cdot 6) + (5 \cdot 6 \cdot 2) = 180$$
$$(A_1(A_2A_3)) = (4 \cdot 6 \cdot 2) + (5 \cdot 4 \cdot 2) = 88$$

There is considerable savings achieved even for this simple example. In general, however, in what order should we multiply a series of matrices $A_1A_2\ldots A_n$. Matrix multiplication is an associative but not commutative operation. We are free to add parenthesis the above multiplication but the order of matrices can not be changed. The *Chain Matrix Multiplication Problem* is stated as follows:

> Given a sequence $A_1, A_2, \ldots, A_n$ and dimensions $p_0, p_1, \ldots, p_n$ where A_i is of dimension $p_{i-1} \times p_i$, determine the order of multiplication that minimizes the number of operations.

We could write a procedure that tries all possible parenthesizations. Unfortunately, the number of ways of parenthesizing an expression is very large. If there are n items, there are $n-1$ ways in which outer most pair of parentheses can placed.

$$\begin{array}{ll} & (A_1)(A_2A_3A_4\ldots A_n) \\ \text{or} & (A_1A_2)(A_3A_4\ldots A_n) \\ \text{or} & (A_1A_2A_3)(A_4\ldots A_n) \\ \ldots & \ldots \\ \text{or} & (A_1A_2A_3A_4\ldots A_{n-1})(A_n) \end{array}$$

Once we split just after the k^{th} matrix, we create two sublists to be parethesized, one with k and other with $n-k$ matrices.

$$(A_1A_2\ldots A_k)\,(A_{k+1}\ldots A_n)$$

We could consider all the ways of parenthesizing these two. Since these are independent choices, if there are L ways of parenthesizing the left sublist and R ways to parenthesize the right sublist, then the total is $L \cdot R$. This suggests the following recurrence for $P(n)$, the number of different ways of parenthesizing n items:

$$P(n) = \begin{cases} 1 & \text{if } n = 1, \\ \sum_{k=1}^{n-1} P(k)P(n-k) & \text{if } n \geq 2 \end{cases}$$

This is related to a famous function in combinatorics called the *Catalan numbers*. Catalan numbers are related the number of different binary trees on n nodes. Catalan number is given by the formula:

$$C(n) = \frac{1}{n+1}\binom{2n}{n}$$

In particular, $P(n) = C(n-1)\ C(n) \in \Omega(4^n/n^{3/2})$ The dominating term is the exponential 4^n thus $P(n)$ will grow large very quickly. So this approach is not practical.

6.4.1 Chain Matrix Multiplication-Dynamic Programming Formulation

The dynamic programming solution involves breaking up the problem into subproblems whose solutions can be combined to solve the global problem. Let $A_{i..j}$ be the result of multiplying matrices i through j. It is easy to see that $A_{i..j}$ is a $p_{i-1} \times p_j$ matrix.

$$\underset{4\times5}{A_3}\quad \underset{5\times2}{A_4}\quad \underset{2\times8}{A_5}\quad \underset{8\times7}{A_6} = \underset{4\times7}{A_{3..6}}$$

At the highest level of parenthesization, we multiply two matrices

$$A_{1..n} = A_{1..k} \cdot A_{k+1..n} \quad 1 \leq k \leq n-1.$$

The question now is: what is the optimum value of k for the split and how do we parenthesis the sub-chains $A_{1..k}$ and $A_{k+1..n}$. We can not use divide and conquer because we do not know what is the optimum k. We will have to consider all possible values of k and take the best of them. We will apply this strategy to solve the subproblems optimally.

We will store the solutions to the subproblem in a table and build the table bottom-up (why)? For $1 \leq i \leq j \leq n$, let $m[i,j]$ denote the minimum number of multiplications needed to compute $A_{i..j}$. The optimum can be described by the following recursive formulation.

- If $i = j$, there is only one matrix and thus $m[i,i] = 0$ (the diagonal entries).
- If $i < j$, the we are asking for the product $A_{i..j}$.
- This can be split by considering each k, $i \leq k < j$, as $A_{i..k}$ times $A_{k+1..j}$.

The optimum time to compute $A_{i..k}$ is $m[i,k]$ and optimum time for $A_{k+1..j}$ is in $m[k+1,j]$. Since $A_{i..k}$ is a $p_{i-1} \times p_k$ matrix and $A_{k+1..j}$ is $p_k \times p_j$ matrix, the time to multiply them is $p_{i-1} \times p_k \times p_j$. This suggests the following recursive rule:

$$\begin{aligned} m[i,i] &= 0 \\ m[i,j] &= \min_{i \leq k < j} (m[i,k] + m[k+1,j] + p_{i-1}p_k p_j) \end{aligned}$$

We do not want to calculate m entries recursively. So how should we proceed? We will fill m along the diagonals. Here is how. Set all $m[i,i] = 0$ using the base condition. Compute cost for multiplication of a sequence of 2 matrices. These are $m[1,2], m[2,3], m[3,4], \ldots, m[n-1,n]$. $m[1,2]$, for example is

$$m[1,2] = m[1,1] + m[2,2] + p_0 \cdot p_1 \cdot p_2$$

For example, for m for product of 5 matrices at this stage would be:

$m[1,1]$	$\leftarrow m[1,2]$ $\downarrow$			
	$m[2,2]$	$\leftarrow m[2,3]$ $\downarrow$		
		$m[3,3]$	$\leftarrow m[3,4]$ $\downarrow$	
			$m[4,4]$	$\leftarrow m[4,5]$ $\downarrow$
				$m[5,5]$

Next, we compute cost of multiplication for sequences of three matrices. These are $m[1,3], m[2,4], m[3,5], \ldots, m[n-2,n]$. $m[1,3]$, for example is

$$m[1,3] = \min \begin{cases} m[1,1] + m[2,3] + p_0 \cdot p_1 \cdot p_3 \\ m[1,2] + m[3,3] + p_0 \cdot p_2 \cdot p_3 \end{cases}$$

We repeat the process for sequences of four, five and higher number of matrices. The final result will end up in $m[1,n]$.

Example: Let us go through an example. We want to find the optimal multiplication order for

$$\underset{(5\times4)}{A_1} \cdot \underset{(4\times6)}{A_2} \cdot \underset{(6\times2)}{A_3} \cdot \underset{(2\times7)}{A_4} \cdot \underset{(7\times3)}{A_5}$$

We will compute the entries of the m matrix starting with the base condition. We first fill that main diagonal:

0				
	0			
		0		
			0	
				0

Next, we compute the entries in the first super diagonal, i.e., the diagonal above the main diagonal:

$$m[1,2] = m[1,1] + m[2,2] + p_0 \cdot p_1 \cdot p_2 = 0 + 0 + 5 \cdot 4 \cdot 6 = 120$$
$$m[2,3] = m[2,2] + m[3,3] + p_1 \cdot p_2 \cdot p_3 = 0 + 0 + 4 \cdot 6 \cdot 2 = 48$$
$$m[3,4] = m[3,3] + m[4,4] + p_2 \cdot p_3 \cdot p_4 = 0 + 0 + 6 \cdot 2 \cdot 7 = 84$$
$$m[4,5] = m[4,4] + m[5,5] + p_3 \cdot p_4 \cdot p_5 = 0 + 0 + 2 \cdot 7 \cdot 3 = 42$$

The matrix m now looks as follows:

0	120			
	0	48		
		0	84	
			0	42
				0

We now proceed to the second super diagonal. This time, however, we will need to try two possible values for k. For example, there are two possible splits for computing $m[1,3]$; we will choose the split that yields the minimum:

$$m[1,3] = m[1,1] + m[2,3] + p_0 \cdot p_1 \cdot p_3 == 0 + 48 + 5 \cdot 4 \cdot 2 = 88$$
$$m[1,3] = m[1,2] + m[3,3] + p_0 \cdot p_2 \cdot p_3 = 120 + 0 + 5 \cdot 6 \cdot 2 = 180$$
$$\text{the minimum } m[1,3] = 88 \text{ occurs with } k = 1$$

Similarly, for $m[2,4]$ and $m[3,5]$:

$$m[2,4] = m[2,2] + m[3,4] + p_1 \cdot p_2 \cdot p_4 = 0 + 84 + 4 \cdot 6 \cdot 7 = 252$$
$$m[2,4] = m[2,3] + m[4,4] + p_1 \cdot p_3 \cdot p_4 = 48 + 0 + 4 \cdot 2 \cdot 7 = 104$$
$$\text{minimum } m[2,4] = 104 \text{ at } k = 3$$

$$m[3,5] = m[3,3] + m[4,5] + p_2 \cdot p_3 \cdot p_5 = 0 + 42 + 6 \cdot 2 \cdot 3 = 78$$
$$m[3,5] = m[3,4] + m[5,5] + p_2 \cdot p_4 \cdot p_5 = 84 + 0 + 6 \cdot 7 \cdot 3 = 210$$
$$\text{minimum } m[3,5] = 78 \text{ at } k = 3$$

With the second super diagonal computed, the m matrix looks as follow:

0	120	88		
	0	48	104	
		0	84	78
			0	42
				0

We repeat the process for the remaining diagonals. However, the number of possible splits (values of k) increases:

$$m[1,4] = m[1,1] + m[2,4] + p_0 \cdot p_1 \cdot p_4 = 0 + 104 + 5 \cdot 4 \cdot 7 = 244$$
$$m[1,4] = m[1,2] + m[3,4] + p_0 \cdot p_2 \cdot p_4 = 120 + 84 + 5 \cdot 6 \cdot 7 = 414$$
$$m[1,4] = m[1,3] + m[4,4] + p_0 \cdot p_3 \cdot p_4 = 88 + 0 + 5 \cdot 2 \cdot 7 = 158$$
$$\text{minimum } m[1,4] = 158 \text{ at } k = 3$$

$$m[2,5] = m[2,2] + m[3,5] + p_1 \cdot p_2 \cdot p_5 = 0 + 78 + 4 \cdot 6 \cdot 3 = 150$$
$$m[2,5] = m[2,3] + m[4,5] + p_1 \cdot p_3 \cdot p_5 = 48 + 42 + 4 \cdot 2 \cdot 3 = 114$$
$$m[2,5] = m[2,4] + m[5,5] + p_1 \cdot p_4 \cdot p_5 = 104 + 0 + 4 \cdot 7 \cdot 3 = 188$$
$$\text{minimum } m[2,5] = 114 \text{ at } k = 3$$

The matrix m at this stage is:

0	120	88	158	
	0	48	104	114
		0	84	78
			0	42
				0

That leaves the $m[1,5]$ which can now be computed:

$$
\begin{aligned}
m[1,5] &= m[1,1] + m[2,5] + p_0 \cdot p_1 \cdot p_5 = 0 + 114 + 5 \cdot 4 \cdot 3 = 174 \\
m[1,5] &= m[1,2] + m[3,5] + p_0 \cdot p_2 \cdot p_5 = 120 + 78 + 5 \cdot 6 \cdot 3 = 288 \\
m[1,5] &= m[1,3] + m[4,5] + p_0 \cdot p_3 \cdot p_5 = 88 + 42 + 5 \cdot 2 \cdot 3 = 160 \\
m[1,5] &= m[1,4] + m[5,5] + p_0 \cdot p_4 \cdot p_5 = 158 + 0 + 5 \cdot 7 \cdot 3 = 263 \\
&\text{minimum } m[1,5] = 160 \text{ at } k = 3
\end{aligned}
$$

We thus have the final cost matrix.

0	120	88	158	***160***
0	0	48	104	114
0	0	0	84	78
0	0	0	0	42
0	0	0	0	0

Here is the order in which m entries are calculated

0	1	5	8	10
0	0	2	6	9
0	0	0	3	7
0	0	0	0	4
0	0	0	0	0

and the split k values that led to a minimum $m[i,j]$ value

0	1	1	3	3
	0	2	3	3
		0	3	3
			0	4
				0

Based on the computation, the minimum cost for multiplying the five matrices is 160 and the optimal order for multiplication is

$$((A_1(A_2A_3))(A_4A_5))$$

This can be represented as a binary tree

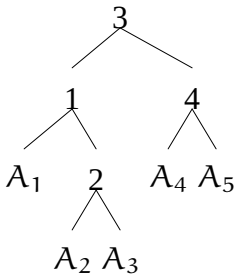


Figure 6.2: Optimum matrix multiplication order for the five matrices example

Here is the dynamic programming based algorithm for computing the minimum cost of chain matrix multiplication.

```
MATRIX-CHAIN(p, N)
 1  for i = 1, N
 2  do m[i, i] ← 0
 3  for L = 2, N
 4  do
 5     for i = 1, n − L + 1
 6     do j ← i + L − 1
 7        m[i, j] ← ∞
 8        for k = 1, j − 1
 9        do t ← m[i, k] + m[k + 1, j] + p_{i−1} · p_k · p_j
10           if (t < m[i, j])
11              then m[i, j] ← t;   s[i, j] ← k
```

Analysis: There are three nested loops. Each loop executes a maximum n times. Total time is thus $\Theta(n^3)$.

The s matrix stores the values k. The s matrix can be used to extracting the order in which matrices are to be multiplied. Here is the algorithm that caries out the matrix multiplication to compute $A_{i..j}$:

```
MULTIPLY(i, j)
1  if (i = j)
2     then return A[i]
3     else k ← s[i, j]
4          X ← MULTIPLY(i, k)
5          Y ← MULTIPLY(k + 1, j)
6          return X · Y
```

6.5 0/1 Knapsack Problem

A thief goes into a jewelry store to steal jewelry items. He has a knapsack (a bag) that he would like to fill up. The bag has a limit on the total weight of the objects placed in it. If the total weight exceeds the limit, the bag would tear open. The value of of the jewelry items varies for cheap to expensive. The thief's goal is to put items in the bag such that the value of the items is maximized and the weight of the items does not exceed the weight limit of the bag. Another limitation is that an item can either be put in the bag or not - fractional items are not allowed. The problem is: what jewelry should the thief choose that satisfy the constraints?

Formally, the problem can be stated as follows: Given a knapsack with maximum capacity W, and a set S consisting of n items Each item i has some weight w_i and value value v_i (all w_i , v_i and W are integer values) How to pack the knapsack to achieve maximum total value of packed items? For example, consider the following scenario:

Item i	Weight w_i	Value v_i
1	2	3
2	3	4
3	4	5
4	5	8
5	9	10

Figure 6.3: Knapsack can hold $W = 20$

The knapsack problem belongs to the domain of optimization problems. Mathematically, the problem is

$$\text{maximize} \sum_{i \in T} v_i$$

$$\text{subject to} \sum_{i \in T} w_i \leq W$$

The problem is called a *"0-1"* problem, because each item must be entirely accepted or rejected. How do we solve the problem. We could try the brute-force solution:

- Since there are n items, there are 2^n possible combinations of the items (an item either chosen or not).
- We go through all combinations and find the one with the most total value and with total weight less or equal to W

Clearly, the running time of such a brute-force algorithm will be $O(2^n)$. Can we do better? The answer is "yes", with an algorithm based on dynamic programming Let us recap the steps in the dynamic programming strategy:

1. **Simple Subproblems**: We should be able to break the original problem to smaller subproblems that have the same structure
2. **Principle of Optimality**: Recursively define the value of an optimal solution. Express the solution of the original problem in terms of optimal solutions for smaller problems.
3. **Bottom-up computation**: Compute the value of an optimal solution in a bottom-up fashion by using a table structure.
4. **Construction of optimal solution**: Construct an optimal solution from computed information.

Let us try this: If items are labelled $1, 2, \ldots, n$, then a subproblem would be to find an optimal solution for

$$S_k = \text{items labelled } 1, 2, \ldots, k$$

This is a valid subproblem definition. The question is: can we describe the final solution S_n in terms of subproblems S_k? Unfortunately, we cannot do that. Here is why. Consider the optimal solution if we can choose items 1 through 4 only.

Solution S_4

- Items chosen are $1, 2, 3, 4$
- Total weight: $2 + 3 + 4 + 5 = 14$
- Total value: $3 + 4 + 5 + 8 = 20$

Item	w_i	v_i
1	2	3
2	3	4
3	4	5
4	5	8
5	9	10

Now consider the optimal solution when items 1 through 5 are available.

Solution S_5

- Items chosen are $1, 3, 4, 5$
- Total weight: $2 + 4 + 5 + 9 = 20$
- Total value: $3 + 5 + 8 + 10 = 26$

S_4 is not part of S_5!!

Item	w_i	v_i
1	2	3
2	3	4
3	4	5
4	5	8
5	9	10

The solution for S_4 ***is not part of the solution for*** S_5. So our definition of a subproblem is flawed and we need another one.

6.5.1 0/1 Knapsack Problem: Dynamic Programming Approach

For each $i \leq n$ and each $w \leq W$, solve the knapsack problem for the first i objects when the capacity is w. Why will this work? Because solutions to larger subproblems can be built up easily from solutions to smaller ones. We construct a matrix $V[0 \ldots n, 0 \ldots W]$. For $1 \leq i \leq n$, and $0 \leq j \leq W$, $V[i, j]$ will store the maximum value of any set of objects $\{1, 2, \ldots, i\}$ that can fit into a knapsack of weight j. $V[n, W]$ will contain the maximum value of all n objects that can fit into the entire knapsack of weight W.

To compute entries of V we will imply an inductive approach. As a basis, $V[0, j] = 0$ for $0 \leq j \leq W$ since if we have no items then we have no value. We consider two cases:

Leave object i**:** If we choose to not take object i, then the optimal value will come about by considering how to fill a knapsack of size j with the remaining objects $\{1, 2, \ldots, i-1\}$. This is just $V[i-1, j]$.

Take object i**:** If we take object i, then we gain a value of v_i. But we use up w_i of our capacity. With the remaining $j - w_i$ capacity in the knapsack, we can fill it in the best possible way with objects $\{1, 2, \ldots, i-1\}$. This is $v_i + V[i-1, j-w_i]$. This is only possible if $w_i \leq j$.

This leads to the following recursive formulation:

$$
\begin{aligned}
V[i, j] &= -\infty \qquad \text{if } j < 0 \\
V[0, j] &= 0 \qquad \text{if } j \geq 0 \\
V[i, j] &= \begin{cases} V[i-1, j] & \text{if } w_i > j \\ \max\{V[i-1, j],\ \ v_i + V[i-1, j-w_i]\} & \text{if } w_i \leq j \end{cases}
\end{aligned}
$$

A naive evaluation of this recursive definition is exponential. So, as usual, we avoid re-computation by making a table.

Example: The maximum weight the knapsack can hold is W is 11. There are five items to choose from. Their weights and values are presented in the following table:

Weight limit (j):	0	1	2	3	4	5	6	7	8	9	10	11
$w_1 = 1\ v_1 = 1$												
$w_2 = 2\ v_2 = 6$												
$w_3 = 5\ v_3 = 18$												
$w_4 = 6\ v_4 = 22$												
$w_5 = 7\ v_5 = 28$												

The $[i, j]$ entry here will be $V[i, j]$, the best value obtainable using the first i rows of items if the maximum capacity were j. We begin by initializating and first row.

Weight limit:	0	1	2	3	4	5	6	7	8	9	10	11
$w_1 = 1\ v_1 = 1$	0	1	1	1	1	1	1	1	1	1	1	1
$w_2 = 2\ v_2 = 6$	0											
$w_3 = 5\ v_3 = 18$	0											
$w_4 = 6\ v_4 = 22$	0											
$w_5 = 7\ v_5 = 28$	0											

Recall that we take $V[i, j]$ to be 0 if either i or j is ≤ 0. We then proceed to fill in top-down, left-to-right always using

$$V[i, j] = \max\{V[i-1, j],\ \ v_i + V[i-1, j-w_i]\}$$

Weight limit:	0	1	2	3	4	5	6	7	8	9	10	11
$w_1 = 1\ v_1 = 1$	0	1	1	1	1	1	1	1	1	1	1	1
$w_2 = 2\ v_2 = 6$	0	1	6	7	7	7	7	7	7	7	7	7
$w_3 = 5\ v_3 = 18$	0											
$w_4 = 6\ v_4 = 22$	0											
$w_5 = 7\ v_5 = 28$	0											

Weight limit:	0	1	2	3	4	5	6	7	8	9	10	11
$w_1 = 1\ v_1 = 1$	0	1	1	1	1	1	1	1	1	1	1	1
$w_2 = 2\ v_2 = 6$	0	1	6	7	7	7	7	7	7	7	7	7
$w_3 = 5\ v_3 = 18$	0	1	6	7	7	18	19	*24*	25	25	25	25
$w_4 = 6\ v_4 = 22$	0											
$w_5 = 7\ v_5 = 28$	0											

As an illustration, the value of $V[3, 7]$ was computed as follows:

$$\begin{aligned} V[3,7] &= \max\{V[3-1,7],\ \ v_3 + V[3-1,7-w_3]\} \\ &= \max\{V[2,7],\ \ 18 + V[2,7-5]\} \\ &= \max\{7,\ \ 18+6\} \\ &= 24 \end{aligned}$$

Weight limit:	0	1	2	3	4	5	6	7	8	9	10	11
$w_1 = 1\ v_1 = 1$	0	1	1	1	1	1	1	1	1	1	1	1
$w_2 = 2\ v_2 = 6$	0	1	6	7	7	7	7	7	7	7	7	7
$w_3 = 5\ v_3 = 18$	0	1	6	7	7	18	19	24	25	25	25	25
$w_4 = 6\ v_4 = 22$	0	1	6	7	7	18	22	24	28	29	29	40
$w_5 = 7\ v_5 = 28$	0											

Finally, we have

Weight limit:	0	1	2	3	4	5	6	7	8	9	10	11
$w_1 = 1\ v_1 = 1$	0	1	1	1	1	1	1	1	1	1	1	1
$w_2 = 2\ v_2 = 6$	0	1	6	7	7	7	7	7	7	7	7	7
$w_3 = 5\ v_3 = 18$	0	1	6	7	7	18	19	24	25	25	25	25
$w_4 = 6\ v_4 = 22$	0	1	6	7	7	18	22	24	28	29	29	40
$w_5 = 7\ v_5 = 28$	0	1	6	7	7	18	22	28	29	34	35	*40*

The maximum value of items in the knapsack is 40, the bottom-right entry). The dynamic programming approach can now be coded as the following algorithm:

```
KNAPSACK(n, W)
1  for w = 0, W
2  do V[0, w] ← 0
3  for i = 0, n
4  do V[i, 0] ← 0
5     for w = 0, W
6     do if (w_i ≤ w & v_i + V[i − 1, w − w_i] > V[i − 1, w])
7           then V[i, w] ← v_i + V[i − 1, w − w_i]
8           else  V[i, w] ← V[i − 1, w]
```

The time complexity is clearly $O(n \cdot W)$. It must be cautioned that as n and W get large, both time and space complexity become significant.

Constructing the Optimal Solution

The algorithm for computing $V[i, j]$ does not keep record of which subset of items gives the optimal solution. To compute the actual subset, we can add an auxiliary boolean array $keep[i, j]$ which is 1 if we decide to take the i^{th} item and 0 otherwise. We will use all the values $keep[i, j]$ to determine the optimal subset T of items to put in the knapsack as follows:

- If $keep[n, W]$ is 1, then $n \in T$. We can now repeat this argument for $keep[n - 1, W - w_n]$.
- If $kee[n, W]$ is 0, the $n \notin T$ and we repeat the argument for $keep[n - 1, W]$.

We will add this to the knapsack algorithm:

```
KNAPSACK(n, W)
 1  for w = 0, W
 2  do V[0, w] ← 0
 3  for i = 0, n
 4  do V[i, 0] ← 0
 5     for w = 0, W
 6     do if (w_i ≤ w & v_i + V[i − 1, w − w_i] > V[i − 1, w])
 7           then V[i, w] ← v_i + V[i − 1, w − w_i]; keep[i, w] ← 1
 8           else V[i, w] ← V[i − 1, w]; keep[i, w] ← 0
 9  // output the selected items
10  k ← W
11  for i = n downto 1
12  do if (keep[i, k] = 1)
13        then output i
14             k ← k − w_i
```

Here is the keep matrix for the example problem.

Weight limit:	0	1	2	3	4	5	6	7	8	9	10	11
$w_1 = 1\ v_1 = 1$	0	1	1	1	1	1	1	1	1	1	1	1
$w_2 = 2\ v_2 = 6$	0	0	1	1	1	1	1	1	1	1	1	1
$w_3 = 5\ v_3 = 18$	0	0	0	0	0	**1**	1	1	1	1	1	1
$w_4 = 6\ v_4 = 22$	0	0	0	0	0	0	1	0	1	1	1	**1**
$w_5 = 7\ v_5 = 28$	0	0	0	0	0	0	0	1	1	1	1	0

When the item selection algorithm is applied, the selected items are 4 and 3. This is indicated by the boxed entries in the table above.

Chapter 7

Greedy Algorithms

An *optimization problem* is one in which you want to find, not just a solution, but the best solution. Search techniques look at many possible solutions. E.g. dynamic programming or backtrack search. A " greedy algorithm" sometimes works well for optimization problems

A greedy algorithm works in phases. At each phase:

- You take the best you can get right now, without regard for future consequences.
- You hope that by choosing a local optimum at each step, you will end up at a global optimum.

For some problems, greedy approach always gets optimum. For others, greedy finds good, but not always best. If so, it is called a greedy heuristic, or approximation. For still others, greedy approach can do very poorly.

7.1 Example: Counting Money

Suppose you want to count out a certain amount of money, using the fewest possible bills (notes) and coins. A greedy algorithm to do this would be: at each step, take the largest possible note or coin that does not overshoot.

```
while (N > 0){
    give largest denomination coin ≤ N
    reduce N by value of that coin
}
```

Consider the currency in U.S.A. There are paper notes for one dollar, five dollars, ten dollars, twenty dollars, fifty dollars and hundred dollars. The notes are also called "bills". The coins are one cent, five cents (called a "nickle"), ten cents (called a "dime") and twenty five cents (a "quarter"). In Pakistan, the currency notes are five rupees, ten rupees, fifty rupees, hundred rupees, five hundred rupees and thousand

rupees. The coins are one rupee and two rupees. Suppose you are asked to give change of $6.39 (six dollars and thirty nine cents), you can choose:

- a $5 note
- a $1 note to make $6
- a 25 cents coin (quarter), to make $6.25
- a 10 cents coin (dime), to make $6.35
- four 1 cents coins, to make $6.39

Notice how we started with the highest note, $5, before moving to the next lower denomination.

Formally, the Coin Change problem is: Given k denominations $d_1, d_2, \ldots, d_k$ and given N, find a way of writing

$$N = i_1 d_1 + i_2 d_2 + \cdots + i_k d_k$$

such that

$$i_1 + i_2 + \cdots + i_k \text{ is minimized.}$$

The "size" of problem is k.

The greedy strategy works for the coin change problem but not always. Here is an example where it fails. Suppose, in some (fictional) monetary system, " krons" come in 1 kron, 7 kron, and 10 kron coins Using a greedy algorithm to count out 15 krons, you would get A 10 kron piece Five 1 kron pieces, for a total of 15 krons This requires six coins. A better solution, however, would be to use two 7 kron pieces and one 1 kron piece This only requires three coins The greedy algorithm results in a solution, but not in an optimal solution

The greedy approach gives us an optimal solution when the coins are all powers of a fixed denomination.

$$N = i_0 D^0 + i_1 D^1 + i_2 D^2 + \cdots + i_k D^k$$

Note that this is N represented in based D. U.S.A coins are multiples of 5: 5 cents, 10 cents and 25 cents.

7.1.1 Making Change: Dynamic Programming Solution

The general coin change problem can be solved using Dynamic Programming. Set up a Table, $C[1..k, 0..N]$ in which the rows denote available denominations, d_i; $(1 \leq i \leq k)$ and columns denote the amount from $0 \ldots N$ units, $(0 \leq j \leq N)$. $C[i, j]$ denotes the minimum number of coins, required to pay an amount j using only coins of denominations 1 to i. C[k, N] is the solution required.

To pay an amount j units, using coins of denominations 1 to i, we have two choices:

1. either chose NOT to use any coins of denomination i,
2. or chose at least one coin of denomination i, and also pay the amount $(j - d_i)$.

To pay $(j - d_i)$ units it takes $C[i, j - d_i]$ coins. Thus,

$$C[i, j] = 1 + C[i, j - d_i]$$

Since we want to minimize the number of coins used,

$$C[i, j] = \min(C[i - 1, j], 1 + C[i, j - d_i])$$

Here is the dynamic programming based algorithm for the coin change problem.

```
COINS(N)
 1  d[1..n] = {1, 4, 6}   // (coinage, for example)
 2  for i = 1 to k
 3  do c[i, 0] ← 0
 4  for i = 1 to k
 5  do for j = 1 to N
 6      do if (i = 1 & j < d[i])
 7            then c[i, j] ← ∞
 8            else if (i = 1)
 9                    then c[i, j] ← 1 + c[1, j − d[1]]
10                    else if (j < d[i])
11                            then c[i, j] ← c[i − 1, j]
12                            else c[i, j] ← min (c[i − 1, j], 1 + c[i, j − d[i]])
13  return c[k, N]
```

7.1.2 Complexity of Coin Change Algorithm

Greedy algorithm (non-optimal) takes $O(k)$ time. Dynamic Programming takes $O(kN)$ time. Note that N can be as large as 2^k so the dynamic programming algorithm is really exponential in k.

7.2 Greedy Algorithm: Huffman Encoding

The Huffman codes provide a method of encoding data efficiently. Normally, when characters are coded using standard codes like ASCII. Each character is represented by a fixed-length codeword of bits, e.g., 8 bits per character. Fixed-length codes are popular because it is very easy to break up a string into its individual characters, and to access individual characters and substrings by direct indexing. However, fixed-length codes may not be he most efficient from the perspective of minimizing the total quantity of data.

Consider the string " abacdaacac". if the string is coded with ASCII codes, the message length would be $10 \times 8 = 80$ bits. We will see shortly that the same string encoded with a variable length Huffman encoding scheme will produce a shorter message.

7.2.1 Huffman Encoding Algorithm

Here is how the Huffman encoding algorithm works. Given a message string, determine the frequency of occurrence (relative probability) of each character in the message. This can be done by parsing the message and counting how many time each character (or symbol) appears. The probability is the number of occurrence of a character divided by the total characters in the message. The frequencies and probabilities for the example string " abacdaacac" are

character	a	b	c	d
frequency	5	1	3	1
probability	0.5	0.1	0.3	0.1

Next, create binary tree (leaf) node for each symbol (character) that occurs with nonzero frequency Set node weight equal to the frequency of the symbol. Now comes the greedy part: Find two nodes with smallest frequency. Create a new node with these two nodes as children, and with weight equal to the sum of the weights of the two children. Continue until we have a single tree.

Finding two nodes with the smallest frequency can be done efficiently by placing the nodes in a heap-based priority queue. The min-heap is maintained using the frequencies. When a new node is created by combining two nodes, the new node is placed in the priority queue. Here is the Huffman tree building algorithm.

```
HUFFMAN(N, symbol[1..N], freq[1..N])
 1  for i = 1 to N
 2  do t ← TreeNode(symbol[i], freq[i])
 3     pq.insert(t, freq[i])  // priority queue
 4  for i = 1 to N − 1
 5  do x = pq.remove(); y = pq.remove()
 6     z ← new TreeNode
 7     z.left ← x;  z.right ← y
 8     z.freq ← x.freq + y.freq
 9     pq.insert(z, z.freq);
10  return pq.remove();  // root
```

Figure 7.1 shows the tree built for the example message "abacdaacac"

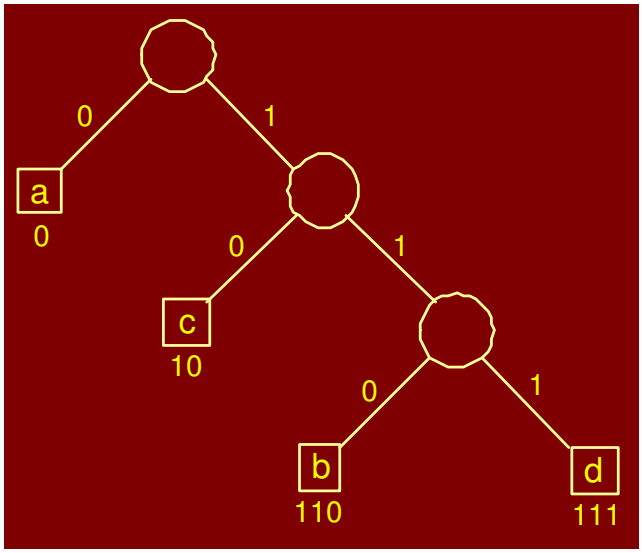


Figure 7.1: Huffman binary tree for the string "abacdaacac"

Prefix Property:

The codewords assigned to characters by the Huffman algorithm have the property that no codeword is a prefix of any other:

character	a	b	c	d
frequency	5	1	3	1
probability	0.5	0.1	0.3	0.1
codeword	0	110	10	111

The prefix property is evident by the fact that codewords are leaves of the binary tree. Decoding a prefix code is simple. We traverse the root to the leaf letting the input 0 or 1 tell us which branch to take.

Expected encoding length:

If a string of n characters over the alphabet $C = \{a, b, c, d\}$ is encoded using 8-bit ASCII, the length of encoded string is $8n$. For example, the string "abacdaacac" will require $8 \times 10 = 80$ bits. The same string encoded with Huffman codes will yield

a	b	a	c	d	a	a	c	a	c
0	110	0	10	111	0	0	10	0	10

This is just 17 bits, a significant saving!. For a string of n characters over this alphabet, the expected encoded string length is

$$n(0.5 \cdot 1 + 0.1 \cdot 3 + 0.3 \cdot 2 + 0.1 \cdot 3) = 1.7n$$

In general, let $p(x)$ be the probability of occurrence of a character, and let $d_T(x)$ denote the length of the codeword relative to some prefix tree T. The expected number of bits needed to encode a text with n characters is given by

$$B(T) = n \sum_{x \in C} p(x) d_T(x)$$

7.2.2 Huffman Encoding: Correctness

Huffman algorithm uses a greedy approach to generate a prefix code T that minimizes the expected length $B(T)$ of the encoded string. In other words, Huffman algorithm generates an optimum prefix code. The question that remains is that *why is the algorithm correct?*

Recall that the cost of any encoding tree T is

$$B(T) = n \sum_{x \in C} p(x) d_T(x)$$

Our approach to prove the correctness of Huffman Encoding will be to show that any tree that differs from the one constructed by Huffman algorithm can be converted into one that is equal to Huffman's tree without increasing its costs. Note that the binary tree constructed by Huffman algorithm is a full binary tree.

Claim:

Consider two characters x and y with the smallest probabilities. Then there is optimal code tree in which these two characters are siblings at the maximum depth in the tree.

Proof:

Let T be any optimal prefix code tree with two siblings b and c at the maximum depth of the tree. Such a tree is shown in Figure 7.2Assume without loss of generality that

$$p(b) \leq p(c) \qquad \text{and} \qquad p(x) \leq p(y)$$

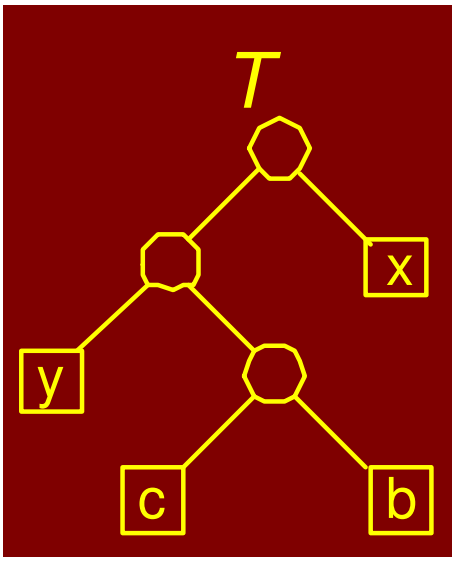


Figure 7.2: Optimal prefix code tree T

Since x and y have the two smallest probabilities (we claimed this), it follows that

$$p(x) \leq p(b) \qquad \text{and} \qquad p(y) \leq p(c)$$

Since b and c are at the deepest level of the tree, we know that

$$d(b) \geq d(x) \qquad \text{and} \qquad d(c) \geq d(y) \qquad (d \text{ is the depth})$$

Thus we have

$$p(b) - p(x) \geq 0$$

and

$$d(b) - d(x) \geq 0$$

Hence their product is non-negative. That is,

$$(p(b) - p(x)) \cdot (d(b) - d(x)) \geq 0$$

Now swap the positions of x and b in the tree

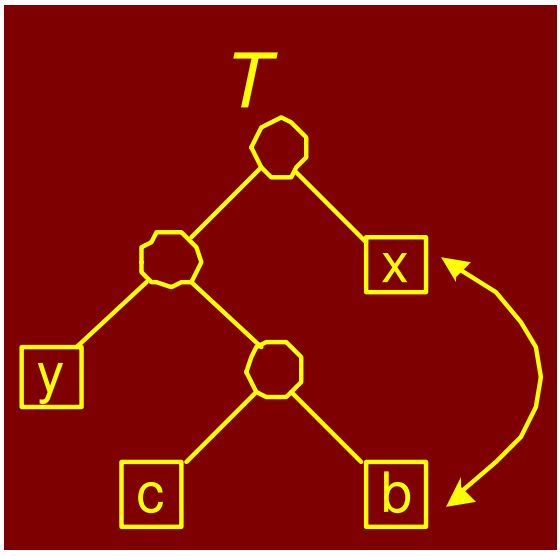


Figure 7.3: Swap x and b in tree prefix tree T

This results in a new tree T'

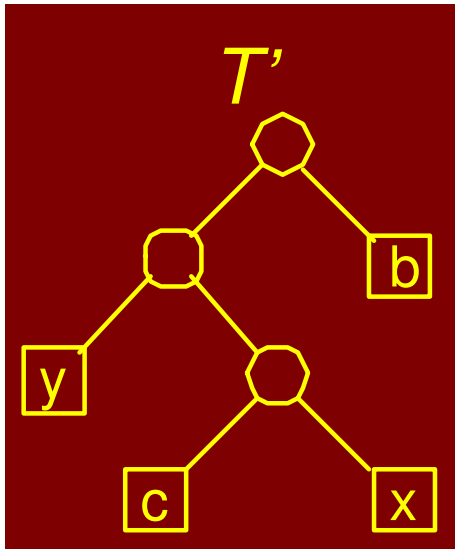


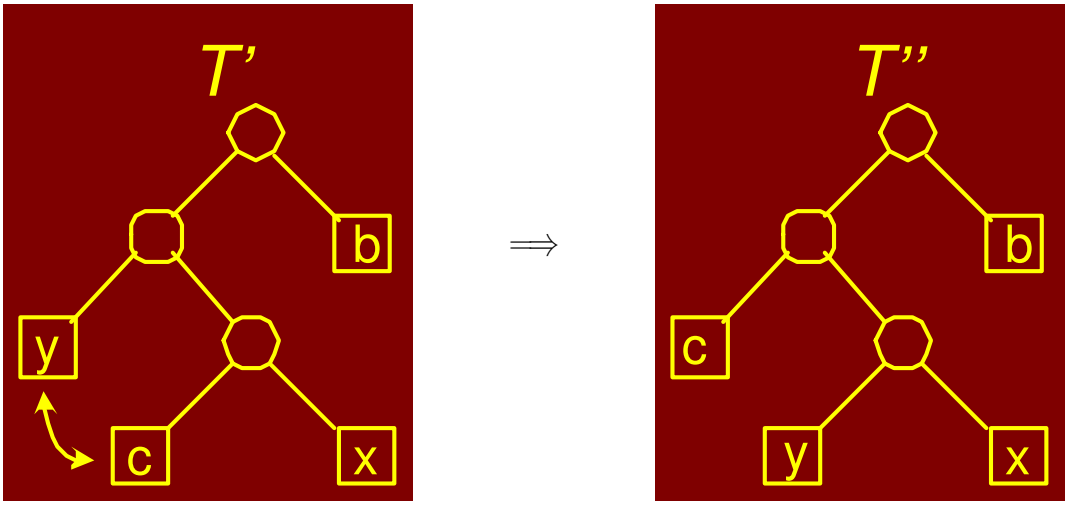
Figure 7.4: Prefix tree T' after x and b are swapped

Let's see how the cost changes. The cost of T' is

$$\begin{aligned} B(T') &= B(T) - p(x)d(x) + p(x)d(b) - p(b)d(b) + p(b)d(x) \\ &= B(T) + p(x)[d(b) - d(x)] - p(b)[d(b) - d(x)] \\ &= B(T) - (p(b) - p(x))(d(b) - d(x)) \\ &\leq B(T) \quad \text{because } (p(b) - p(x))(d(b) - d(x)) \geq 0 \end{aligned}$$

Thus the cost does not increase, implying that T' is an optimal tree.

By switching y with c we get the tree T''. Using a similar argument, we can show that T'' is also optimal.



The final tree T'' satisfies the claim we made earlier, i.e., consider two characters x and y with the

smallest probabilities. Then there is optimal code tree in which these two characters are siblings at the maximum depth in the tree.

The claim we just proved asserts that the first step of Huffman algorithm is the proper one to perform (the greedy step). The complete proof of correctness for Huffman algorithm follows by induction on n.

Claim: Huffman algorithm produces the optimal prefix code tree.

Proof: The proof is by induction on n, the number of characters. For the basis case, $n = 1$, the tree consists of a single leaf node, which is obviously optimal. We want to show it is true with exactly n characters.

Suppose we have exactly n characters. The previous claim states that two characters x and y with the lowest probability will be siblings at the lowest level of the tree. Remove x and y and replace them with a new character z whose probability is $p(z) = p(x) + p(y)$. Thus $n - 1$ characters remain.

Consider any prefix code tree T made with this new set of $n - 1$ characters. We can convert T into prefix code tree T' for the original set of n characters by replacing z with nodes x and y. This is essentially undoing the operation where x and y were removed an replaced by z. The cost of the new tree T' is

$$\begin{aligned} B(T') &= B(T) - p(z)d(z) + p(x)[d(z) + 1] + p(y)[d(z) + 1] \\ &= B(T) - (p(x) + p(y))d(z) + (p(x) + p(y))[d(z) + 1] \\ &= B(T) + (p(x) + p(y))[d(z) + 1 - d(z)] \\ &= B(T) + p(x) + p(y) \end{aligned}$$

The cost changes but the change depends in no way on the structure of the tree T (T is for $n - 1$ characters). Therefore, to minimize the cost of the final tree T', we need to build the tree T on $n - 1$ characters optimally. By induction, this is exactly what Huffman algorithm does. Thus the final tree is optimal.

7.3 Activity Selection

The activity scheduling is a simple scheduling problem for which the greedy algorithm approach provides an optimal solution. We are given a set $S = \{a_1, a_2, \ldots, a_n\}$ of n activities that are to be scheduled to use some resource. Each activity a_i must be started at a given start time s_i and ends at a given finish time f_i.

An example is that a number of lectures are to be given in a single lecture hall. The start and end times have be set up in advance. The lectures are to be scheduled. There is only one resource (e.g., lecture hall) Some start and finish times may overlap. Therefore, not all requests can be honored. We say that two activities a_i and a_j are non-interfering if their start-finish intervals do not overlap. I.e, $(s_i, f_i) \cap (s_j, f_j) = \varnothing$. The activity selection problem is to select a maximum-size set of mutually non-interfering activities for use of the resource.

So how do we schedule the largest number of activities on the resource? Intuitively, we do not like long

activities Because they occupy the resource and keep us from honoring other requests. This suggests the greedy strategy: Repeatedly select the activity with the smallest duration $(f_i - s_i)$ and schedule it, provided that it does not interfere with any previously scheduled activities. Unfortunately, this turns out to be non-optimal

Here is a simple greedy algorithm that works: Sort the activities by their finish times. Select the activity that finishes first and schedule it. Then, among all activities that do not interfere with this first job, schedule the one that finishes first, and so on.

```
SCHEDULE(a[1..N])
1   sort a[1..N] by finish times
2   A ← {a[1]};   // schedule activity 1 first
3   prev ← 1;   // most recently scheduled
4   for i = 2 to N
5   do if (a[i].start ≥ a[prev].finish)
6         then A ← A ∪ a[i];  prev ← i
```

Figure 7.5 shows an example of the activity scheduling algorithm. There are eight activities to be scheduled. Each is represented by a rectangle. The width of a rectangle indicates the duration of an activity. The eight activities are sorted by their finish times. The eight rectangles are arranged to show the sorted order. Activity a_1 is scheduled first. Activities a_2 and a_3 interfere with a_1 so they ar not selected. The next to be selected is a_4. Activities a_5 and a_6 interfere with a_4 so are not chosen. The last one to be chosen is a_7. Eventually, only three out of the eight are scheduled.

Timing analysis: Time is dominated by sorting of the activities by finish times. Thus the complexity is $O(N \log N)$.

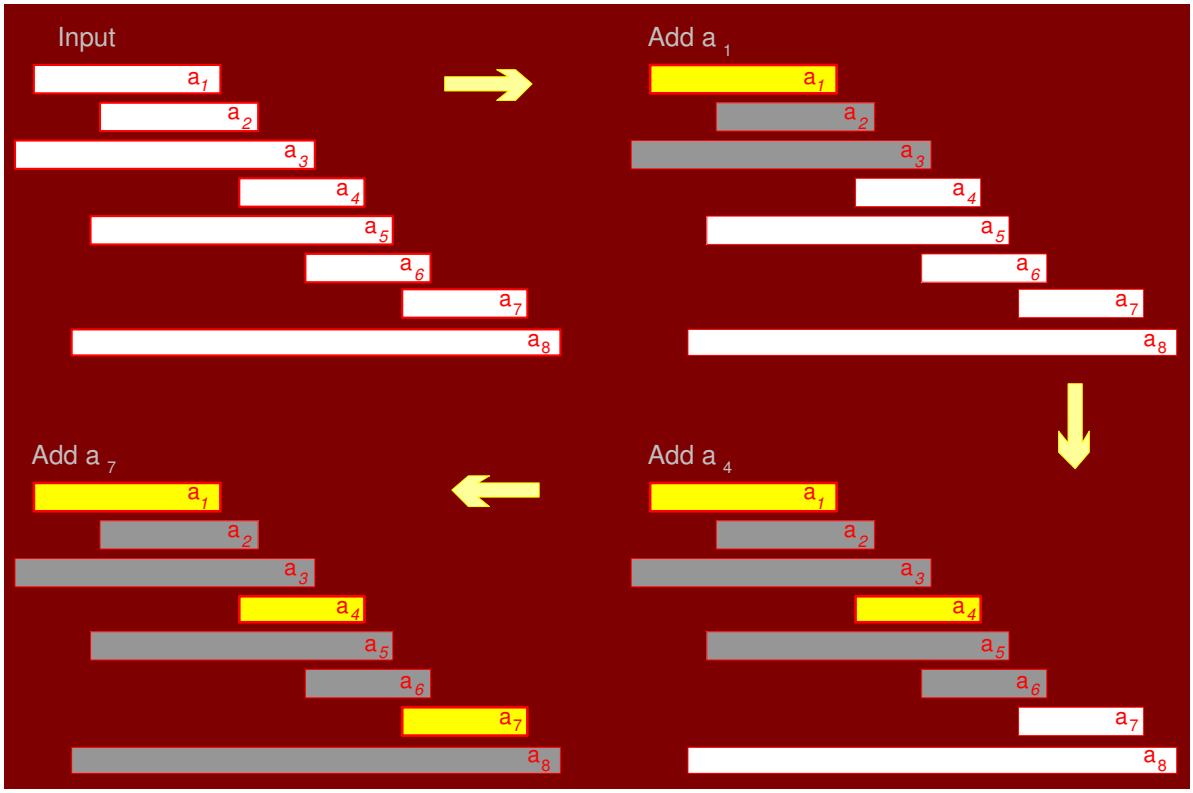


Figure 7.5: Example of greedy activity scheduling algorithm

7.3.1 Correctness of Greedy Activity Selection

Our proof of correctness is based on showing that the first choice made by the algorithm is the best possible. And then using induction to show that the algorithm is globally optimal. The proof structure is noteworthy because many greedy correctness proofs are based on the same idea: Show that any other solution can be converted into the greedy solution without increasing the cost.

Claim:

Let $S = \{a_1, a_2, \ldots, a_n\}$ of n activities, sorted by increasing finish times, that are to be scheduled to use some resource. Then there is an optimal schedule in which activity a_1 is scheduled first.

Proof:

Let A be an optimal schedule. Let x be the activity in A with the smallest finish time. If $x = a_1$ then we are done. Otherwise, we form a new schedule A' by replacing x with activity a_1.

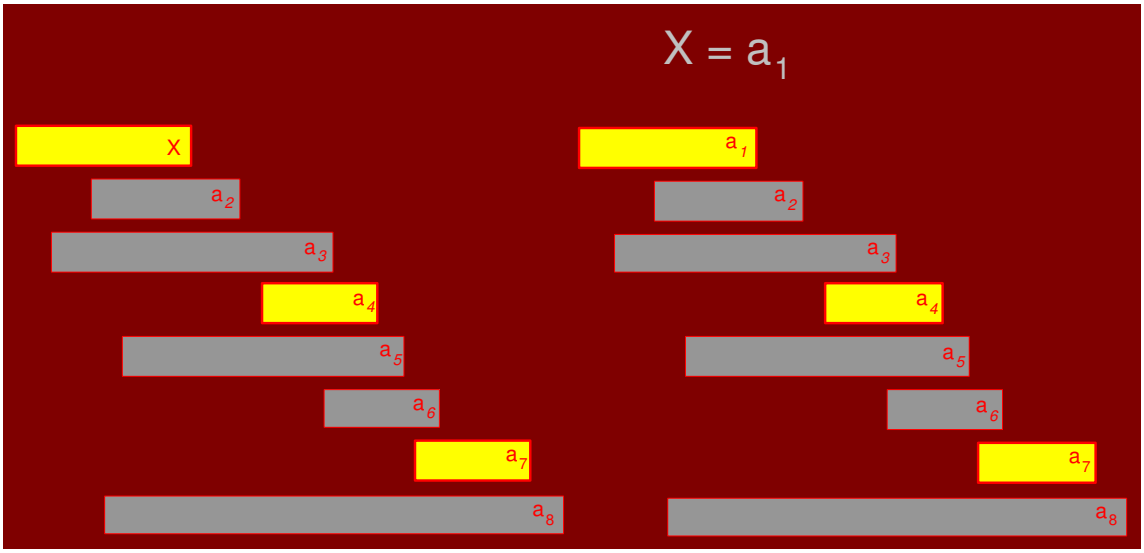


Figure 7.6: Activity $X = a_1$

We claim that $A' = A - \{x\} \cup \{a_1\}$ is a feasible schedule, i.e., it has no interfering activities. This because $A - \{x\}$ cannot have any other activities that start before x finishes, since otherwise, these activities will interfere with x.

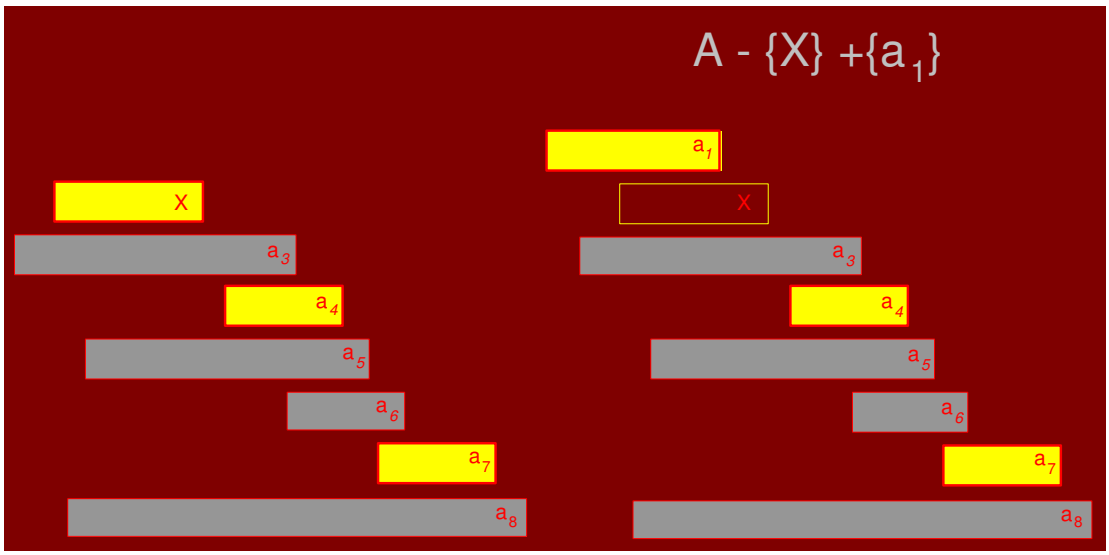


Figure 7.7: New schedule A' by replacing x with ctivity a_1.

Since a_1 is by definition the first activity to finish, it has an earlier finish time than x. Thus a_1 cannot interfere with any of the activities in $A - \{x\}$. Thus, A' is a feasible schedule. Clearly A and A' contain the same number of activities implying that A' is also optimal.

Claim:

The greedy algorithm gives an optimal solution to the activity scheduling problem.

Proof:

The proof is by induction on the number of activities. For the basis case, if there are no activities, then the greedy algorithm is trivially optimal. For the induction step, let us assume that the greedy algorithm is optimal on any set of activities of size strictly smaller than $|S|$ and we prove the result for S. Let S' be the set of activities that do not interfere with activity a_1, That is

$$S' = \{a_i \in S | s_i \geq f_1\}$$

Any solution for S' can be made into a solution for S by simply adding activity a_1, and vice versa. Activity a_1 is in the optimal schedule (by the above previous claim). It follows that to produce an optimal schedule for the overall problem, we should first schedule a_1 and then append the optimal schedule for S'. But by induction (since $|S'| < |S|$), this exactly what the greedy algorithm does.

7.4 Fractional Knapsack Problem

Earlier we saw the 0-1 knapsack problem. A knapsack can only carry W total weight. There are n items; the i^{th} item is worth v_i and weighs w_i. Items can either be put in the knapsack or not. The goal was to maximize the value of items without exceeding the total weight limit of W. In contrast, in the fractional knapsack problem, the setup is exactly the same. But, one is allowed to take *fraction* of an item for a fraction of the weight and fraction of value. The 0-1 knapsack problem is hard to solve. However, there is a simple and efficient greedy algorithm for the fractional knapsack problem.

Let $\rho_i = v_i/w_i$ denote the *value per unit weight* ratio for item i. Sort the items in decreasing order of ρ_i. Add items in decreasing order of ρ_i. If the item fits, we take it all. At some point there is an item that does not fit in the remaining space. We take as much of this item as possible thus filling the knapsack completely.

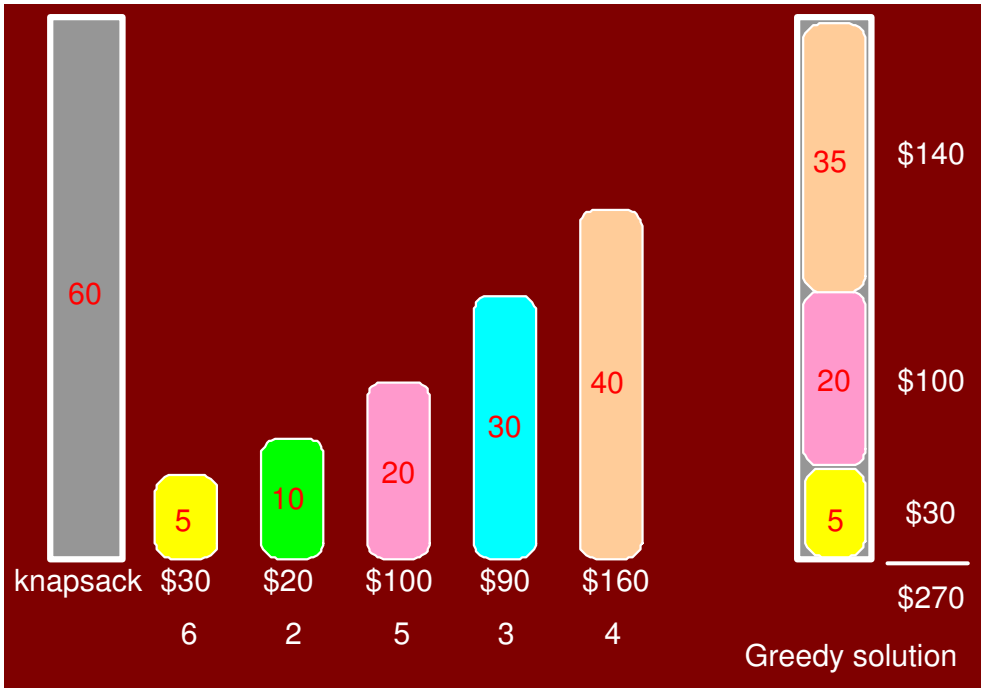


Figure 7.8: Greedy solution to the fractional knapsack problem

It is easy to see that the greedy algorithm is optimal for the fractional knapsack problem. Given a room with sacks of gold, silver and bronze, one (thief?) would probably take as much gold as possible. Then take as much silver as possible and finally as much bronze as possible. It would never benefit to take a little less gold so that one could replace it with an equal weight of bronze.

We can also observe that the greedy algorithm is not optimal for the 0-1 knapsack problem. Consider the example shown in the Figure 7.9. If you were to sort the items by ρ_i , then you would first take the items of weight 5, then 20, and then (since the item of weight 40 does not fit) you would settle for the item of weight 30, for a total value of \$30 + \$100 + \$90 = \$220. On the other hand, if you had been less greedy, and ignored the item of weight 5, then you could take the items of weights 20 and 40 for a total value of \$100+\$160 = \$260. This is shown in Figure 7.10.

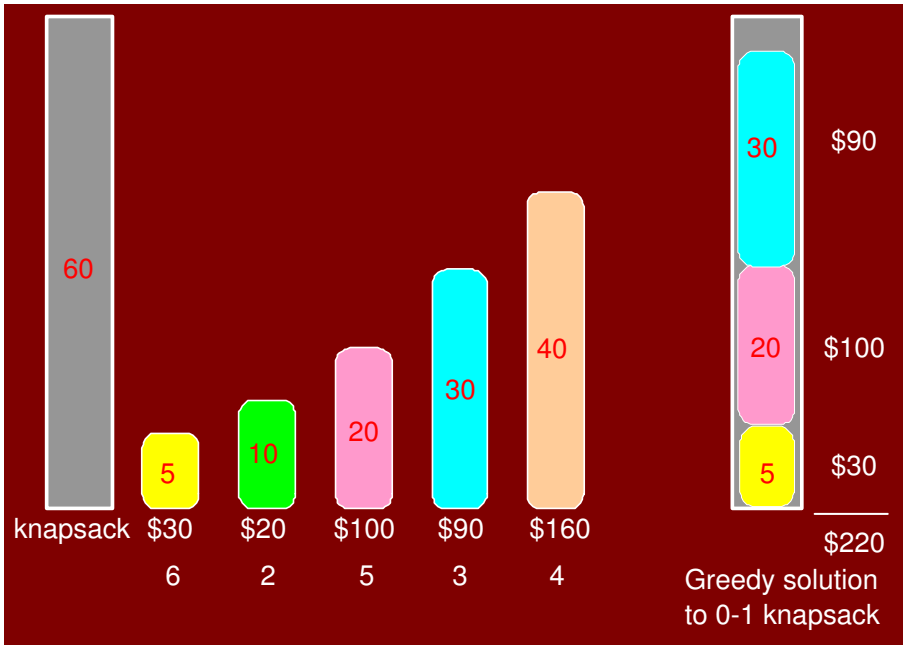


Figure 7.9: Greedy solution for the 0-1 knapsack problem (non-optimal)

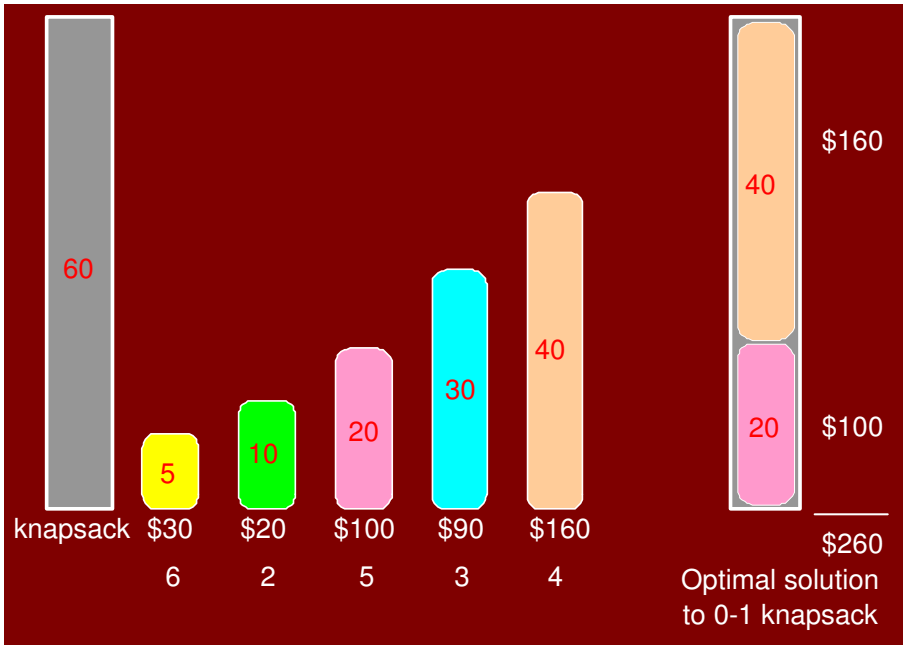


Figure 7.10: Optimal solution for the 0-1 knapsack problem

Chapter 8

Graphs

We begin a major new topic: Graphs. Graphs are important discrete structures because they are a flexible mathematical model for many application problems. Any time there is a set of objects and there is some sort of "connection" or "relationship" or "interaction" between pairs of objects, a graph is a good way to model this. Examples of this can be found in computer and communication networks transportation networks, e.g., roads VLSI, logic circuits surface meshes for shape description in computer-aided design and GIS precedence constraints in scheduling systems.

A *graph* $G = (V, E)$ consists of a finite set of *vertices* V (or nodes) and E, a binary relation on V called *edges*. E is a set of pairs from V. If a pair is *ordered*, we have a *directed* graph. For *unordered* pair, we have an *undirected* graph.

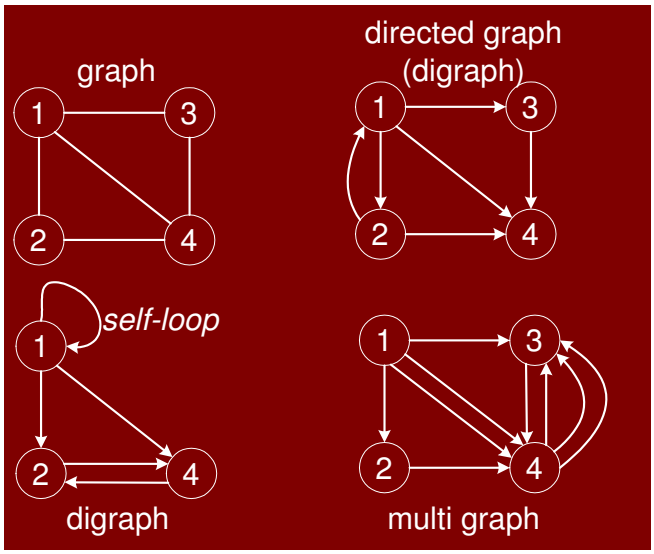


Figure 8.1: Types of graphs

A vertex w is *adjacent* to vertex v if there is an edge from v to w.

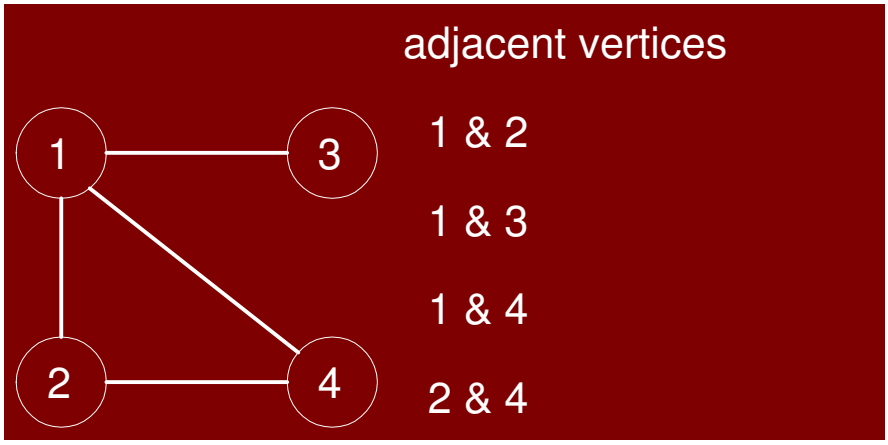


Figure 8.2: Adjacent vertices

In an undirected graph, we say that an edge is *incident* on a vertex if the vertex is an endpoint of the edge. of the edge

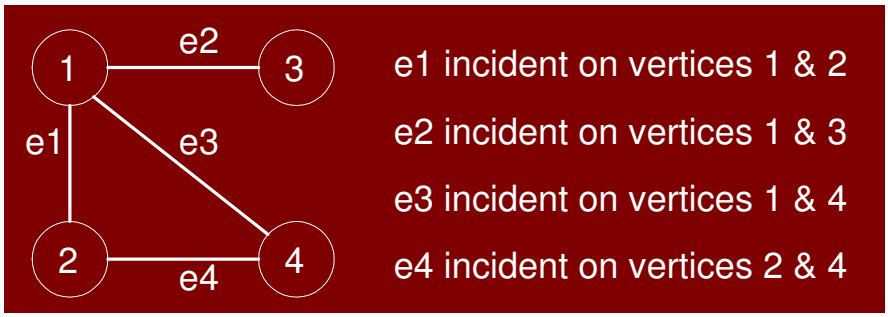


Figure 8.3: Incidence of edges on vertices

In a digraph, the number of edges coming out of a vertex is called the *out-degree* of that vertex. Number of edges coming in is the *in-degree*. In an undirected graph, we just talk of degree of a vertex. It is the number of edges incident on the vertex.

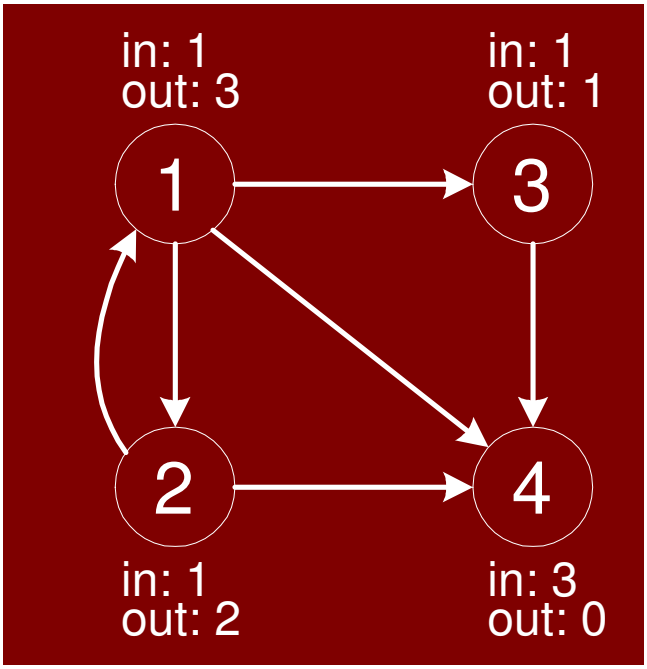


Figure 8.4: In and out degrees of vertices of a graph

For a digraph $G = (V, E)$,

$$\sum_{v \in V} \texttt{in-degree}(v) = \sum_{v \in V} \texttt{out-degree}(v) = |E|$$

where $|E|$ means the cardinality of the set E, i.e., the number of edges.

For an undirected graph $G = (V, E)$,

$$\sum_{v \in V} \texttt{degree}(v) = 2|E|$$

where $|E|$ means the cardinality of the set E, i.e., the number of edges.

A *path* in a directed graphs is a sequence of vertices $\langle v_0, v_1, \ldots, v_k \rangle$ such that (v_{i-1}, v_i) is an edge for $i = 1, 2, \ldots, k$. The *length* of the paths is the number of edges, k. A vertex w is *reachable* from vertex u is there is a path from u to w. A path is simple if all vertices (except possibly the fist and last) are distinct.

A *cycle* in a digraph is a path containing at least one edge and for which $v_0 = v_k$. A *Hamiltonian* cycle is a cycle that visits every vertex in a graph exactly once. A *Eulerian* cycle is a cycle that visits every edge of the graph exactly once. There are also "path" versions in which you do not need return to the starting vertex.

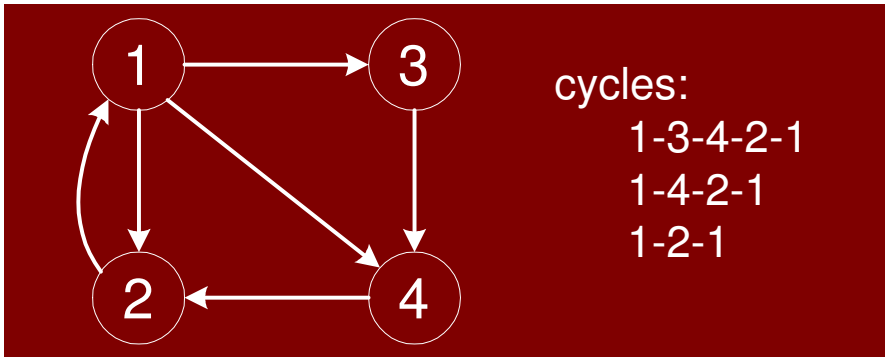


Figure 8.5: Cycles in a directed graph

A graph is said to be *acyclic* if it contains no cycles. A graph is *connected* if every vertex can reach every other vertex. A directed graph that is acyclic is called a *directed acyclic graph (DAG)*.

There are two ways of representing graphs: using an adjacency matrix and using an adjacency list. Let $G = (V, E)$ be a digraph with $n = |V|$ and let $e = |E|$. We will assume that the vertices of G are indexed $\{1, 2, \ldots, n\}$.

An *adjacency matrix* is a $n \times n$ matrix defined for $1 \leq v, w \leq n$.

$$A[v, w] = \begin{cases} 1 & \text{if } (v, w) \in E \\ 0 & \text{otherwise} \end{cases}$$

An *adjacency list* is an array $Adj[1..n]$ of pointers where for $1 \leq v \leq n$, $Adj[v]$ points to a linked list containing the vertices which are adjacent to v

Adjacency matrix requires $\Theta(n^2)$ storage and adjacency list requires $\Theta(n + e)$ storage.

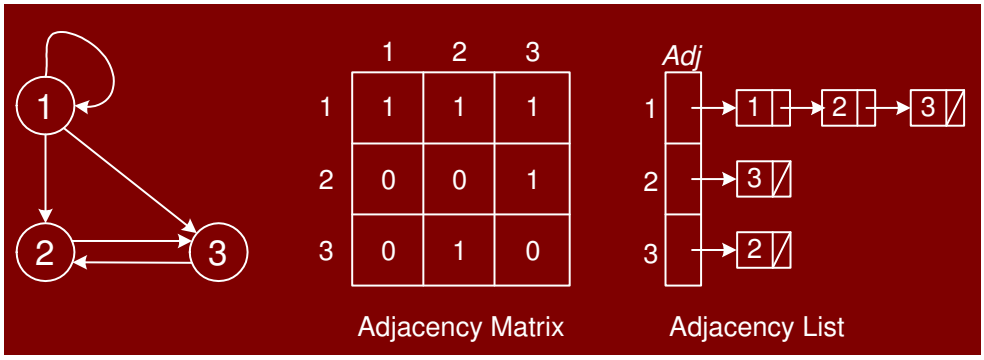


Figure 8.6: Graph Representations

8.1 Graph Traversal

To motivate our first algorithm on graphs, consider the following problem. We are given an undirected graph $G = (V, E)$ and a *source vertex* $s \in V$. The *length* of a path in a graph is the number of edges on

the path. We would like to find the shortest path from s to each other vertex in the graph. The final result will be represented in the following way. For each vertex $v \in V$, we will store $d[v]$ which is the *distance* (length of the shortest path) from s to v. Note that $d[s] = 0$. We will also store a predecessor (or parent) pointer $\pi[v]$ which is the first vertex along the shortest path if we walk from v backwards to s. We will set $\pi[s] = \mathrm{Nil}$.

There is a simple brute-force strategy for computing shortest paths. We could simply start enumerating all simple paths starting at s, and keep track of the shortest path arriving at each vertex. However, there can be as many as $n!$ simple paths in a graph. To see this, consider a fully connected graph shown in Figure 8.7

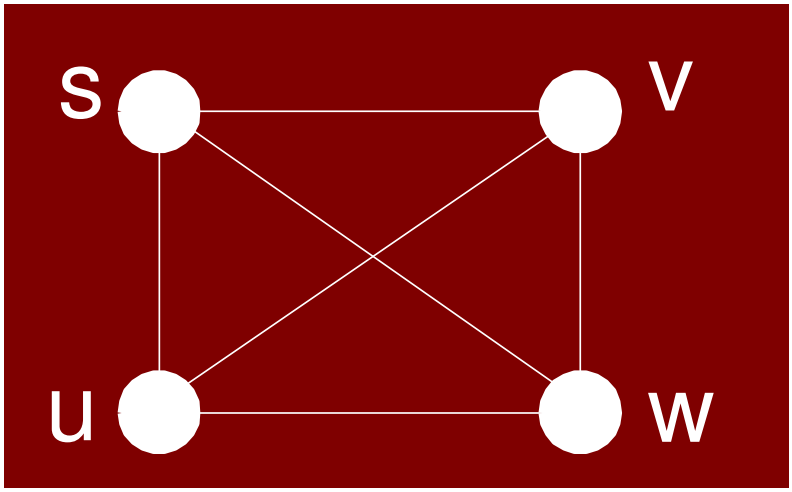


Figure 8.7: Fully connected graph

There n choices for source node s, $(n-1)$ choices for destination node, $(n-2)$ for first hop (edge) in the path, $(n-3)$ for second, $(n-4)$ for third down to $(n-(n-1))$ for last leg. This leads to $n!$ simple paths. Clearly this is not feasible.

8.1.1 Breadth-first Search

Here is a more efficient algorithm called the *breadth-first search* (BFS) Start with s and visit its adjacent nodes. Label them with distance 1. Now consider the neighbors of neighbors of s. These would be at distance 2. Now consider the neighbors of neighbors of neighbors of s. These would be at distance 3. Repeat this until no more unvisited neighbors left to visit. The algorithm can be visualized as a *wave front* propagating outwards from s visiting the vertices in bands at ever increasing distances from s.

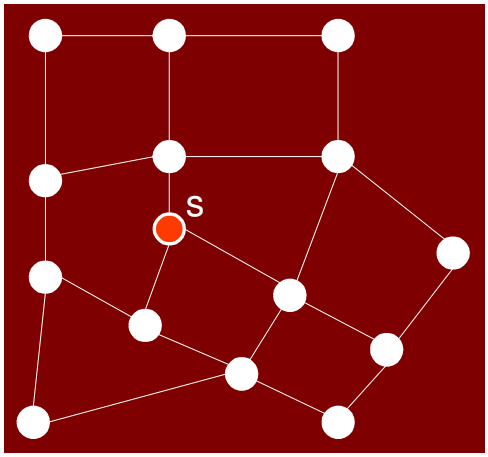


Figure 8.8: Source vertex for breadth-first-search (BFS)

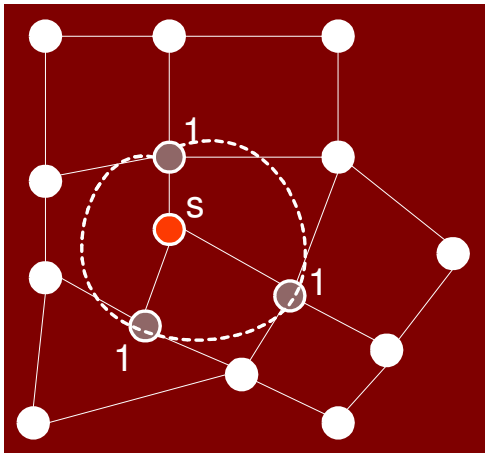


Figure 8.9: Wave reaching distance 1 vertices during BFS

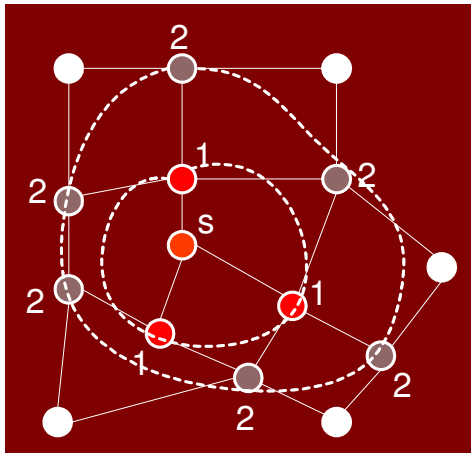


Figure 8.10: Wave reaching distance 2 vertices during BFS

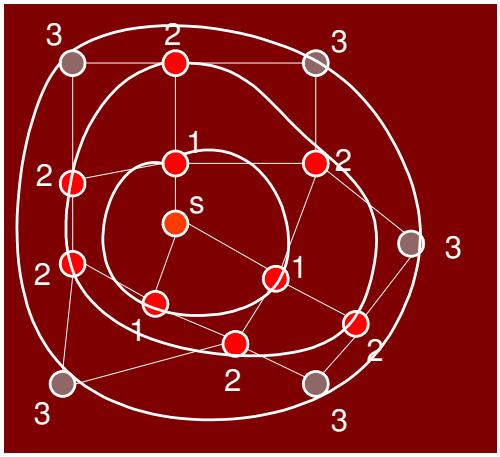


Figure 8.11: Wave reaching distance 3 vertices during BFS

8.1.2 Depth-first Search

Breadth-first search is one instance of a general family of *graph traversal algorithms*. Traversing a graph means visiting every node in the graph. Another traversal strategy is *depth-first search* (DFS). DFS procedure can be written recursively or non-recursively. Both versions are passed s initially.

```
RECURSIVEDFS(v)
1  if (v is unmarked )
2    then mark v
3         for each edge (v,w)
4         do RECURSIVEDFS(w)
```

```
ITERATIVEDFS(s)
1  PUSH(s)
2  while stack not empty
3  do v ← POP()
4     if v is unmarked
5       then mark v
6            for each edge (v,w)
7            do PUSH(w)
```

8.1.3 Generic Graph Traversal Algorithm

The *generic graph traversal* algorithm stores a set of candidate edges in some data structures we'll call a "*bag*". The only important properties of the "bag" are that we can put stuff into it and then later take stuff

back out. Here is the generic traversal algorithm.

```
TRAVERSE(s)
1  put (∅, s) in bag
2  while bag not empty
3  do take (p, v) from bag
4     if (v is unmarked)
5        then mark v
6             parent (v) ← p
7             for each edge (v, w)
8             do put (v, w) in bag
```

Notice that we are keeping edges in the bag instead of vertices. This is because we want to remember, whenever we visit v for the first time, which previously-visited vertex p put v into the bag. The vertex p is call the *parent of* v.

The running time of the traversal algorithm depends on how the graph is represented and what data structure is used for the bag. But we can make a few general observations.

- Since each vertex is visited at most once, the for loop in line 7 is executed at most V times.
- Each edge is put into the bag exactly twice; once as (u, v) and once as (v, u), so line 8 is executed at most $2E$ times.
- Finally, since we can't take out more things out of the bag than we put in, line 3 is executed at most $2E + 1$ times.
- Assume that the graph is represented by an adjacency list so the overhead of the for loop in line 7 is constant per edge.

If we implement the bag by using a *stack*, we have *depth-first* search (DFS) or traversal.

```
TRAVERSE(s)
1  push(∅, s)
2  while stack not empty
3  do pop(p, v)
4     if (v is unmarked)
5        then mark v
6             parent (v) ← p
7             for each edge (v, w)
8             do push(v, w)
```

Figures 8.12 to 8.20 show a trace of the DFS algorithm applied to a graph. The figures show the content of the stack during the execution of the algorithm.

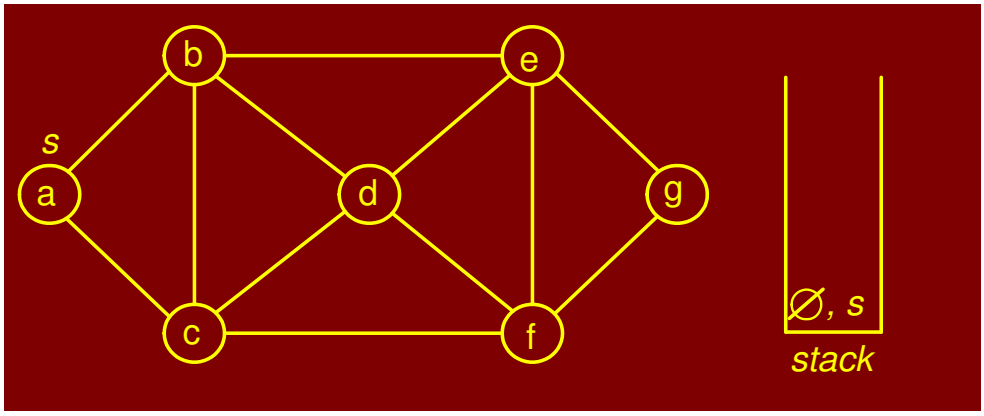


Figure 8.12: Trace of Depth-first-search algorithm: source vertex ‘s’

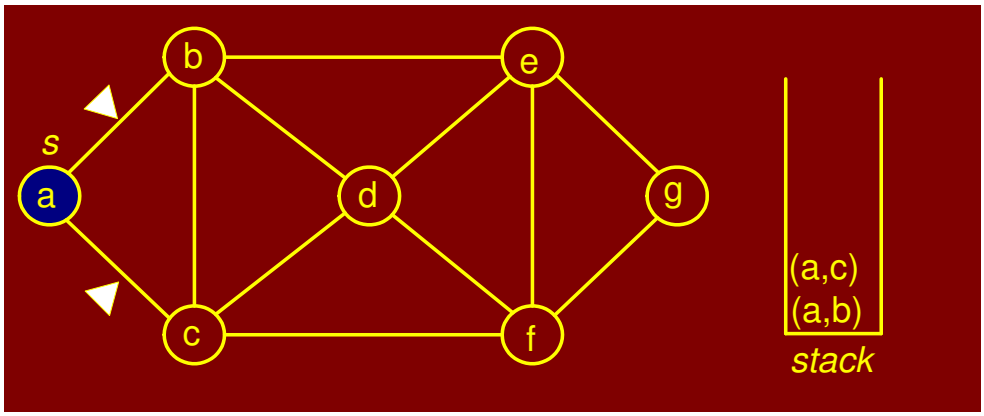


Figure 8.13: Trace of DFS algorithm: vertex ‘a’ popped

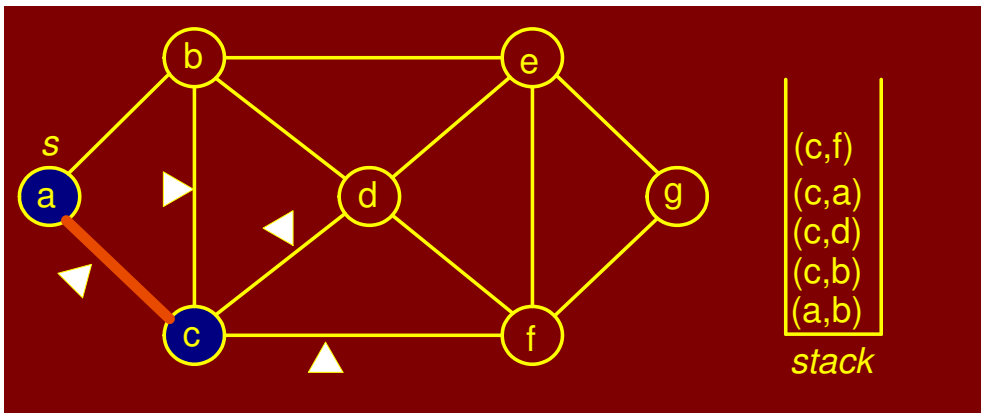


Figure 8.14: Trace of DFS algorithm: vertex ‘c’ popped

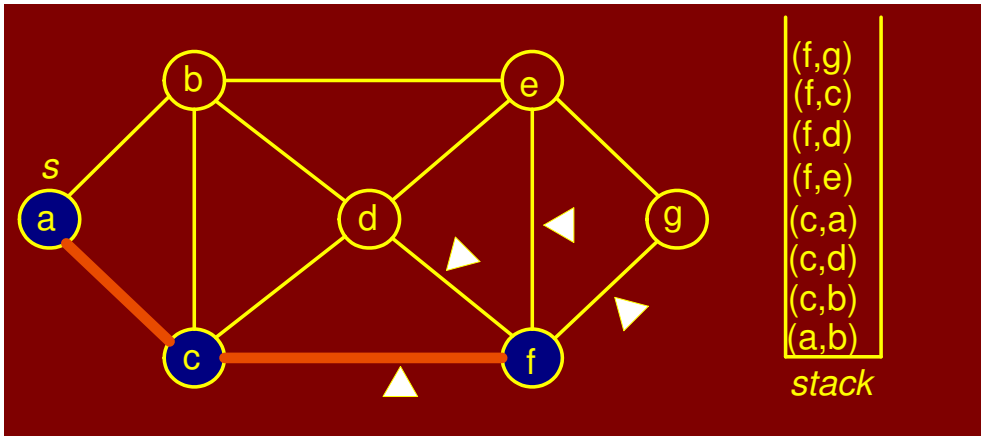


Figure 8.15: Trace of DFS algorithm: vertex ‘f’ popped

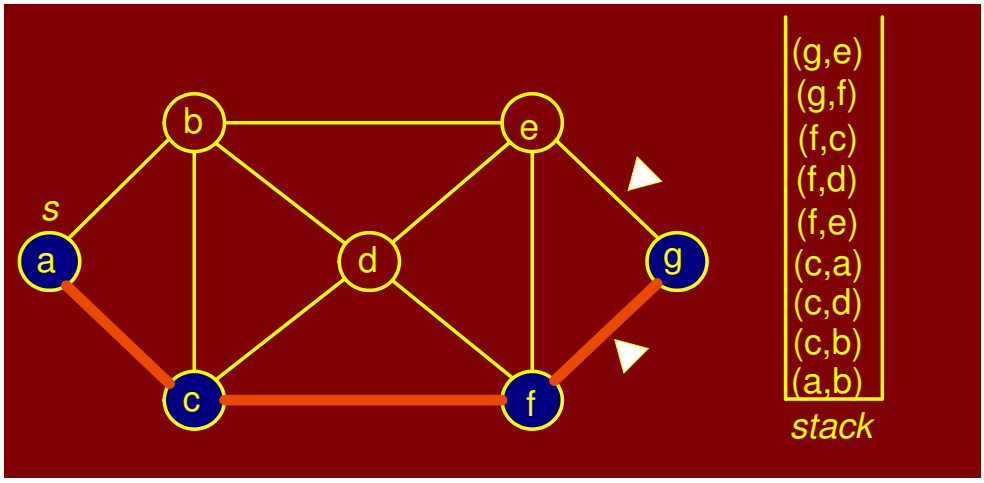


Figure 8.16: Trace of DFS algorithm: vertex ‘g’ popped

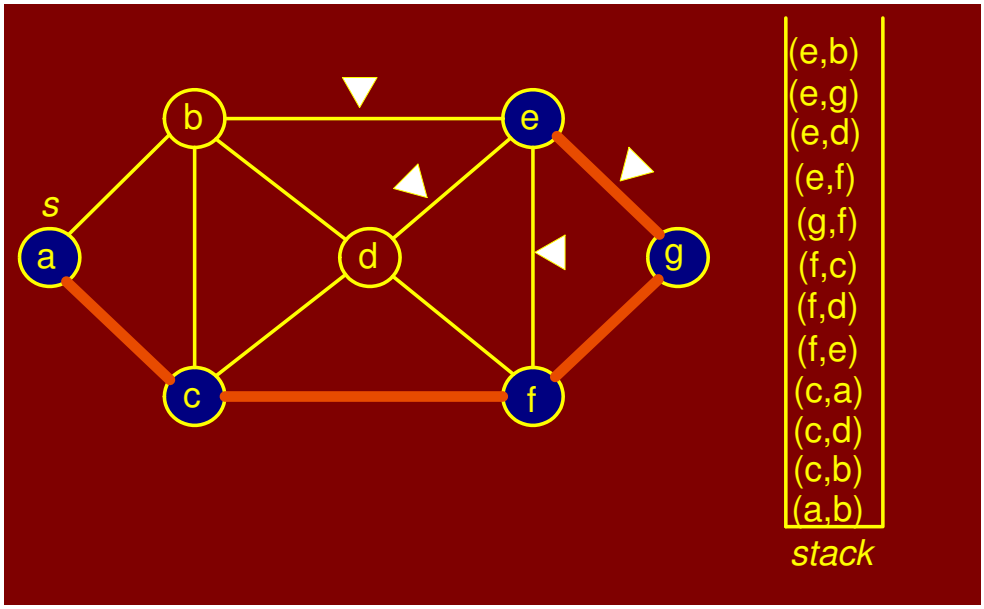


Figure 8.17: Trace of DFS algorithm: vertex ‘e’ popped

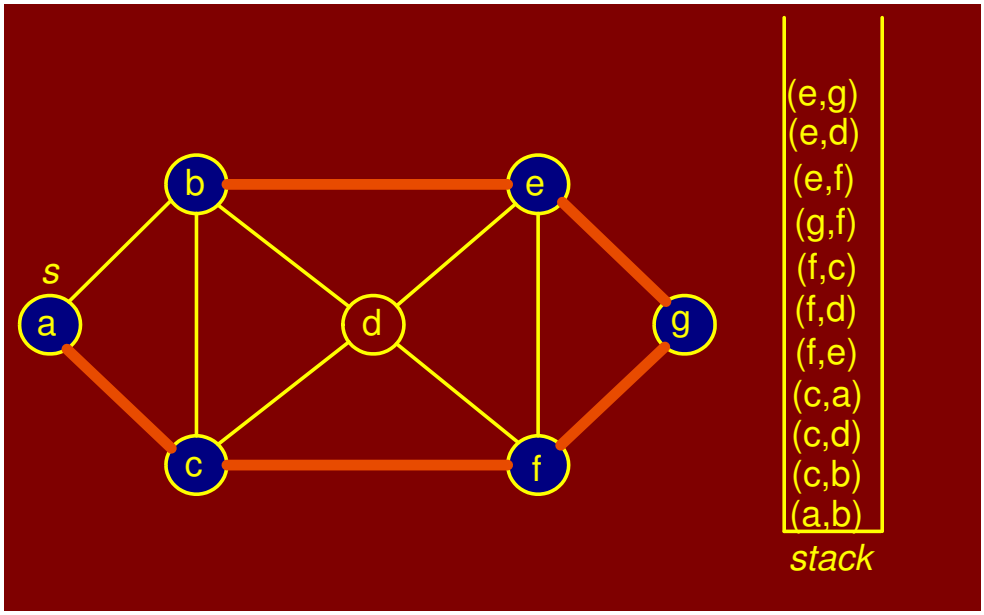


Figure 8.18: Trace of DFS algorithm: vertex ‘b’ popped

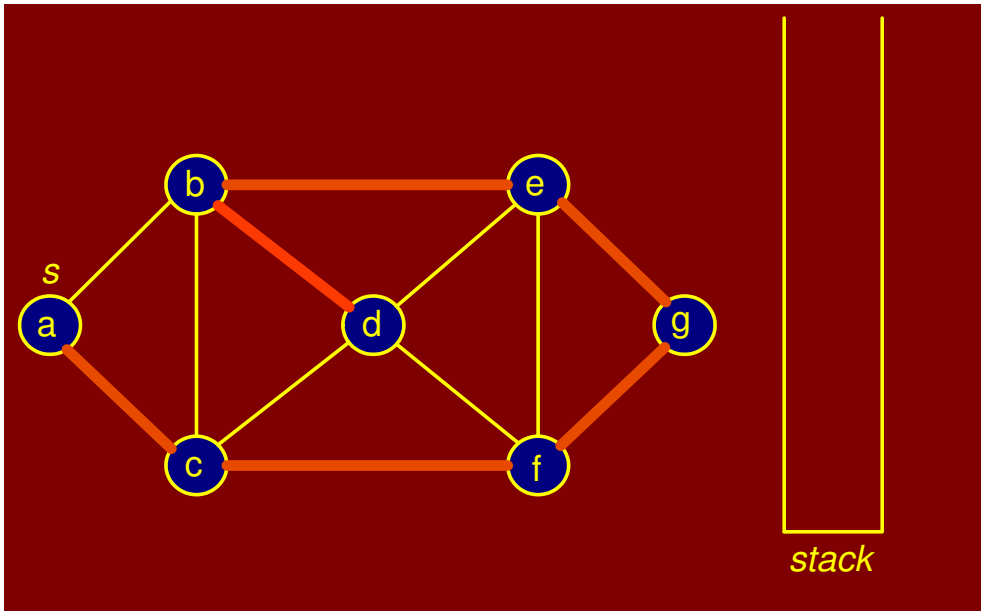


Figure 8.19: Trace of DFS algorithm: vertex ‘d’ popped

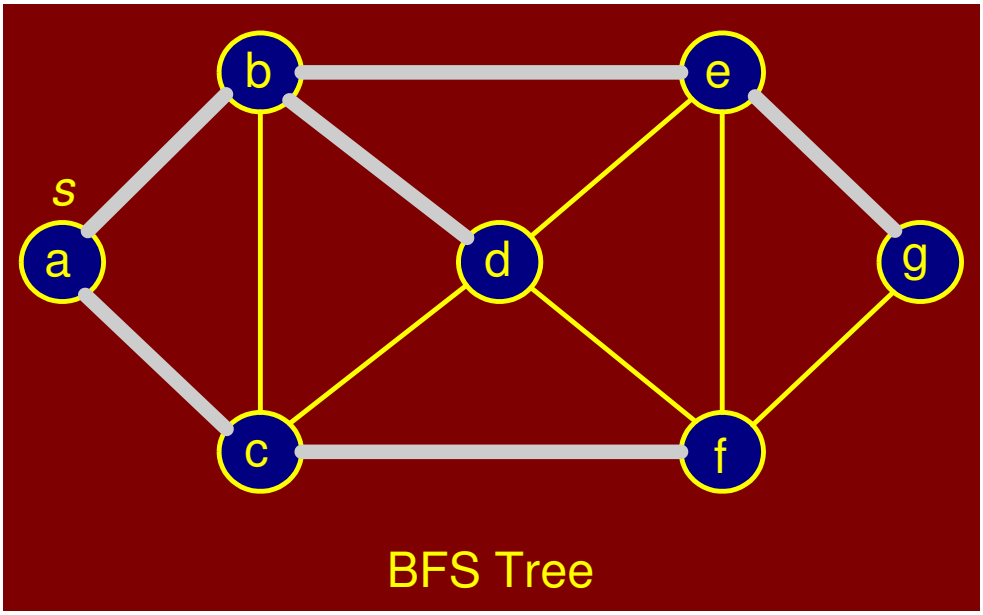


Figure 8.20: Trace of DFS algorithm: the final DFS tree

Each execution of line 3 or line 8 in the TRAVERSE-DFS algorithm takes constant time. So the overall running time is $O(V + E)$. Since the graph is connected, $V \leq E + 1$, this is $O(E)$.

If we implement the bag by using a *queue*, we have *breadth-first* search (BFS). Each execution of line 3

or line 8 still takes constant time. So overall running time is still $O(E)$.

```
TRAVERSE(s)
1  enqueue(∅, s)
2  while queue not empty
3  do dequeue(p, v)
4     if (v is unmarked)
5        then mark v
6             parent (v) ← p
7             for each edge (v, w)
8             do enqueue(v, w)
```

If the graph is represented using an *adjacency matrix*, the finding of all the neighbors of vertex in line 7 takes $O(V)$ time. Thus depth-first and breadth-first take $O(V^2)$ time overall.

Either DFS or BFS yields a spanning tree of the graph. The tree visits every vertex in the graph. This fact is established by the following lemma:

Lemma:

The generic TRAVERSE(S) marks every vertex in any connected graph exactly once and the set of edges $(v, parent(v))$ with $parent(v) \neq \varnothing$ form a spanning tree of the graph.

Proof:

First, it should be obvious that no vertex is marked more than once. Clearly, the algorithm marks s. Let $v \neq s$ be a vertex and let $s \rightarrow \cdots \rightarrow u \rightarrow v$ be a path from s to v with the minimum number of edges.

Since the graph is connected, such a path always exists. If the algorithm marks u, then it must put (u, v) into the bag, so it must take (u, v) out of the bag at which point v must be marked. Thus, by induction on the shortest-path distance from s, the algorithm marks every vertex in the graph.

Call an edge $(v, parent(v))$ with $parent(v) \neq \varnothing$, a *parent edge*. For any node v, the path of parent edges $v \rightarrow parent(v) \rightarrow parent(parent(v)) \rightarrow \ldots$ eventually leads back to s. So the set of parent edges form a connected graph.

Clearly, both end points of every parent edge are marked, and the number of edges is exactly one less than the number of vertices. Thus, the parent edges form a *spanning tree*.

8.1.4 DFS - Timestamp Structure

As we traverse the graph in DFS order, we will associate two numbers with each vertex. When we first discover a vertex u, store a counter in $d[u]$. When we are finished processing a vertex, we store a counter in $f[u]$. These two numbers are *time stamps*.

Consider the *recursive* version of depth-first traversal

```
DFS(G)
1  for (each u ∈ V)
2  do color[u] ← white
3     pred[u] ← nil
4  time ← 0
5  for each u ∈ V
6  do if (color[u] = white)
7        then DFSVISIT(u)
```

The DFSVISIT routine is as follows:

```
DFSVISIT(u)
1  color[u] ← gray;  // mark u visited
2  d[u] ←++ time
3  for (each v ∈ Adj[u])
4  do if (color[v] = white)
5        then pred[v] ← u
6             DFSVISIT(v)
7  color[u] ← black;  // we are done with u
8  f[u] ←++ time;
```

Figures 8.21 through 8.25 present a trace of the execution of the time stamping algorithm. Terms like "2/5" indicate the value of the counter (time). The number before the "/" is the time when a vertex was discovered (colored gray) and the number after the "/" is the time when the processing of the vertex finished (colored black).

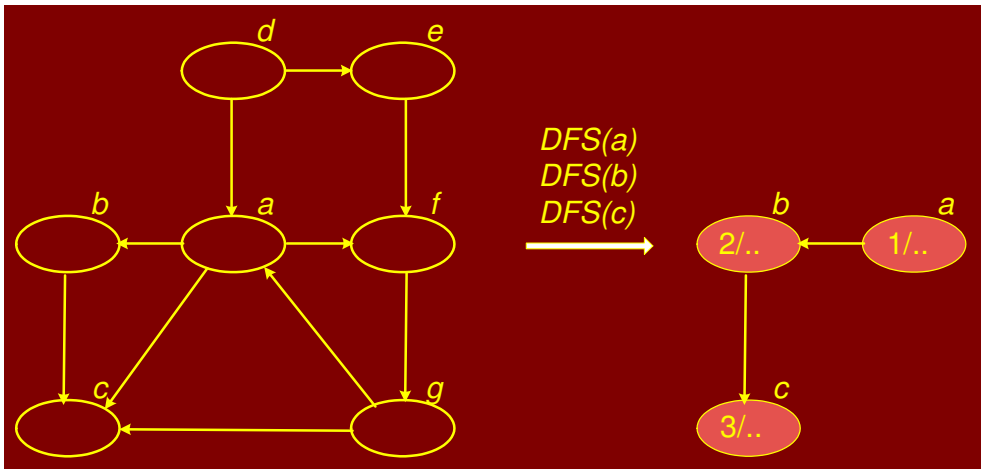


Figure 8.21: DFS with time stamps: recursive calls initiated at vertex 'a'

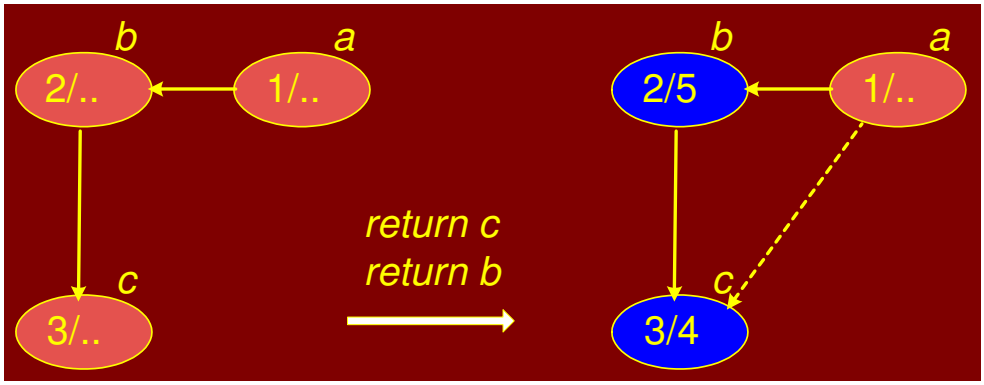


Figure 8.22: DFS with time stamps: processing of ‘b’ and ‘c’ completed

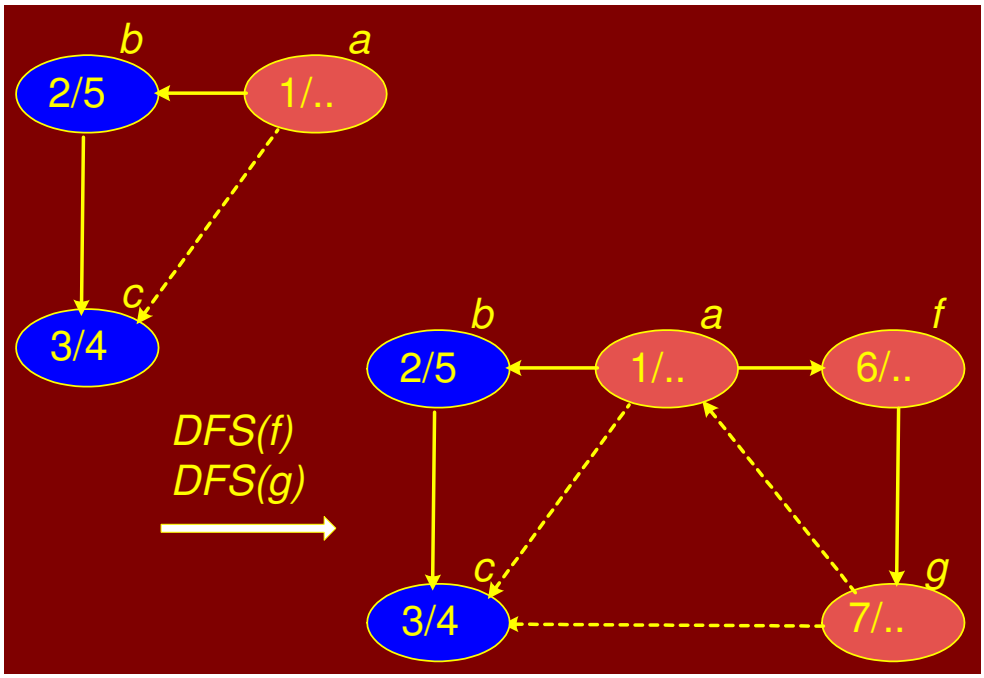


Figure 8.23: DFS with time stamps: recursive processing of ‘f’ and ‘g’

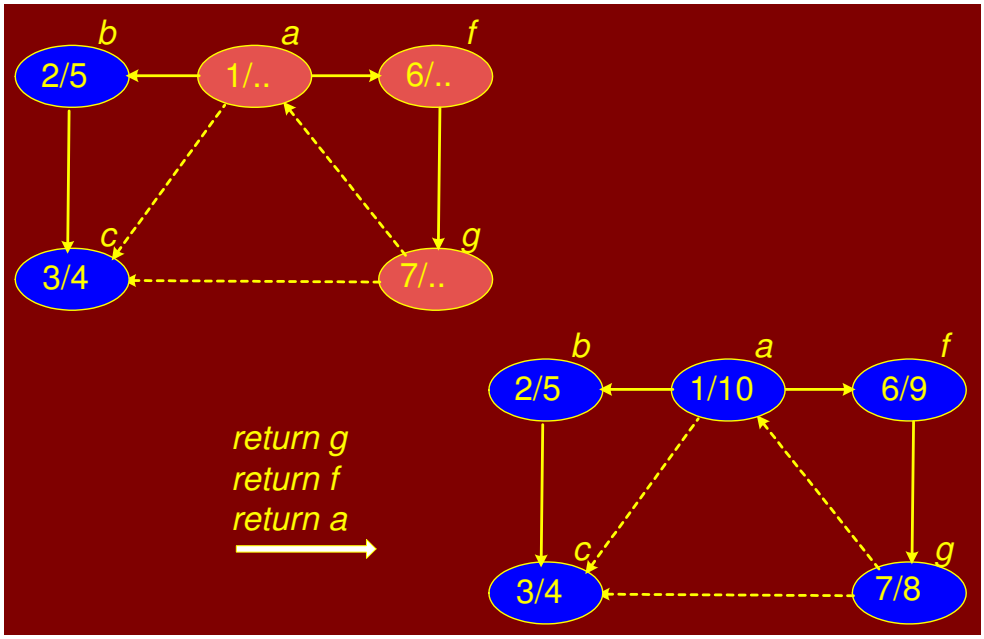


Figure 8.24: DFS with time stamps: processing of ‘f’ and ‘g’ completed

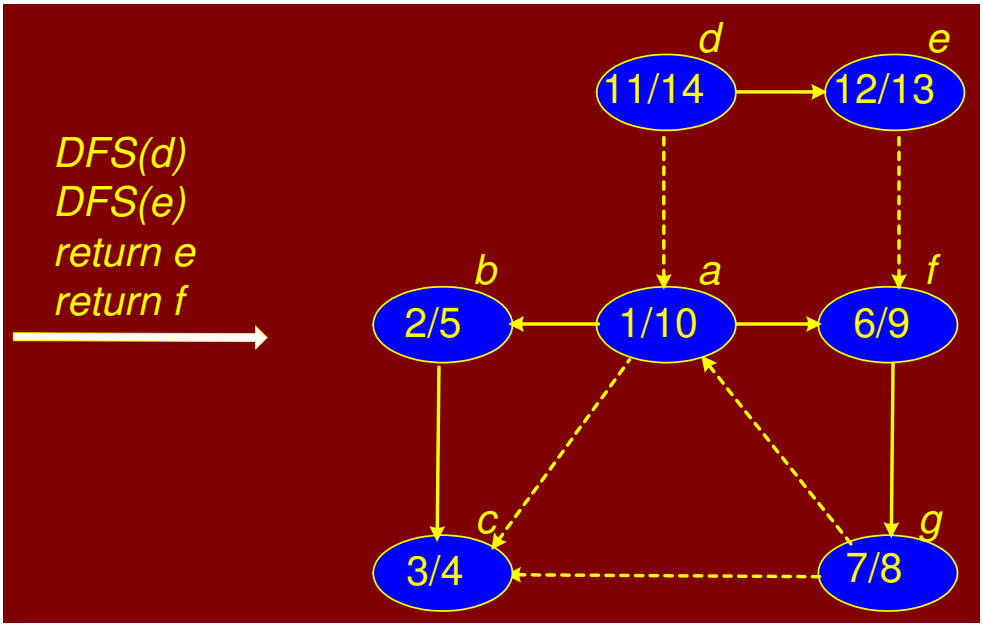


Figure 8.25: DFS with time stamps: processing of ‘d’ and ‘e’

Notice that the DFS tree structure (actually a collection of trees, or a forest) on the structure of the graph is just the recursion tree, where the edge (u, v) arises when processing vertex u we call DFSVISIT(V) for some neighbor v. For *directed graphs* the edges that are not part of the tree (indicated as dashed edges in Figures 8.21 through 8.25) edges of the graph can be classified as follows:

Back edge: (u, v) where v is an ancestor of u in the tree.

Forward edge: (u, v) where v is a proper descendent of u in the tree.

Cross edge: (u, v) where u and v are not ancestor or descendent of one another. In fact, the edge may go between different trees of the forest.

The ancestor and descendent relation can be nicely inferred by the *parenthesis* lemma. u is a descendent of v if and only if $[d[u], f[u]] \subseteq [d[v], f[v]]$. u is a ancestor of v if and only if $[d[u], f[u]] \supseteq [d[v], f[v]]$. u is unrelated to v if and only if $[d[u], f[u]]$ and $[d[v], f[v]]$ are disjoint. The is shown in Figure 8.26. The width of the rectangle associated with a vertex is equal to the time the vertex was discovered till the time the vertex was completely processed (colored black). Imagine an opening parenthesis '(' at the start of the rectangle and and closing parenthesis ')' at the end of the rectangle. The rectangle (parentheses) for vertex 'b' is completely enclosed by the rectangle for 'a'. Rectangle for 'c' is completely enclosed by vertex 'b' rectangle.

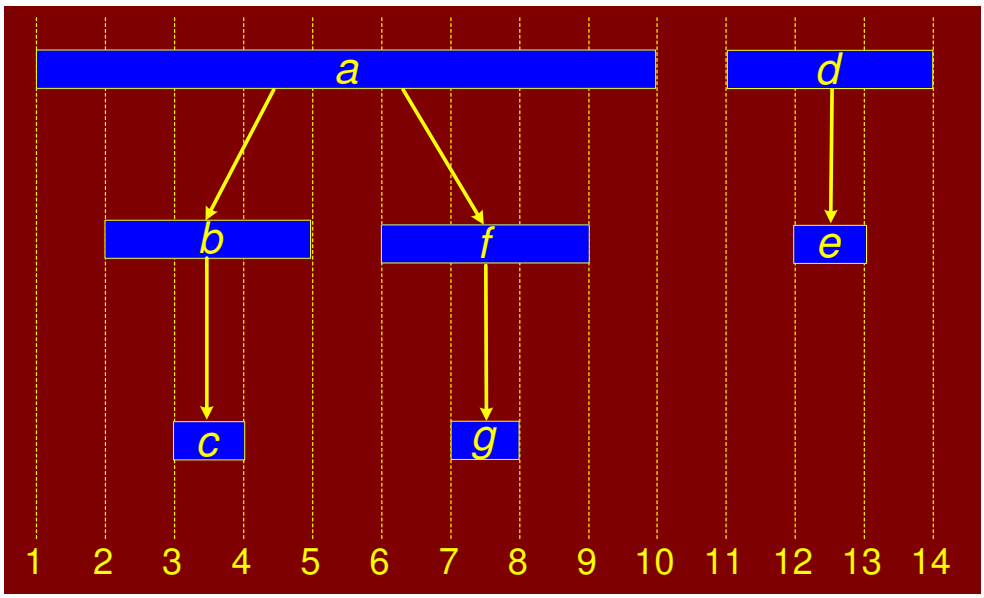


Figure 8.26: Parenthesis lemma

Figure 8.27 shows the classification of the non-tree edges based on the parenthesis lemma. Edges are labelled 'F', 'B' and 'C' for forward, back and cross edge respectively.

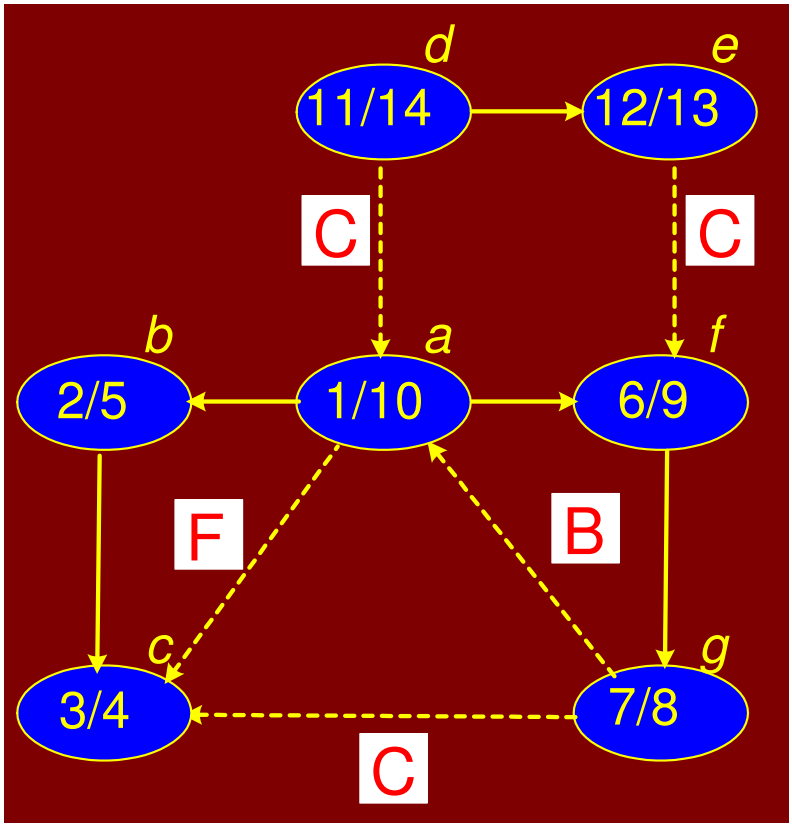


Figure 8.27: Classfication of non-tree edges in the DFS tree for a graph

For *undirected* graphs, there is no distinction between forward and back edges. By convention they are all called back edges. Furthermore, there are no cross edges (can you see why not?)

8.1.5 DFS - Cycles

The time stamps given by DFS allow us to determine a number of things about a graph or digraph. For example, we can determine whether the graph contains any *cycles*. We do this with the help of the following two lemmas.

Lemma: Given a digraph $G = (V, E)$, consider any DFS forest of G and consider any edge $(u, v) \in E$. If this edge is a tree, forward or cross edge, then $f[u] > f[v]$. If this edge is a back edge, then $f[u] \leq f[v]$.

Proof: For the non-tree forward and back edges the proof follows directly from the parenthesis lemma. For example, for a forward edge (u, v), v is a descendent of u and so v's start-finish interval is contained within u's implying that v has an earlier finish time. For a cross edge (u, v) we know that the two time intervals are disjoint. When we were processing u, v was not white (otherwise (u, v) would be a tree edge), implying that v was started before u. Because the intervals are disjoint, v must have also finished before u.

Lemma: Consider a digraph $G = (V, E)$ and any DFS forest for G. G has a cycle if and only if the DFS forest has a *back* edge.

Proof: If there is a back edge (u, v) then v is an ancestor of u and by following tree edge from v to u, we get a cycle.

We show the contrapositive: suppose there are no back edges. By the lemma above, each of the remaining types of edges, tree, forward, and cross all have the property that they go from vertices with higher finishing time to vertices with lower finishing time. Thus along any path, finish times decrease monotonically, implying there can be no cycle.

The DFS forest in Figure 8.27 has a back edge from vertex 'g' to vertex 'a'. The cycle is 'a-g-f'.

Beware: No back edges means no cycles. But you should not infer that there is some simple relationship between the number of back edges and the number of cycles. For example, a DFS tree may only have a single back edge, and there may anywhere from one up to an exponential number of simple cycles in the graph.

A similar theorem applies to undirected graphs, and is not hard to prove.

8.2 Precedence Constraint Graph

A *directed acyclic graph* (DAG) arise in many applications where there are precedence or ordering constraints. There are a series of tasks to be performed and certain tasks must precede other tasks. For example, in construction, you have to build the first floor before the second floor but you can do electrical work while doors and windows are being installed. In general, a *precedence constraint graph* is a DAG in which vertices are tasks and the edge (u, v) means that task u must be completed before task v begins.

For example, consider the sequence followed when one wants to dress up in a suit. One possible order and its DAG are shown in Figure 8.28. Figure 8.29 shows the DFS with time stamps of the DAG.

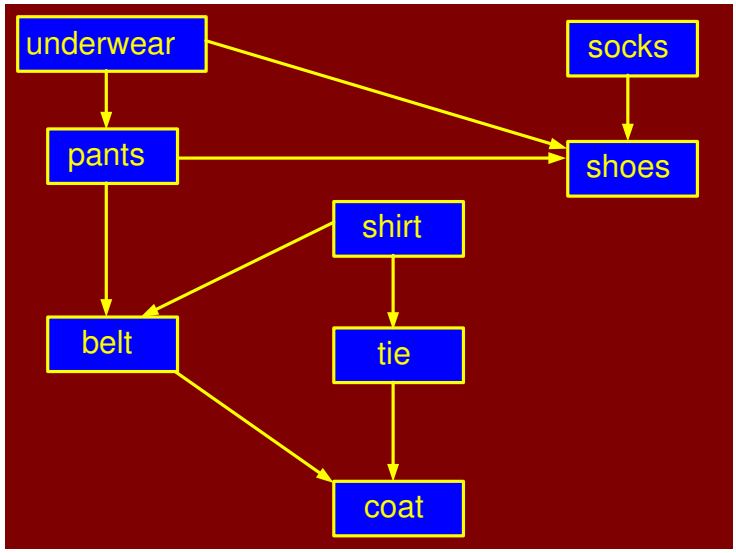


Figure 8.28: Order of dressing up in a suit

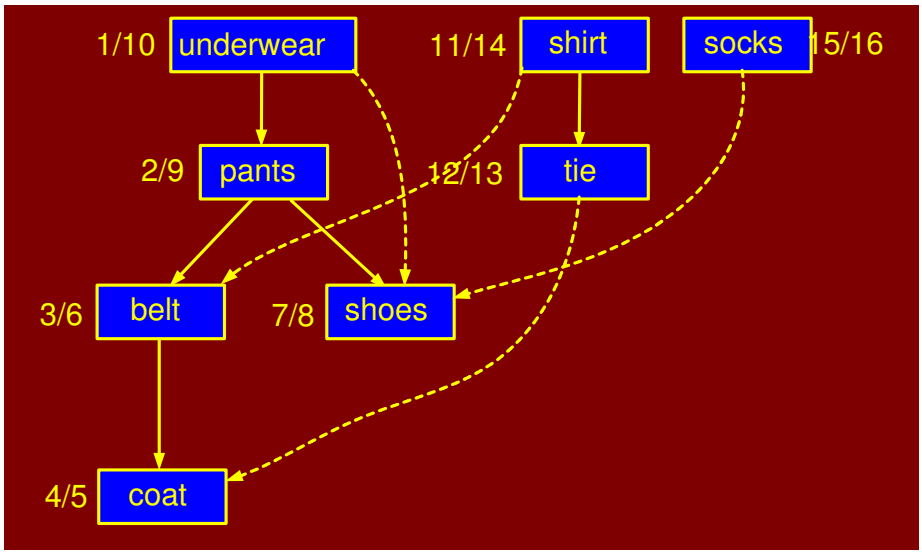


Figure 8.29: DFS of dressing up DAG with time stamps

Another example of precedence constraint graph is the sets of prerequisites for CS courses in a typical undergraduate program.

C1	Introduction to Computers	
C2	Introduction to Computer Programming	
C3	Discrete Mathematics	
C4	Data Structures	C2
C5	Digital Logic Design	C2
C6	Automata Theory	C3
C7	Analysis of Algorithms	C3, C4
C8	Computer Organization and Assembly	C2
C9	Data Base Systems	C4, C7
C10	Computer Architecture	C4, C5,C8
C11	Computer Graphics	C4,C7
C12	Software Engineering	C7,C11
C13	Operating System	C4,C7,C11
C14	Compiler Construction	C4,C6,C8
C15	Computer Networks	C4,C7,C10

Table 8.1: Prerequisites for CS courses

The prerequisites can be represented with a precedence constraint graph which is shown in Figure 8.30

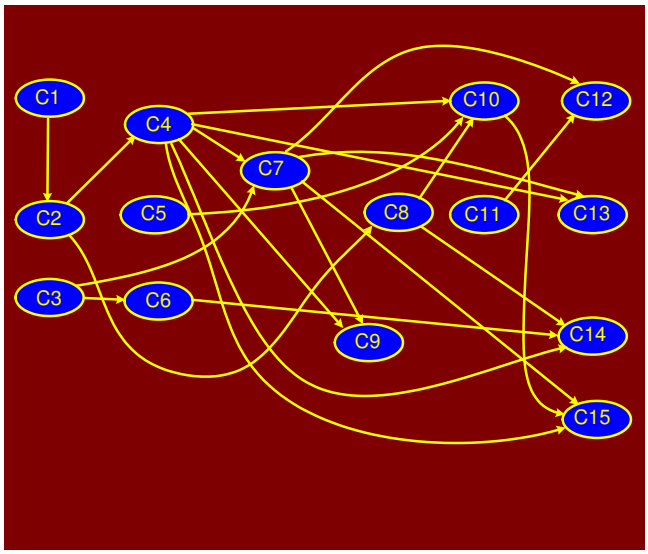


Figure 8.30: Precedence constraint graph for CS courses

8.3 Topological Sort

A topological sort of a DAG is a linear ordering of the vertices of the DAG such that for each edge (u, v), u appears before v in the ordering.

Computing a topological ordering is actually quite easy, given a DFS of the DAG. For every edge (u, v) in a DAG, the finish time of u is greater than the finish time of v (by the lemma). Thus, it suffices to output the vertices in the reverse order of finish times.

We run DFS on the DAG and when each vertex is finished, we add it to the front of a linked. Note that in general, there may be many legal topological orders for a given DAG.

```
TOPOLOGICALSORT(G)
1  for (each u ∈ V)
2  do color[u] ← white
3  L ← new LinkedList()
4  for each u ∈ V
5  do if (color[u] = white)
6        then TOPVISIT(u)
7  return L
```

```
TOPVISIT(u)
1  color[u] ← gray;  // mark u visited
2  for (each v ∈ Adj[u])
3  do if (color[v] = white)
4        then TOPVISIT(v)
5  Append u to the front of L
```

Figure 8.31 shows the linear order obtained by the topological sort of the sequence of putting on a suit. The DAG is still the same; it is only that the order in which the vertices of the graph have been laid out is special. As a result, all directed edges go from left to right.

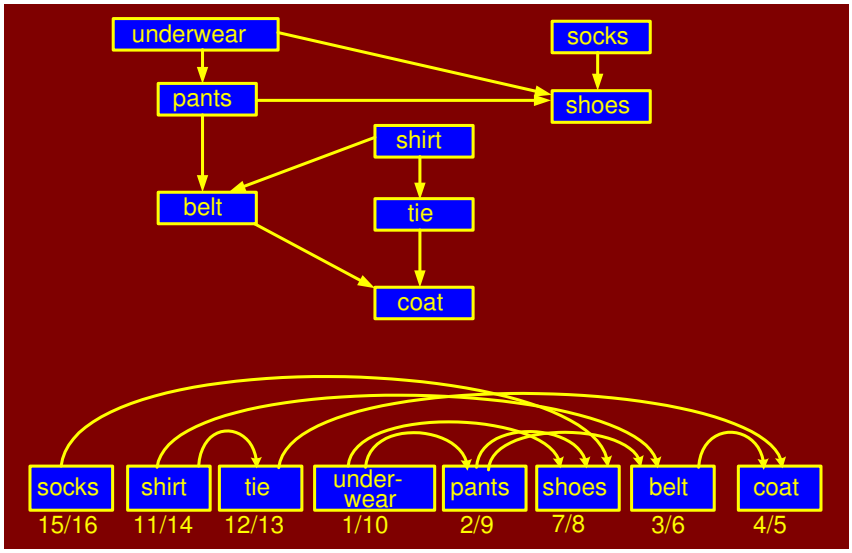


Figure 8.31: Topological sort of the dressing up sequence

This is a typical example of how DFS used in applications. The running time is $\Theta(V + E)$.

8.4 Strong Components

We consider an important connectivity problem with digraphs. When diagraphs are used in communication and transportation networks, people want to know that their networks are *complete*. Complete in the sense that that it is possible from any location in the network to reach any other location in the digraph.

A digraph is *strongly connected* if for every pair of vertices $u, v \in V$, u can reach v and vice versa. We would like to write an algorithm that determines whether a digraph is strongly connected. In fact, we will solve a generalization of this problem, of computing the *strongly connected components* of a digraph.

We partition the vertices of the digraph into subsets such that the induced subgraph of each subset is strongly connected. We say that two vertices u and v are *mutually reachable* if u can reach v and vice versa. Consider the directed graph in Figure 8.32. The strong components are illustrated in Figure 8.33.

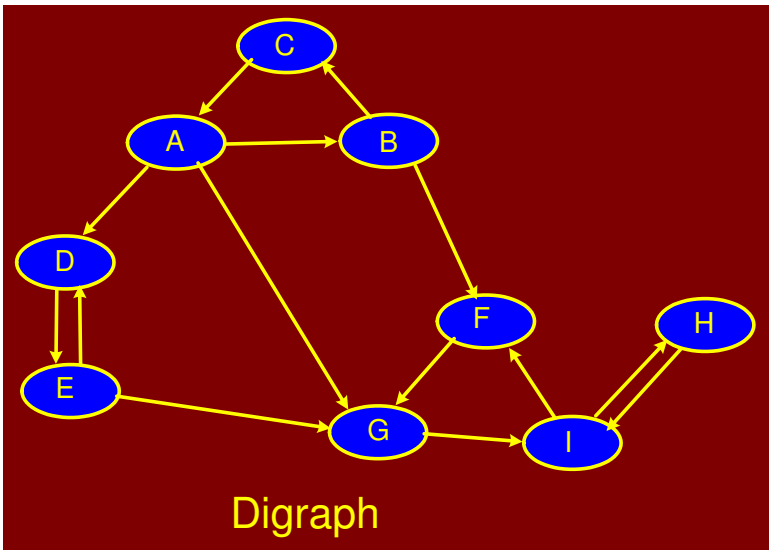


Figure 8.32: A directed graph

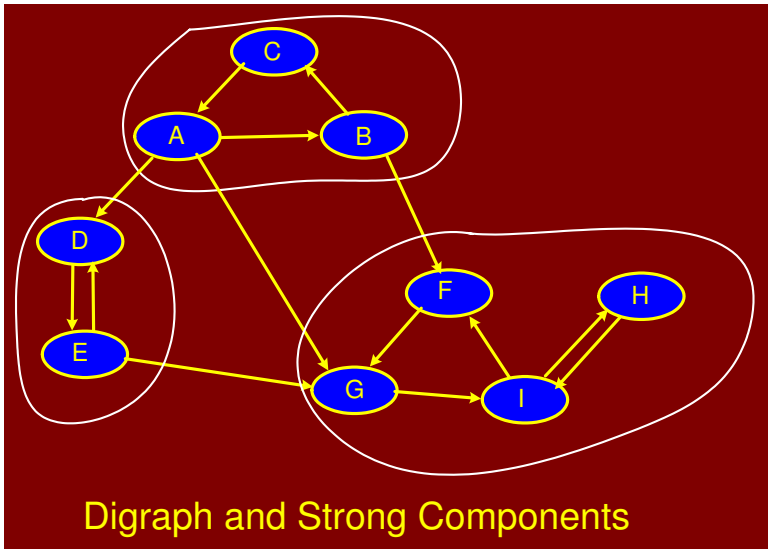


Figure 8.33: Digraph with strong components

It is easy to see that mutual reachability is an *equivalence relation*. This equivalence relation partitions the vertices into equivalence classes of mutually reachable vertices and these are the strong components.

If we merge the vertices in each strong component into a single *super vertex*, and join two super vertices (A, B) if and only if there are vertices $u \in A$ and $v \in B$ such that $(u, v) \in E$, then the resulting digraph is called the *component digraph*. The component digraph is necessarily acyclic. The is illustrated in Figure 8.34.

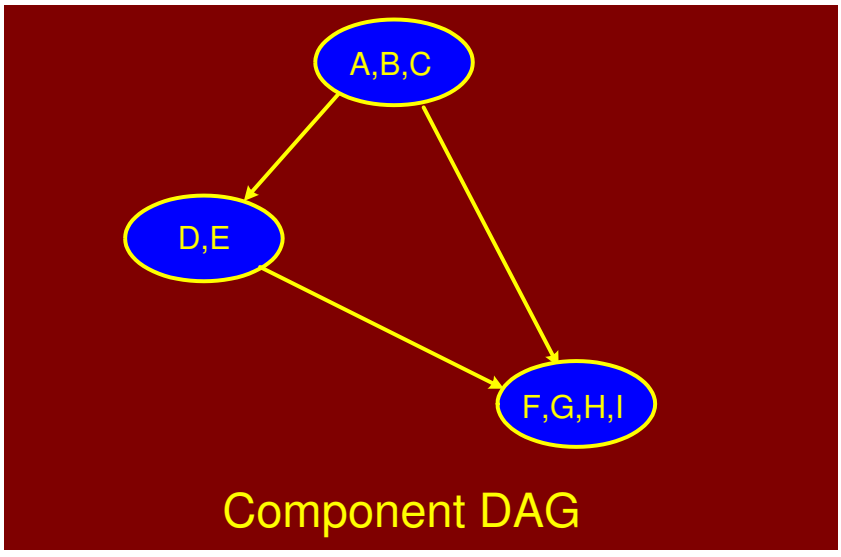


Figure 8.34: Component DAG of super vertices

8.4.1 Strong Components and DFS

Consider DFS of a digraph given in Figure **??**. Once you enter a strong component, every vertex in the component is reachable. So the DFS does not terminate until all the vertices in the component have been visited. Thus all vertices in a strong component must appear in the same tree of the DFS forest.

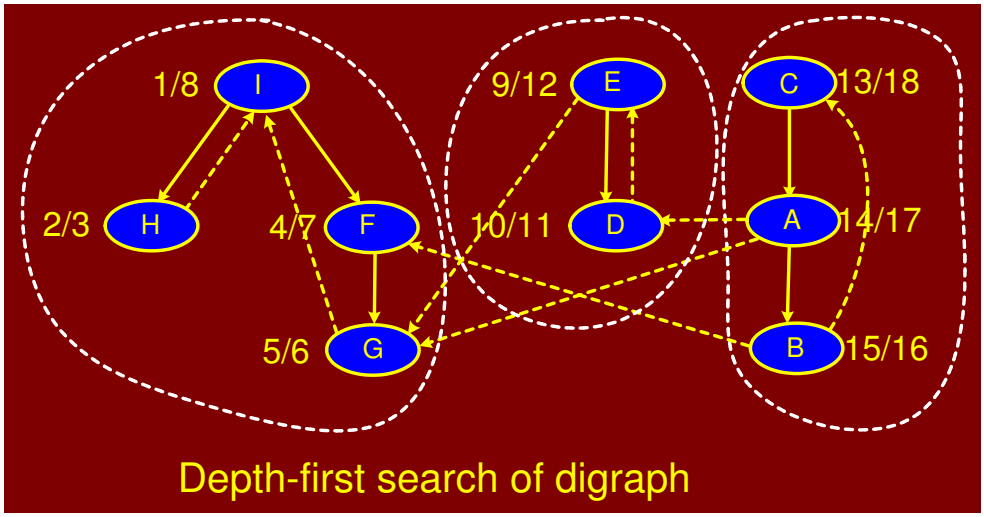


Figure 8.35: DFS of a digraph

fig:dfsofdigraph

Observe that each strong component is a subtree in the DFS forest. Is it always true for any DFS? The answer is “no”. In general, many strong components may appear in the same DFS tree as illustrated in Figure 8.36

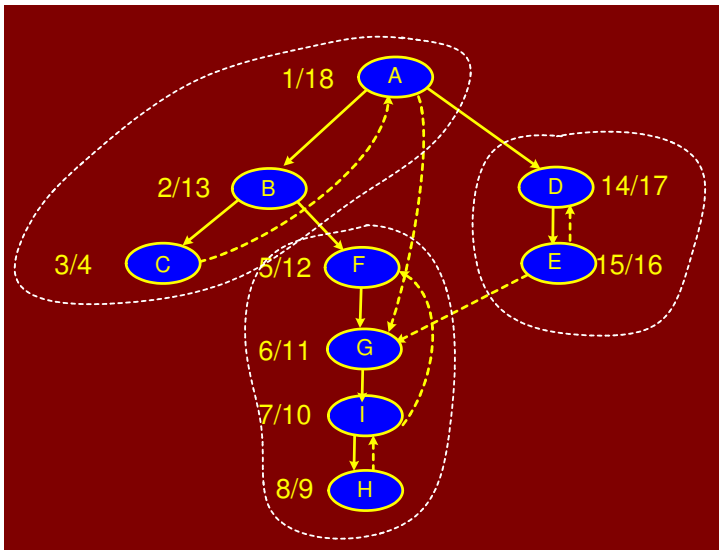


Figure 8.36: Another DFS tree of the digraph

Is there a way to order the DFS such that it true? Fortunately, the answer is "yes". Suppose that you knew the component DAG in advance. (This is ridiculous, because you would need to know the strong components and this is the problem we are trying to solve.) Further, suppose that you computed a *reversed topological order* on the component DAG. That is, for edge (u, v) in the component DAG, then v comes before u. This is presented in Figure 8.37. Recall that the component DAG consists of super vertices.

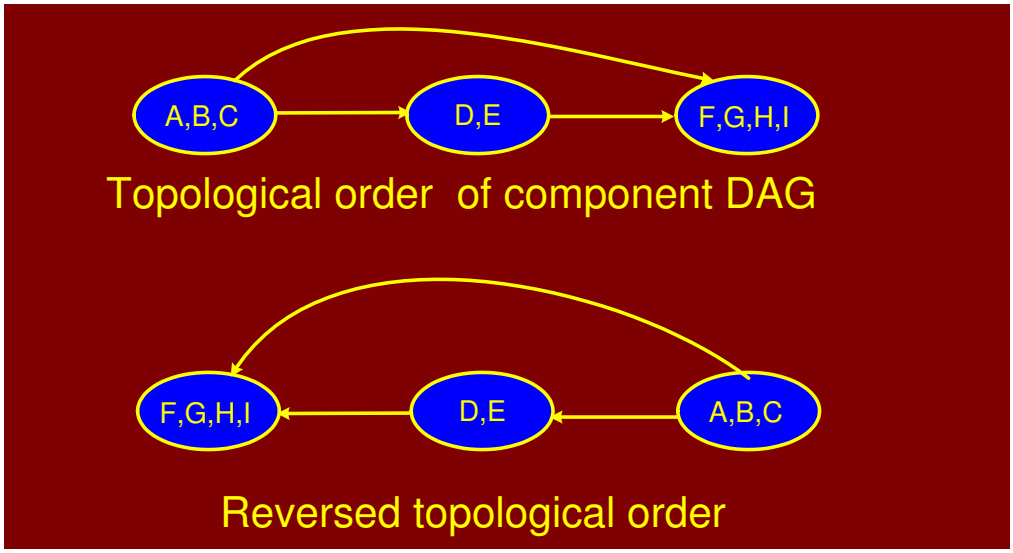


Figure 8.37: Reversed topological sort of component DAG

Now, run DFS, but every time you need a new vertex to start the search from, select the next available vertex according to this reverse topological order of the component digraph. Here is an informal justification. Clearly once the DFS starts within a given strong component, it must visit every vertex within the component (and possibly some others) before finishing. If we do not start in reverse topological, then the search may "leak out" into other strong components, and put them in the same DFS tree. For example, in the Figure 8.36, when the search is started at vertex 'a', not only does it visit its component with 'b' and 'c', but it also visits the other components as well. However, by visiting components in reverse topological order of the component tree, each search cannot "leak out" into other components, because other components would have already have been visited earlier in the search.

This leaves us with the intuition that if we could somehow order the DFS, so that it hits the strong components according to a reverse topological order, then we would have an easy algorithm for computing strong components. However, we do not know what the component DAG looks like. (After all, we are trying to solve the strong component problem in the first place). The trick behind the strong component algorithm is that we can find an ordering of the vertices that has essentially the necessary property, without actually computing the component DAG.

We will discuss the algorithm without proof. Define G^T to be the digraph with the same vertex set at G but in which all edges have been reversed in direction. This is shown in Figure 8.38. Given an adjacency list for G, it is possible to compute G^T in $\Theta(V + E)$ time.

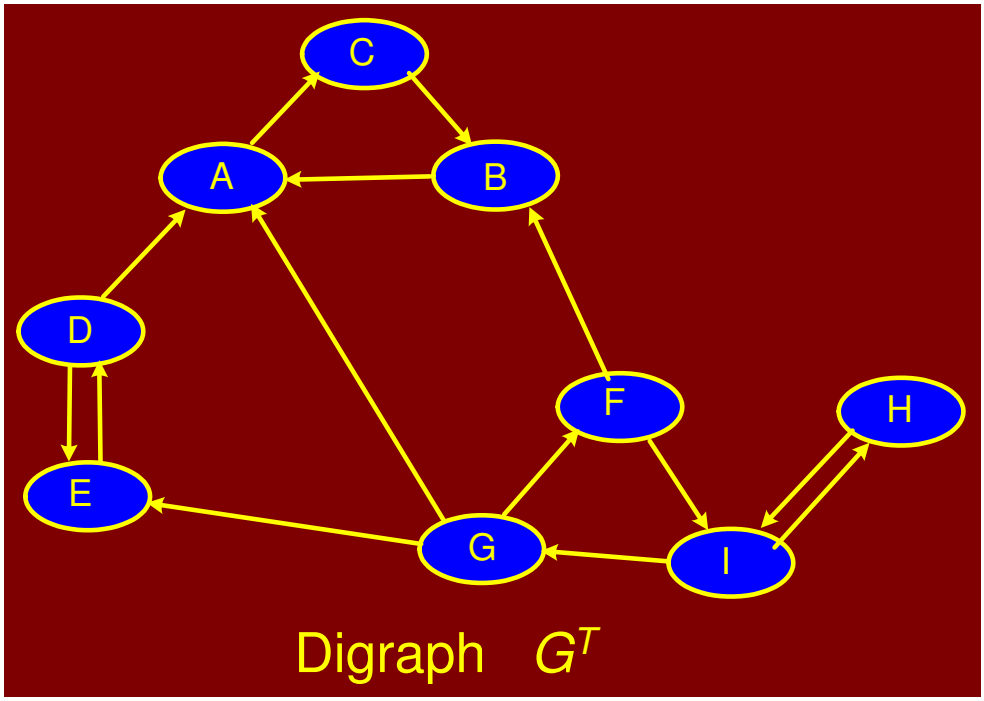


Figure 8.38: The digraph G^T

Observe that the strongly connected components are not affected by reversal of all edges. If u and v are mutually reachable in G, then this is certainly true in G^T. All that changes is that the component DAG is completely reversed. The ordering trick is to order the vertices of G according to their finish times in a DFS. Then visit the nodes of G^T in decreasing order of finish times. All the steps of the algorithm are quite easy to implement, and all operate in $\Theta(V + E)$ time. Here is the algorithm:

```
STRONGCOMPONENTS(G)
1  Run DFS(G) computing finish times f[u]
2  Compute G^T
3  Sort vertices of G^T in decreasing f[u]
4  Run DFS(G^T) using this order
5  Each DFS tree is a strong component
```

The execution of the algorithm is illustrated in Figures 8.39, 8.40 and 8.41.

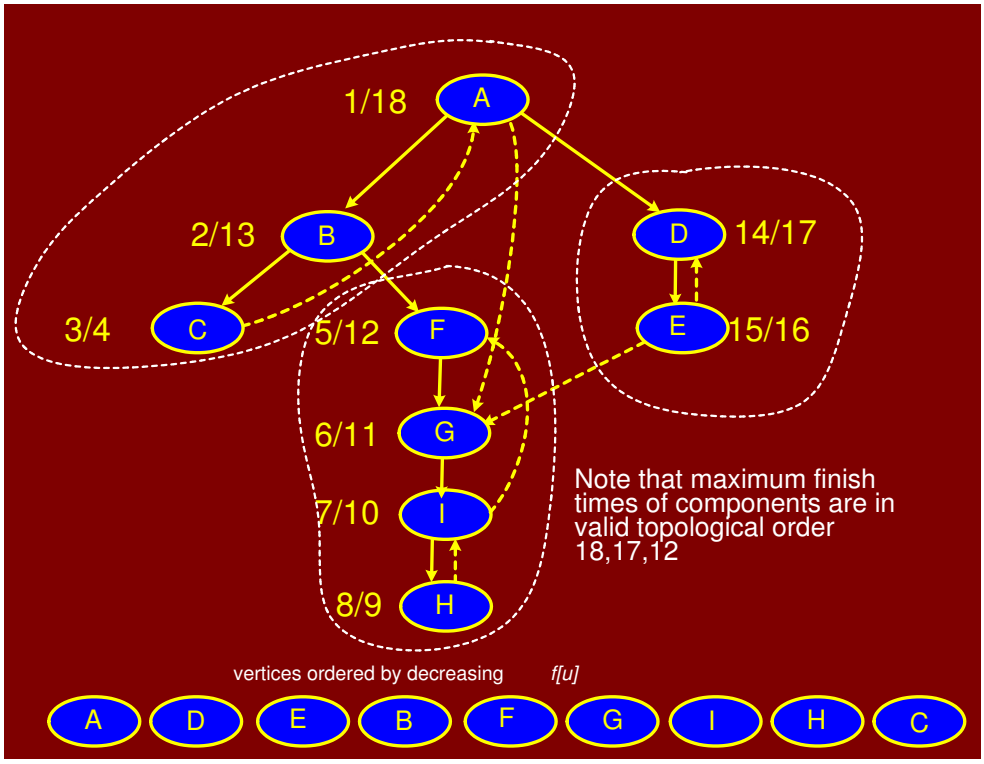


Figure 8.39: DFS of digraph with vertices in descending order by finish times

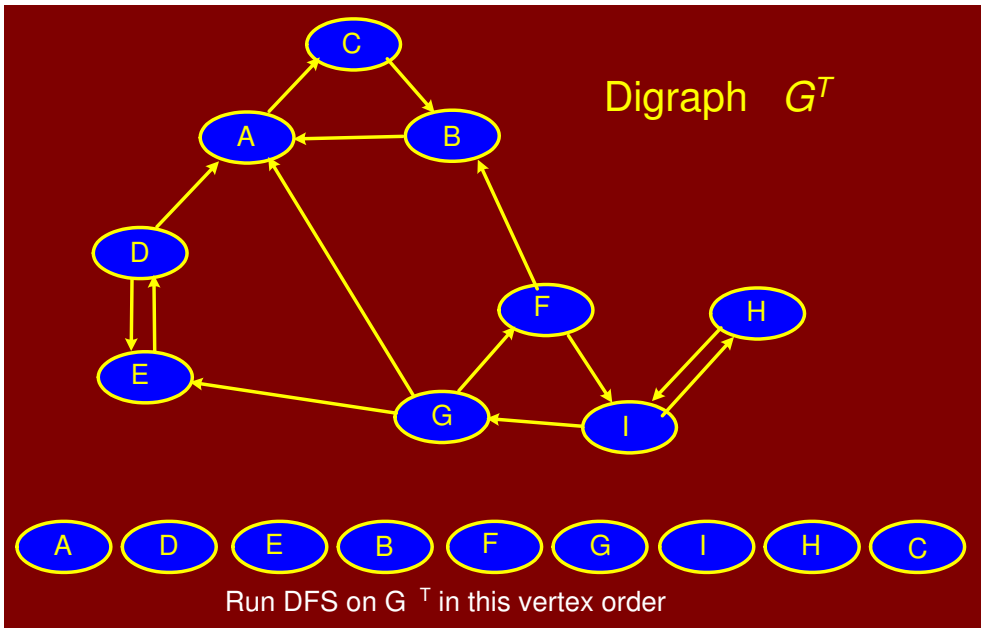


Figure 8.40: Digraph G^T and the vertex order for DFS

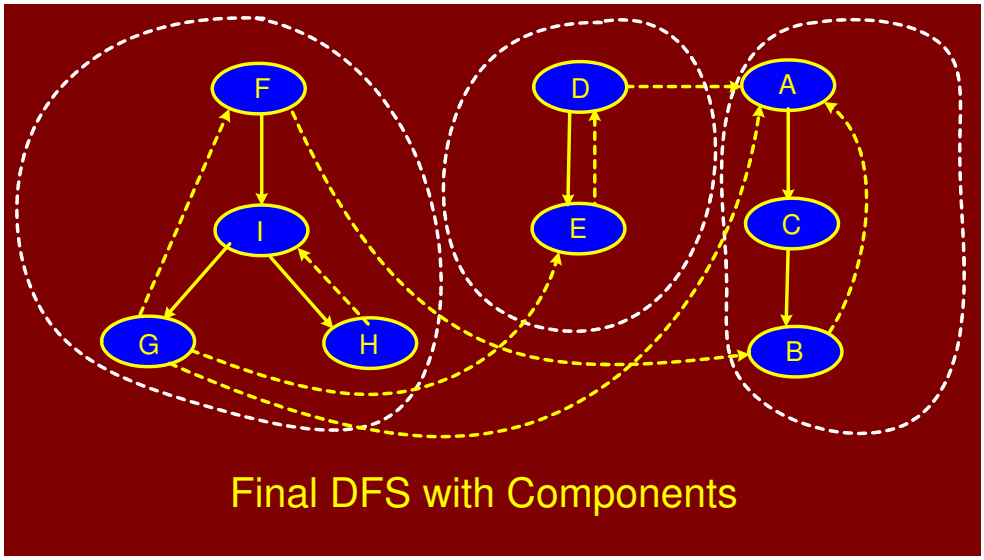


Figure 8.41: Final DFS with strong components of G^T

The complete proof for why this algorithm works is in CLR. We will discuss the intuition behind why the algorithm visits vertices in decreasing order of finish times and why the graph is revered. Recall that the main intent is to visit the strong components in a reverse topological order. The problem is how to order the vertices so that this is true. Recall from the topological sorting algorithm, that in a DAG, finish times occur in reverse topological order (i.e., the first vertex in the topological order is the one with the highest finish time). So, if we wanted to visit the components in reverse topological order, this suggests that we should visit the vertices in increasing order of finish time, starting with the lowest finishing time.

This is a good starting idea, but it turns out that it doesn't work. The reason is that there are many vertices in each strong component, and they all have different finish times. For example, in Figure 8.36, observe that in the first DFS, the lowest finish time (of 4) is achieved by vertex 'c', and its strong component is first, not last, in topological order.

However, there is something to notice about the finish times. If we consider the *maximum finish time* in each component, then these are related to the topological order of the component graph. In fact it is possible to prove the following (but we won't).

Lemma: Consider a digraph G on which DFS has been run. Label each component with the maximum finish time of all the vertices in the component, and sort these in decreasing order. Then this order is a topological order for the component digraph.

For example, in Figure 8.36, the maximum finish times for each component are 18 (for $\{a, b, c\}$), 17 (for $\{d,e\}$), and 12 (for $\{f,g,h,i\}$). The order (18, 17, 12) is a valid topological order for the component digraph. The problem is that this is not what we wanted. We wanted a reverse topological order for the component digraph. So, the final trick is to reverse the digraph. This does not change the component graph, but it reverses the topological order, as desired.

8.5 Minimum Spanning Trees

A common problem is communications networks and circuit design is that of connecting together a set of nodes by a network of total minimum length. The length is the sum of lengths of connecting wires. Consider, for example, laying cable in a city for cable t.v.

The computational problem is called the *minimum spanning tree* (MST) problem. Formally, we are given a connected, undirected graph $G = (V, E)$ Each edge (u, v) has numeric weight of cost. We define the cost of a spanning tree T to be the sum of the costs of edges in the spanning tree

$$w(T) = \sum_{(u,v)\in T} w(u, v)$$

A minimum spanning tree is a tree of minimum weight.

Figures **??**, **??** and **??** show three spanning trees for the same graph. The first is a spanning tree but is not a MST; the other two are.

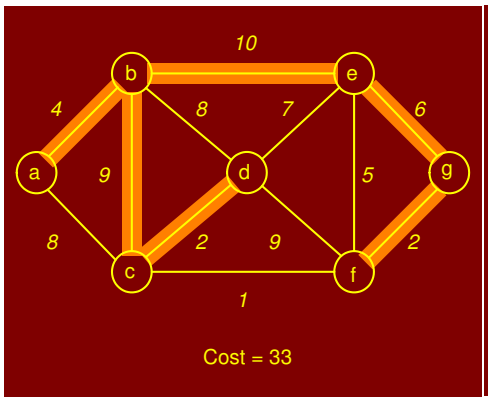


Figure 8.42: A spanning tree that is *not* MST

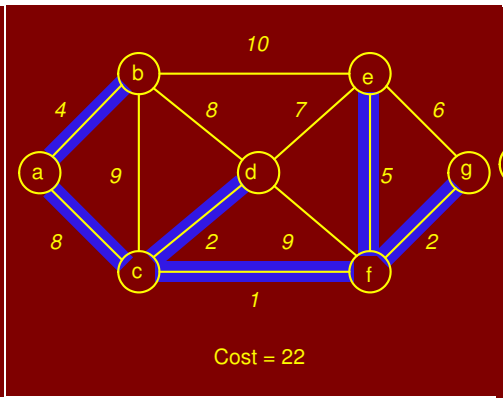


Figure 8.43: A minimum spanning tree

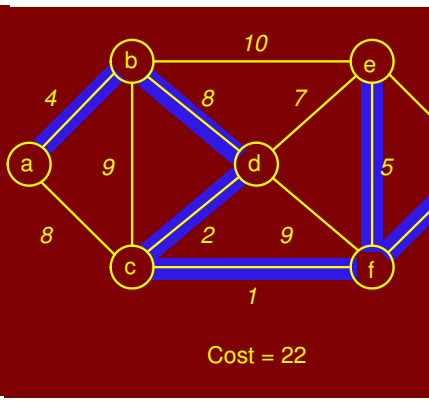


Figure 8.44: Another minimum spanning tree

We will present two *greedy* algorithms (Kruskal's and Prim's) for computing MST. Recall that a greedy algorithm is one that builds a solution by repeatedly selecting the cheapest among all options at each stage. Once the choice is made, it is never undone.

Before presenting the two algorithms, let us review facts about *free trees*. A free tree is a tree with no vertex designated as the root vertex. A free tree with n vertices has exactly $n - 1$ edges. There exists a unique path between any two vertices of a free tree. Adding any edge to a free tree creates a unique cycle. Breaking any edge on this cycle restores the free tree. This is illustrated in Figure 8.45. When the edges (b, e) or (b, d) are added to the free tree, the result is a cycle.

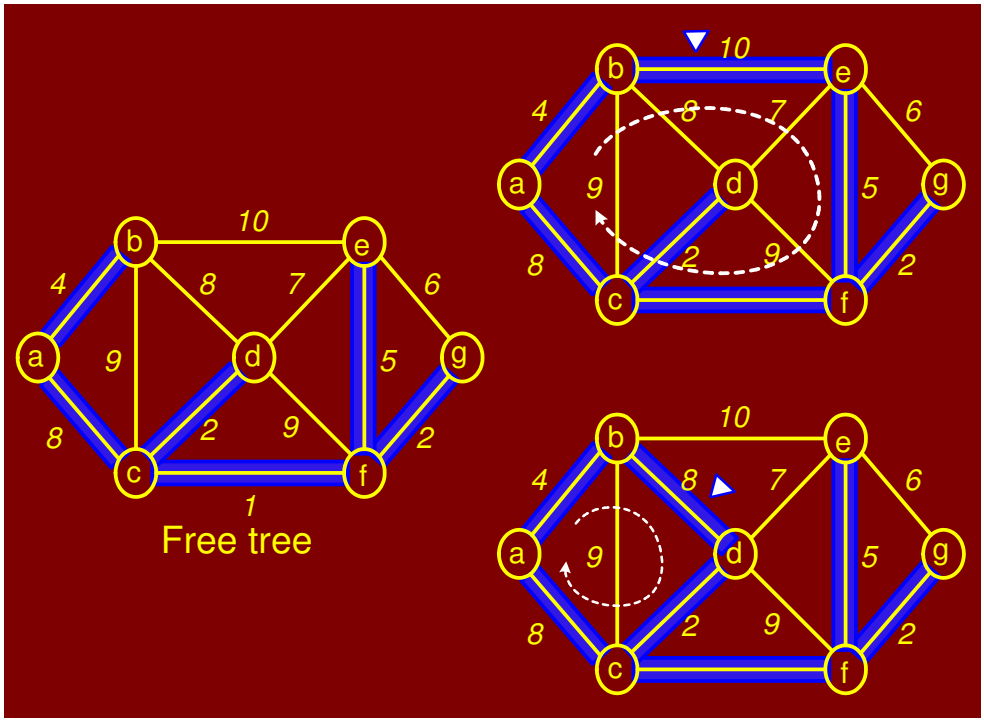


Figure 8.45: Free tree facts

8.5.1 Computing MST: Generic Approach

Let $G = (V, E)$ be an undirected, connected graph whose edges have numeric weights. The intuition behind greedy MST algorithm is simple: we maintain a subset of edges E of the graph . Call this subset A. Initially, A is empty. We will add edges one at a time until A equals the MST.

A subset $A \subseteq E$ is *viable* if A is a subset of edges of *some* MST. An edge $(u, v) \in E - A$ is *safe* if $A \cup \{(u, v)\}$ is viable. In other words, the choice (u, v) is a safe choice to add so that A can still be extended to form a MST.

Note that if A is viable, it cannot contain a cycle. A generic greedy algorithm operates by repeatedly adding any *safe* edge to the current spanning tree.

When is an edge safe? Consider the theoretical issues behind determining whether an edge is safe or not. Let S be a subset of vertices $S \subseteq V$. A *cut* $(S, V - S)$ is just a partition of vertices into two disjoint subsets. An edge (u, v) *crosses* the cut if one endpoint is in S and the other is in $V - S$.

Given a subset of edges A, a cut *respects* A if no edge in A crosses the cut. It is not hard to see why respecting cuts are important to this problem. If we have computed a partial MST and we wish to know which edges can be added that *do not* induce a cycle in the current MST, any edge that crosses a respecting cut is a possible candidate.

8.5.2 Greedy MST

An edge of E is a *light edge* crossing a cut if among all edges crossing the cut, it has the minimum weight. Intuition says that since all the edges that cross a respecting cut do not induce a cycle, then the lightest edge crossing a cut is a natural choice. The main theorem which drives both algorithms is the following:

MST Lemma: Let $G = (V, E)$ be a connected, undirected graph with real-valued weights on the edges. Let A be a viable subset of E (i.e., a subset of some MST). Let $(S, V - S)$ be any cut that respects A and let (u, v) be a light edge crossing the cut. Then the edge (u, v) is *safe* for A. This is illustrated in Figure 8.46.

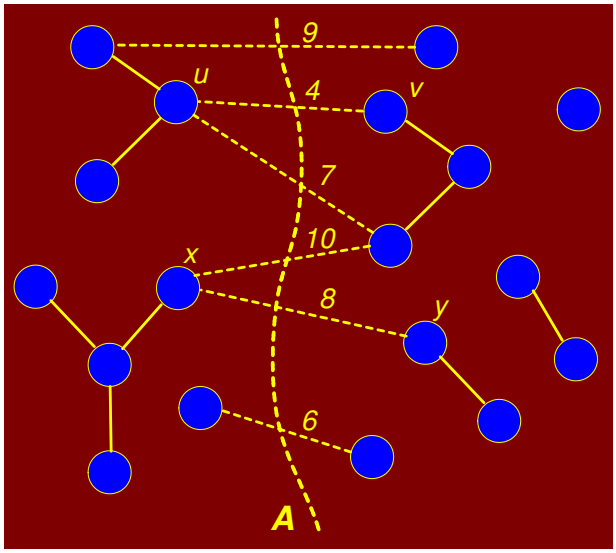


Figure 8.46: Subset A with a cut (wavy line) that respects A

MST Proof: It would simplify the proof if we assume that all edge weights are distinct. Let T be any MST for G. If T contains (u, v) then we are done. This is shown in Figure 8.47 where the lightest edge (u, v) with cost 4 has been chosen.

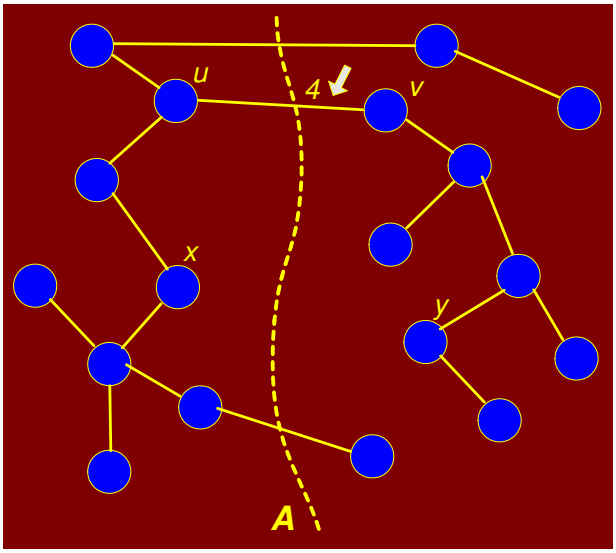


Figure 8.47: MST T which contains light edge (u, v)

Suppose no MST contains (u, v). Such a tree is shown in Figure 8.48. We will derive a contradiction.

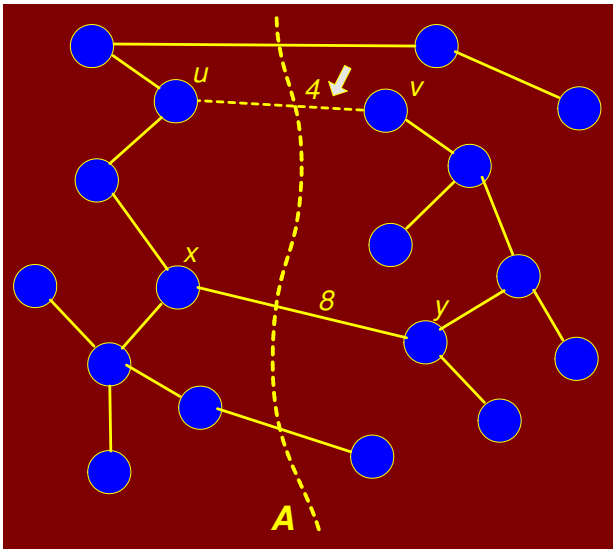


Figure 8.48: MST T which *does not* contains light edge (u, v)

Add (u, v) to T thus creating a cycle as illustrated in Figure 8.49.

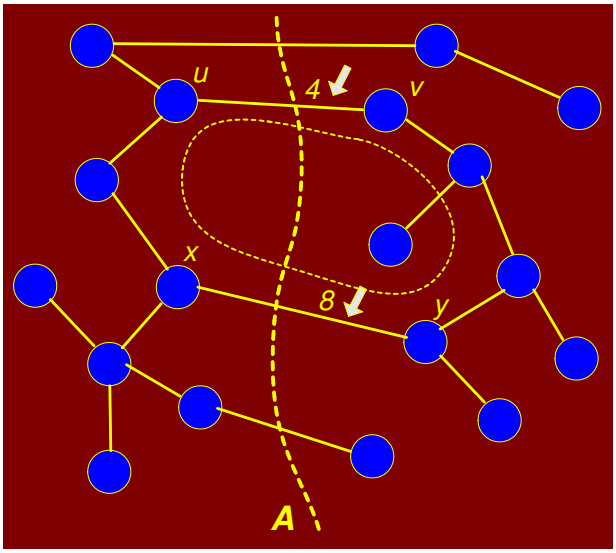


Figure 8.49: Cycle created due to $T + (u, v)$

Since u and v are on opposite sides of the cut, and any cycle must cross the cut an even number of times, there must be at least one other edge (x, y) in T that crosses the cut. The edge (x, y) is not in A because the cut respects A. By removing (x, y) we restore a spanning tree, call it T'. This is shown in Figure 8.50

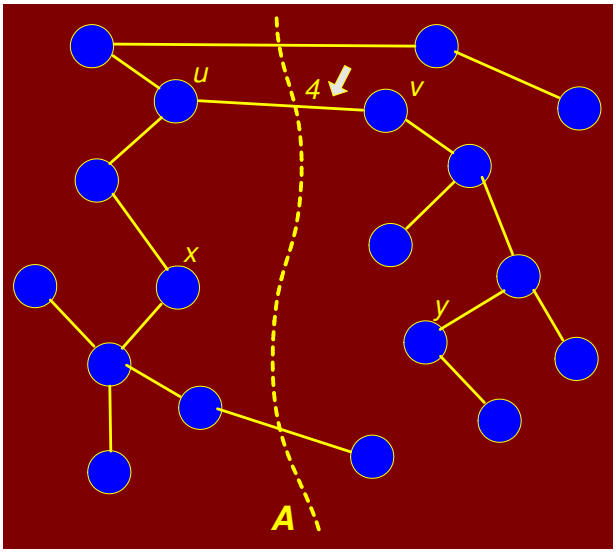


Figure 8.50: Tree $T' = T - (x, y) + (u, v)$

We have $w(T') = w(T) - w(x, y) + w(u, v)$. Since (u, v) is the lightest edge crossing the cut we have $w(u, v) < w(x, y)$. Thus $w(T') < w(T)$ which contradicts the assumption that T was an MST.

8.5.3 Kruskal's Algorithm

Kruskal's algorithm works by adding edges in increasing order of weight (lightest edge first). If the next edge does not induce a cycle among the current set of edges, then it is added to A. If it does, we skip it and consider the next in order. As the algorithm runs, the edges in A induce a *forest* on the vertices. The trees of this forest are eventually merged until a single tree forms containing all vertices.

The tricky part of the algorithm is how to detect whether the addition of an edge will create a cycle in A. Suppose the edge being considered has vertices (u, v). We want a fast test that tells us whether u and v are in the same tree of A. This can be done using the *Union-Find* data structure which supports the following $O(\log n)$ operations:

Create-set(u): Create a set containing a single item u.

Find-set(u): Find the set that contains u

Union(u,v): merge the set containing u and set containing v into a common set.

In Kruskal's algorithm, the vertices will be stored in sets. The vertices in each tree of A will be a set. The edges in A can be stored as a simple list. Here is the algorithm: Figures 8.51 through **??** demonstrate the algorithm applied to a graph.

```
KRUSKAL(G = (V, E))
1  A ← {}
2  for ( each  u ∈ V)
3  do create_set(u)
4  sort E in increasing order by weight w
5  for ( each (u, v) in sorted edge list)
6  do if (find(u) ≠ find(v))
7        then add (u, v) to A
8             union(u, v)
9  return A
```

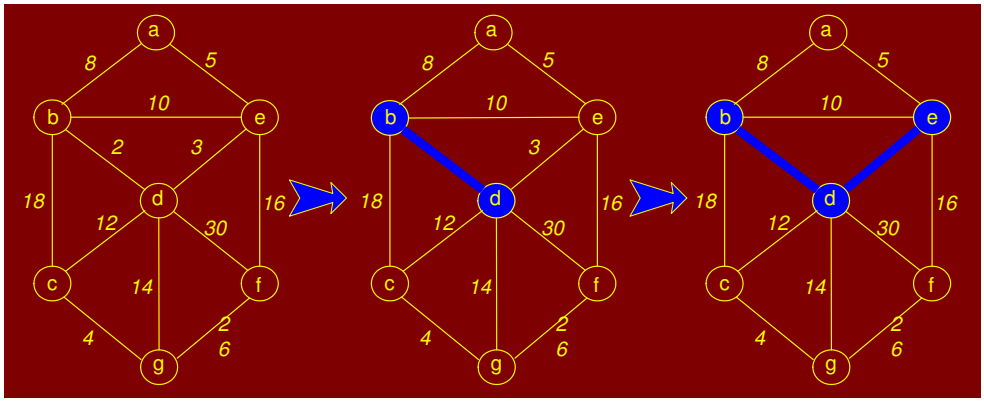


Figure 8.51: Kruskal algorithm: (b, d) and (d, e) added

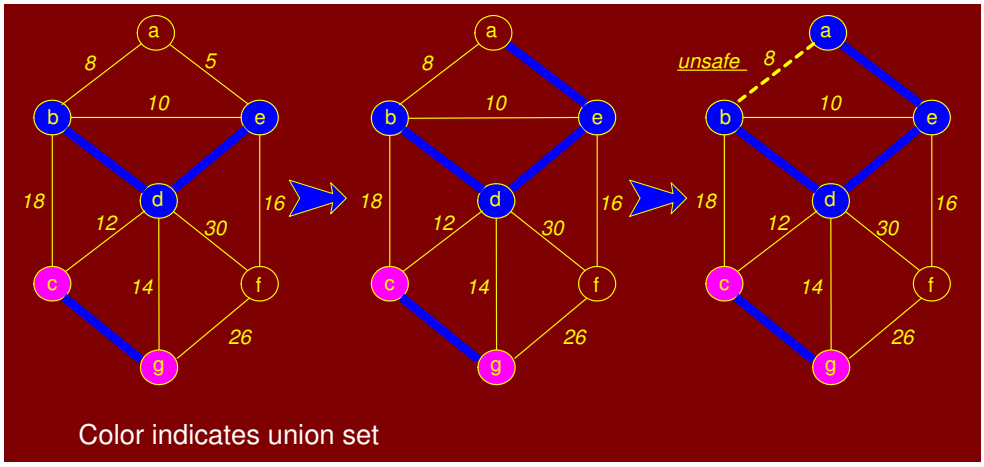


Figure 8.52: Kruskal algorithm: (c, g) and (a, e) added

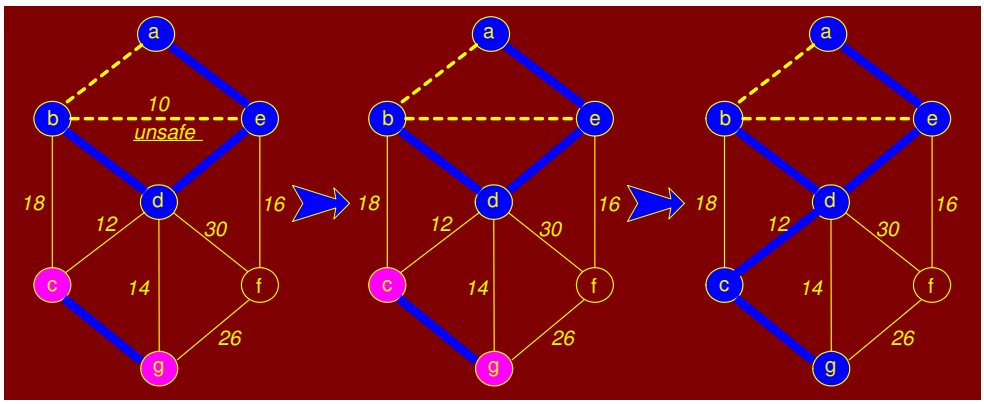


Figure 8.53: Kruskal algorithm: unsafe edges

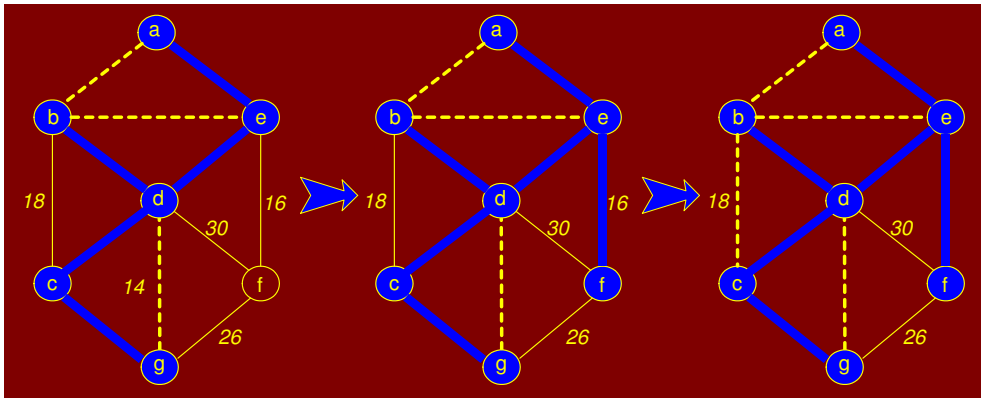


Figure 8.54: Kruskal algorithm: (e, f) added

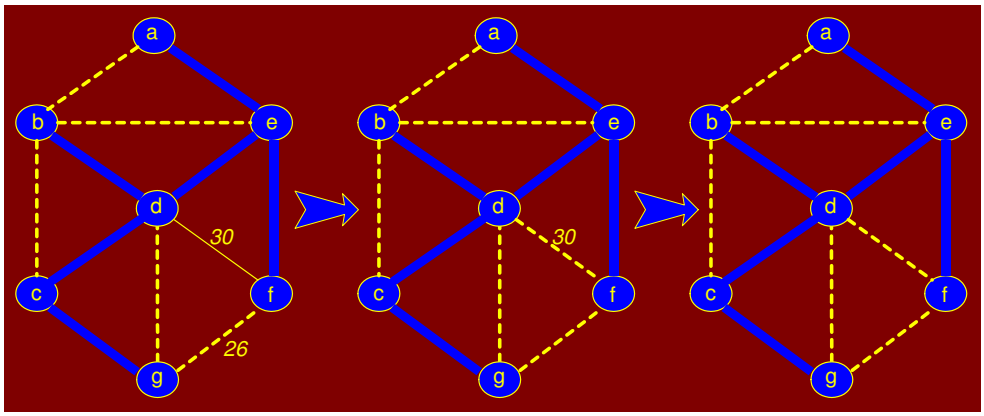


Figure 8.55: Kruskal algorithm: more unsafe edges and final MST

Analysis:

Since the graph is connected, we may assume that $E \geq V - 1$. Sorting edges (*line 4*) takes $\Theta(E \log E)$. The for loop (*line 5*) performs $O(E)$ find and $O(V)$ union operations. Total time for union $-$ find is $O(E\alpha(V))$ where $\alpha(V)$ is the inverse Ackerman function. $\alpha(V) < 4$ for V less the number of atoms in the entire universe. Thus the time is dominated by sorting. Overall time for Kruskal is $\Theta(E \log E) = \Theta(E \log V)$ if the graph is sparse.

8.5.4 Prim's Algorithm

Kruskal's algorithm worked by ordering the edges, and inserting them one by one into the spanning tree, taking care never to introduce a cycle. Intuitively Kruskal's works by merging or splicing two trees together, until all the vertices are in the same tree.

In contrast, Prim's algorithm builds the MST by adding leaves one at a time to the current tree. We start with a root vertex r; it can be any vertex. At any time, the subset of edges A forms a single tree (in Kruskal's, it formed a forest). We look to add a single vertex as a leaf to the tree.

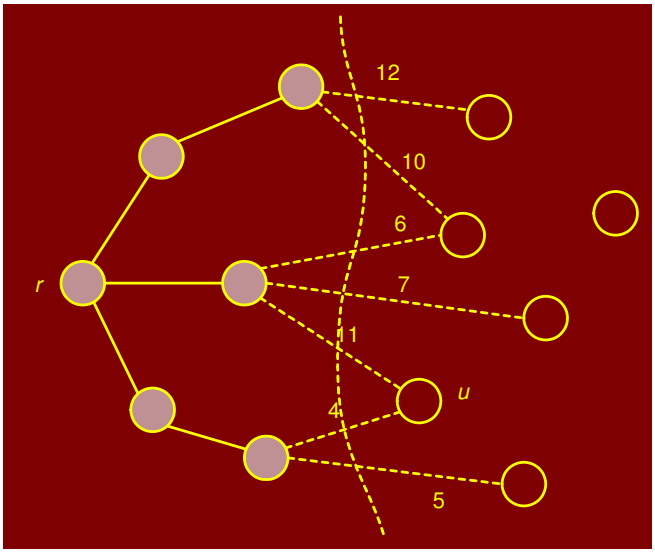


Figure 8.56: Prim's algorithm: a *cut* of the graph

Consider the set of vertices S currently part of the tree and its complement $(V - S)$ as shown in Figure 8.56. We have *cut* of the graph. Which edge should be added next? The greedy strategy would be to add the lightest edge which in the figure is edge to 'u'. Once u is added, Some edges that crossed the cut are no longer crossing it and others that were not crossing the cut are as shown in Figure 8.57

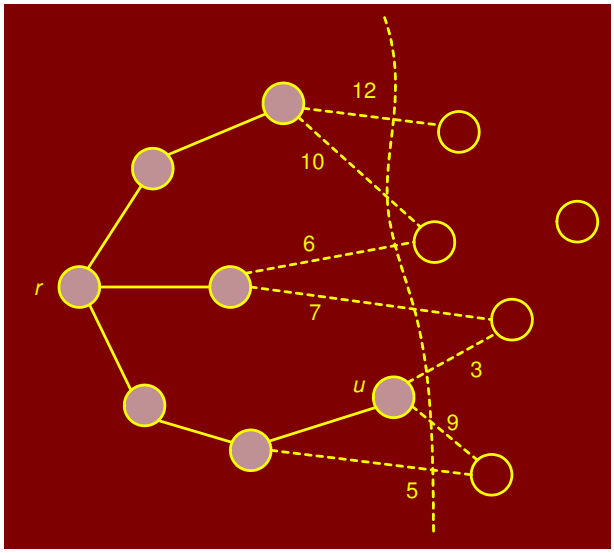


Figure 8.57: Prim's algorithm: u selected

We need an efficient way to update the cut and determine the light edge quickly. To do this, we will make use of a *priority queue*. The question is what do we store in the priority queue? It may seem logical that edges that cross the cut should be stored since we choose light edges from these. Although possible, there is more elegant solution which leads to a simpler algorithm.

For each vertex $u \in (V - S)$ (not part of the current spanning tree), we associate a key $key[u]$. The $key[u]$ is the weight of the lightest edge going from u to any vertex in S. If there is no edge from u to a vertex in S, we set the key value to ∞. We also store in $pred[u]$ the end vertex of this edge in S. We will also need to know which vertices are in S and which are not. To do this, we will assign a color to each vertex. If the color of a vertex is black then it is in S otherwise not. Here is the algorithm:

```
PRIM((G, w, r))
 1  for ( each  u ∈ V)
 2  do  key [u] ← ∞;   pq.insert (u, key[u])
 3      color [u] ←  white
 4   key [r] ← 0;   pred [r] ← nil;  pq.decrease_key (r,  key [r]);
 5  while ( pq.not_empty ())
 6  do u ←  pq.extract_min ()
 7     for ( each  u ∈  adj [u])
 8     do if ( color [v] ==  white )and( w (u, v) <  key [v])
 9           then  key [v] =  w (u, v)
10                 pq.decrease_key (v,  key [v])
11                 pred [v] = u
12      color [u] =  black
```

Figures 8.58 through 8.60 illustrate the algorithm applied to a graph. The contents of the priority queue are shown as the algorithm progresses. The arrows indicate the predecessor pointers and the numeric labels in each vertex is its key value.

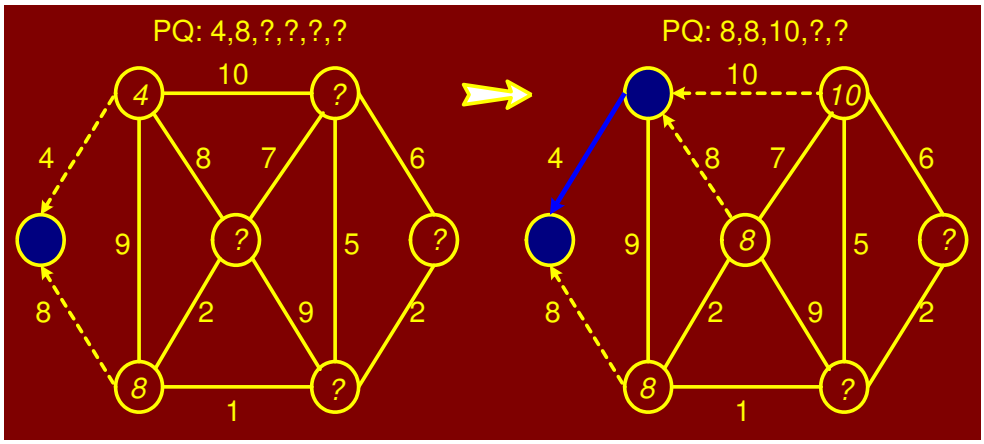


Figure 8.58: Prim's algorithm: edge with weight 4 selected

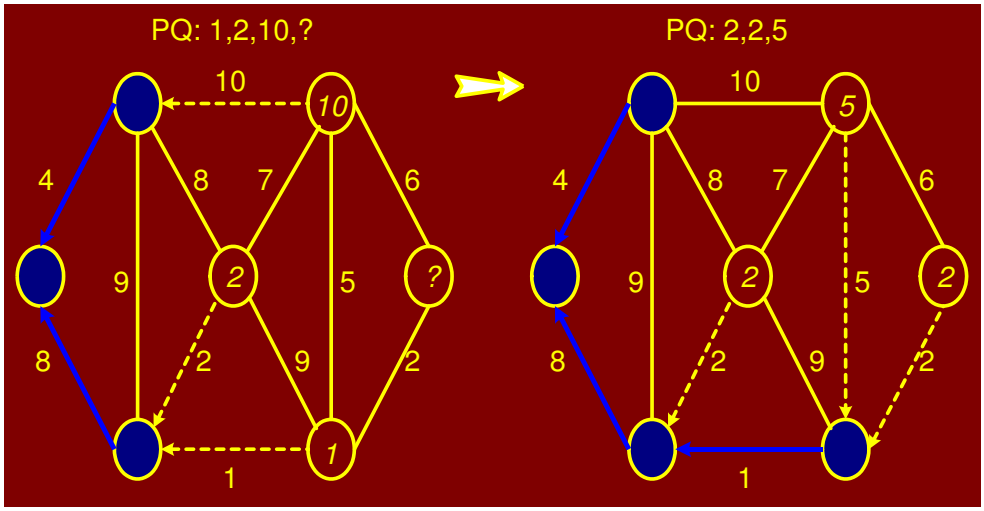


Figure 8.59: Prim's algorithm: edges with weights 8 and 1 selected

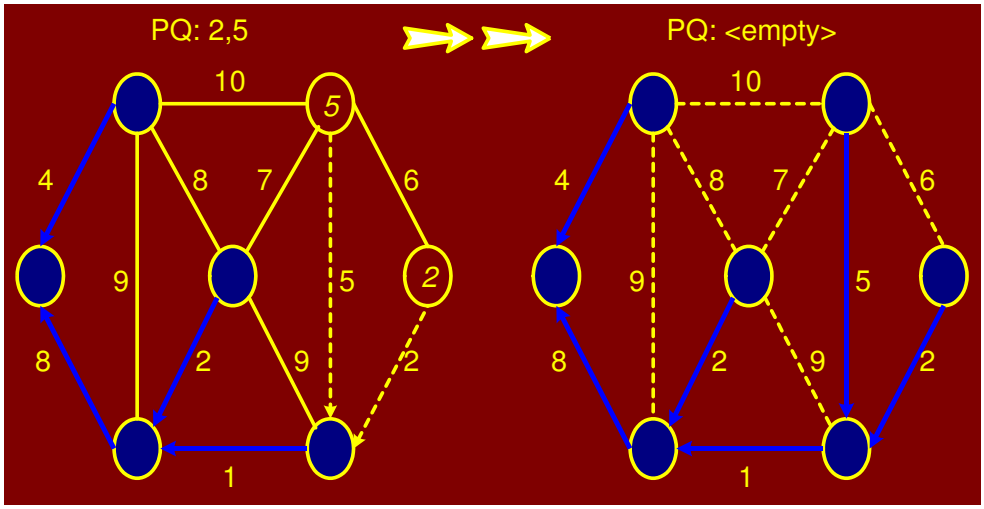


Figure 8.60: Prim's algorithm: the final MST

Analysis:

It takes $O(\log V)$ to extract a vertex from the priority queue. For each incident edge, we spend potentially $O(\log V)$ time decreasing the key of the neighboring vertex. Thus the total time is $O(\log V + \deg(u) \log V)$. The other steps of update are constant time.

So the overall running time is

$$\begin{aligned} T(V,E) &= \sum_{u \in V} (\log V + \deg(u) \log V) \\ &= \log V \sum_{u \in V} (1 + \deg(u)) \\ &= (\log V)(V + 2E) = \Theta((V + E) \log V) \end{aligned}$$

Since G is connected, V is asymptotically no greater than E so this is $\Theta(E \log V)$, same as Kruskal's algorithm.

8.6 Shortest Paths

A motorist wishes to find the shortest possible route between Peshawar and Karachi. Given a road map of Pakistan on which the distance between each pair of adjacent cities is marked Can the motorist determine the shortest route?

In the *shortest-paths problem* We are given a weighted, directed graph $G = (V, E)$ The weight of path $p =< v_0, v_1, \ldots, v_k >$ is the sum of the constituent edges:

$$w(p) = \sum_{i=1}^{k} w(v_{i-1}, v_i)$$

We define the *shortest-path weight* from u to v by

$$\delta(u, v) = \begin{cases} \min\{w(p) : u \overset{p}{\rightsquigarrow} v\} & \text{if there is a path from } u \text{ to } v \\ \infty & \text{otherwise} \end{cases}$$

Problems such as shortest route between cities can be solved efficiently by modelling the road map as a graph. The nodes or vertices represent cities and edges represent roads. Edge weights can be interpreted as distances. Other metrics can also be used, e.g., time, cost, penalties and loss.

Similar scenarios occur in computer networks like the Internet where data packets have to be routed. The vertices are *routers*. Edges are communication links which may be be wire or wireless. Edge weights can be distance, link speed, link capacity link delays, and link utilization.

The breadth-first-search algorithm we discussed earlier is a shortest-path algorithm that works on un-weighted graphs. An un-weighted graph can be considered as a graph in which every edge has weight one unit.

There are a few variants of the shortest path problem. We will cover their definitions and then discuss algorithms for some.

Single-source shortest-path problem: Find shortest paths from a given (single) *source* vertex $s \in V$ to every other vertex $v \in V$ in the graph G.

Single-destination shortest-paths problem: Find a shortest path to a given destination vertex t from each vertex v. We can reduce the this problem to a single-source problem by reversing the direction of each edge in the graph.

Single-pair shortest-path problem: Find a shortest path from u to v for given vertices u and v. If we solve the single-source problem with source vertex u, we solve this problem also. No algorithms for this problem are known to run asymptotically faster than the best single-source algorithms in the worst case.

All-pairs shortest-paths problem: Find a shortest path from u to v for *every pair* of vertices u and v. Although this problem can be solved by running a single-source algorithm once from each vertex, it can usually be solved faster.

8.6.1 Dijkstra's Algorithm

Dijkstra's algorithm is a simple *greedy* algorithm for computing the **single-source shortest-paths** to all other vertices. Dijkstra's algorithm works on a weighted directed graph $G = (V, E)$ in which all edge weights are non-negative, i.e., $w(u, v) \geq 0$ for each edge $(u, v) \in E$.

Negative edges weights maybe counter to intuition but this can occur in real life problems. However, we will *not allow negative cycles* because then there is no shortest path. If there is a negative cycle between, say, s and t, then we can always find a shorter path by going around the cycle one more time.

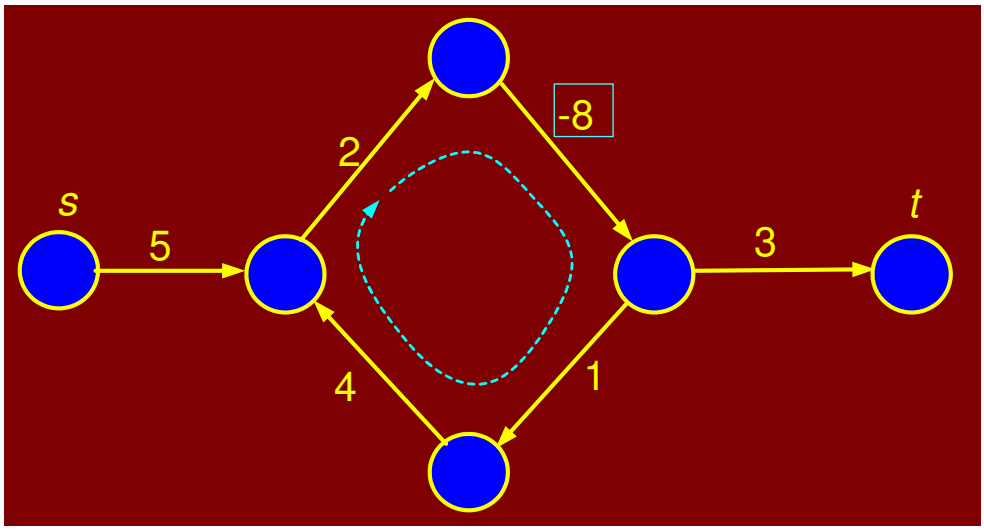


Figure 8.61: Negative weight cycle

The basic structure of Dijkstra's algorithm is to maintain an *estimate* of the shortest path from the source vertex to each vertex in the graph. Call this estimate $d[v]$. Intuitively, $d[v]$ will the length of the shortest path *that the algorithm knows of* from s to v. This value will always be greater than or equal to the true shortest path distance from s to v. I.e., $d[v] \geq \delta(u, v)$. Initially, we know of no paths, so $d[v] = \infty$. Moreover, $d[s] = 0$ for the source vertex.

As the algorithm goes on and sees more and more vertices, it attempts to update $d[v]$ for each vertex in the graph. The process of updating estimates is called *relaxation*. Here is how relaxation works.

Intuitively, if you can see that your solution is not yet reached an optimum value, then push it a little closer to the optimum. In particular, if you discover a path from s to v shorter than $d[v]$, then you need to update $d[v]$. This notion is common to many optimization algorithms.

Consider an edge from a vertex u to v whose weight is $w(u, v)$. Suppose that we have already computed current estimates on $d[u]$ and $d[v]$. We know that there is a path from s to u of weight $d[u]$. By taking

this path and following it with the edge (u,v) we get a path to v of length $d[u] + w(u,v)$. If this path is better than the existing path of length $d[v]$ to v, we should take it. The relaxation process is illustrated in the following figure. We should also remember that the shortest way back to the source is through u by updating the predecessor pointer.

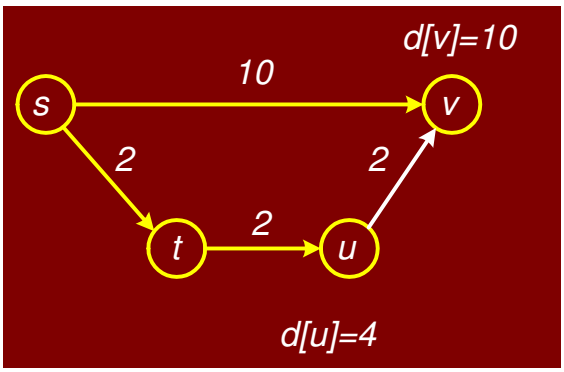


Figure 8.62: Vertex u relaxed

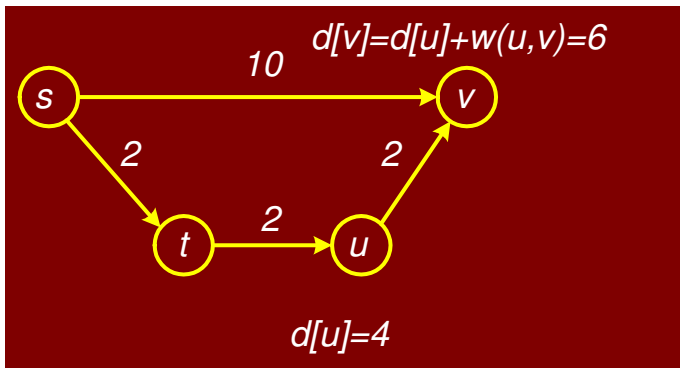


Figure 8.63: Vertex v relaxed

```
RELAX((u,v))
1  if (d[u] + w(u,v) < d[v])
2     then d[v] ← d[u] + w(u,v)
3          pred[v] = u
```

Observe that whenever we set $d[v]$ to a finite value, there is always evidence of a path of that length. Therefore $d[v] \geq \delta(s,v)$. If $d[v] = \delta(s,v)$, then further relaxations cannot change its value.

It is not hard to see that if we perform RELAX(U,V) repeatedly over all edges of the graph, the $d[v]$ values will eventually converge to the final true distance value from s. The cleverness of any shortest path algorithm is to perform the updates in a judicious manner, so the convergence is as fast as possible.

Dijkstra's algorithm is based on the notion of performing repeated relaxations. The algorithm operates by maintaining a subset of vertices, $S \subseteq V$, for which we claim we *know* the true distance, $d[v] = \delta(s,v)$.

Initially $S = \emptyset$, the empty set. We set $d[u] = 0$ and all others to ∞. One by one we select vertices from $V - S$ to add to S.

How do we select which vertex among the vertices of $V - S$ to add next to S? Here is *greediness* comes in. For each vertex $u \in (V - S)$, we have computed a distance estimate $d[u]$.

The greedy thing to do is to take the vertex for which $d[u]$ is minimum, i.e., take the unprocessed vertex that is closest by our estimate to s. Later, we justify why this is the proper choice. In order to perform

this selection efficiently, we store the vertices of $V - S$ in a *priority queue*.

```
DIJKSTRA((G, w, s))
 1  for ( each  u ∈ V)
 2  do d[u] ← ∞
 3      pq.insert (u, d[u])
 4  d[s] ← 0;  pred [s] ← nil;  pq.decrease_key (s, d[s]);
 5  while ( pq.not_empty ())
 6  do u ←  pq.extract_min ()
 7      for ( each  v ∈ adj[u])
 8      do if (d[u] + w(u, v) < d[v])
 9            then d[v] = d[u] + w(u, v)
10                 pq.decrease_key (v, d[v])
11                 pred [v] = u
```

Note the similarity with Prim's algorithm, although a different key is used here. Therefore the running time is the same, i.e., $\Theta(E \log V)$.

Figures 8.64 through **??** demonstrate the algorithm applied to a directed graph with no negative weight edges.

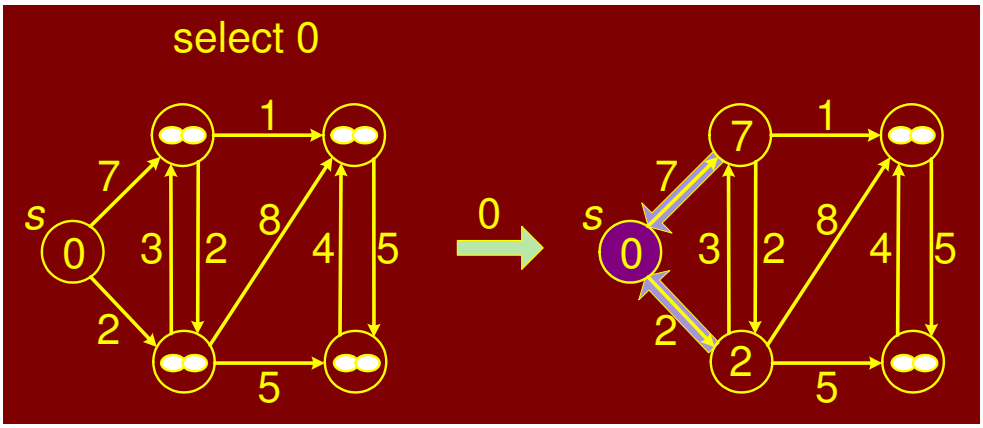


Figure 8.64: Dijkstra's algorithm: select 0

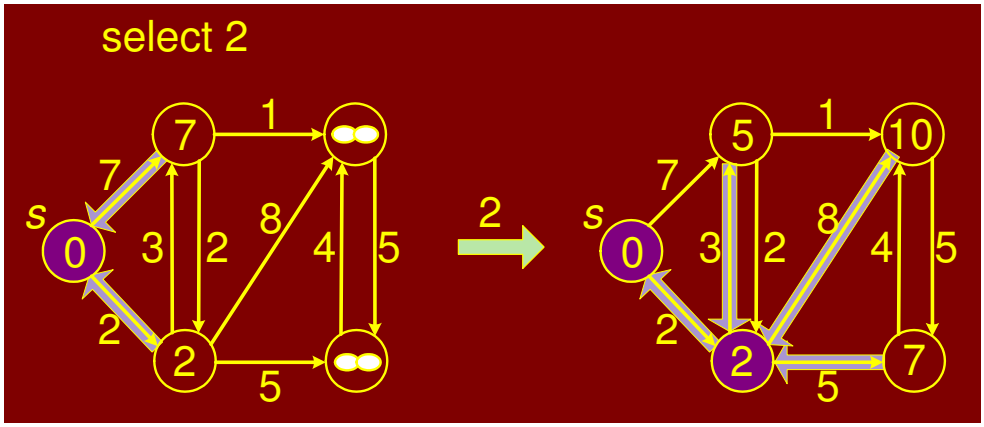


Figure 8.65: Dijkstra's algorithm: select 2

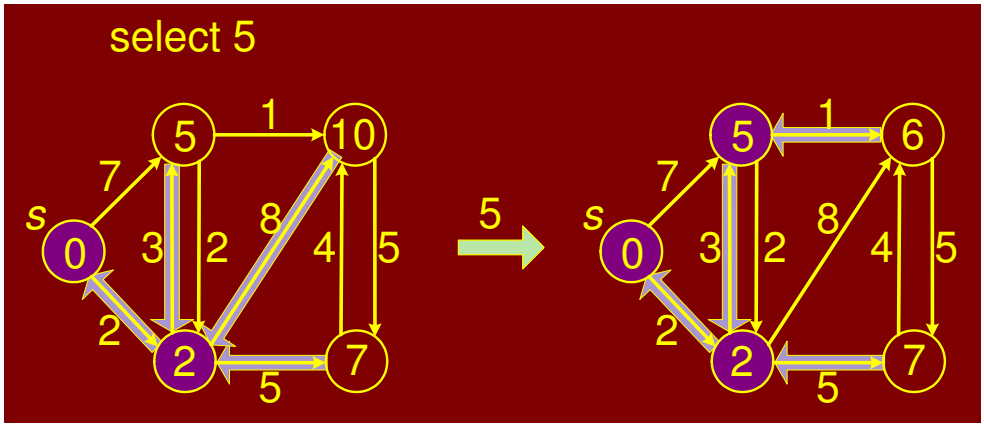


Figure 8.66: Dijkstra's algorithm: select 5

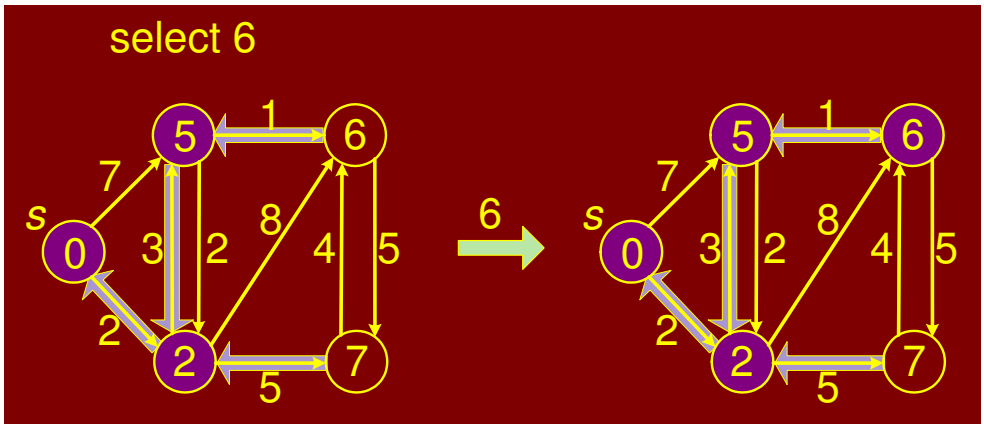


Figure 8.67: Dijkstra's algorithm: select 6

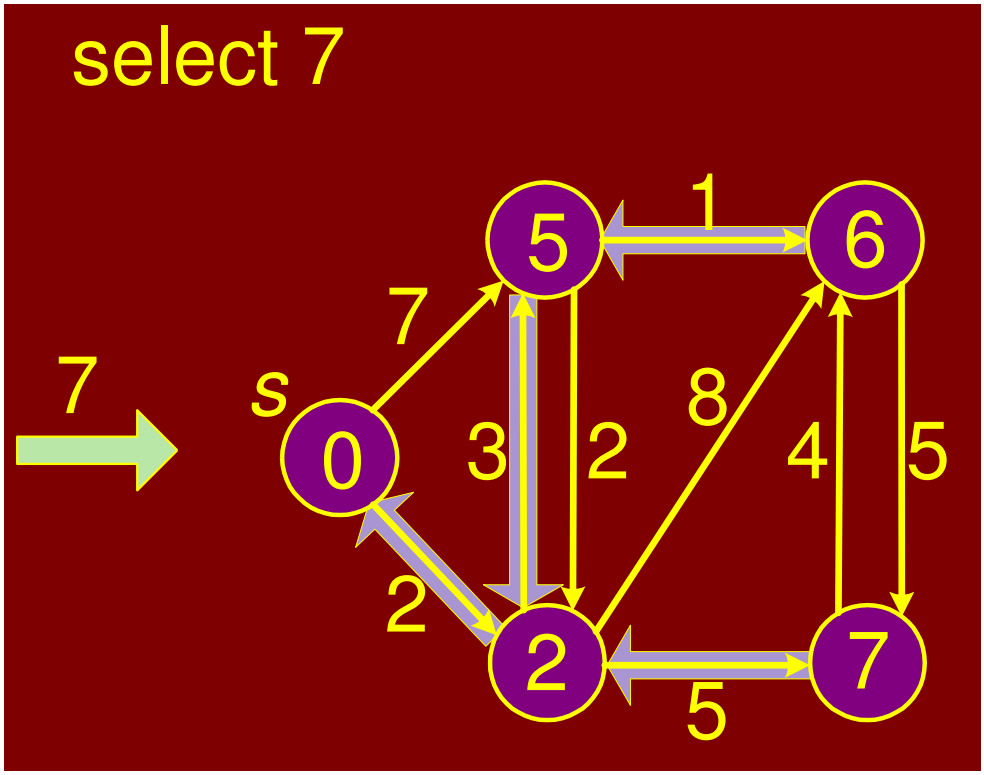


Figure 8.68: Dijkstra's algorithm: select 7

fig:dijlast

8.6.2 Correctness of Dijkstra's Algorithm

We will prove the correctness of Dijkstr's algorithm by Induction. We will use the definition that $\delta(s, v)$ denotes the minimal distance from s to v.

For the base case

1. $S = \{s\}$
2. $d(s) = 0$, which is $\delta(s, s)$

Assume that $d(v) = \delta(s, v)$ for every $v \in S$ and all neighbors of v have been relaxed. If $d(u) \leq d(u')$ for every $u' \in V$ then $d(u) = \delta(s, u)$, and we can transfer u from V to S, after which $d(v) = \delta(s, v)$ for every $v \in S$.

We do this as a proof by contradiction. Suppose that $d(u) > \delta(s, u)$. The shortest path from s to u, $p(s, u)$, must pass through one or more vertices exterior to S. Let x be the last vertex inside S and y be the first vertex outside S on this path to u. Then $p(s, u) = p(s, x) \cup \{(x, y)\} \cup p(y, u)$.

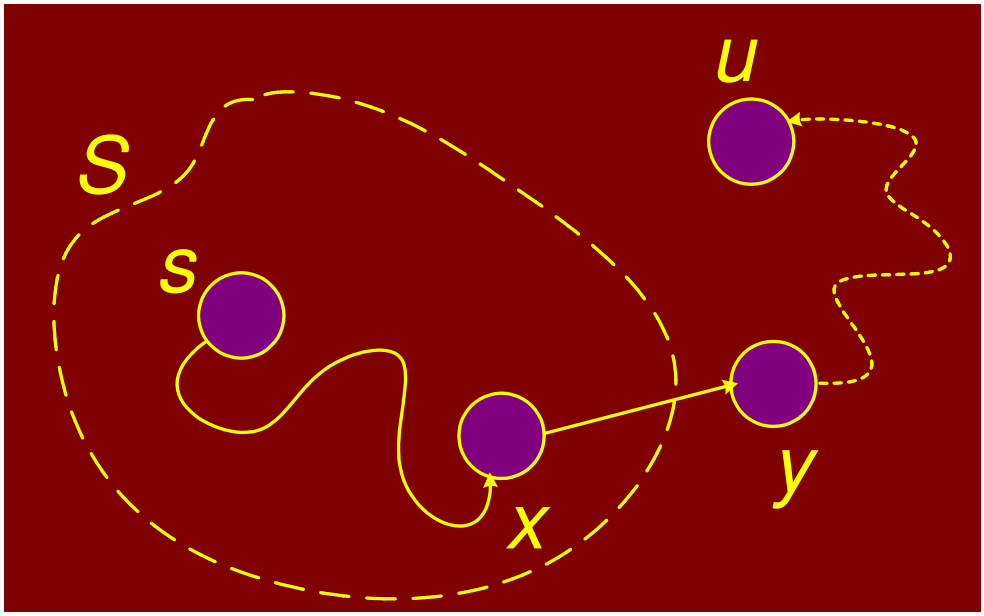


Figure 8.69: Correctness of Dijkstra's algorithm

The length of $p(s,u)$ is $\delta(s,u) = \delta(s,y) + \delta(y,u)$. Because y was relaxed when x was put into S, $d(y) = \delta(s,y)$ by the convergence property. Thus $d(y) \leq \delta(s,u) \leq d(u)$. But, because $d(u)$ is the smallest among vertices not in S, $d(u) \leq d(y)$. The only possibility is $d(u) = d(y)$, which requires $d(u) = \delta(s,u)$ contradicting the assumption.

By the upper bound property, $d(u) \geq \delta(s,u)$. Since $d(u) > \delta(s,u)$ is false, $d(u) = \delta(s,u)$, which is what we wanted to prove. Thus, if we follow the algorithm's procedure, transferring from V to S, the vertex in V with the smallest value of $d(u)$ then all vertices in S have $d(v) = \delta(s,v)$

8.6.3 Bellman-Ford Algorithm

Dijkstra's single-source shortest path algorithm works if all edges weights are non-negative and there are no negative cost cycles. Bellman-Ford *allows* negative weights edges and no negative cost cycles. The algorithm is slower than Dijkstra's, running in $\Theta(VE)$ time.

Like Dijkstra's algorithm, Bellman-Ford is based on performing repeated relaxations. Bellman-Ford applies relaxation to *every edge* of the graph and repeats this $V - 1$ times. Here is the algorithm; its is

illustrated in Figure 8.70.

```
BELLMAN-FORD(G, w, s)
1  for ( each u ∈ V)
2  do d[u] ← ∞
3     pred[u] = nil
4
5  d[s] ← 0;
6  for i = 1 to V − 1
7  do for ( each (u, v) in E )
8     do RELAX(u, v)
```

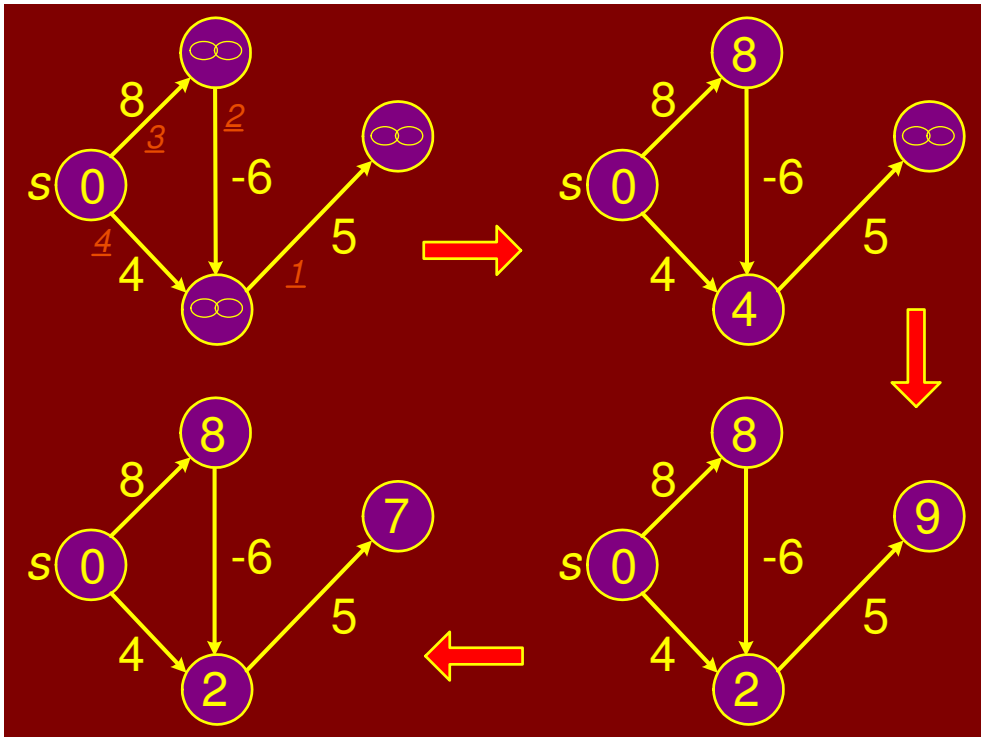


Figure 8.70: The Bellman-Ford algorithm

8.6.4 Correctness of Bellman-Ford

Think of Bellman-Ford as a sort of bubble-sort analog for shortest path. The shortest path information is propagated sequentially along each shortest path in the graph. Consider any shortest path from s to some other vertex u: $\langle v_0, v_1, \ldots, v_k \rangle$ where $v_0 = s$ and $v_k = u$.

Since a shortest path will never visit the same vertex twice, we know that $k \leq V - 1$. Hence the path consists of at most $V - 1$ edges. Since this a shortest path, it is $\delta(s, v_i)$, the true shortest path cost from s

to v_i that satisfies the equation:

$$\delta(s, v_i) = \delta(s, v_{i-1}) + w(v_{i-1}, v_i)$$

Claim: We assert that after the i^{th} pass of the "for-i" loop, $d[v_i] = \delta(s, v_i)$.

Proof: The proof is by induction on i. Observe that after the initialization (pass 0), $d[v_1] = d[s] = 0$.

In general, prior to the i^{th} pass through the loop, the induction hypothesis tells us that $d[v_{i-1}] = \delta(s, v_{i-1})$. After the i^{th} pass, we have done relaxation on the edge (v_{i-1}, v_i) (since we do relaxation along all edges). Thus after the i^{th} pass we have

$$\begin{aligned} d[v_i] &\leq d[v_{i-1}] + w(v_{i-1}, v_i) \\ &= \delta(s, v_{i-1}) + w(v_{i-1}, v_i) \\ &= \delta(s, v_i) \end{aligned}$$

Recall from Dijkstra's algorithm that $d[v_i]$ is never less than $\delta(s, v_i)$. Thus, $d[v_i]$ is in fact equal to $\delta(s, v_i)$. This completes the induction proof.

In summary, after i passes through the for loop, all vertices that are i edges away along the shortest path tree from the source have the correct values stored in $d[u]$. Thus, after the $(V-1)^{st}$ iteration of the for loop, all vertices u have correct distance values stored in $d[u]$.

8.6.5 Floyd-Warshall Algorithm

We consider the generalization of the shortest path problem: to compute the shortest paths between all pairs of vertices. This is called the all-pairs shortest paths problem.

Let $G = (V, E)$ be a directed graph with edge weights. If $(u, v) \in E$ is an edge then $w(u, v)$ denotes its weight. $\delta(u, v)$ is the distance of the minimum cost path between u and v. We will allow G to have negative edges weights but will not allow G to have negative cost cycles. We will present an $\Theta(n^3)$ algorithm for the all pairs shortest path. The algorithm is called the *Floyd-Warshall algorithm* and is based on *dynamic programming*.

We will use an adjacency matrix to represent the digraph. Because the algorithm is matrix based, we will employ the common matrix notation, using i, j and k to denote vertices rather than u, v and w.

The input is an $n \times n$ matrix of edge weights:

$$w_{ij} = \begin{cases} 0 & \text{if } i = j \\ w(i, j) & \text{if } i \neq j \text{ and } (i, j) \in E \\ \infty & \text{if } i \neq j \text{ and } (i, j) \notin E \end{cases}$$

The output will be an $n \times n$ distance matrix $D = d_{ij}$, where $d_{ij} = \delta(i, j)$, the shortest path cost from vertex i to j.

The algorithm dates back to the early 60's. As with other dynamic programming algorithms, the genius of the algorithm is in the clever recursive formulation of the shortest path problem. For a path $p = \langle v_1, v_2, \ldots, v_l$, we say that the vertices $v_2, v_3, \ldots, v_{l-1}$ are the *intermediate vertices* of this path.

Formulation: Define $d_{ij}^{(k)}$ to be the shortest path from i to j such that any intermediate vertices on the path are chosen from the set $\{1, 2, \ldots, k\}$. The path is free to visit any subset of these vertices and in any order. How do we compute $d_{ij}^{(k)}$ assuming we already have the previous matrix $d^{(k-1)}$? There are two basic cases:

1. Don't go through vertex k at all.

2. Do go through vertex k.

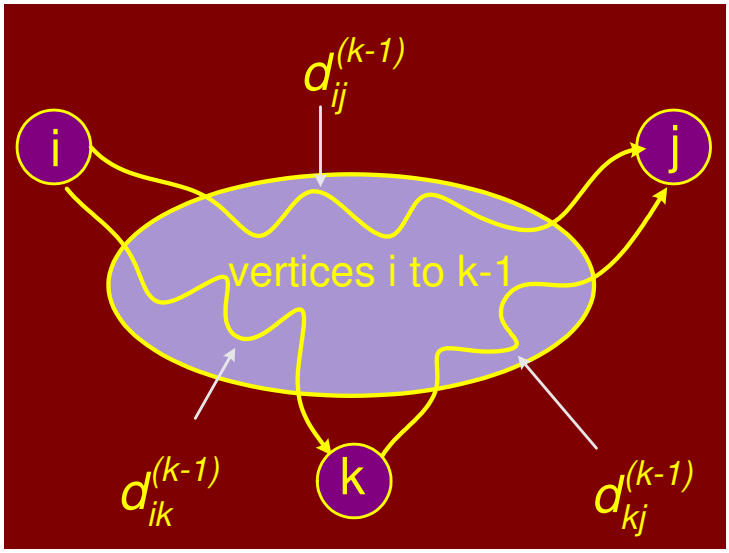


Figure 8.71: Two cases for all-pairs shortest path

Don't go through k at all

Then the shortest path from i to j uses only intermediate vertices $\{1, 2, \ldots, k-1\}$. Hence the length of the shortest is $d_{ij}^{(k-1)}$

Do go through k

First observe that a shortest path does not go through the same vertex twice, so we can assume that we pass through k exactly once. That is, we go from i to k and then from k to j. In order for the overall path to be as short as possible, we should take the shortest path from i to k and the shortest path from k to j. Since each of these paths uses intermediate vertices $\{1, 2, \ldots, k-1\}$, the length of the path is $d_{ik}^{(k-1)} + d_{kj}^{(k-1)}$.

The following illustrate the process in which the value of $d_{3,2}^{k}$ is updated as k goes from 0 to 4.

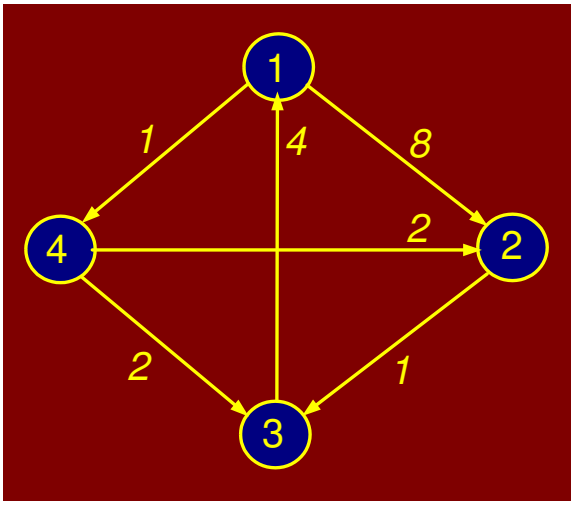


Figure 8.72: $k = 0,\ d^{(0)}_{3,2} = \infty$ (no path)

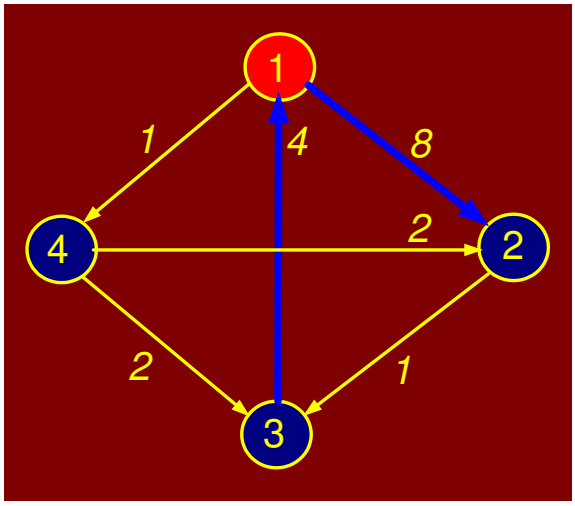


Figure 8.73: $k = 1,\ d^{(1)}_{3,2} = 12\ \ (3 \to 1 \to 2)$

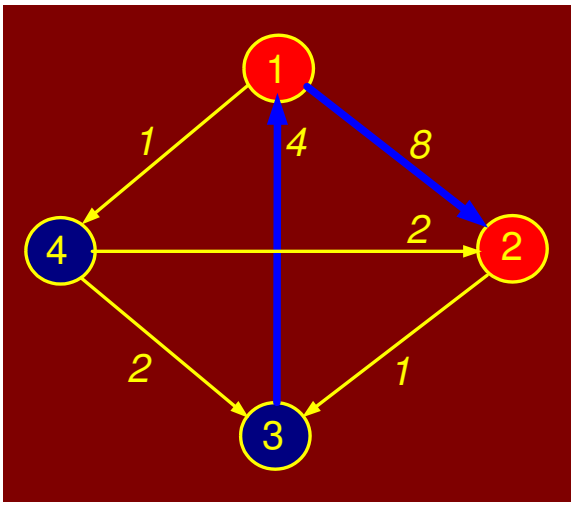


Figure 8.74: $k = 2,\ d^{(2)}_{3,2} = 12\ \ (3 \to 1 \to 2)$

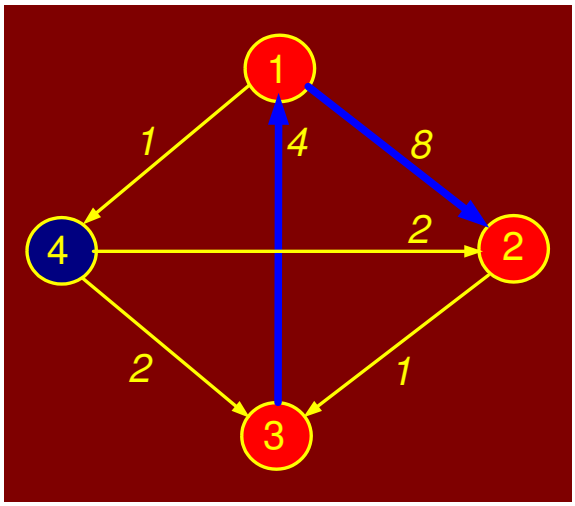


Figure 8.75: $k = 3,\ d^{(3)}_{3,2} = 12\ \ (3 \to 1 \to 2)$

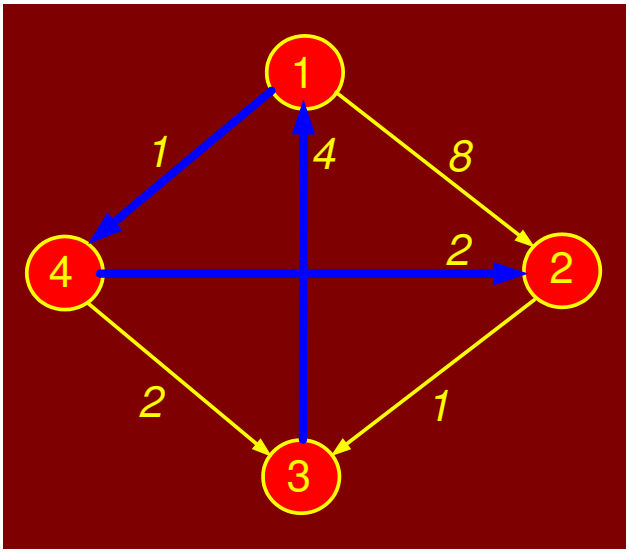


Figure 8.76: $k = 4,\ d^{(4)}_{3,2} = 7\ \ (3 \to 1 \to 4 \to 2)$

This suggests the following recursive (DP) formulation:

$$d^{(0)}_{ij} = w_{ij}$$
$$d^{(k)}_{ij} = \min\left(d^{(k-1)}_{ij}, d^{(k-1)}_{ik} + d^{(k-1)}_{kj}\right)$$

The final answer is $d^{(n)}_{ij}$ because this allows all possible vertices as intermediate vertices.

As is the case with DP algorithms, we will avoid recursive evaluation by generating a table for $d^{(k)}_{ij}$. The algorithm also includes mid-vertex pointers stored in $mid[i, j]$ for extracting the final path.

```
FLOYD-WARSHALL(n, w[1..n, 1..n])
1  for (i = 1, n)
2  do for (j = 1, n)
3      do d[i, j] ← w[i, j]; mid[i, j] ← null
4  for (k = 1, n)
5  do for (i = 1, n)
6      do for (j = 1, n)
7          do if (d[i, k] + d[k, j] < d[i, j])
8                then d[i, j] = d[i, k] + d[k, j]
9                     mid[i, j] = k
```

Clearly, the running time is $\Theta(n^3)$. The space used by the algorithm is $\Theta(n^2)$.

Figure 8.77 through 8.81 demonstrate the algorithm when applied to a graph. The matrix to left of the graph contains the matrix d entries. A circle around an entry $d_{i,k}$ indicates that it was updated in the current k iteration.

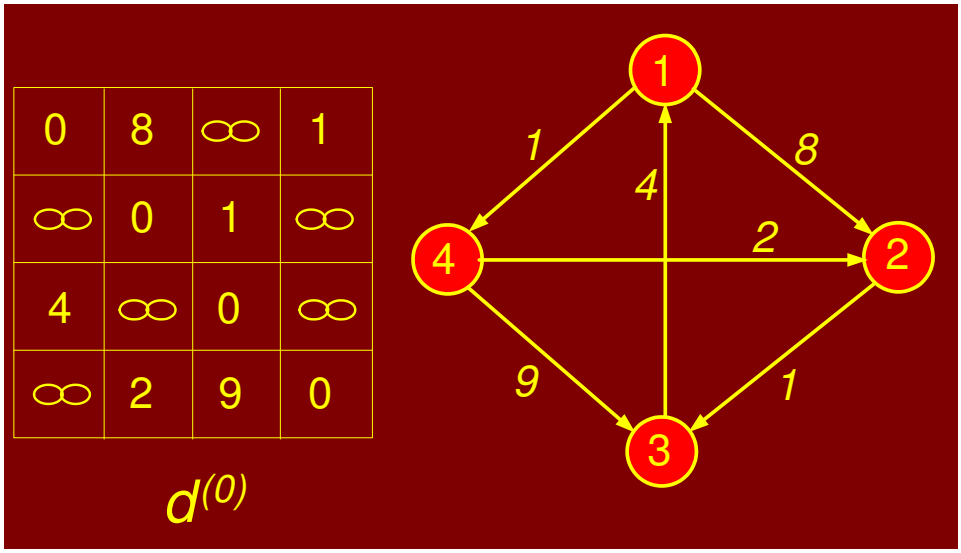


Figure 8.77: Floyd-Warshall Algorithm example: $d^{(0)}$

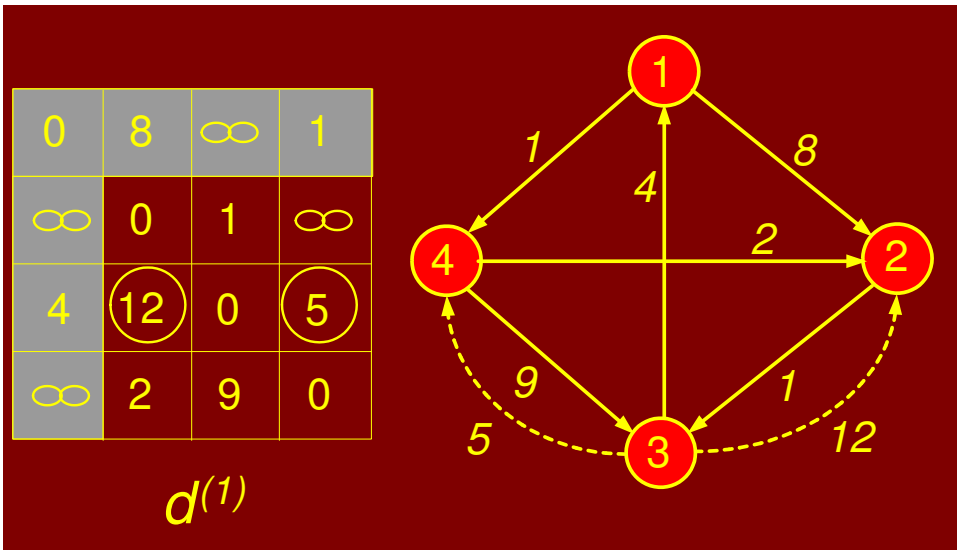


Figure 8.78: Floyd-Warshall Algorithm example: $d^{(1)}$

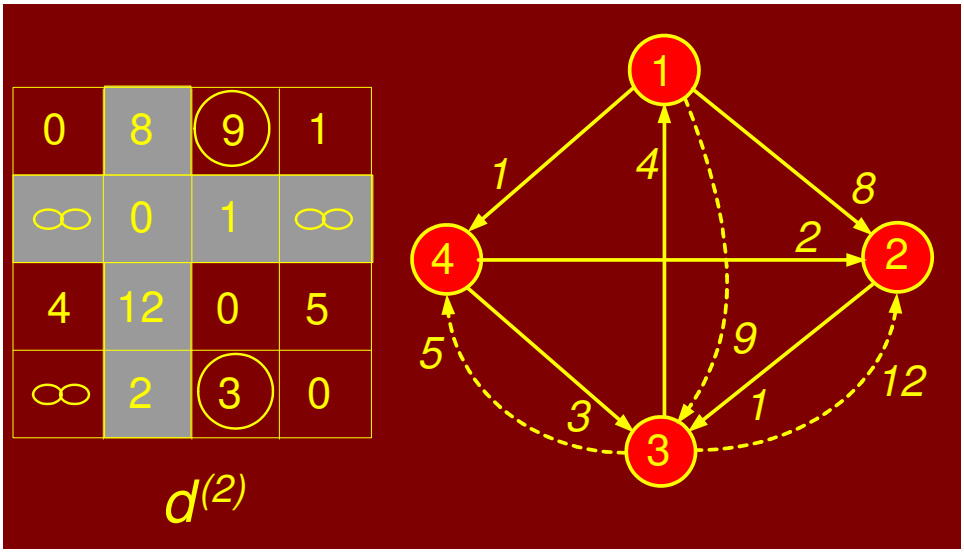


Figure 8.79: Floyd-Warshall Algorithm example: $d^{(2)}$

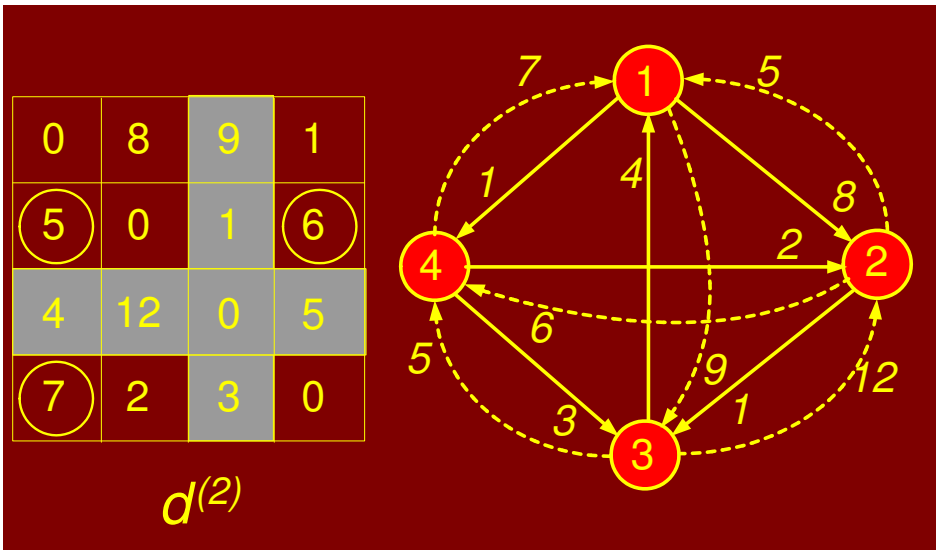


Figure 8.80: Floyd-Warshall Algorithm example: $d^{(3)}$

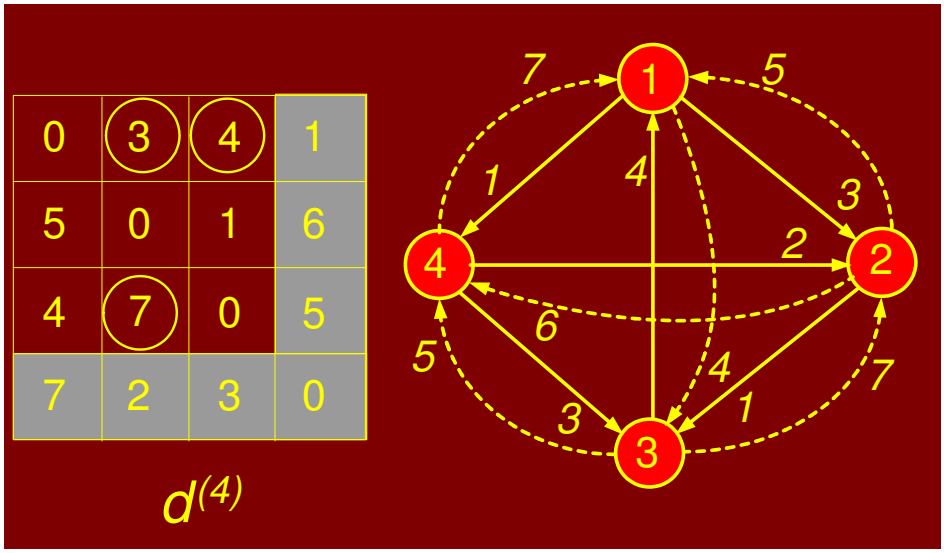


Figure 8.81: Floyd-Warshall Algorithm example: $d^{(4)}$

Extracting Shortest Path:

The matrix d holds the final shortest distance between pairs of vertices. In order to compute the shortest path, the mid-vertex pointers $mid[i,j]$ can be used to extract the final path. Whenever we discovered that the shortest path from i to j passed through vertex k, we set $mid[i,j] = k$. If the shortest path did not pass through k then $mid[i,j] = null$.

To find the shortest path from i to j, we consult $mid[i,j]$. If it is null, then the shortest path is just the edge (i,j). Otherwise we recursively compute the shortest path from i to $mid[i,j]$ and the shortest path from $mid[i,j]$ to j.

```
PATH(i, j)
1  if (mid[i, j] == null)
2     then output(i, j)
3     else PATH(i, mid[i, j])
4          PATH(mid[i, j], j)
```

Chapter 9

Complexity Theory

So far in the course, we have been building up a "bag of tricks" for solving algorithmic problems. Hopefully you have a better idea of how to go about solving such problems. What sort of design paradigm should be used: divide-and-conquer, greedy, dynamic programming.

What sort of data structures might be relevant: trees, heaps, graphs. What is the running time of the algorithm. All of this is fine if it helps you discover an acceptably efficient algorithm to solve your problem.

The question that often arises in practice is that you have tried every trick in the book and nothing seems to work. Although your algorithm can solve small problems reasonably efficiently (e.g., $n \leq 20$), for the really large problems you want to solve, your algorithm never terminates. When you analyze its running time, you realize that it is running in exponential time, perhaps $n^{\sqrt{n}}$, or 2^n, or 2^{2^n}, or $n!$ or worse!.

By the end of 60's, there was great success in finding efficient solutions to many combinatorial problems. But there was also a growing list of problems for which there seemed to be no known efficient algorithmic solutions.

People began to wonder whether there was some unknown paradigm that would lead to a solution to there problems. Or perhaps some proof that these problems are inherently hard to solve and no algorithmic solutions exist that run under exponential time.

Near the end of the 1960's, a remarkable discovery was made. Many of these hard problems were interrelated in the sense that if you could solve any one of them in polynomial time, then you could solve all of them in polynomial time. this discovery gave rise to the notion of NP-completeness.

This area is a radical departure from what we have been doing because the emphasis will change. The goal is no longer to prove that a problem *can* be solved efficiently by presenting an algorithm for it. Instead we will be trying to show that a problem *cannot* be solved efficiently.

Up until now all algorithms we have seen had the property that their worst-case running time are bounded above by some *polynomial* in n. A *polynomial time algorithm* is any algorithm that runs in $O(n^k)$ time. A problem is solvable in polynomial time if there is a polynomial time algorithm for it.

Some functions that do not look like polynomials (such as $O(n \log n)$ are bounded above by polynomials (such as $O(n^2)$). Some functions that do look like polynomials are not. For example, suppose you have

an algorithm that takes as input a graph of size n and an integer k and run in $O(n^k)$ time.

Is this a polynomial time algorithm? No, because k is an input to the problem so the user is allowed to choose $k = n$, implying that the running time would be $O(n^n)$. $O(n^n)$ is surely not a polynomial in n. The important aspect is that the exponent must be a constant independent of n.

9.1 Decision Problems

Most of the problems we have discussed involve optimization of one form of another. Find the shortest path, find the minimum cost spanning tree, maximize the knapsack value. For rather technical reasons, the NP-complete problems we will discuss will be phrased as *decision problems*.

A problem is called a *decision problem* if its output is a simple "yes" or "no" (or you may this of this as true/false, 0/1, accept/reject.) We will phrase may optimization problems as decision problems. For example, the MST decision problem would be: Given a weighted graph G and an integer k, does G have a spanning tree whose weight is at most k?

This may seem like a less interesting formulation of the problem. It does not ask for the weight of the minimum spanning tree, and it does not even ask for the edges of the spanning tree that achieves this weight. However, our job will be to show that certain problems cannot be solved efficiently. If we show that the simple decision problem cannot be solved e¡ciently, then the more general optimization problem certainly cannot be solved efficiently either.

9.2 Complexity Classes

Before giving all the technical definitions, let us say a bit about what the general classes look like at an intuitive level.

Class P: This is the set of all decision problems that can be *solved* in polynomial time. We will generally refer to these problems as being "easy" or "efficiently solvable".

Class NP: This is the set of all decision problems that can be *verified* in polynomial time. This class contains P as a subset. It also contains a number of problems that are believed to be very " hard" to solve.

Class NP: The term "NP" does not mean "not polynomial". Originally, the term meant " non-deterministic polynomial" but it is a bit more intuitive to explain the concept from the perspective of verification.

Class NP-hard: In spite of its name, to say that a problem is NP-hard does not mean that it is hard to solve. Rather, it means that if we could solve this problem in polynomial time, then we could solve *all* NP problems in polynomial time. Note that for a problem to NP-hard, it does not have to be in the class NP.

Class NP-complete: A problem is NP-complete if (1) it is in NP and (2) it is NP-hard.

The Figure 9.1 illustrates one way that the sets P, NP, NP-hard, and NP-complete (NPC) might look. We say might because we do not know whether all of these complexity classes are distinct or whether they are all solvable in polynomial time. The Graph Isomorphism, which asks whether two graphs are identical up to a renaming of their vertices. It is known that this problem is in NP, but it is not known to be in P. The other is QBF, which stands for Quantified Boolean Formulas. In this problem you are given a boolean formula and you want to know whether the formula is true or false.

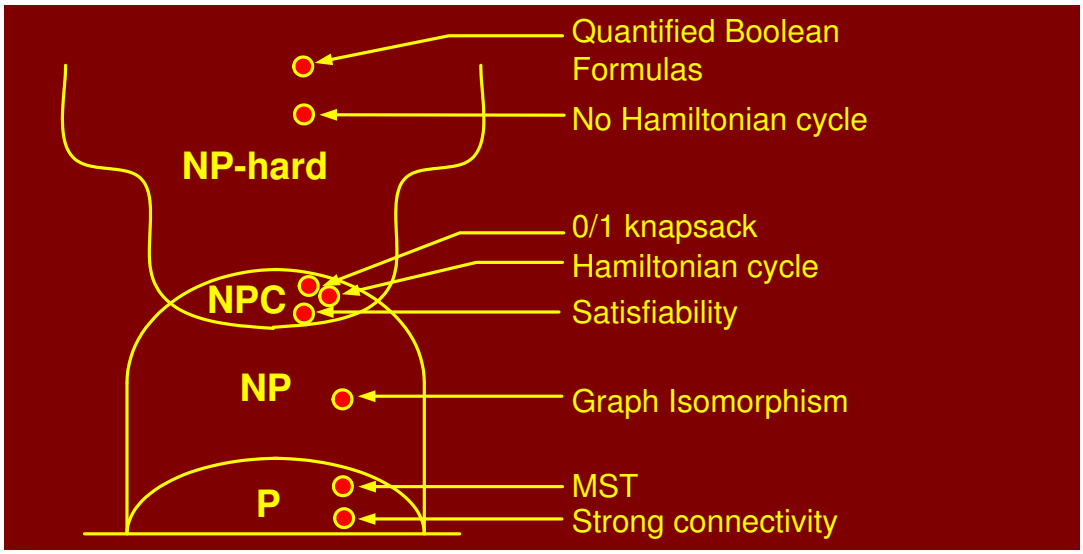


Figure 9.1: Complexity Classes

9.3 Polynomial Time Verification

Before talking about the class of NP-complete problems, it is important to introduce the notion of a *verification algorithm*. Many problems are hard to solve but they have the property that it easy to verify the solution if one is provided. Consider the Hamiltonian cycle problem.

Given an undirected graph G, does G have a cycle that visits every vertex exactly once? There is no known polynomial time algorithm for this problem.

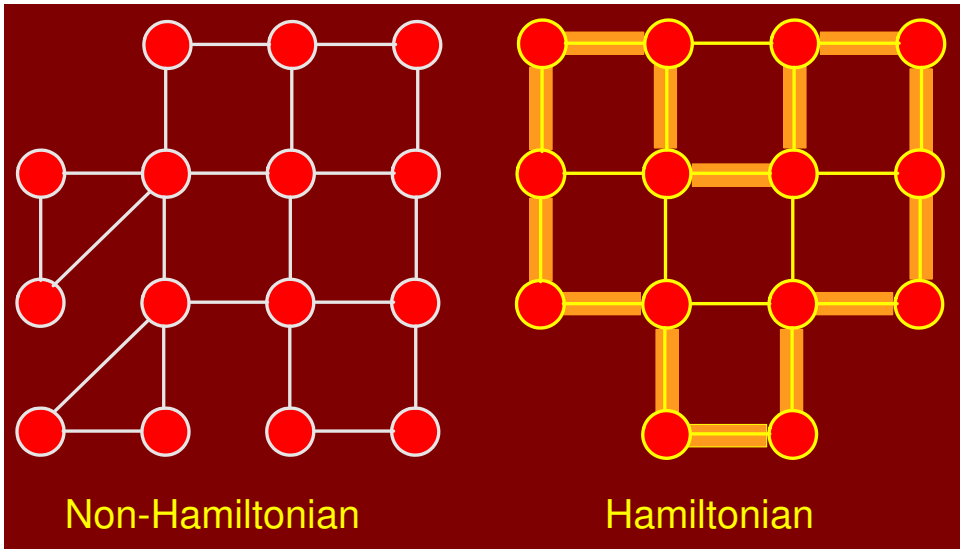


Figure 9.2: Hamiltonian Cycle

However, suppose that a graph did have a Hamiltonian cycle. It would be easy for someone to convince of this. They would simply say: "the cycle is $\langle v_3, v_7, v_1, \ldots, v_13\rangle$ We could then inspect the graph and check that this is indeed a legal cycle and that it visits all of the vertices of the graph exactly once. Thus, even though we know of no efficient way to *solve* the Hamiltonian cycle problem, there is a very efficient way to *verify* that a a given cycle is indeed a Hamiltonian cycle.

The piece of information that allows verification is called a *certificate*. Note that not all problems have the property that they are easy to verify. For example, consider the following two:

1. $\mathrm{UHC} = \{(G)|G \text{ has a unique Hamiltonian cycle}\}$
2. $\overline{\mathrm{HC}} = \{(G)|G \text{ has no Hamiltonian cycle}\}$

Suppose that a graph G is in UHC. What information would someone give us that would allow us to verify this? They could give us an example of the unique Hamiltonian cycle and we could verify that it is a Hamiltonian cycle. But what sort of certificate could they give us to convince us that this is the *only* one?

They could give another cycle that is *not* Hamiltonian. But this does not mean that there is not another cycle somewhere that is Hamiltonian. They could try to list every other cycle of length n, but this is not efficient at all since there are $n!$ possible cycles in general. Thus it is hard to imagine that someone could give us some information that would allow us to efficiently verify that the graph is in UHC.

9.4 The Class NP

The class NP is a set of all problems that can be verified by a polynomial time algorithm. Why is the set called "NP" and not "VP"? The original term NP stood for *non-deterministic polynomial time*. This

referred to a program running on a non-deterministic computer that can make guesses. Such a computer could non-deterministically guess the value of the certificate. and then verify it in polynomial time. We have avoided introducing non-determinism here; it is covered in other courses such as automata or complexity theory.

Observe that $\mathsf{P} \subseteq \mathsf{NP}$. In other words, if we can solve a problem in polynomial time, we can certainly verify the solution in polynomial time. More formally, we do not need to see a certificate to solve the problem; we can solve it in polynomial time anyway.

However, it is not known whether $\mathsf{P} = \mathsf{NP}$. It seems unreasonable to think that this should be so. Being able to verify that you have a correct solution does not help you in finding the actual solution. The belief is that $\mathsf{P} \neq \mathsf{NP}$ but no one has a proof for this.

9.5 Reductions

The class NP-complete (NPC) problems consists of a set of decision problems (a subset of class NP) that no one knows how to solve efficiently. But if there were a polynomial solution for even a single NP-complete problem, then ever problem in NPC will be solvable in polynomial time. For this, we need the concept of *reductions*.

Consider the question: Suppose there are two problems, A and B. You know (or you strongly believe at least) that it is impossible to solve problem A in polynomial time. You want to prove that B cannot be solved in polynomial time. We want to show that

$$(\mathsf{A} \notin \mathsf{P}) \Rightarrow (\mathsf{B} \notin \mathsf{P})$$

How would you do this? Consider an example to illustrate reduction: The following problem is well-known to be NPC:

3-color: Given a graph G, can each of its vertices be labelled with one of 3 different colors such that two adjacent vertices have the same label (color).

Coloring arises in various partitioning problems where there is a constraint that two objects cannot be assigned to the same set of partitions. The term “coloring” comes from the original application which was in map drawing. Two countries that share a common border should be colored with different colors.

It is well known that planar graphs can be colored (maps) with *four colors*. There exists a polynomial time algorithm for this. But determining whether this can be done with 3 colors is hard and there is no polynomial time algorithm for it. In Figure 9.3, the graph on the left can be colored with 3 colors while the graph on the right cannot be colored.

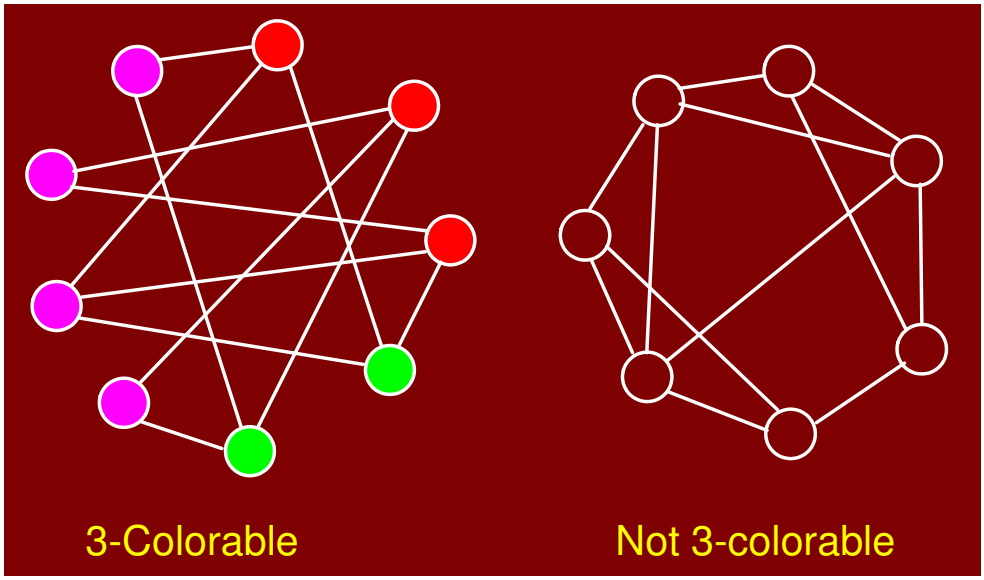


Figure 9.3: Examples of 3-colorable and non-3-colorable graphs

Example1: Fish tank problem

Consider the following problem than can be solved with the graph coloring approach. A tropical fish hobbyist has six different types of fish designated by A, B, C, D, E, and F, respectively. Because of predator-prey relationships, water conditions and size, some fish can be kept in the same tank. The following table shows which fish cannot be together:

Type	Cannot be with
A	B, C
B	A ,C, E
C	A, B, D, E
D	C, F
E	B, C, F
F	D, E

These constraints can be displayed as a graph where an edge between two vertices exists if the two species cannot be together. This is shown in Figure 9.4. For example, A cannot be with B and C; there is an edge between A and B and between A and C.

Given these constraints, What is the *smallest* number of tanks needed to keep all the fish? The answer can be found by coloring the vertices in the graph such that no two adjacent vertices have the same color. This particular graph is 3-colorable and therefore, 3 fish tanks are enough. This is depicted in Figure 9.5. The 3 fish tanks will hold fish as follows:

Tank 1	Tank 2	Tank 3
A, D	F, C	B, E

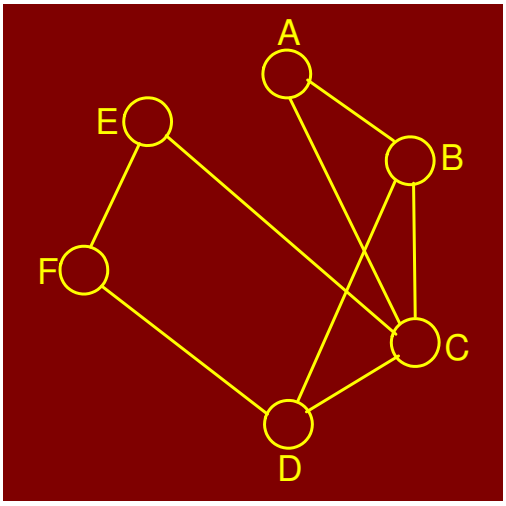


Figure 9.4: Graph representing constraints between fish species

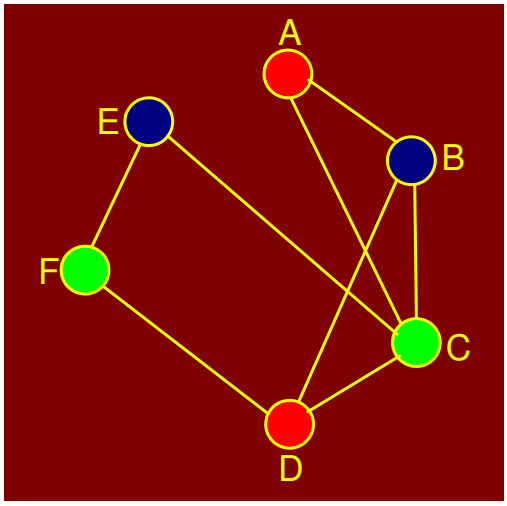


Figure 9.5: Fish tank graph colored with 3 colors

The 3-color (3Col) problem will the play the role of A, which we strongly suspect to not be solvable in polynomial time. For our problem B, consider the following problem: Given a graph $G = (V, E)$, we say that a subset of vertices $V' \subseteq V$ forms a *clique* if for every pair of vertices $u, v \in V'$, the edge $(u, v) \in V'$ That is, the subgraph induced by V' is a complete graph.

Clique Cover: Given a graph G and an integer k, can we find k subsets of vertices $V_1, V_2, \ldots, V_k$, such that $\bigcup_i V_i = V$, and that each V_i is a clique of G.

The following figure shows a graph that has a clique cover of size 3. There are three subgraphs that are complete.

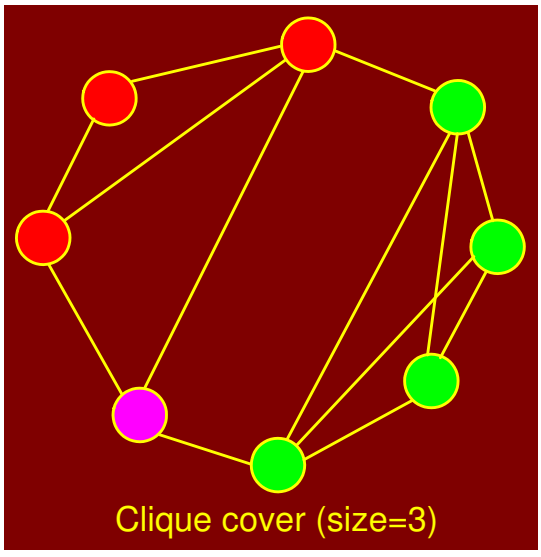


Figure 9.6: Graph with clique cover of size 3

The clique cover problem arises in applications of clustering. We put an edge between two nodes if they are similar enough to be clustered in the same group. We want to know whether it is possible to cluster all the vertices into k groups.

Suppose that you want to solve the CCov problem. But after a while of fruitless effort, you still cannot find a polynomial time algorithm for the CCov problem. How can you prove that CCov is likely to not have a polynomial time solution?

You know that 3Col is NP-complete and hence, experts believe that $3\text{Col} \notin P$. You feel that there is some connection between the CCov problem and the 3Col problem. Thus, you want to show that

$$(3\text{Col} \notin P) \Rightarrow (\text{CCov} \notin P)$$

Both problems involve partitioning the vertices into groups. In the clique cover problem, for two vertices to be in the same group, they must be adjacent to each other. In the 3-coloring problem, for two vertices to be in the same color group, they must not be adjacent. In some sense, the problems are almost the same but the adjacency requirements are exactly reversed.

We claim that we can reduce the 3-coloring problem into the clique cover problem as follows: Given a graph G for which we want to determine its 3-colorability, output the pair $(\overline{G}, 3)$ where $\overline{G}$ denotes the complement of G. Feed the pair $(\overline{G}, 3)$ into a routine for clique cover.

For example, the graph G in Figure 9.7 is 3-colorable and its complement $(\overline{G}, 3)$ is coverable by 3 cliques. The graph G in Figure 9.8 is not 3-colorable; it is also not coverable by cliques.

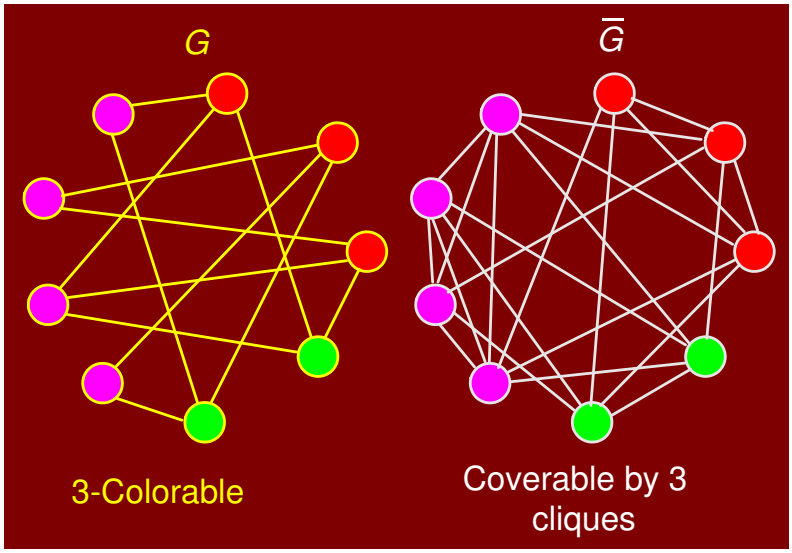


Figure 9.7: 3-colorable G and clique coverable $(\overline{G}, 3)$

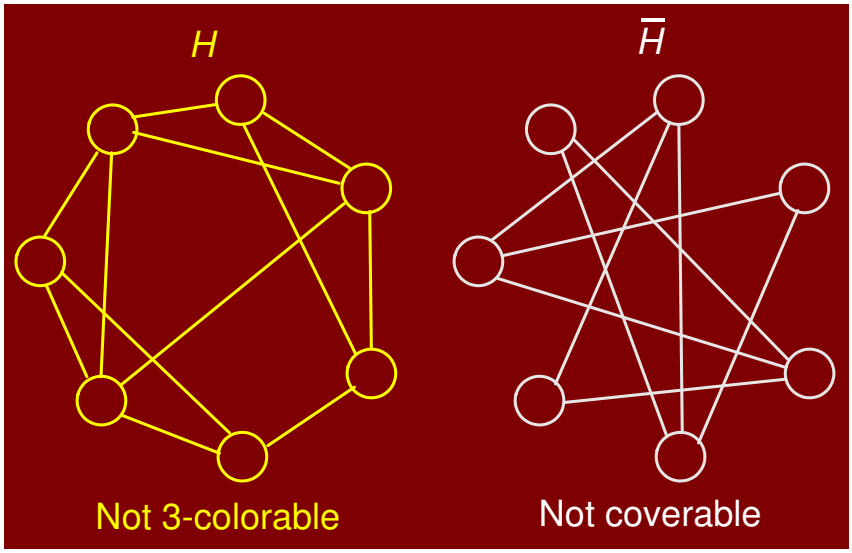


Figure 9.8: G is not 3-colorable and $(\overline{G}, 3)$ is not clique coverable

9.6 Polynomial Time Reduction

Definition: We say that a decision problem L_1 is polynomial-time reducible to decision problem L_2 (written $L_1 \leq_p L_2$) if there is polynomial time computable function f such that for all x, $x \in L_1$ if and only if $f(x) \in L_2$.

In the previous example we showed that

$$3\text{Col} \leq_P \text{CCov}$$

In particular, we have $f(G) = (\overline{G}, 3)$. It is easy to complement a graph in $O(n^2)$ (i.e., polynomial time). For example, flip the 0's and 1's in the adjacency matrix.

The way this is used in NP-completeness is that we have strong evidence that L_1 is not solvable in polynomial time. Hence, the reduction is effectively equivalent to saying that "since L_1 is not likely to be solvable in polynomial time, then L_2 is also not likely to be solvable in polynomial time.

9.7 NP-Completeness

The set of NP-complete problems is all problems in the complexity class NP for which it is known that if any one is solvable in polynomial time, then they all are. Conversely, if any one is not solvable in polynomial time, then none are.

Definition: A decision problem L is NP-Hard if

$$L' \leq_P L \quad \text{for all } L' \in \text{NP}.$$

Definition: L is NP-complete if

1. $L \in$ NP and
2. $L' \leq_P L$ for some known NP-complete problem L'.

Given this formal definition, the complexity classes are:

P: is the set of decision problems that are solvable in polynomial time.

NP: is the set of decision problems that can be verified in polynomial time.

NP-Hard: L is NP-hard if for all $L' \in$ NP, $L' \leq_P L$. Thus if we could solve L in polynomial time, we could solve all NP problems in polynomial time.

NP-Complete L is NP-complete if

1. $L \in$ NP and
2. L is NP-hard.

The importance of NP-complete problems should now be clear. If any NP-complete problem is solvable in polynomial time, then every NP-complete problem is also solvable in polynomial time. Conversely, if we can prove that any NP-complete problem cannot be solved in polynomial time, the every NP-complete problem cannot be solvable in polynomial time.

9.8 Boolean Satisfiability Problem: Cook's Theorem

We need to have at least one NP-complete problem to start the ball rolling. Stephen Cook showed that such a problem existed. He proved that the *boolean satisfiability problem* is NP-complete. A boolean formula is a logical formulation which consists of variables x_i. These variables appear in a logical expression using logical operations

1. negation of x: $\overline{x}$
2. boolean or: $(x \vee y)$
3. boolean and: $(x \wedge y)$

For a problem to be in NP, it must have an efficient verification procedure. Thus virtually all NP problems can be stated in the form, "does there exists X such that $P(X)$", where X is some structure (e.g. a set, a path, a partition, an assignment, etc.) and $P(X)$ is some property that X must satisfy (e.g. the set of objects must fill the knapsack, or the path must visit every vertex, or you may use at most k colors and no two adjacent vertices can have the same color). In showing that such a problem is in NP, the certificate consists of giving X, and the verification involves testing that $P(X)$ holds.

In general, any set X can be described by choosing a set of objects, which in turn can be described as choosing the values of some boolean variables. Similarly, the property $P(X)$ that you need to satisfy, can be described as a boolean formula. Stephen Cook was looking for the most general possible property he could, since this should represent the hardest problem in NP to solve. He reasoned that computers (which represent the most general type of computational devices known) could be described entirely in terms of boolean circuits, and hence in terms of boolean formulas. If any problem were hard to solve, it would be one in which X is an assignment of boolean values (true/false, 0/1) and $P(X)$ could be any boolean formula. This suggests the following problem, called the *boolean satisfiability problem*.

SAT: Given a boolean formula, is there some way to assign truth values (0/1, true/false) to the variables of the formula, so that the formula evaluates to true?

A boolean formula is a logical formula which consists of variables x_i , and the logical operations $\overline{x}$ meaning the *negation* of x, *boolean-or* $(x \vee y)$ and *boolean-and* $(x \wedge y)$. Given a boolean formula, we say that it is satisfiable if there is a way to assign truth values (0 or 1) to the variables such that the final result is 1. (As opposed to the case where no matter how you assign truth values the result is always 0.) For example

$$(x_1 \wedge (x_2 \vee \overline{x_3})) \wedge ((\overline{x_2} \wedge \overline{x_3}) \vee \overline{x_1})$$

is satisfiable, by the assignment $x_1 = 1$, $x_2 = 0$ and $x_3 = 0$. On the other hand,

$$(\overline{x_1} \vee (x_2 \wedge x_3)) \wedge (x_1 \vee (\overline{x_2} \wedge \overline{x_3})) \wedge (x_2 \vee x_3) \wedge (\overline{x_2} \vee \overline{x_3})$$

is not satisfiable. Such a boolean formula can be represented by a logical circuit made up of OR, AND and NOT gates. For example, Figure 9.9 shows the circuit for the boolean formula

$$((x_1 \wedge x_4) \vee x_2) \wedge ((x_3 \wedge \overline{x_4}) \vee \overline{x_2}) \wedge \overline{x_5}$$

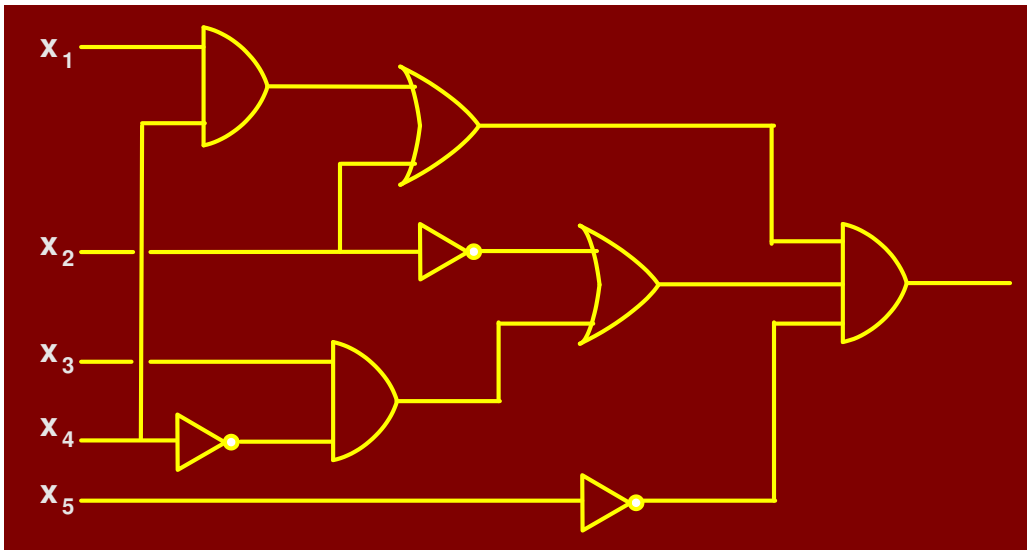


Figure 9.9: Logical circuit for a boolean formula

Cook's Theorem: SAT is NP-complete

We will not prove the theorem; it is quite complicated. In fact, it turns out that a even more restricted version of the satisfiability problem is NP-complete.

A *literal* is a variable x or its negation $\overline{x}$. A boolean formula is in *3-Conjunctive Normal Form* (3-CNF) if it is the boolean-and of clauses where each clause is the boolean-or of exactly three literals. For example,

$$(x_1 \vee x_2 \vee \overline{x_3}) \wedge (\overline{x_1} \vee x_3 \vee x_4) \wedge (x_2 \vee \overline{x_3} \vee \overline{x_4})$$

is in 3-CNF form. 3SAT is the problem of determining whether a formula is 3-CNF is satisfiable. 3SAT is NP-complete. We can use this fact to prove that other problems are NP-complete. We will do this with the *independent set problem*.

Independent Set Problem: Given an undirected graph $G = (V, E)$ and an integer k, does G contain a subset V' of k vertices such that no two vertices in V' are adjacent to each other.

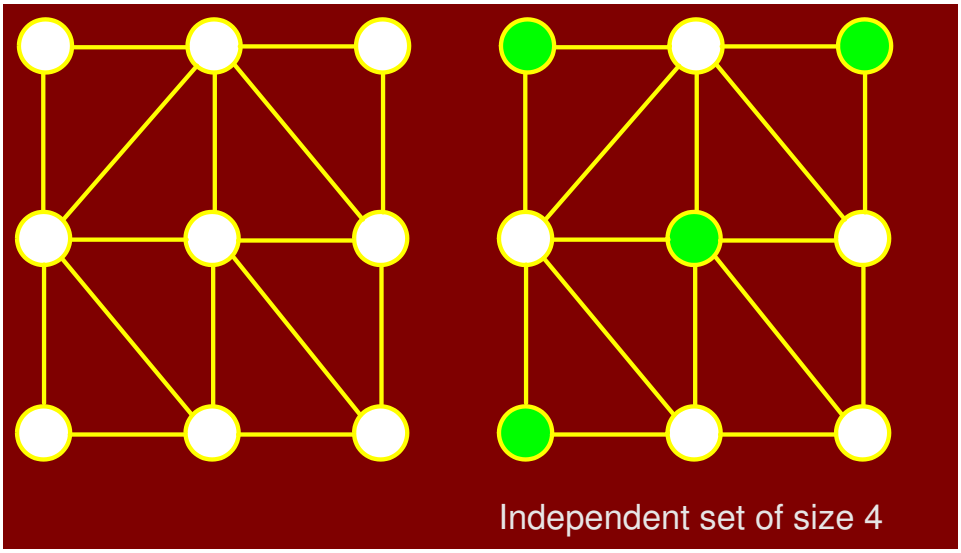


Figure 9.10:

The independent set problem arises when there is some sort of selection problem where there are mutual restrictions pairs that cannot both be selected. For example, a company dinner where an employee and his or her immediate supervisor cannot both be invited.

Claim: IS is NP-complete

The proof involves two parts. First, we need to show that IS $\in$ NP. The certificate consists of k vertices of V'. We simply verify that for each pair of vertices $u, v \in V'$, there is no edge between them. Clearly, this can be done in polynomial time, by an inspection of the adjacency matrix.

Second, we need to establish that IS is NP-hard This can be done by showing that some known NP-compete (3SAT) is polynomial-time reducible to IS. That is, 3SAT $\leq_P$ IS.

An important aspect to reductions is that we do not attempt to solve the satisfiability problem. Remember: It is NP-complete, and there is not likely to be any polynomial time solution. The idea is to translate the similar elements of the satisfiable problem to corresponding elements of the independent set problem.

What is to be selected?

3SAT: Which variables are to be assigned the value true, or equivalently, which literals will be true and which will be false.

IS: Which vertices will be placed in V'.

Requirements:

3SAT: Each clause must contain at least one true valued literal.

IS: V' must contain at least k vertices.

Restrictions:

3SAT: If x_i is assigned true, then $\overline{x_i}$ must be false and vice versa.

IS: If u is selected to be in V' and v is a neighbor of u then v cannot be in V'.

We want a function which given any 3-CNF boolean formula F, converts it into a pair (G, k) such that the above elements are translated properly. Our strategy will be to turn each literal into a vertex. The vertices will be in clause clusters of three, one for each clause. Selecting a true literal from some clause will correspond to selecting a vertex to add to V'. We will set k equal to the number of clauses, to force the independent set subroutine to select one true literal from each clause. To keep the IS subroutine from selecting two literals from one clause and none from some other, we will connect all the vertices in each clause cluster with edges. To keep the IS subroutine from selecting a literal and its complement to be true, we will put an edge between each literal and its complement.

A formal description of the reduction is given below. The input is a boolean formula F in 3-CNF, and the output is a graph G and integer k.

3SAT-TO-IS(F)
1 $k \leftarrow$ number of clauses in F
2 **for** (each clause C in F)
3 **do** create a clause cluster of
4 3 vertices from literals of C
5 **for** (each clause cluster (x_1, x_2, x_3))
6 **do** create an edge (x_i, x_j) between
7 all pairs of vertices in the cluster
8 **for** (each vertex x_i)
9 **do** create an edge between x_i and
10 all its complement vertices $\overline{x_i}$

11 **return** (G, k) *// output is graph* G *and integer* k

If F has k clauses, then G has exactly 3k vertices. Given any reasonable encoding of F , it is an easy programming exercise to create G (say as an adjacency matrix) in polynomial time. We claim that F is satisfiable if and only if G has an independent set of size k.

Example: Suppose that we are given the 3-CNF formula:

$$(x_1 \vee \overline{x_2} \vee \overline{x_3}) \wedge (\overline{x_1} \vee x_2 \vee x_3) \wedge (\overline{x_1} \vee x_2 \vee \overline{x_3}) \wedge (x_1 \vee \overline{x_2} \vee x_3)$$

The following series of figures show the reduction which produces the graph and sets k = 4. First, each of the four literals is converted into a three-vertices graph. This is shown in Figure 9.11

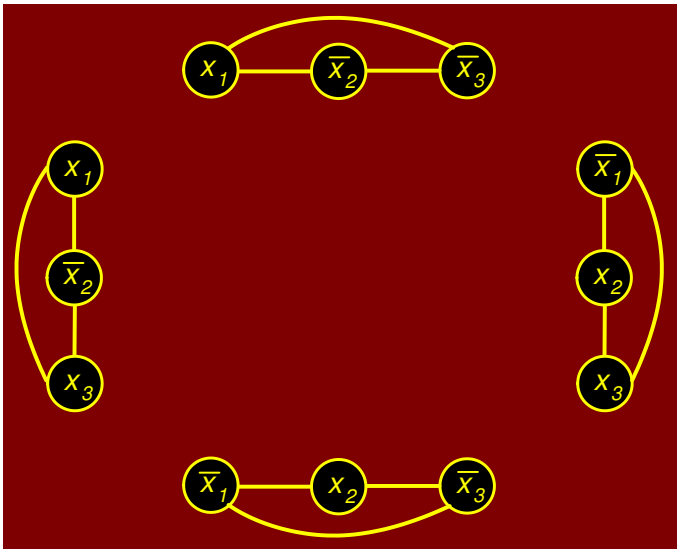


Figure 9.11: Four graphs, one for each of the 3-terms literal

Next, each term is connected to its complement. This is shown in Figure 9.12.

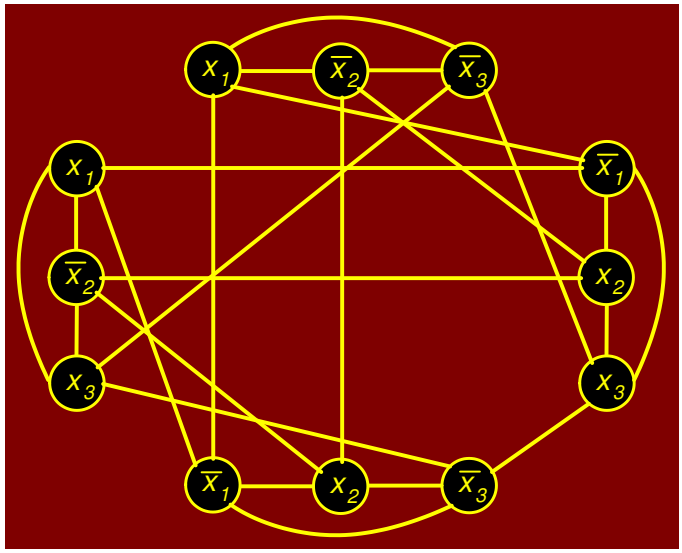


Figure 9.12: Each term is connected to its complement

The boolean formula is satisfied by the assignment $x_1 = 1,\ x_2 = 1\ x_3 = 0$. This implies that the first literal of the first and last clauses are 1, the second literal of the second clause is 1, and the third literal of the third clause is 1. By selecting vertices corresponding to true literals in each clause, we get an independent set of size $k = 4$. This is shown in Figure 9.13.

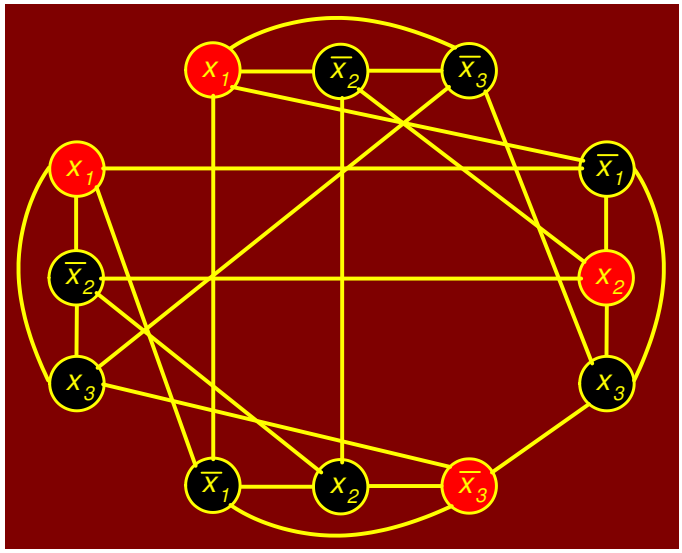


Figure 9.13: Independent set corresponding to $x_1 = 1,\ x_2 = 1\ x_3 = 0$

Correctness Proof:

We claim that F is satisfiable if and only if G has an independent set of size k. If F is satisfiable, then each of the k clauses of F must have at least one true literal. Let V' denote the corresponding vertices from each of the clause clusters (one from each cluster). Because we take vertices from each cluster, there are

no inter-cluster edges between them, and because we cannot set a variable and its complement to both be true, there can be no edge of the form $(x_i, \overline{x_i})$ between the vertices of V'. Thus, V' is an independent set of size k.

Conversely, G has an independent set V' of size k. First observe that we must select a vertex from each clause cluster, because there are k clusters, and we cannot take two vertices from the same cluster (because they are all interconnected). Consider the assignment in which we set all of these literals to 1. This assignment is logically consistent, because we cannot have two vertices labelled x_i and $\overline{x_i}$ in the same cluster.

Finally the transformation clearly runs in polynomial time. This completes the NP-completeness proof.

Also observe that the the reduction had no knowledge of the solution to either problem. Computing the solution to either will require exponential time. Instead, the reduction simple translated the input from one problem into an equivalent input to the other problem.

9.9 Coping with NP-Completeness

With NP-completeness we have seen that there are many important optimization problems that are likely to be quite hard to solve exactly. Since these are important problems, we cannot simply give up at this point, since people do need solutions to these problems. Here are some strategies that are used to cope with NP-completeness:

Use brute-force search: Even on the fastest parallel computers this approach is viable only for the smallest instance of these problems.

Heuristics: A heuristic is a strategy for producing a valid solution but there are no guarantees how close it to optimal. This is worthwhile if all else fails.

General search methods: Powerful techniques for solving general combinatorial optimization problems. Branch-and-bound, A*-search, simulated annealing, and genetic algorithms

Approximation algorithm: This is an algorithm that runs in polynomial time (ideally) and produces a solution that is within a guaranteed factor of the optimal solution.

MIDTERM EXAMINATION
Spring 2010
CS502- Fundamentals of Algorithms (Session - 6)

Time: 60 min
Marks: 38

Question No: 1 (Marks: 1) - Please choose one

Random access machine or RAM is a/an

- ▶ Machine build by Al-Khwarizmi
- ▶ Mechanical machine
- ▶ Electronics machine
- ▶ **Mathematical model**

Question No: 2 (Marks: 1) - Please choose one

_____ is a graphical representation of an algorithm

- ▶ Σ notation
- ▶ Θ notation
- ▶ **Flowchart**
- ▶ Asymptotic notation

Question No: 3 (Marks: 1) - Please choose one

A RAM is an idealized machine with _____ random-access memory.

- ▶ 256MB
- ▶ 512MB
- ▶ **an infinitely large**
- ▶ 100GB

Question No: 4 (Marks: 1) - Please choose one

What type of instructions Random Access Machine (RAM) can execute? Choose best answer

- ▶ Algebraic and logic
- ▶ Geometric and arithmetic
- ▶ **Arithmetic and logic**
- ▶ Parallel and recursive

Question No: 5 (Marks: 1) - Please choose one

What will be the total number of max comparisons if we run brute-force maxima algorithm with n elements?

- ▶ n^2
- ▶ $n^{\frac{n}{2}}$
- ▶ n

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- n^8

Question No: 6 (Marks: 1) - Please choose one

What is the solution to the recurrence $T(n) = T(n/2)+n$.

- $O(\log n)$
- **$O(n)$**
- **$O(n \log n)$**
- $O(n^2)$

Question No: 7 (Marks: 1) - Please choose one

Consider the following code:

```
For(j=1; j<n;j++)
        For(k=1; k<15;k++)
                For(l=5; l<n; l++)
                {
                        Do_something_constant();
                }
```

What is the order of execution for this code.

- **$O(n)$**
- $O(n^3)$
- $O(n^2 \log n)$
- $O(n^2)$

Question No: 8 (Marks: 1) - Please choose one

Consider the following Algorithm:

```
Factorial (n){
  if (n=1)
    return 1
  else
     return (n * Factorial(n-1))
{
```

Recurrence for the following algorithm is:

- $T(n) = T(n-1) +1$
- $T(n) = nT(n-1) +1$
- $T(n)= T(n-1) +n$
- **$T(n)=T(n(n-1)) +1$**

Question No: 9 (Marks: 1) - Please choose one

What is the total time to heapify?

- **O(log n)**
- O(n log n)
- $O(n^2 \log n)$
- $O(\log^2 n)$

Question No: 10 (Marks: 1) - Please choose one

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When we call heapify then at each level the comparison performed takes time

- **It will take Θ (1)**
- Time will vary according to the nature of input data
- It can not be predicted
- It will take Θ (log n)

Question No: 11 (Marks: 1) - Please choose one

In Quick sort, we don't have the control over the sizes of recursive calls

- **True**
- False
- Less information to decide
- Either true or false

Question No: 12 (Marks: 1) - Please choose one

Is it possible to sort without making comparisons?

- **Yes**
- No

Question No: 13 (Marks: 1) - Please choose one

If there are Θ (n^2) entries in edit distance matrix then the total running time is

- Θ (1)
- **Θ (n^2)**
- Θ (n)
- Θ (n log n)

Question No: 14 (Marks: 1) - Please choose one

For Chain Matrix Multiplication we can not use divide and conquer approach because,

- We do not know the optimum k
- **We use divide and conquer for sorting only**
- We can easily perform it in linear time
- Size of data is not given

Question No: 15 (Marks: 1) - Please choose one

The Knapsack problem belongs to the domain of _____ problems.

- **Optimization**
- NP Complete
- Linear Solution
- Sorting

Question No: 16 (Marks: 1) - Please choose one

Suppose we have three items as shown in the following table, and suppose the capacity of the knapsack is 50 i.e. W = 50.

Item	Value	Weight
1	60	10
2	100	20

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3	120	30

The optimal solution is to pick

- ► Items 1 and 2
- ► Items 1 and 3
- ► **Items 2 and 3**
- ► None of these

Question No: 17 (Marks: 2)

Describe an efficient algorithm to find the ***median*** of a set of 10^6 integers; it is known that there are fewer than 100 distinct integers in the set

Question No: 18 (Marks: 2)

How we can avoid unnecessary repetitions for recursive calls?

We can avoid this unnecessary repetitions by writing down the results of recursive calls and looking them up again if we need them later. This process is called *memoization*

Question No: 19 (Marks: 2)

Draw the cost table for chain matrix multiplication problem with initial state.

$(A_1)(A_2A_3A_4 \ldots A_n)$
or $(A_1A_2)(A_3A_4 \ldots A_n)$
or $(A_1A_2A_3)(A_4 \ldots A_n)$
. . . . . .
or $(A_1A_2A_3A_4 \ldots A_{n-1})(A_n)$

Question No: 20 (Marks: 3)

Solve it,

$$T(n) = \frac{1}{2}\sum_{q=1}^{2}(T(q-1) + T(2-q) + 2)$$

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Question No: 21 (Marks: 3)

What are Catalan numbers? Give the formula.

PAGE NO. 85 ON PDF

Question No: 22 (Marks: 5)

What is the effect of calling Max-Heapify(A, i) when i > heap-size[A]/2?

Question No: 23 (Marks: 5)

Write the pseudo code for 0/1 knapsack algorithm developed using dynamic programming technique.

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AA
Dear fellows
My todays papre of algo(CS502)

write these word 3 different format
"MATHS and ARTS" 3marks

explain the average and worst case of quick sort. 2marks

how many combination of ..... (sorry i forgot) 2marks

define the cost table of initial stae of 3marks

suggest and describe one modification of implmenting quick sort 5marks

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CS502 QUIZ 1 SOLVED

Provide by VU Answer

1. DFS or BFS yields a ___ of the graph.

Traversed Tree

b) Spanning tree

c. Simple Tree

d. Free Tree

2. In Knapsack Problem, the goal is to put items in the knapsack such that thevalue of the items is_subject to weight limit of knapsack.

a) Minimized

b) Decreased

c) Maximized

d) None of the given options

3. Using ASCII code, each character is represented by a fixed-length code of

_____bits per character.

a) 4

b) 6

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c) 8

d) 10

4. Which type of algorithm is harder to prove the correctness?

a) Greedy

b) Brute Force

c) Dynamic Programming

d) Divide and Conquer

5. Consider the string "abcdaacac". if the string is coded with ASCII codes, the message length would be_.

a) 70 bits

b) 60 bits

c) 90 bits

d) 75 bits **8*9=72 bits are the right answer**

6. In__algorithm, you hope that by choosing a local optimum at eachstep, you will end up at a global optimum.

a) Simple

b) Divide and conquer

c) Greedy

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d) Brute force

7. For a digraph G = (V, E), Sum of in-degree(v) _____

a) Not equal to Sum of out-degree(V)

b) =Sum of out-degree(v)

c) < Sum of out-degree(v)

d) > Sum of out-degree(v)

8. In inductive approach of knapsack problem, we consider 2 cases, _____ Or_____.

a) Median, Mode

b) Recursive, Iterative

c) Leave object, Take object

d. Sequentially, Parallel

9. A/an_____is one in which you want to find, not just a solution, but thebest solution.

a) Optimization Problem

b) Divide and Conquer

c) NP Complete problem

d) Best Problem

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10. If the graph is represented using an **adjacency matrix,** then **Breadth-firstsearch** takes_time.

a) O(E+1)

b) O(V^2)

c) O(V)

d) O(E)

11. If the graph is represented using an **adjacency list**, then **Breadth-firstsearch** takes_time.

a) O(V^2)

b) O(V)

c) Co(V + E)

d. O(E + 1)

12. A graph is_____if every vertex can reach every other vertex

a) Connected

b) Cycle

c) Acyclic

d) Loop

13. The number of edges that come out of a vertex is called the __of thatvertex in a digraph.

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a) post-degree

b) in-degree

c) out-degree

d) pre-degree

14. Huffman algorithm produces the____prefix code tree

a) Optimal

b) Best

c) Worst

d) Better

15. graph is said to be acyclic if it contains ____

a) at least one cycle

b) exactly one cycle

c) always more than one cycle

d) no cycles

16. Fractional Knapsack is founded on__method.

a) Greedy

b) Recursive

c) Dynamic Programming

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d) Divide and Conquer

17. If Matrix-A has dimensions "3x2" and Matrix-B has dimensions "2x3", then multiplication of Matrix-A and Matrix-B will result a new Matrix-C having dimensions

a) 3x2

b) 2x3

c) 2x2

d) 3x3

18. Some places a person wants to visit Starting from a vertex, he wants to visit every place linked to this vertex, and so on. What algorithm does he need to use?

a) Depth First Search

b) Breadth First Search

c) Greedy Algorithm

d) Divide and Conquer

19. Greedy algorithm can NOT be used to solve all the_____ problems.

a) Dynamic programming

b) Memorization programming

c) Edit distance programming

d) Storing value programing

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20. In Huffman encoding, the____is the number of occurrences of a characterdivided by the total characters in the message.

a) Counting

b) Parsing

c) Relative Probability

d) Weight

21. In Brute Force solution of knapsack Problem, there are__possiblecombinations for n items.

a) 2

b) n

c) 2^n (2 raise to the power n)

d. 3^n (3 raise to the power n)

22. The Binary Tree constructed by Huffman Encoding is a:

a) Full binary tree

b) Partial binary tree

c) Incomplete binary tree

d) None of the given options

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23. In Huffman encoding when a new node is created by combining two nodes,the new node is placed in the _____.

a) Priority queue

b) Linked list

c) Min heap tree

d) Graph traversal

24. In general, the Activity Selection Problem is to select a__.

a) Minimum-size set of interfering activities

b) Maximum-size set of mutually non-interfering activities

c) Maximum-size set of interfering activities

d) Minimum-size set of mutually non-interfering activities

26.In Activity selection problem, intuitively_____.

a) There are always short activities as input

b) Short activities are not attractive

c) Duration of the activities does not matter

d) We do not like long activities

27. Which of the following ways can be used to represent a graph?

a) Adjacency List

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b) Incidence Matrix

c) Adjacency Matrix Adjacency List

d) No way to represent

28. Which of the following algorithms solves the fractional knapsack problemmost effectively?

a) Divide and conquer

b) Greedy algorithm

c) Dynamic programming

d) Backtracking

29. The prefix code generated by Huffman algorithm _____the expectedlength of the encoded string.

a) Minimizes

b) Balances

c) Maximizes

d) Keeps Constant

30. In context of activity selection algorithm, time is dominated by sorting of theactivities by ___.

a) Start Times

b) Finish Times

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c) Average Times

d) CPU Burst Times

31. Huffman algorithm generates an optimum _____code.

a) Prefix

b) Postfix

c) Infix

d) None of the given options

32. If a matrix has three rows and two columns, then dimensions of matrix willbe:

a) 3 x 2

b) 2 x 3

c) 3 x 3

d) 2 x 2

33. In greedy algorithm, at each phase, you take the _____you can get rightnow, without regard for future consequences.

a) Worst

b) Minimum

c) Good

d) Best

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34. Which of the following algorithms provides an optimal solution for theactivity selection problem?

a) Divide and Conquer

b) Brute Force

c) Greddy

d) Recursive

35. Suppose you are given infinite coins of 1, 2, 3, and 4. Select the ways of the minimum number of coins that required to achieve a sum of 6:

a) 1

b) 2 (4+2=6)

c) 3

d) 4

36. Using ASCII standard the string "greedy" will be encoded with

a) 48 bits 8n=8(6)=48 bits ascii encode

b) 120 bits

c) 44 bits

d) 40 bits

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37. For graph traversal, Breadth-first search strategy _____

a) is always recursive

b) Cannot be recursive

c) Cannot be non-recursive

d) Can be both recursive and non-recursive

38. The greedy approach gives us an optimal solution when the coins are allpowers of a____denomination.

b) Fixed

c) Variable

d) Constant

c) Static

39. In Activity Selection, we say that two activities are non-interfering if theirstart-finish interval_____overlap.

a) Do

b) Do not

c) Sometimes

d) Once

40. Bag is a__.

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a) type of algorithm

b) data structure

c) program

d) compiler

41. The running time of brute-force algorithm to solve Knapsack problem is _____.

a) O(n)

b) O(n3)

c) O(n!)

d) O(2^n)

42. In Activity scheduling algorithm, the width of a rectangle__.

a) Is always ignored

b) Directs towards recursion

c) Should be maximized

d) Indicates the duration of an activity

43. In a digraph, the number of edges coming out of a vertex is called the of that vertex.

a) In-degree

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b) Complete degree

c) Node

d) Out-degree

44. How many steps are involved to design the dynamic programming strategy?

a) 2

b) 3

c) 1

d) 4

45. The Huffman codes provide a method of__data efficiently.

a) Reading/Writing

b) Encoding/Decoding

c) Divide/Conquer

d) Inserting/Deleting

46. Time complexity of the "0 - 1" knapsack algorithm depends on_.

a) Number of items

b) Capacity of the knapsack

c) Size of the table

d) Number of items and capacity of knapsack

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47. If the string "lmncde" is coded with ASCII code, the message length wouldbe_ bits.

a) 24

b) 36

c) 48 8*6=48

d) 60

48. Each time we traverse graph by Breadth-first search algorithm, we count the distance from _____.

a) Starting node

b) neighbors of the starting node

c) right most node

d) Left most node

49. The coin change problem is to find the minimum number of coins requiredto get the sum S of infinite coins of denominations al, a2, a3, , an. This problem can be solved using_.

a) Greedy algorithm

b) Dynamic programming

c) Divide and conquer

d) Backtracking

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50. Each vertex can reach every other vertex is called.

a) Unordered graph

b) Unconnected graph

c) Connected graph

d) Ordered graph

51. ___data structure is used to perform the Depth-first search.

a) Array

b) Queue

c) Stack

d) Tree

52. In general, a graph G = (V, E) consists of a and E, a binary relationon V called edges.

a) Infinite set of vertices V

b) Infinite set of nodes

c) Finite set of vertices V

d) Infinite set of objects

53. The general coin change problem can be solved using___.

a) Dynamic programming

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b) Greedy Algorithm

c) Divide and Conquer

d) Recursion

54. For traversing the graphs,___can be visualized as a wave frontpropagating inward towards root node.

a) Depth-first Search

b) Breadth-first search

c) Binary search

d) Linear search

55. Counting Money problem:

a) Can be optimally solved by greedy algorithm

b) cannot be optimally solved by greedy algorithm

c) Cannot be solved by brute force algorithm

d) Can be solved by simple recursive algorithm

56. The activity scheduling is a simple scheduling problem for which the greedy algorithm approach provides a/an _____solution.

a) Simple

b) Sub optimal

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c) Optimal

d) Non optimal

57. Which of the following is true about Huffman Coding?

a) Huffman Codes may not be optimal lossless codes in some cases

b) Huffman coding may become Lossy in some cases

c) Huffman coding is not an efficient coding way

d) In Huffman coding, no code is prefix of any other code

58. One of the limitations in 0/1 knapsack is that an item can either be ___ in the bag or not.

a) Use

b) Put

c) Move

d) Store

59. In_involves breaking up the problem into sub problems whosesolutions can be combined to solve the global problem.

a) Complexity Theory

b) Dynamic programming solution

c) Divide and Conquer Strategy

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d) Greedy Algorithms

60. The time complexity of activity selection algorithm is____.

a) O(NlogN)

b) O(N)

c) O(logN)

d) O(NlogN)

61. The_____is a problem for which the greedy algorithm approachprovides an optimal solution.

a) Activity selection

b) Dynamic programming

c) Knapsack Problem

d) NP complete problem

62. Which of the following is true about graph?

a) A graph may contain no edges and many vertices

b) A graph may contain many edges and no vertices

c) A graph may contain no edges and no vertices

d) A graph may contain no vertices and many edges

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63. In Huffman encoding fixed-length codes are popular because it is very easyto break up a__into its individual___.

a) Number, string

b) Character, Number

c) String, Characters

d) Floating number, string

64. In general in comparison with Fractional Knapsack problem, ___

a) 0 – 1 knapsack problem is very easy to solve

b) 0 – 1 knapsack problem is hard to solve

c) Both are easy to solve

d) We cannot compare them

65. In an undirected graph, we say that an edge is __on a vertex if thevertex is an endpoint of the edge.

a) Incident

b) Directed

c) Undirected

d) Tree

66. In Huffman Encoding, the characters with smallest probabilities are placed atthe _____depth of the tree

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a) Minimum

b) Average

c) Maximum

d) Root

67. The probability in Huffman encoding is the number of occurrences of acharacter divided by the total_in the message

a) Numbers

b) Frequencies

c) Strings

d) Characters

68. The Knapsack problem belongs to the domain of_problems

a) Optimization

b) Sorting

c) Linear solution

d) Searching

69. In recursive formulation of Knapsack Problem:

V[0, j] =_____for j>=0

a) -1

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b) 0

c) 1

d) 2

70. An optimization problem is one in which you want to find the ___ solution

a) Simple

b) Good

c) Best

d) Worst

71. If we implement the bag by using a queue, we have____.

a) BFS

b) DFS

c) Graph

d) Loop

72. Which graph traversal algorithm uses a stack to keep track of vertices?

a) Divide and Conquer

b) Breadth First Search

c) Greedy Algorithm

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d) Depth First Search

73. In Dynamic Programming based solution of Knapsack Problem, if we decideto take an object ‘i’, then we gain_.

a) W (Total Weight of Knapsack)

b) V (Total Value of all items)

c) vi (Value of object i)

d) None of the given option

74. If there are no cycles in the graph, what is the maximum number of edgespresent in a simple directed graph with 5 vertices?

a) 3

b) 4

c) 5

d) 6

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CS502 grand quiz answers

) In heap sort algorithm, we build _____ for ascending sort.

- Max Heap
- Min Heap

2) Quicksort is _____ based on the divide and conquer strategy.

- Not
- Always (Pg#46)
- Sometime
- Occasionally

3) _____ overcomes the limitations of _____ by working as per positional notation of numbers.

- Counting Sort, Radix Sort
- Radix Sort, Counting Sort

4) In heap sort algorithm, every time the maximum element is removed _____.

- We called build heap procedure.
- Heap sort algorithm terminate., We call heapify procedure,
- Nothing happens.

5) Rank of an element can be defined as _____.

- One minus the number of element that can be smaller.

- Two plus the number of element that can be greater. One plus the number of element that can be smaller. (Pg#34) Two plus the number of element that can be smaller.

6) Radix sort is not a non-comparative integer sorting algorithm.

- True
- False

7) In Dynamic Programming, our approach is to_____

- Develop the solution in a top down fashion
- Express the Problems Non-Recursively
- Build the solution in a bottom-up fashion.(pg#79) Input several sub problems.

8) Efficient algorithm requires less computational...

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- Memory
- Running Time
- Memory and Running Time (Pg#9)
- Energy

9) If pj dominates pi and pi dominates ph then pj also dominate ph it means dominant relation is.

- Transitive
- Non Transitive
- Equation
- Symbolic

10) Sorting algorithm having O (orderly) _____ running time is considered to be slow ones.

- N
- N2 (pg#39)
- N3
- Log(n)

11) In plane sweep approach, a vertical line is swept across the 2d-plane from.....

- Right to Left
- Left to Right
- Top to Bottom
- Bottom to Top

12) If we associate (x, y) integers pair to cars where x is the speed of car and y is the negation of the price. High y value for a car means a .....car.

- Fast
- Slow
- Expensive
- Cheap

13)If the indices passed to merge sort algorithm are ... Then this means that there is only one element to sort.

- Small
- Large
- Equal
- Not Equal

14) Merge sort is based on.....

- Brute-Force
- Plane-sweep
- Axis-sweep
- Divide and Conquer

15) In asymptotical analysis of n*(5+2)-3, as n becomes large, the dominant (fastest growing) term is some constant times...

- n-1
- n
- n +1
- n*n

16) The process of .....ends when you are left with such tiny pieces remaining that it is trivial to solve them.

- Brute-force
- Plane-sweep
- Divide and conquer
- Axis-sweep

17) The sequence of merge sort algorithm is:

- Divide-Combine-Conquer Conquer-Divide-Combine
- Divide-Conquer-Combine
- Combine- Conquer-Combine

18)While solving Selection problem, in Sieve technique we partition input data...

- In increasing order
- In decreasing order
- According to pivot

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- Randomly

19) Result of asymptotical analysis of n (n – 3) and 4n*n is that .....

- n(n – 1) is asymptotically Less
- n(n – 1) is asymptotically Greater
- Both are asymptotically Not equivalent
- Both are asymptotically Equivalent

20) In Heap Sort algorithm, we build....for ascending sort.

- Max heap
- Min heap

21) Quick sort is a not a partitioning based algorithm

- True
- False

22) In 2d-maxima problem, a point is said to be _____if it is not dominated by any other point in that space.

- Moniter
- Minimal
- Maximal
- Joine

23) The definition of theta notation is refer to proving _____satisfied asymptotic bound.

- Over
- Lower
- Upper
- Both lower& upper

24)Sieve technique is very important special case of divide-and-conquer strategy.

- True
- False

25) Boolean operation in a_____ operation on an ideal led RAM model of computation.

- Starting
- Basic
- Advance
- Normal

26) While solving selection problem in sieve technique choose pivot.

- Minimum element
- Maximum element
- Randomly

27) If the indicate passed the merge sort algorithm are _____ then this merges that there is only one elemental sort.

- Small
- Large
- Equal
- Non equal

28) The running time of quick sort algorithm.

- Is importable to compute
- Has nothing to do with pivot selection
- Is randomly upon each execution
- Greatly influenced by the selection of pivot

29) In heap sort algorithm, we build for according sort.

- Max heap
- Min heap

30) In merge sort algorithm, we split the array account five _____ index q

- Entering
- Mid
- existing

- starting

31) Dynamic Programming strategy is useful when sub-problems are independent

- True (PAGE 75)
- False

32) In Plane sweep approach, a vertical line is swept across the 2d plane from

- Right to left
- Left to right (PAGE 18)
- Top to bottom
- Bottom to top

33) Bubble sort is not an in-place sorting algorithm.

- True
- False (page 39)

34) Boolean operation is a _____ operation on an idealized RAM model of computation.

- Starting
- Basic
- Advance (page 11)
- Normal

35) Algorithm is a sequence of computational steps that _____ the input into output

- Merge
- Assign
- Transform (page 7)
- Integrate

36) In quick sort algorithms, pivots form _____

- Stack
- Queue
- Binary search tree (page 49)
- Graph

37) In selection algorithm, we assume pivot selection take theta _____ running time.

- n (page 37)
- n2
- n3
- log(n)

38) Brute force algorithm for 2d maxima is operated by comparing ____ pairs of points.

a) two b) some

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a) most d) all (page 18)

b)

39) in generating Fibonacci sequence, we can avoid un necessary repetitions by ____ process

a) tokenization b) memorization (page 74)

b) randomization d) memorization

40) the sieve technique is a special case, where the number of sub problems is just _____ a) 1 (page 34) b) 2

b) 3 d) 4

25) Time complexity of dynamic programming based algorithm for computing the minimum cost of chain matrix multiplication is _____

- Logn
- N
- n^2 (n square)
- n^3(n cube) (page 90)

26) Dynamic programming strategy is useful when sub problems are independent

- true
- false

27) We can make _____ recursive calls in Fibonacci sequence

- Infinite (page74)
- Finite

28) Counting sort is suitable to sort the elements in range 1 to k

- k is large
- k is small (page 57)
- k may be large or small
- none

29) Dynamic programming comprises of _____

- Recursion only
- Repetition only
- Recursion and repetition
- no repetition but recursion only 9page 75)

30) As per algorithm of dynamic programming we need to store the results of _____

- First sub problem only
- Best situation only
- Intermediate sub problems (page 75)
- final situation only

31) Bubble pace is not an in-place sorting problem.

- True
- False (page 54)

32) Dynamic programming is a problem solving approach in which _____

- Problem is solved in zero time
- Solution is developed only at final stage
- Both are incorrect (page 73)
- Both are correct

33) Counting sort is suitable for sorting the elements within range 1 to P where, _____

- p is large
- p is small (page 57)
- p is very large
- p is undetermined

34) Memorization is a part of Dynamic Programming strategy.

- True
- False

35) Dynamic Programming is a problem-solving approach in which...

- Problem is solved in zero time
- Solution is developed only at final stage
- Both are correct
- Both are incorrect (pg#75)

36) Quicksort is a/an .....and .....sorting algorithm.

- Not in place, not stable one
- In place, not stable one
- In place, stable one (pg#54)
- Not in place, stable one

37) Insertion sort is an in-place sorting algorithm.

- True (pg#54)
- False

38) An in-place sorting algorithm is one that.....uses additional array for storage.

- Always
- Permanently
- Does not (pg#54)
- Sometime

39) In Brute Force solution of Knapsack Problem, there are ....possible combinations for n times.

- 2
- n
- 2^n(2 raise to the power n) (pg#92)
- 3^n(3 raise to the power n)

40) ..... Overcomes the limitations of .....by working as per positional notations of numbers.

- Counting sort, Radix sort
- Radix sort, Counting sort (pg#71)

41) Identify the TRUE statement.

- The Knapsack problem does not belong to the domain of optimization problems.
- The Knapsack problem belongs to the domain of optimization problems.

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- The knapsack problem cannot be solved by using dynamic programming The knapsack problem is optimally solved by using brute force algorithm.

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42) Selection sort is not an in-place sorting algorithm

- True
- False (pg#54)

43) Bubble sort is not an in-place sorting algorithm.

- True
- False (pg#54)

44) In plane sweep approach of solving geometric problems, a __is swept across the plan.

- Line
- Plan
- Cube
- Box

45) Quick sort algorithm was developed by____

- Alfred Aho
- Sedgewich
- John Vincent Atanasoff
- Tony Hoare

46) In max heap (for Heap Sort algorithm), when every time maximum element is removed from top we replace it with _____ leaf in the tree.

- Second last
- Last
- First
- Any

47) In Quick Sort algorithm, ____ decides nature of Binary Search Tree formed by Pivots.

- No one
- Smallest element from input
- Rank of the Pivot
- Largest element from Pivot

48) While analyzing Selection algorithm, we make a number of passes, in fact it could be as many as.

- n(n+1)
- log(n)
- n/3
- n/4

49) While solving Selection problem, in Sieve technique we choose pivot

- Minimum element
- Maximum element
- Randomly
- Average element

50) Boolean operation is an _____ operation on an idealized RAM model of computation.

- Starting
- Basic
- Advance
- Normal

51) If the indices passed to merge sort algorithm are _____then this means that there is only one element to sort.

- Small
- Large
- Equal
- Not Equal

52) To find maximal points in brute-force algorithm, each point of the space is composed against _____ of that space.

- One other point
- All other points
- Few other points
- Most of the other points

53) In Heap Sort algorithm (using max heap), when every time maximum element is removed from top _____.

- We call merge Sort algorithm
- It became Order n2 Algorithm
- Divide and Conquer strategy helps us
- We are left with a hole

54) While analyzing selection algorithm, we make a number of passes infect it could be as many as;

- N(n+1)
- Log(n)
- n/3
- n/4

55) Sorting can be in_____

- Increasing order only
- Decreasing order only
- Both increasing and decreasing order
- random

56) In the statement

*if p(i)x<p[j]x)&(p[i]y<p[j]ym

The number of times elements of P are accessed is_____

- 1
- 2
- 3
- 4

57) In selection problem the rank of an element will be a_____ position if we sort the input data.

- First
- Final (p34)
- Second last
- Last

58) Merge sort is based on_____

- Basic force
- Plan sweep
- axis sweep
- divide and conquer (p 28)

59) There are so hard formal rules to the syntax of the _____ code.

- Basic
- Programming
- Pseudo(not sure)
- assembly

76)The heap sort algorithm the total running time for heavily procedure is_____

- Theta(log n) p(40)
- Order(log n)
- Omega(log n)
- O1iii constraint time

77) The brute-force algorithm for 2-D maxima runs in order O (___) time

- (n) (p 13)
- N(logn)
- N*n
- N(o)

78) In sorting the key value of attribute ___an order domain

- Must be
- Not always

79) Rank of an element can be defined as____

- The number of element that are smaller
- Two plus the number of elements that are greater(not sure)
- One plus the number of element that are smaller

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- Two minus the number of elements that are smaller

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80) Unlike programs, algorithm to be understood primarily by _____ and _____.

- Machines not people
- Mathematical expression, not algebraic expression
- Programmers ,not machines
- RAM, not programmer

81) Recurrences are useful for analyzing

- Recursive Algorithms
- Simple Algorithms
- Parallel Algorithms
- Parallel Algorithms & Recursive Algorithms

82)Divide and conquer involve breaking the problem into small number of

- Pivot
- Sub problems
- Selection
- Sieve

83) In which order we can sort?

- Increasing or decreasing order
- Both at the same time
- Increasing order only
- Decreasing order only

84) comparison based sorting algorithms cannot run faster than

- (n log n)
- (n log n)
- ( )*n*2
- ( )*n*2

85) In Quick sort, we don't have the control over the sizes of recursive calls

- True
- False
- Less information to decide
- Either true or false

86) Who invented Quick sort procedure?

- Hoare
- Segewick Mellroy
- Coreman

87) Counting sort assumes that the numbers to be sorted are in the range 1 to where k is

- Small
- Large
- No restriction on k
- None of these

88)If there are ( )*n*2 entries in edit distance matrix then the total running time is

- (1)
- ( )*n*2
- ( )*n*
- (*n*log*n*)

89) When the recursive algorithm revisits the same problem over and over again, we say that the optimization problem has ___ sub-problems.

- Overlapping
- Over costing
- Optimized
- None of these

90) A $p \; q$ matrix A can be multiplied with a $q \; r$ matrix B. The result will be a $p \; r$ matrix C.in particular ,for $1 \; i \; p$ and $1 \; j \; r$

- $C[i, j] = \sum_{k=1}^{q} A[i, k] B[k, j]$
- $C[i, j] = \sum_{k=1}^{q} A[k, i] B[k, j]$
- $C[i, j] = \sum_{k=1}^{q} A[k, i] B[j, k]$
- None of these

91) We can find the product $A \; B$ of matrices A and B ,only if they are compatible which means,

- No of Columns of A must be equal to No of Rows of B

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- No of Columns of A must be equal to No of Columns of B
- No of Rows of A must be equal to No of Rows of B
- Order of A must be equal to order of B

92)Time complexity of chain matrix multiplication is $( )n3$ and space complexity is

- $( )n2$
- $( )n3$
- $(n\log n)$
- $(\log n)$

93) Computational model of sequential RAM is:

- Parallel machine may be expensive to model and have more computational power than sequential RAM.
- Computational power of sequential RAM is same as that of parallel machines.
- Both first and second option are true
- Less powerful Computational wise than parallel machines

94) In Sieve Technique ,we know

- Item of internet
- Order of items
- complexity of items
- all items are interest

95) In Random access machine, instructions are executed .

- five at a time
- infinite instruction at a time
- one-by-one
- parallel

96) Which type of instruction RAM can execute?

- Algebraic and logic
- Geometric and arithmetic
- Arithmetic and logic
- Parallel and recursive

97)Which of the following is calculated with Big O notation?

- Medium bounds
- Lower bounds
- Upper bounds
- Both lower and upper

98) Which of the following functions grows faster as n gets larger?

- $n_{10}2_n$
- $n_8 3_n$
- $n_5 5_n$
- $n_{3\,2}2_{\ n}$

99) The worst case running time algorithm is given below is:

```
MAXIMA(int n, Point  P[1...n])
1        for i  1 to n  n times
2        do maximal  true
3        for j  1 to n  n times
4        do
5        if  (i   i)&( ].P i x   P j x[ ]. )&( ].P i y   P j y[ ].)   4 accesses
6        then maximal  false  break
7        if maximal,
8        then  output P[i].x,P[i].y   2 accesses
```

- $(\ )n6^{2n}$
- $(n\ 6\ )$
- $(\ )n2$
- $(2n\log 6)$

100) Al-Khwarizmi's work was written in a book titled.....

- Al Kitab al-mukhatasar fi hisab al-jabrwa'l-muqabalah (pg#7)
- Calculation by Completion
- Al Kitab
- Al-jabrwa'l-muqabalah

101) What is the asymptotic growth of $\frac{4n3+15n2+11n}{6}$?

- $\theta\left(\frac{4n3+15n2+11n}{6}\right)$
- $\theta\ (4n3+15n2+11n)$
- $\theta(15n2)$
- $\theta(n3)$ (Note: Read Asymptotic Notation on pg#23 to understand this.)

102) The reason for introducing Sieve Technique algorithm is that it illustrates a very important special case of:

- Divide-and-conquer (pg#34)
- Decrease and conquer
- Greedy nature
- 2-dimension maxima

103) Strictly increasing arrays are..... input for quick sort.

- The worst-case (not sure)
- The best-case
- Average case
- None of the given

104) The running time of quick sort depends heavily on the selection of

- Required Output
- Input and output
- Pivot (pg#49)
- Input data only

105) Quick sort is based on divide and conquer paradigm. In the Combine step of divide and Conquer process:

- No work is needed to combine the sub-arrays, the array is already sorted
- Work is needed to combine the sub-arrays; the array is not already sorted
- Merging the arrays
- Dividing the arrays (not sure)

106) In in-place sorting algorithm is one that uses no...Array for storage.

- Two dimensional
- Three dimensional
- N dimensional
- Additional (pg#54)

107) Bubble sort and insertion sort can be implemented as stable algorithms but..... cannot be.

- Selection sort (pg#54)
- Bubble sort
- Merge sort
- Stable sort

108) Catalan number is given by formula:

- C(n) = $\frac{1}{n+1}\binom{2n}{n}$ (pg#85)
- C(n) = $\frac{1}{n-1}\binom{2n}{n}$
- C(n) = $\frac{1}{n+1}\binom{n}{2n}$ C(n) = $\frac{1}{n-1}\binom{2n}{n}$

109) We can find the product A×B of matrices A and B, only if they are compatible which means:

- No of column of A must be equal to No of Rows of B
- No of column of A must be equal to No of Column of B
- No of Rows of A must be equal to No of Rows of B
- Order of A must be equal to order of B

110) Matrix – Chain- Order is..... than the exponential time method of enumerating all possible parent hesitations and checking each one.

- Much more efficient
- Only fractional efficient(is ka answer given file mein b mark ni hy)
- Worst
- Too slow

111) Time complexity of chain matrix multiplication is $\theta(n3)$ and space complexity is:

- $\theta(n2)$
- $\theta(n3)$
- $\theta(\text{nlog } n)$
- $\theta(\log n)$

112) The Knapsack problem belongs to the domain of ..... problems.

- Optimization (pg#91)
- NP Complete
- Linear solution
- Sorting

113) The recurrence relation of Hanoi is given below:

$$T(n)= \begin{cases} 1 & if\ n=1 \\ 2T(n-1)+1 & if\ n>1 \end{cases}$$

In order to move a tower of 4 rings from one peg to another, how many ring moves are required?

- 15
- 7 (is ka answer given file mein b mark ni hy)
- 12
- 32

114) Which type of instruction Random Access Machine (RAM) can execute? Choose best answer.

- Algebraic and logic
- Geometric and arithmetic
- Arithmetic and logic (pg#10)
- Parallel and recursive

115) Algorithm's essential elements are:

- Step wise solution
- Stepwise solution and finite time
- Step wise solution finite inputs (not sure)
- Stepwise approach in which time and memory does not matter.

116) Which one is the correct conversion of given algorithm to sigma notation?

MAXIMA(int n, Point[1...n])

1 for i ← 1to n n times(ye question full ni tha meray pass)

2 do maximal ← true (see page #14 for remaining question

- and answer)

- n
-

117) Which of the following is calculated with Big Theta notation?

- Lower bounds
- Upper bounds
- Both upper and lower bounds
- Medium bounds

118) Suppose we have an algorithm that carries out N^2 operations for an input of size N. Let us say that a computer takes 1 microsecond (1/1000000 second) to carry out one operation. How long does the algorithm run for an input of size 3000?

- 90 seconds (for N input, an algorithm carries out N^2 In given question input size is 3000, so carried 9 seconds out operations are $(3000)^2=9000000$. Time for 0.9 seconds 1operation is 1 microsecond (1/1000000).
- 0.09 seconds time for input size n = 9000000/1000000 =9)

119) Which one is the best algorithm from the following with respect to running time?

- $\theta(n)$
- $\theta(n2)$
- $\theta(n3)$
- $\theta(\text{nlog } n)$

120) The word algorithm comes from the name of the Muslim author

- Abu Ja Far Mohammad ibn Mussal- Khwarizmi or Al Khwarizmi(p7)
- Khawarizm
- Kheva

- Uz bekistan

121) AL-Khwarizmi's work was written in a book titled_____

- Al kitab al- mukhtasar fi hisab al-jabrwa'l muqabalah (p7)
- Calculation by completion
- Al kitab
- Al jabrwa al- muqabalah

122) _____ is a graphic representation of an algorithm

- ∑ notation
- Θ notation
- Flowchart(not sure)
- Asymptotic notation

123) What is the asymptotic growth of $n^2 + \frac{4}{16}n^{15} - 11n$?

- Θ(n^2)(not sure)
- Θ(n)

124) Recurrences are useful for analyzing

- Recursive algorithm
- Simple algorithm
- Parallel algorithm
- Parallel algorithm& recursive algorithm

125) The sieve technique is a special case, where the number of sub problem is _____just

- 5
- Many
- 1(p34
- Few

126) What is the total time of heapify

- o(log n)(p43
- o(n log n)
- o(n^2 log n)
- o($\log^2 n$)

127) comparison based sorting algorithm cannot run faster than

- Ω(n log n)
- O(n log n)(p40
- Ω(n^2)
- O(n^2)

128) After partitioning array in quick sort , pivot is placed in a position such that

- Value smaller than pivot are on left and larger than pivot are on right
- Value larger than pivot are on left and smaller than pivot are on right
- Pivot is the first element of array
- Pivot is the last element of array(p47

129) The running time of quick sort depends heavily on the selection of

- Required output
- Input and output
- Pivot(p49
- Input data only

130) Memorization is a process

- To avoid unnecessary repetition by written down the result of recursive calls and looking them again if needed later
- To avoid repeated data by writing down the result of input array and looking them again if needed later
- None of these
- All option are correct

131) A problem exhibits _____ if an optimal solution to the problem contains within it optimal solution to sub problem

- Optimal structure
- Efficient structure
- Inefficient structure
- Unknown behavior

132) A p x q matrix can be multiplied with a q x r matrix B. The result will be a p x r matrix C. There are (p.r) total entries in C and each takes _____

- O(q) (p84
- O(1)
- O(n^2)
- O(n^3)

133) Catalan number of given by the formula

1 1. $C(n)= \frac{1}{n+1}\binom{2n}{n}$

2. $C(n)= \binom{2n}{n}$

3. C(n)=

4. C(n)=

134) For chain matrix multiplication we cannot use divide and conquer approach because

- We do not know the optimum k(p86)
- We use divide and conquer for sorting only
- We can easily perform it in linear time
- Size of data is not given

135) Time complexity of chain matrix multiplication is $\Theta(n^3)$and space complexity is

- $\Theta(n^2)$(p84)
- $\Theta(n^3)$
- $\Theta(n \log n)$
- $\Theta(\log n)$

136) Computational model of sequential RAM is:

- Parallel machines may be expensive to model and have more computational power than sequential RAM
- computational power OF sequential RAM is same is that of parallel machine only time efficiency is achieved with parallel machine
- both first and second option are true
- less powerful computational wise than parallel machine(p10) 137) what is the correct order of operation in merge sort.
- Conquer → divide → combine
- Divide → conquer → combine(p27)
- Conquer → divide → split
- Split → conquer → destroy

138) What is the average case time for merge sort to sort an array of n element

- O(logn)
- O(n)
- O(n log n)(p31)

- O(n2)

139) What is the solution to the recurrence *T(n) = T(n/2)+n ,T(1)=1?*

- *O(logn)*
- *O(n)*
- *O(n log n)*
- *O(n2)*

140) Al khawarzmi work was written in a book titled

- Al kitab al-mukhastar fi hisab al jabrwa'l muqabalah (page 7)
- Calculation by completion Al kitab
- Al jabrawa'l muqabalah

141) Unlike programs, algorithms to be understood primarily by ____ and ____

- Machines, not people
- Mathematical expressions not algebraic expressions
- Programmers, not machines (page 10)
- RAM, not programmer

142) Which formula is used for calculating worst case running time?

- Tworst(n) =max –n T(1) (page30)
- Tworst(n) = max+n T(1)
- Tworst(n)= max-n T(n)
- Tworst (n) = max+T(n)

143) Analysis of selection algorithm ends up with

- ϴ (n) (page 37)
- ϴ (1+1/n)
- ϴ (n/2)
- ϴ (n/2+2)

144) A (an) is a left complete binary tree that confirms to the *heap order*

- Heap (page 40)
- Binary tree
- Binary search tree
- Array

145) Principal of operation for maintaining the heap property is called *heapify* , it is also called

- Shifting down (page 43)
- Shifting up
- Shifting left
- Shifting right

146) What is common between bubble sort, insertion sort, quick sort and heap sort?

- All are in place algorithms note:(see articles of algorithms)
- All are stable algorithms
- None of these
- All are unstable algorithms

147) When primary memory storage is at a premium, a (an) ____ algorithm will be preferred?

- In place
- External (not sure)
- Stable
- None of these

148) If there are ϴ(n^2) entries in edit distance matrix then the total running time is

- Θ(1)
- Θ(n^2) page 26
- Θ(n)
- Θ(n log n)

149) Catalan number is given by the formula

- C(n)=1/n+1(2n n) page 85
- C(n) =1/n-1(2n n)
- C(n) = 1/n+1(n 2n)
- Cn= 1/n-1(n 2n)

150) For chain matrix multiplication, we cannot use divide and conquer approach because

- We do not have the optimum k (page 86)
- We use divide and conquer for sorting only
- We can easily perform it in linear time'
- Size of data is not give

151) We can find the product A*B of matrices A and B , only if they are compatible which means

- No of columns of A must be equal to no of rows B note: basic rule of mathematics
- No of columns of A must be equal to no of columns B
- NO of rows of A must be equal to no of rows of B
- Order of A must be equal to order of B

152) Time complexity of chain matrix multiplication is Θ(n^3) and space complexity is

Θ(n^2) note:- because in general we take 2D Dimensions, but if the dimension is 3D
THEN IT WILL BE OPTION B

- Θ(n^3)
- Θ(n log n)
- Θ(log n)

153) The worst running time of algorithm is

- Θ(n^6)
- Θ(n*2n/6)
- Θ(n^2) (page 169)
- Θ(2n log 6)

154) In sieve technique, we know _____

- Item of interest
- Order of items
- Complexity of items
- All items are of interest (page 34)

155) In RAM, instructions are executed

- Five at a time
- Infinite instructions at a time
- One by one (note: by using FIFO Rule)
- Parallel

156) Which of the following is calculated by *big theta notation*?

- Lower bounds
- Upper bounds (page 24)
- Both lower and upper bounds
- Medium bounds

157) Suppose we have an algorithm which carries out N^2 operation for an input of size N. let us say that a computer takes 1 microsecond to carry out one operation. How does the algorithm run for an input of size 3000?

- 90 seconds
- 9 seconds by using formula O(n n)
  O(n 3000)
  O (8.006)
  =9
- 0.9 seconds

- 0.09 seconds

158) Which one is the best algorithm from the following with respect to running time?

- Θ (n)
- Θ(n^2) repeated mcqs
- Θ(n log n)
- Θ(n^3)

159) Which of the following is true?

- The worst case time complexity of the quick sort is NlogN
- The best time complexity of the merge sort is logN
- the worst case time complexity of the selection sort is N^2 AS REPEATED IN MANY MCQS. For example Θ is any number which is N so its complexity is its square
- the worst case time complexity of the merge sort is logN

160) We analuze an algorithm to check

It's running time

$8n2 \quad n$

161) What is the asymptotic growth of

$3 \quad 5$

(n2)

162) What will be result of the following recurrence relation?

$4 \qquad \text{if } n \ 1$

$T\,n \quad \{ \quad 2$

$T\,n \quad /5 \quad 3n \quad \text{if } n \text{ isdivisible by } 5$

then $T \quad 5$ *is*

75

163) Divide and conquer involves breaking the problem into a small number of

Sub Problems

164) The sieve technique is a special case where the number of sub problems is just

1

165) When we call heapify then at each level the comparison performed takes time

It will take o(1)

166) In quick sort we don't have the control over the sizes of recursion calls

True

The running time of quick sort depends heavily on the selction of

Pivot

167) Any binary tree of height T(n) has at most $2T\,n$ leaves. Thus a comparison base sorting algorithm can distinguish between at most _____different final outcomes.

2_{Th}

168) If there are o($n2$) entries in edit distance matrix then each entry E (i,j) takes _____ time to compute.

o(1)

169) If there are o($n2$) entries in edit distance matrix then the total running time is >>>>>> o($n2$)

170) When a recursive algorithm revisits the same problem over and over again, we say that the optimization problem has ___>>>>>

Overlapping

171) A problem exhibits optimal structure if an optimal solution to the problem contains with in it optimal solution to____

Sub problems

172) The Knapsack problem belongs to the domain of _____ problems

Optimization

173) Computational model of sequential RAM is

Less powerful computation wise then parallel machines

174) Merge sort require

Extra Storage

175) The worst case running time of the algorithm given below is

```
MAXIMA (int n, point P[1...n])
1  For i  1 to n
2  do maximal  true
3  do
4  if  i  j  &  p i x  . p j x  .  &  P i y  . p j y  .
5  Then maximal  false break
6  if maximal
7  then output P[i].x, P[i].y
```

n^2

176) Which one is the correct conversion of the given algorithm to sigma notation?

```
1  For i  1 to n
2  do maximal  true
3  for j  1 to n
4  do
5  if  i  j  &  p i x  . p j x  .  &  P i y  . p j y  .
6  Then maximal  false break
7  if maximal
8  then output P[i].x, P[i].y
```

$\sum_{i=1}^{n} 4n - 2$

177) when this relation will be true? F(n)= O(f(n)2)

Always True

178) when of the following function grows fastest as n grows larger?

n^2

Cs502 Solved MCQS for Final Term Including Current Quizzes July 2013

Question

Bellman Ford Algorithm applies relaxations to every _____

Vertex of ...

Edge of Graph and repeats this V-1 times (p159)

Question

Floyd-Warshall Algorithm is _____

Based on dynamic programming (p161)

Question

Running time of Floyd-Warshall Algorithm is _____

Θ n2

Θ n4

Θ n3 (p164)

3n

Question

Space used by Floyd-Warshall Algorithm is _____

Θ n2 (p164)

Θ n4

Θ n3

3n

Question

Huffman encoding Algorithm is a ___

Greedy Algorithm (p99)

Dynamic and Greedy

Question

Graphs cannot be traversed by brute-force technique.

True (p117) False

Question

Brute-force technique is always optimal for Graph traversal.

True False (p117)

Question

For graph traversal, Breadth-first search strategy _____ is always recursive Cannot be recursive

Cannot be non-recursive

Can be both recursive and non-recursive

Question

Greedy algorithm can do very poorly for some problems.

True (p97) False

Question

A greedy algorithm sometimes works well for optimization problems.

True (p97) False

Question

A greedy algorithm does not work in phases.

True False (p97)

Question

Greedy algorithm approach solves both 0/1 Knapsack and Fractional Knapsack problems.

True False

Question

Dynamic Programming approach solves both 0/1 Knapsack and Fractional Knapsack problems.

True False

Question

In _____ based solution of knapsack problem, we consider 2 cases. Leave object Or Take object.

Brute force

Dynamic programming (p93)

Question

In brute force based solution of knapsack problem, we consider 2 cases. Leave object Or Take object.

True False (p93)

Question

In DP based solution of knapsack problem, V[0, j] = _____ if j>=0 -1
0 (p93)

1

2

Question

In DP based solution of knapsack problem, V[0, j] = 1 if j>=0

True

False (p93)

Question

In DP based solution of knapsack problem, to compute entries of V we will imply a/an _____ approach. Subjective

Inductive (p93)

Brute force

Combination

Question

In DP based solution of knapsack problem, if we decide to take an object i, then we gain a value of ____. w i-1
vi-1 w i
vi (p93)

Question

In Fractional Knapsack problem, one is allowed to take fraction of an item for _____ for
fraction of the weight for fraction of the value
Both, fraction of the weight and value (p109)

None

Question

_____ approach is optimal for the fractional knapsack problem.

Divide and Conquer

Dynamic Programming

Greedy algorithm (p110)

Brute force

Question

In ___ Knapsack Problem, limitation is that an item can either be put in the bag or not-fractional items are not allowed.

0 1 0/1 Fractional

Question

In Fractional Knapsack problem, the goal is to _____

minimize the value of items without exceeding the total weight limit of W

maximize the value of items without exceeding the total weight limit of W (p109)
minimize the value of items even if exceeding the total weight limit of W maximize the
value of items even if exceeding the total weight limit of W

Question

In solving Fractional Knapsack problem, we do not need sorting. True
False (p109)

Question

Suppose that a graph G = (V,E) is implemented using adjacency lists. What is the complexity of a breadth-first traversal of G? Select correct option:

O(|V |^2)

O(|V | |E|)

O(|V |^2|E|)

O(|V | + |E|) (p116)

Question

For traversing graphs, Breadth-first search can be visualized as a wave front propagating inwards towards root (or source) node. True
False (p117)

Question

Breadth-first search is not a traversal strategy for graphs.

True

False (p119)

Question

Breadth-first search is not a popular algorithm technique used for traversing graphs. True
False (p119)

Question

Breadth-first search begins at a root node and inspects all the nodes except neighboring ones. True
False (117)

Question

Each time we traverse graph by Breath-first search algorithm, we count the distance from _____

Starting node (p117)
neighbors of the starting node
right most node left most node

Question

Suppose we are given a set S = {a1, a2, ... an} of n activities that are to be scheduled to use some resource. Each activity ai must be started at a given start time si and ends at a given finish time fi. We want to schedule the largest number of activities on the resource. It is an example of _____.

Searching

Dynamic Programming

Activity selection (p105)

Divide and Conquer

Question

In general, the Activity selection problem is to select a _____ minimum-size set of interfering activities
maximum-size set of mutually non-interfering activities (p105) maximum-size set of interfering activities

minimum-size set of mutually non-interfering activities

Question

In Activity selection (using Greedy approach), intuitively _____

Short activities are not preferable

There are always short activities as input

We do not like long activities (p105)

It does not matter about the length of activities

Question

The activity schedule is a simple scheduling problem for which the ___ approach provides an optimal solution.
Dynamic Programming

Graph

Divide and Conquer

Greedy Algorithm (p105)

Question

The activity scheduling is a simple scheduling problem for which the greedy algorithm approach provides a/an _____solution.

Simple

Sub optimal

Optimal (p105)

Non optimal

Question

A number of lectures are to be given in a single lecture hall. Optimum scheduling for this is an example of Activity selection.

True (p105)

False

Question

In Activity scheduling algorithm, each activity is represented by a _____ Circle

Square

Triangle

Rectangle (p106)

Question

In Activity scheduling algorithm, the width of a rectangle _____ Is always
ignored

Directs towards recursion

Should be maximized

Indicates the duration of an activity (p106)

Question

In Activity scheduling algorithm, the time is dominated by sorting of the activities by _____ start times
finish times (p106)

Question

The complexity of Activity scheduling algorithm is _____

O (n)

O log n

O (n) log n (p106)

O n^2

Question

In Activity scheduling algorithm, as base case if there are no activities then Greedy algorithm ____
cannot be optimized is solved using Recursion
is transformed into Dynamic Programming is
trivially optimal (p109)

Question # Bag is a _____ type of algorithm
data structure (p119)
program compiler
Question #
For _____ problems, greedy approach always gets optimum. Select correct option:

Two

Some (p97)

All

None of the

Question

Fixed-length codes may not be efficient from the perspective of _____the total quantity of data. Minimizing (p99)

Averaging

Maximizing

Summing

Question

There are _____ nested loops in DP based algorithm for computing the minimum cost of chain matrix multiplication.

2

3 (p90)

4

5

Question

Time complexity of DP based algorithm for computing the minimum cost of chain matrix Multiplication is ----. log n n n2 n3 (p90)

Question

In greedy algorithm, at each phase, you take the_____ you can get right now, without regard for future consequences.

Worst Minimum Good Best (p97)

Question

An optimization problem is one in which you want to find, not just a solution, but the _____ solution.

Simple Good Best (p97) Worst

Question

In _____ algorithm, you hope that by choosing a local optimum at each step, you will end up at a global optimum.

Simple Non-Greedy. Greedy (p97) Brute force

Question

The string |xyz|, if coded with ASCII code, the message length would be 24 bits.

True False

Question

If the string |lmncde| is coded with ASCII code, the message length would be --- bits.

24 36 48 60

Question

If there are n items, there are _____ possible combinations of the items.

2 n 2n (p92) 3n

Question

In Huffman encoding, for a given message string, the frequency of occurrence (relative probability) of each character in the message is determined last.

True False (p100)

Question

In DP based solution of knapsack problem, to compute entries of V we will imply a/an _____ approach.

Subjective Inductive (p93) Brute force Combination

Question

Inductive approach to compute entries of V is implied in _____ based solution of knapsack problem.

Brute force Dynamic programming (p93)

Question

In brute force based solution of knapsack problem, we consider 2 cases, Leave object Or Take object.

True (p93) False

Question

There are _____ nested loops in DP based algorithm for computing the minimum cost of chain matrix multiplication.

2

3

4

5 (not confirm page 90)

Question

Those problems in which Greedy finds good, but not always best is called a greedy_____. Algorithm

Solution

Heuristic (p97)

Result

Question

The Huffman codes provide a method of _____ data efficiently. Reading
Encoding (p99)

Decoding

Printing

Question

Fixed-length codes are known for easy break up of a string into its individual characters.

True (p99) False

Question

The Huffman codes provide a method of _____ data efficiently.

Reading Encoding (p99) Decoding Printing

Question

In Huffman encoding, the _____is the number of occurrence of a character divided by the total characters in the message.

Counting

Parsing

Probability (p100)

Weight

Question

In Huffman encoding, for a given message string, the frequency of occurrence (relative probability) of each _____in the message is done first.

Word Space Digit Character (p100)

Question

The Huffman algorithm finds a (n) _____ solution.

Optimal (Reference) Non-optimal Exponential Polynomial

Question #

The Huffman algorithm finds an exponential solution True False (Reference)

Question

The Huffman algorithm finds a polynomial solution True False (Reference)

Prefix Property:
The codewords assigned to characters by the Huffman algorithm have the property that no codeword is a prefix of any other:

Question

Non-optimal or greedy algorithm for money change takes_____

O(k) (p99) O(kN) O(2k) O(N)

Question

The Huffman codes provide a method of encoding data inefficiently when coded using ASCII standard.

True False (p99) ○ The Huffman codes provide a method of encoding data efficiently.

Question

The codeword assigned to characters by the Huffman algorithm have the property that no codeword is the postfix of any other. True False

Question

The codeword assigned to characters by the Huffman algorithm have the property that no codeword is the prefix of any other. True (p101) False

Question

Using ASCII standard the string “abacdaacacwe” will be encoded with _____ bits Select correct option:

64

128

96 (p101 12x8=96)

120

Question

Using ASCII standard the string abacdaacac will be encoded with _____ bits.

80 (p99) 160 320 100 ○ Consider the string " abacdaacac". if the string is coded with ASCII codes, the message length would be ○ 10 × 8 = 80 bits.

Question

Using ASCII standard the string abacdaacac will be encoded with 160 bits. True False (p99)

Question #

Using ASCII standard the string abacdaacac will be encoded with 320 bits. True False (p99)

Question #

Using ASCII standard the string abacdaacac will be encoded with 100 bits. True False (p99)

Question #

Using ASCII standard the string abacdaacac will be encoded with 32 bytes. True False (p99)

Question

The greedy part of the Huffman encoding algorithm is to first find two nodes with smallest frequency.

True (p100) False

Question

The greedy part of the Huffman encoding algorithm is to first find two nodes with character frequency.
True False (p100)

Question

The greedy part of the Huffman encoding algorithm is to first find two nodes with larger frequency. True
False

Question

Huffman algorithm uses a greedy approach to generate an antefix code T that minimizes the expected length B (T) of the encoded string. True False (p102)

Question

Huffman algorithm uses a greedy approach to generate a postfix code T that minimizes the expected length B (T) of the encoded string. True False (p102)

Question

Huffman algorithm uses a greedy approach to generate a prefix code T that minimizes the expected length B (T) of the encoded string. True (p102) False

Question

Huffman algorithm generates an optimum prefix code.

True (p102) False

Question

A graph is said to acyclic if it contains _____ at
least one cycle exactly one cycle
always more than one cycle no
cycle (p116)

Question

Graph is not a good way to model some sort of "connection" or "relationship" or "interaction" between pairs of objects taken from a set of objects.

True False (p113)

Question

A graph is not connected if every vertex can reach every other vertex.

True False (p116)

Question

Traversing a graph means visiting _____ in the graph. no node
at least one more than
one node

every node (p119)

Question

Graphs are important _____ model for many application problems.

Unsystematic

Mathematical (p113)

Haphazard

Unpredictable

Question

In a digraph, the number of edges coming in of a vertex is not called the in-degree of that vertex.

True False (p114)

Question

In a digraph, the number of edges coming out of a vertex is not called the out-degree of that vertex.

True False (p114)

Question

For a digraph G = (V, E), Sum of in-degree(v) _____

Not equal to Sum of out-degree(v)

= Sum of out-degree(v) (p115)

< Sum of out-degree(v)

> Sum of out-degree(v)

Question

Graphs cannot be represented by an adjacency matrix.

True False (p116)

Question

A vertex a is not adjacent to vertex b if there is an edge from a to b.

True False (p113)

Question

A dense undirected graph is:

Select correct option:

A graph in which E = O (V^2) (Reference)

A graph in which E = O (V)

A graph in which E = O (log V)

All items above may be used to characterize a dense undirected graph

Question

What algorithm technique is used in the implementation of Kruskal solution for the MST?

Greedy Technique (p142)

Divide-and-Conquer Technique

Dynamic Programming Technique

The algorithm combines more than one of the above techniques

Question

A digraph is strongly connected under what condition?

A digraph is strongly connected if for every pair of vertices u, v e V, u can reach v .

A digraph is strongly connected if for every pair of vertices u, v e V, u can reach v and vice versa. (p135) A digraph is strongly connected if for at least one pair of vertex u, v e V, u can reach v and vice versa.

A digraph is strongly connected if at least one third pair of vertices u, v e V, u can reach v and vice versa.

Question

The relationship between number of back edges and number of cycles in DFS is, Both are equal

Back edges are half of cycles

Back edges are one quarter of cycles

There is no relationship between no. of edges and cycles (p131)

Question # Back edge is?

(u, v) where v is an ancestor of u in the tree. (p128) (u,v) where u is an ancestor of v in the tree.

(u, v) where v is an predecessor of u in the tree.

None of above

Question

Forward edge is?

(u, v) where u is a proper descendent of v in the tree.

(u, v) where v is a proper descendent of u in the tree. (p129) (u, v) where v is a proper ancestor of u in the tree.

(u, v) where u is a proper ancestor of v in the tree.

Question

Edge (u, v) is a forward edge if u is a proper descendant of v in the tree v is a proper descendant of u in the tree

None of these

Question # Cross edge is?

(u, v) where u and v are not ancestor of one another

(u, v) where u is ancestor of v and v is not descendent of u.

(u, v) where u and v are not ancestor or descendent of one another (p129) (u, v) where u and v are either ancestor or descendent of one another.

Question

If you find yourself in maze the better traversel approach will be? Select correct option:

BFS

BFS and DFS both are valid (p119)

Level order

DFS

Question

In digraph G=(V,E) ;G has cycle if and only if Select correct option:

The DFS forest has forward edge.

The DFS forest has back edge (p131)

The DFS forest has both back and forward edge

BFS forest has forward edge

Question

What is generally true of Adjacency List and Adjacency Matrix representations of graphs?

Select correct option:

Lists require less space than matrices but take longer to find the weight of an edge (v1,v2)

Lists require less space than matrices and they are faster to find the weight of an edge (v1, v2)

Lists require more space than matrices and they take longer to find the weight of an edge (v1, v2) Lists require more space than matrices but are faster to find the weight of an edge (v1, v2)

Question

There is relationship between number of back edges and number of cycles in DFS Select correct option:

Both are equal.

Cycles are half of back edges.

Cycles are one fourth of back edges.

There is no relationship between back edges and number of cycle (p131)

Question

You have an adjacency list for G, what is the time complexity to compute Graph transpose G^T.? Select correct option:

(V+E) (Reference)

V.E V

E

Question

Although it requires more complicated data structures, Prim's algorithm for a minimum spanning tree is better than Kruskal's when the graph has a large number of vertices.

True False

Question

Maximum number of vertices in a Directed Graph may be $|V^2|$

True False

Question

Dijkstra's algorithm:

Has greedy approach to find all shortest paths

Has both greedy and dynamic approach to find all shortest paths

Has greedy approach to compute single source shortest paths to all other vertices (p154)

Has both greedy and dynamic approach to compute single source shortest paths to all other vertices.

Question

Dijkstra's single source shortest path algorithm works if all edges weights are non-negative and there are negative cost cycles. True False

Question

here re no Dijkstra s single source shortest path algorithm works if all edges weights are non-negative and negative cost cycles. True (p159) False

Question

Dijkstra s single source shortest path algorithm works if all edges weights are negative and there are no negative cost cycles.

True False

Question

Dijkstra's algorithm is operates by maintaining a subset of vertices. True
False

Question

Floyd-Warshall algorithm is a dynamic programming algorithm; the genius of the algorithm is in the clever recursive formulation of the shortest path problem. True (p162) False

Question

Floyd-Warshall algorithm, as in the case with DP algorithms, we avoid recursive evaluation by generating a table for $d_{ij(k)}$

True (p164) False

Question #

The term coloring came from the original application which was in map drawing. True (p173) False

Question #

The term "coloring" came from the original application which was in architectural design. True False

Question

In the clique cover problem, for two vertices to be in the same group, they must be _____ each other. Apart from Far from Near to Adjacent to (p176)

Question

In the clique cover problem, for two vertices to be in the same group, they must be adjacent to each other.

True (p176) False

Question

In the clique cover problem, for two vertices to be in the same group, they must be apart from each other.

True False (p176)

Question

The difference between Prims algorithm and Dijkstra s algorithm is that Dijkstra s algorithm uses a different key.

True (p156) False

Question

The difference between Prim s algorithm and Dijkstra s algorithm is that Dijkstra s algorithm uses a same key.
True False (p156)

Question

You have an adjective list for G, what is the time complexity to computer graph transpose G^T.? (V + E
) (p138)

Question

Given an adjacency list for G, it is possible to compute G T in Θ(V + E) time. It takes
O(log V) to extract a vertex from the priority queue.

Question #

Overall time for Kruskal is Θ(E log E) = Θ(E log V) if the graph is sparse. True (p149)

Question

An optimization problem is one in which you want to find,

Not a solution

An algorithm

Good solution

The best solution (p97)

Question

Although it requires more complicated data structures, Prim's algorithm for a minimum spanning tree is better than Kruskal's when the graph has a large number of vertices.

True False

Question

If a graph has v vertices and e edges then to obtain a spanning tree we have to delete
v edges.
v – e + 5 edges v
+ e edges. None of
these

Question

Bellman-Ford allows negative weights edges and negative cost cycles.

True False (p159)
○ Bellman-Ford allows negative weights edges and no negative cost cycles.

Question # We do
sorting to,
keep elements in random positions keep
the algorithm run in linear order keep the
algorithm run in (log n) order
keep elements in increasing or decreasing order (p39)

Question

After partitioning array in Quick sort, pivot is placed in a position such that

Values smaller than pivot are on left and larger than pivot are on right (Ref) Values larger than pivot are on left and smaller than pivot are on right

Pivot is the first element of array

Pivot is the last element of array

Question

Dynamic programming algorithms need to store the results of intermediate sub-problems. True
False

Question

Which statement is true?

If a dynamic-programming problem satisfies the optimal-substructure property, then a locally optimal solution is globally optimal.

If a greedy choice property satisfies the optimal-substructure property, then a locally optimal solution is globally optimal.

Both of above None of above

Question

What general property of the list indicates that the graph has an isolated vertex?

There is Null pointer at the end of list.

The Isolated vertex is not handled in list.

Only one value is entered in the list.

There is at least one null list.

Question

Depth first search is shortest path algorithm that works on un-weighted graphs.
True False (p153) ○ The breadth-first-search algorithm is a shortest-path algorithm that works on un-weighted graphs.

Question

Which is true statement?

Select correct option:

Breadth first search is shortest path algorithm that works on un-weighted graphs

(p153) Depth first search is shortest path algorithm that works on un-weighted graphs. Both of above are true.

None of above are true.

Question

Shortest path problems can be solved efficiently by modeling the road map as a graph.

True False

Question

Which is true statement in the following?

Kruskal algorithm is multiple source technique for finding MST.

Kruskal's algorithm is used to find minimum spanning tree of a graph, time complexity of this algorithm is O(EV) Both of above

Kruskal's algorithm (choose best non-cycle edge) is better than Prim's (choose best tree edge) when the graph has relatively few edges.

Question

Kruskal's algorithm (choose best non-cycle edge) is better than Prim's (choose best tree edge) when the graph has relatively few edges.

True False

Question

What is the time complexity to extract a vertex from the priority queue in Prim's algorithm? Select correct option:

O (log E)

(V)

(V+E)

O (log V) (p152)

Question

In Prim's algorithm, the additional information maintained by the algorithm is the length of the shortest edge from vertex v to points already in the tree.

a) True b) False c) unknown

Question

Although it requires more complicated data structures, Prim's algorithm for a minimum spanning tree is better than Kruskal's when the graph has a large number of vertices.

a) True b) False c) unknown

Question

Which statement is true?

(i) The running time of Bellman-Ford algorithm is T (VE)

(ii) Both Dijkstra's algorithm and Bellman-Ford are based on performing repeated relaxations

(iii) The 0-1 knapsack problem is hard to solve

Only i Only iii Both i and iii All of these

Question

Which of the following arrays represent descending (max) heaps?

i. [10,7,7,2,4,6] ii. [10,7,6,2,4,7] iii. [10,6,7,2,4,6] iv. [6,6,7,2,4,10]

Only ii Only iv Both ii and iv Both i and iii

Question

Which of the following statement(s) is/are correct?

(a) O(n log n + n2) = O(n2). (b) O(n log n + n2) = O(n2 log 2n)

(c) O(c n2) = O(n2) where c is a constant. (d) O(c n2) = O(c) where c is a constant. (e) O(c) = O(1) where c is a constant.

Only (a) & (e) Both (c) and (e)

Question

Which of the shortest path algorithms would be most appropriate for finding paths in the graph with negative edge weights and cycles?

i. Dijkstra's Algorithm ii. Bellman-Ford Algorithm.

iii. Floyd Warshall Algorithm

Only ii Only iii

Both ii & iii

Question

If, in a DFS forest of digraph G = (V, E), f[u] = f[v] for an edge (u, v) ? E then the edge is called Back edge Forward edge Cross Edge Tree Edge None of these

Question #

Best and worst case times of an algorithm may be same. True False

Question #

Can an adjacency matrix for a directed graph ever not be square in shape? Yes No

Question

Consider the following Huffman Tree. The binary code for the string TEA is:

10 00 010

011 00 010

10 00 110

11 10 110

String	Encoding
TEA	10 00 010
SEA	011 00 010
TEN	10 00 110

Question

Can an adjacency matrix for a directed graph ever not be square in shape? Yes No ○ No. since we want to describe the relationship between each node and each other node, we need precisely n^2 matrix entries.

Question

Suppose we have two problems A and B .Problem A is polynomial-time reducible and problem B is NP-complete. If we reduce problem A into B then problem A becomes NP-complete Yes No

Question

If a problem is NP-complete, it must also be in NP.

a) True b) False c) unknown

Question

If a problem is in NP, it must also be in P.

False (p178) True unknown

Question

If a problem is in NP-complete, it must also be in NP.

True False

Cs502 Solved MCQS for MidTerm

Including Current Quizes

26 May 2013

Question # 1

Word Algorithm comes from the name of the Muslim author:

Abu Ja'far Mohammad ibn Musaal-Khowarizmi. (p7)

Question # 2

Al-Khwarizmi's work was written in a book titled

al Kitab al-mukhatasar fi hisab al-jabr wa'l-muqabalah (p7)

Question # 3

Analysis of Selection algorithm ends up with, Select correct option:

T(n) (p37) T(1 / 1 + n) T(n / 2) T((n / 2) + n)

Question # 4

The function f(n)= n(logn+1)/2 is asymptotically equivalent to n log n. Hre Upper Bound means the function f(n) grows asymptotically _____ faster than n log n.

Select correct option:

More Quiet Not (p24) At least

Question # 5

The running time of an algorithm would not depend upon the optimization by the compiler but that of an implementation of the algorithm would depend on it.

Select correct option:

True False (p13)

Question # 6

After sorting in merge sort algorithm, merging process is invoked.

Select correct option:

True (p27) False

Question # 7

Asymptotic growth rate of the function is taken over_____ case running time. Select correct option:

Best

Average

Worst (p14)

Normal

Question # 8

In analysis of f(n)=n(n/5)+n-10 log n, f(n) is asymptotically equivalent to _____.

n 2n
n+1
n2 (p23)

Question # 9 f(n) and g(n) are asymptotically equivalent. This means that they have essentially the same _____ for large n.

Results Variables Size Growth rates (p23)

Question # 10

Algorithm is concerned with ____ issues.

Macro Micro 1 Both Macro & Micro (p8) Normal

Question # 11

We cannot make any significant improvement in the running time which is better than that of brute-force algorithm.

True False (p18, p92)

Question # 12

In addition to passing in the array itself to Merge Sort algorithm, we will pass in _____other arguments which are indices.

Two (p28) Three Four Five

Question # 13

For the worst-case running time analysis, the nested loop structure containing one "for" and one "while" loop, might be expressed as a pair of _____nested summations.

1 2 (p14) 3 4

Question # 14

The O-notation is used to state only the asymptotic _____bounds.

Two Lower Upper (p25) Both lower & upper

Question # 15

Which of the following is calculated with big O notation?

Lower bounds Upper bounds (p25)

Both upper and lower bound Medium bounds

Question # 16

The Ω-notation allows us to state only the asymptotic _____ bounds.

Lower (p25) Upper

Both upper and lower bound Medium bounds

Question # 17

The time assumed for each basic operation to execute on RAM model of computation is ____

Infinite Continuous Constant (p10) Variable

Question # 18A

If the indices passed to merge sort algorithm are not equal, the algorithm may return immediately.

True False (p28)

Question #18B

If the indices passed to merge sort algorithm are _____,then this means that there is only one element to sort.

Small Large Equal (p28) Not Equal

Question # 19

Brute-force algorithm uses no intelligence in pruning out decisions.

True (p18) False

Question # 20

In analysis, the Lower Bound means the function grows asymptotically at least as fast as its largest term.

True (p24) False

Question # 21

In analysis, the Upper Bound means the function grows asymptotically no faster than its largest term.

True (p24) False

Question # 22

For small values of n, any algorithm is fast enough. Running time does become an issue when n gets large.

True (p14) Fast

Question # 23

The array to be sorted is not passed as argument to the merge sort algorithm.

True False (p28)

Question # 24

In simple brute-force algorithm, we give no thought to efficiency.

True (p11) False

Question # 25

The ancient Roman politicians understood an important principle of good algorithm design that is plan-sweep algorithm.

True False (p27) [Divide and Conquer]

Question # 26

In 2d-space a point is said to be _____if it is not dominated by any other point in that space.

Select correct option:

Member Minimal Maximal (p11) Joint

Question # 27

An algorithm is a mathematical entity that is dependent on a specific programming language. Select correct option:

True False (p7)

Question # 28

Fibonacci sequence was posted by ____

Leonardo Pisano (p73)

Question # 29

$8n^2 + 2n - 3$ will eventually exceed $c_2(n)$ no matter how large we make c2.

True (p25) False

Question # 30

If we associate (x, y) integers pair to cars where x is the speed of the car and y is the negation of the price. High y value for a car means a _____ car.

Fast Slow Expensive Cheap (p11)

Question # 31

The running time of quick sort depends heavily on the selection of:

No of inputs Arrangement of elements in array

Size of elements Pivot element (p49)

Question # 32

Which sorting algorithm is faster?

O(n^2) O(nlogn) O(n+k) (p58) O(n^3)

Question # 33

Q knapsack problem is called a "0-1" problem, because

Each item must be entirely accepted or rejected (p92)

Question # 34

Which statement is true?

Select correct option:

If a dynamic-programming problem satisfies the optimal-substructure property, then a locally optimal solution is globally optimal.

If a greedy choice property satisfies the optimal-substructure property, then a locally optimal solution is globally optimal.

Both of above (p77, p98) None of above

Question # 35

Merge sort is stable sort, but not an in-place algorithm

True (p54) False

Question # 36

In counting sort, once we know the ranks, we simply _____ numbers to their final positions in an output array.
Delete copy (p57) Mark arrange

Question # 37

Dynamic programming algorithms need to store the results of intermediate sub-problems. True (p75) False

Question # 38

A p × q matrix A can be multiplied with a q × r matrix B. The result will be a p × r matrix C. There are (p . r) total entries in C and each takes _____ to compute.

O (q) O (1) (p84) O (n2) O (n3)

Question # 39

Due to left-complete nature of binary tree, heaps can be stored in

Link list Structure Array (p40) None of above

Question # 40

Efficient algorithm requires less computational ____

Memory Running Time Memory and Running Time (p9) Energy

Question # 41

Merge sort makes two recursive calls. Which statement is true after these recursive calls finish, but before the merge step?

The array elements form a heap

Elements in each half of the array are sorted amongst themselves

Elements in the first half of the array are less than or equal to elements in the second half of the array

None of the above

Question # 42

Who invented Quick sort procedure?

Hoare Sedgewick Mellroy Coreman

Question # 43

Merge sort requires extra array storage. True (p54) False o Mergesort is a stable algorithm but not an in-place algorithm. It requires extra array storage.

Question # 44

In which order we can sort?

Increasing order only decreasing order only

Increasing order or decreasing order both at the same time

Question # 45

In the analysis of Selection algorithm, we make a number of passes, in fact it could be as many as,
T(n) T(n / 2) log n (p37) n / 2 + n / 4

Question # 46

How much time merge sort takes for an array of numbers?

T(n^2) T(n) T(log n) T(n log n) (p40)

Question # 47A

Sieve Technique applies to problems where we are interested in finding a single item from a larger set of ___ n items (p34) phases pointers constant

Question # 47B

The sieve technique works in phases as follows. It applies to problems where we are interested in finding a single item from a larger set of n items. (p34)

Question # 47C

The sieve technique works where we have to find _____ item(s) from a large input.

Single (p34) Two Three Similar

Question # 48

The sieve technique works in _____ as follows

Phases (p34) numbers integers routines

Question # 49

For the heap sort, access to nodes involves simple _____ operations.

Arithmetic (p41) binary algebraic logarithmic

Question # 50

The analysis of Selection algorithm shows the total running time is indeed _____in n,

Arithmetic geometric linear (p39) orthogonal

Question # 51

Slow sorting algorithms run in,

T(n^2) (p39) T(n) T(log n) T(n log n)

Question # 52

A heap is a left-complete binary tree that conforms to the

Increasing order only decreasing order only heap order (log n) order

Question # 53

For the heap sort we store the tree nodes in

Level-order traversal (p40) in-order traversal pre-order traversal post-order traversal

Question # 54A

In Sieve Technique we do not know which item is of interest True (p34) False

Question # 54B

In Sieve Technique, we know the item of interest.

True False (p34)

Question # 55

Divide-and-conquer as breaking the problem into a small number of
Pivot Sieve smaller sub problems (p34) Selection

Question # 56

For the sieve technique we solve the problem,

Recursively (p34) mathematically precisely accurately

Question # 57

The reason for introducing Sieve Technique algorithm is that it illustrates a very important special case of, Divide-and-conquer (p34) decrease and conquer greedy nature 2-dimension Maxima

Question # 58

Sorting is one of the few problems where provable ____ bonds exits on how fast we can sort, Upper
Lower (p39) Average log n

Question # 59

Sieve Technique can be applied to selection problem? True (p35) False

Question # 60

One of the clever aspects of heaps is that they can be stored in arrays without using any_____ Pointers (p40) constants variables functions

Question # 61

Heaps can be stored in arrays without using any pointers; this is due to the ___ nature of the binary tree, Left-complete (p40) right-complete tree nodes tree leaves

Question # 62

How many elements do we eliminate in each time for the Analysis of Selection algorithm? n / 2 elements (p36) (n / 2) + n elements n / 4 elements 2 n elements

Question # 63

The sieve technique is a special case, where the number of sub problems is just

5 Many 1 (p34) few

Question # 64

For the Sieve Technique we take time

T(nk) (p34) T(n / 3) n^2 n/3

Question # 65

The number of nodes in a complete binary tree of height h is Select correct option:

2^(h+1) – 1

2 * (h+1) – 1

2 * (h+1)

((h+1) ^ 2) – 1

Question # 66

Random access machine or RAM is a/an ____

Machine build by Al-Khwarizmi Mechanical machine

Electronics machine Mathematical model (p10)

Question # 67

Analysis of Selection algorithm ends up with,

T(n)

T(1 / 1 + n)

T(n / 2)

T((n / 2) + n)

Question # 68

Continuation sort is suitable to sort the elements in range 1 to k

1. K is Large 2. K is not known 3. K may be small or large 4. K is small (p57)

Question # 69

Counting sort is suitable to sort the elements in range 1 to k:

K is large K is small (p57) K may be large or small None

Question # 70

Continuing sort has time complexity of?

1. O (n) (p58) 2. O(n+k) 3. O(nlogn)

Question # 71

Counting sort has time complexity:

O (n) (p58) O(n+k) O(k) O(nlogn)

Question # 72

Memoization is ____:

-To store previous results for further use.

-To avoid unnecessary repetitions by writing down the results of recursive calls and looking them again if needed later. (p74)

-To make the process accurate -None of the above

Question # 73

In Quick sort algorithm, constants hidden in T(n lg n) are:

Large Medium Not known Small (Ref)

Question # 74

Quick sort is based on divide and conquer paradigm; we divide the problem on base of pivot element and:

There is explicit combine process as well to conquer the solution. No work is needed to combine the sub-arrays, the array is already sorted

Merging the subarrays None of above.

Question # 75

In RAM model instructions are executed ____

In Random Access Machine, instructions are executed____

One after another (p10) Parallel Concurrent Random

Question # 76

In the analysis of Selection algorithm, we eliminate a constant fraction of the array with each phase; we get the convergent _____ series in the analysis,

linear arithmetic geometric (p37) exponent

Question # 77

_____ is a graphical representation of an algorithm

Σ notation Θnotation Flowchart (Ref) Asymptotic notation

Question # 78

A RAM is an idealized machine with _____ random-access memory.

256MB 512MB An infinitely large (p10) 100GB

Question # 79

What type of instructions Random Access Machine (RAM) can execute? Choose best answer

Algebraic and logic Geometric and arithmetic

Arithmetic and logic (p10) Parallel and recursive

Question # 80

What will be the total number of max comparisons if we run brute-force maxima algorithm with n elements?

n^2 (p14) 2n/n n 8n

Question # 81

What is the solution to the recurrence T(n) = T(n/2)+n .

O(logn) O(n) O(nlogn) O(n2)

Question # 82

Consider the following code:

```
For(j=1; j<n;j++)
    For(k=1; k<15;k++)
        For(l=5; l<n; l++)
        {
            Do_something_constant();
        }
```

What is the order of execution for this code.

O(n)

O(n3)

O(n2 log n)

O(n2)

Question # 83

Consider the following Algorithm: Fun(n){ if (n=1) return 1 else return (n * Fun(n-1)) } Recurrence for the above algorithm is: nT(n-1)+1 2T(n-1)+1

T(n-1)+cn

T(n-1)+1

Question # 84

Consider the following Algorithm:
Factorial (n){ if (n=1) return 1

```
else
    return (n * Factorial(n-1))  {
```

Recurrence for the following algorithm is:

T(n) = T(n-1) +1

T(n) = nT(n-1) +1

T(n)= T(n-1) +n

T(n)=T(n(n-1)) +1

Question # 85

What is the total time to heapify?

O(log n) (p43) O(n log n) O(n2 log n) O(log2 n)

Question # 86

When we call heapify then at each level the comparison performed takes time

It will take Θ (1) (p43)

Time will vary according to the nature of input data

It cannot be predicted

It will take Θ (log n)

Question # 87

In Quick sort, we don't have the control over the sizes of recursive calls

-True (p49) -False -Less information to decide -Either true or false

Question # 88

Is it possible to sort without making comparisons? Yes (p57) No

Question # 89

If there are Θ (n2) entries in edit distance matrix then the total running time is

Θ (1) Θ (n2) (p84) Θ (n) Θ (n log n)

Question # 90

For Chain Matrix Multiplication we cannot use divide and conquer approach because,

We do not know the optimum k (p86)

We use divide and conquer for sorting only

We can easily perform it in linear time

Size of data is not given

Question # 91

The Knapsack problem belongs to the domain of _____ problems.

Optimization (p91) NP Complete Linear Solution Sorting

Question # 92

Suppose we have three items as shown in the following table, and suppose the capacity of the knapsack is 50 i.e. W = 50.

Item	Value	Weight
1	60	10
2	100	20
3	120	30

The optimal solution is to pick

Items 1 and 2

Items 1 and 3

Items 2 and 3

None of these

Question # 93

Theta asymptotic notation for T (n):

Select correct option:

Set of functions described by: c1g(n)<=f(n) for c1 some constant and n=n0

Set of functions described by c1g(n)>=f(n) for c1 some constant and n=n0

Theta for T(n)is actually upper and worst case complexity of the code

Set of functions described by: c1g(n)<=f(n)<=c2g(n) for c1 and c2 some constants and n=n0

Question # 94

A (an) _____ is a left-complete binary tree that conforms to the heap order.

Heap (page 40) Binary tree Binary search tree Array

Question # 95

A heap is a left-complete binary tree that conforms to the _____

increasing order only decreasing order only heap order (p40) (log n) order

Question # 96

Consider the following Algorithm: Fun(n){ if (n=1) return 1 else return (n * Fun(n-1)) } Recurrence for the above algorithm is: Select correct option: nT(n-1)+1 2T(n-1)+1

T(n-1)+cn

T(n-1)+1

Question # 97

The recurrence relation of Tower of Hanoi is given below T(n)={1 if n=1 and 2T(n-1) if n >1 In order to move a tower of 5 rings from one peg to another, how many ring moves are required?

Select correct option: 16 10 32 (p30) 31

Question # 98

In Quick Sort Constants hidden in T(n log n) are

1. Large 2. Medium 3. Small (Reference) 4. Not Known

Question # 99

In stable sorting algorithm:

One array is used

In which duplicating elements are not handled.

More than one arrays are required.

Duplicating elements remain in same relative position after sorting. (p54)

Question # 100

In in-place sorting algorithm is one that uses arrays for storage:

An additional array

No additional array (p54)

Both of above may be true according to algorithm

More than 3 arrays of one dimension

Question # 101

Which may be a stable sort?

1. Merger 2. Insertion 3. Both above (p54) 4. None of the above

Question # 102

An in place sorting algorithm is one that uses _____ arrays for storage

1. Two dimensional arrays 2. More than one array

3. No Additional Array (p54) 4. None of the above

Question # 103

Quick sort is ____

Stable and In place Not stable but in place (p54)

Stable and not in place Sometime in place and sometime stable

Question # 104

One Example of in place but not stable sort is

Quick (p54) Heap Merge Bubble

Question # 105

Which may be stable sort?

Bubble sort Insertion sort both of above (p54)

Question # 106

Merge sort is stable sort, but not an in-place algorithm True (p54) False

A sorting algorithm is stable if duplicate elements remain in the same relative position after sorting. (p54)

Inplace (SBHIQ)	Stable (MCBI)
Selection Sort	Merge Sort
Bubble Sort	Counting Sort
Heap Sort	Bubble Sort
Insertion Sort	
Quick Sort	

Question # 107

$8n^2 + 2n - 3$ grows asymptotically at least as fast as n^2.

Question # 108

Given a set of points P = {p1, p2, . . . , pn} in 2-space, output the set of maximal points of P, i.e., those points pi such that pi is not dominated by any other point of P.
True (p11) False

Question # 109

While Sorting, the ordered domain means for any two input elements x and y _____ satisfies only. x < y
x > y x = y
All of the above (p34)

Question # 110

Brute-force algorithm for 2D-Maxima is operated by comparing _____ pairs of points.

Two Some Most All (p18)

Question # 111
Merge sort is based on ____

Brute-force Plan-sweep Axis-sweep Divide and Conqer (p27)

Question # 112

In the statement "if (P[i].x < P[j].x) & (P[i].y < P[j].y)", the number of times elements of P are accessed is ___

1 2 3 4 (p14)

Question # 113

Al-Khawarizmi was a/an....

Artist Mathematician Astronomer Khalifah

Question # 114

In Sorting the key value or attribute ___ from an ordered domain.

Must be Not always

✓ The key value need not be a number. It can be any object from a totally ordered domain. (p39)

Question # 115

In Selection problem, the Sieve technique works in ___

Non-recursive manner Constant time

Phases (p34) One complete go

Question # 116

In max heap (for Heap Sort algorithm), when every time maximum element is removed from top we replace it with ___ leaf in the tree.

Second last Last (p41) First Any

Question # 117

If input "n" is odd, the median will be ___

(n+1)/2 (p34) n/2

Question # 118

The function f(n)= n(logn+1)/2 is asymptotically equivalent to n log n. Hre Upper Bound means the function f(n) grows asymptotically _____ faster than n log n.

More Quiet Not (p23) At least

Question # 119

In asymptotical analysis of n(n - 3) and 4n*n, as n becomes large, the dominant (fastest growing) term is some constant times ___.

n+1 n-1 n
n*n (p23)

Question # 120

In asymptotical analysis of n*(5 + 2)-3 , as n becomes large, the dominant (fastest growing) term is some constant times ___.

n+1 n-1 n

n*n

Question # 121

A point p in 2-dimensional space is usually given by its integer coordinate(s)_____ Select correct option:

p.x only

p.y only

p.x & p.z

p.x & p.y (p10)

Question # 122

Algorithm analysts know for sure about efficient solutions for NP-complete problems.

True False (p9)

Question # 123

In _____ we have to find rank of an element from given input.

Select correct option:

Merge sort algorithm Selection problem (p34)

Brute force technique Plane Sweep algorithm

Question # 124

In Heap Sort algorithm, if heap property is violated _____ Select correct option:

We call Build heap procedure We call Heapify procedure (p41) We ignore

Heap property can never be violated

Question # 125

Upper bound requires that there exist positive constants c2 and n0 such that f(n) ___ c2n for all n <= n0 Select correct option:

Less than

Equal to or Less than (p25 => for all n >= n0)

Equal or Greater than

Greater than

Question # 126

A RAM is an idealized algorithm with takes an infinitely large random-access memory.

True False (p10 => A RAM is an idealized machine)

Question # 127

In simple brute-force algorithm, we give no thought to efficiency.

True (p11) False

Question # 128

_____ is one of the few problems, where provable lower bounds exist on how fast we can sort. Select correct option:

Searching Sorting (p39)

Both Searching & Sorting Graphing

Question # 129

The total no of arguments passed to merge sort algorithm is _____ 2

3 (p28 => Array itself + two indices)

4

5

Question # 130

While solving Selection problem, in Sieve technique we partition input data _____

In increasing order In decreasing order According to Pivot (p35) Randomly

Question # 131

The number of accesses to any element of space is not counted for the running time calculation of algorithm. True

False (p14)

Question # 132

In merge sort algorithm, to merge two lists of size n/2 to a list of size n, takes _____ time.

Theta (n) (p30) Theta log(n) Theta log2(n) Theta nlog(n)

Question # 133

In Heap Sort algorithm, Heapify procedure is _____ in nature.

Recursive (p43) Non-Recursive

Question # 134

In Heap Sort algorithm, we call Build-heap procedure _____

Only once Twice Thrice As many times as we need

Question # 135

In merge sort algorithm, we split the array around the _____ index q.

Entring Mid (p28) Exiting Summing

Question # 136

The Iteration method is used for _____

Comparing sorting algorithms only Solving Recurrence relations (p31)

Merging elements in Merge sort Dividing elements in Merge sort

Question # 137

Rank of an element can be defined as _____

One minus the number of elements that are smaller

Two plus the number of elements that are greater

One plus the number of elements that are smaller (p34) Two minus the number of elements that are smaller

Question # 138 f(n) is a set of functions such that there exist positive constants c1, c2 and n0 such that 0 <= c1g(n) <= f(n) <= c2g(n) for all n >= n0 Then f(n) is asymptotically _____ g(n).

Select correct option:

Less than Greater than

Equivalent to (p23) Not equivalent to

Question # 139

Floor and ceiling are _____ to calculate while analyzing algorithms.

Select correct option:

Very easy Usually considered difficult (p31)

Question # 140

In Heap Sort algorithm, the maximum levels an element can move upward is _____ Select correct option:

Theta (log n) (p43) Order (log n) Omega (log n) O (1) i.e. Constant time

Question # 141

In Heap Sort algorithm, the total running time for Heapify procedure is _____ Select correct option:

Theta (log n) Order (log n) (p43) Omega (log n) O (1) i.e. Constant time

Question # 142

In pseudo code, the level of details depends on intended audience of the algorithm.

True (p12) False

Question # 143

Counting sort assumes that the numbers to be sorted are in the range ___ 1 to k where k is small (p57)

Cs502 Quiz No1 Complete Solved File

Pseudo code of algorithms are to be read by _____.

Select correct option

Select correct option half of the input values all possible random values Pivot is input separately values greater than 5

Insertion sort is an in-place sorting algorithm.

Select correct option

True

False

Divide and Conquer

While solving Selection problem, in Sieve technique we partition input data _____

Select correct option

In increasing order

In decreasing order

According to Pivot

Randomly

Call Heapify procedure

Find the number of input elements

In Heap Sort algorithm (using max heap), when every time maximum element is removed from top _____

Select correct option

Has nothing to do with pivot selection

Is Random upon each execution

Greatly influenced by the selection of pivot

n_1

n

n+1

n*n

Select correct option

Quadratic

Linear

Geometric

Space

Execution time Inputs & outputs

Execution time and memory

f(n) ___ c2n for all n <= n0

Select correct option

Less than

Equal to or Less than

Different than Algorithms having O (order) _____ running time are considered to be slow ones.

Select correct option

n

n2

n3

Basic

Advance

Normal

The main purpose of mathematical analysis is measuring the _____required by the algorithm.

into _____

Select correct option

Equal no. of pieces as of input elements

Single piece exactly

Two nearly equal pieces

Select correct option

Comparing sorting algorithms only Solving Recurrence relations

Merging elements in Merge sort

Line

K is large

K is small

K may be large or small

Rank of an element can be defined as _____
Select correct option

One minus the number of elements that are smaller

Two plus the number of elements that are greater

2

3

4

5

In Heap Sort algorithm, the total running time for Heapify procedure is _____
Select correct option

O(n) algorithm

Stable

Complex

Al-Khwarizmi was a/an.....

Select correct option

Sorting

Both Searching & Sorting

Graphing

Recursive

Non-Recursive

While solving Selection problem, in Sieve technique we choose pivot _____

Select correct option

Middle of the array

Array (containing input elements)

Start of the array

(n+1)/2

n/2

.....of reference is an important fact of current processor technology.

Select correct option

x < y

x > y

x = y

All of the above

item(s) from a large input.

Select correct option Single

Two

n/3

n/4

The sequence of merge sort algorithm is:

Select correct option Divide-Combine-Conquer

Conquer-Divide-Combine

not

occasionally

In Heap Sort algorithm, we call Build-heap procedure _____ Select correct option

Only once

P is very large

P is undetermined f(n) and g(n) are asymptotically equivalent. This means that they have essentially the same _____ for large n.

Select correct option

Usually considered difficult

Sorting is one of the _____ problems in Computer Sciences.

Select correct option

Is quadratic

Is always Exponential

Is linear

The definition of Theta-notation relies on proving _____asymptotic bound.

One

Not

Sometimes

Sometimes not

The running time of Quick sort algorithm _____

Memory

Running Time

Memory and Running Time

Energy

We can improve the performance of Quick sort if we can be able

Very easy

Usually considered difficult

We do not need to mathematically prove that for comparisonbased sorting algorithms always takes Omega nlog(n) time.

Select correct option

Plane Sweep algorithm

In Quick sort algorithm, we choose pivot _____

Select correct option

Always smallest element

n

n*n

n3

In Heap Sort algorithm, we build _____ for ascending sort.

Select correct option

Two

Three

Four

Five

n n2

n3

1

2

3

4

Merge sort is based on_____.
Select correct option

Brute-force

Plan-sweep

2

3

4

5

In selection problem, the rank of an element will be its _____ position if we sort the input data.
Select correct option

first

final

Last

1

2

3

4

The O-notation is used to state only the asymptotic _____bounds.

Select correct option

Two

Constant time

Phases

One complete go

the running time.

Select correct option

Inputs

Outputs

Steps

Tony Hoare

In asymptotical analysis of n*(5 + 2) – 3 , as n becomes large, the dominant (fastest growing) term is some constant times _____ Select correct option

n_1

n

n+1

n*n

Yes

No

Skip input elements somehow

Select two or more pivots

Skip any sub-array completely Eliminate recursive calls

Result of asymptotical analysis of n(n - 3) and 4n*n is that _____.

Select correct option n(n-1) is asymptotically Less n(n-1) is asymptotically Greater

Both are asymptotically Not equivalent

Both are asymptotically Equivalent

Median is not useful measure of central tendency of given input set especially when the distribution of values is highly skewed.

Select correct option

True

False

In max heap (for Heap Sort algorithm), when every time maximum element is removed from top we replace it with _____ leaf in the tree.

Select correct option

Second last

Last

First

Any

In Heap Sort algorithm, every time the maximum element is removed _____

Select correct option

Do not uses Divide and Conquer approach

Eliminates undesired data items each time

Bubble sort is not an in-place sorting algorithm.

Select correct option

Mid

Selection algorithm takes theta _____

Select correct option

(n2)

(n)

False

Upper bound requires that there exist positive constants c2 and n0 such that

f(n) ___ c2n for all n <= n0

Select correct option

Equation

Symbolic

For average-case time analysis of Quick sort algorithm, Pivot selection is on average basis from _____

Select correct option

Smallest element from input

Rank of the pivot

Largest element from input

The sieve technique works where we have to find _____

Two

half of the input values

all possible random values

Pivot is input separately

values greater than 5

Sorting can be in _____

Select correct option

Increasing order only Decreasing order only

Both Increasing and Decreasing order

Quick sort is a recursive algorithm.

Select correct option

True

False

We call merge Sort algorithm It becomes Order n2 Algorithm

Divide and Conquer strategy helps us

Transitive

Non Transitive

Equation

due to _____

Select correct option

Using Decrease and Conquer strategy Avoiding to sort all input data

Eliminating Rank of an element

Largest element from input

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Cs502 Quiz No1 Complete Solved File

Pseudo code of algorithms are to be read by _____.

Select correct option

People

RAM

Computer

Compiler

Approach of solving geometric problems by sweeping a line across the plane is called _____sweep.

Select correct option

Line

Plane

Cube

Box

For average-case time analysis of Quick sort algorithm, Pivot selection is on average basis from _____

Select correct option

half of the input values

all possible random values

Pivot is input separately

Maham Shah Zell

values greater than 5

Insertion sort is an in-place sorting algorithm.

Select correct option

True

False

Heap sort is a/an and _ sorting algorithm.

Select correct option

Not in-place, not stable one

In-place, stable one

In-place, not stable one

Not in-place, stable one

Merge sort is based on_____.

Select correct option

Brute-force

Plan-sweep

Axis-sweep

Divide and Conquer

Maham Shah Zell

While solving Selection problem, in Sieve technique we partition input data _____

Select correct option

In increasing order

In decreasing order

According to Pivot

Randomly

Quick sort is a not a partitioning-based algorithm.

Select correct option

True

False

In Heap Sort algorithm, the first step is to _____

Select correct option

Call Build-Heap procedure

Sort the array in descending order

Call Heapify procedure

Maham Shah Zell

Find the number of input elements

In Heap Sort algorithm (using max heap), when every time maximum element is removed from top _____

Select correct option

We call merge Sort algorithm

It becomes Order n2 Algorithm

Divide and Conquer strategy helps us

We are left with a hole

The running time of Quick sort algorithm _____

Select correct option

Is impossible to compute

Has nothing to do with pivot selection

Maham Shah Zell

Is Random upon each execution

Greatly influenced by the selection of pivot

In asymptotical analysis of n*(5 + 2) – 3 , as n becomes large, the dominant (fastest growing) term is some constant times _____

Select correct option

n_1

n

n+1

n*n

The total running time for Selection algorithm is _____

Select correct option

Quadratic

Maham Shah Zeh

Linear

Geometric

Exponential

Quick sort is a not a partitioning-based algorithm.

Select correct option

True

False

The main purpose of mathematical analysis is measuring the _____ required by the algorithm.

Select correct option

Space

Execution time

Maham Shahzell

Inputs & outputs

Execution time and memory

Result of asymptotical analysis of n(n - 3) and 4n*n is that _____.

Select correct option

n(n-1) is asymptotically Less

n(n-1) is asymptotically Greater

Both are asymptotically Not equivalent

Both are asymptotically Equivalent

Upper bound requires that there exist positive constants c2 and n0 such that

f(n) ____ c2n for all n <= n0

Select correct option

Waham Shah Zeli

Less than

Equal to or Less than

Equal or Greater than

Greater than

Merge sort have running time.....running time of Heap sort.

Select correct option

Greater than

Less than

Equal to

Different than

Maham Shah Zell

Algorithms having O (order) _____ running time are considered to be slow ones.

Select correct option

n

n2

n3

log (n)

Boolean operation is a _____ operation on an idealized RAM model of computation.

Select correct option

Starting

Basic

Advance

Maham Shah Zell

Normal

The main purpose of mathematical analysis is measuring the _____required by the algorithm.

Select correct option

Space

Execution time

Inputs & outputs

Execution time and memory

In average-case time analysis of Quick sort algorithm, the most balanced case for partition is when we divide the list of elements into _____

Select correct option

Equal no. of pieces as of input elements

Maham Shah Zell

Single piece exactly

Two nearly equal pieces

Three nearly equal pieces

Algorithm is a sequence of computational steps that .....the input into output.

Select correct option

Merge

Assign

Transform

Integrate

The Iteration method is used for _____

Select correct option

Comparing sorting algorithms only

Maham Shah Zeil

Solving Recurrence relations

Merging elements in Merge sort

Dividing elements in Merge sort

In plane sweep approach of solving geometric problems, a _____is swept across the plane.

Select correct option

Line

Plane

Cube

Box

Counting sort is suitable to sort the elements in range 1 to k:

Select correct option

K is large

Maham Shah Zell

K is small

K may be large or small

None

In _____ we have to find rank of an element from given input.

Select correct option

Merge sort algorithm

Selection problem

Brute force technique

Plane Sweep algorithm

Rank of an element can be defined as _____

Select correct option

Maham Shah Zeil

One minus the number of elements that are smaller

Two plus the number of elements that are greater

One plus the number of elements that are smaller

Two minus the number of elements that are smaller

The total no of arguments passed to merge sort algorithm is _____

Select correct option

2

3

4

5

Maham Shah Zell

In Heap Sort algorithm, the total running time for Heapify procedure is _____

Select correct option

Theta (log n)

Order (log n)

Omega (log n)

O (1) i.e. Constant time

A sorting algorithm is called as _____ if duplicate elements remain in the same relative position after sorting.

Select correct option

Parallel

O(n) algorithm

Stable

Maham Shah Zell

Complex

Al-Khwarizmi was a/an.....

Select correct option

Artist

Mathematician

Astronomer

Khalifah

_____ is one of the few problems, where provable lower bounds exist on how fast we can sort.

Select correct option

Searching

Sorting

Maham Shah Zell

Both Searching & Sorting

Graphing

The worst case running time of Quick sort algorithm _____

Select correct option

Cannot be quadratic

Is quadratic

Is always Exponential

Is linear

In Heap Sort algorithm, Heapify procedure is _____ in nature.

Select correct option

Recursive

Maham Shah Zell

Non-Recursive

While solving Selection problem, in Sieve technique we choose pivot _____

Select correct option

Minimum element

Maximum element

Randomly

Average element

Which one is not passed as parameter in Quick sort algorithm?

Select correct option

End of the array

Middle of the array

Maham Shah Zell

Array (containing input elements)

Start of the array

In plane sweep approach, a vertical line is swept across the 2d-plane from_____.

Select correct option

Right to Left

Left to Right

Top to Bottom

Bottom to Top

If input “n” is odd, the median will be _____

Select correct option

(n+1)/2

Maham Shah Zeli

n/2

.....of reference is an important fact of current processor technology.

Select correct option

Defining

Assigning

Formality

Locality

While Sorting, the ordered domain means for any two input elements x and y _____ satisfies only.

Select correct option

x < y

x > y

Maham Shah Zell

x = y

All of the above

Counting sort is suitable for sorting the elements within range 1 to P, where _____

Select correct option

P is large

P is small

P is very large

P is undetermined

The sieve technique works where we have to find _____ item(s) from a large input.

Select correct option

Maham Shah Zeil

Single

Two

Three

Similar

While analyzing Selection algorithm, we make a number of passes, in fact it could be as many as _____

Select correct option

n(n+1)

log(n)

n/3

n/4

The sequence of merge sort algorithm is:

Select correct option

Maham Shah Zell

Divide-Combine-Conquer

Conquer-Divide-Combine

Divide-Conquer-Combine

Combine-Divide-Conquer

Quick sort is _____ based on Divide and Conquer strategy.

Select correct option

not

always

sometimes

occasionally

In Heap Sort algorithm, we call Build-heap procedure _____

Maham Shah Zell

Select correct option

Only once

Twice

Thrice

As many times as we need

Counting sort is suitable for sorting the elements within range 1 to P, where _____

Select correct option

P is large

P is small

P is very large

P is undetermined

Maham Shah Zell

f(n) and g(n) are asymptotically equivalent. This means that they have essentially the same _____ for large n.

Select correct option

Results

Variables

Size

Growth rates

Floor and ceiling are _____ to calculate while analyzing algorithms.

Select correct option

Very easy

Usually considered difficult

Maham Shah Zell

Sorting is one of the _____ problems in Computer Sciences.

Select correct option

Easiest

Solution-less

Least popular

Most popular

The worst case running time of Quick sort algorithm _____

Select correct option

Cannot be quadratic

Is quadratic

Is always Exponential

Maham Shah Zell

Is linear

The definition of Theta-notation relies on proving _____asymptotic bound.

Select correct option

One

Lower

Upper

Both lower & upper

For comparison-based sorting algorithms, it is _____ possible to sort more efficiently than Omega nlog(n) time.

Select correct option

Always

Not

Maham Shah Zell

Sometimes

Sometimes not

The running time of Quick sort algorithm _____

Select correct option

Is impossible to compute

Has nothing to do with pivot selection

Is Random upon each execution

Greatly influenced by the selection of pivot

Efficient algorithm requires less computational .....

Select correct option

Memory

Running Time

Maham Shah Zell

Memory and Running Time

Energy

We can improve the performance of Quick sort if we can be able to _____

Select correct option

Skip input elements somehow

Select two or more pivots

Skip any sub-array completely

Eliminate recursive calls

Floor and ceiling are _____ to calculate while analyzing algorithms.

Select correct option

Very easy

Usually considered difficult

Maham Shah Zell

We do not need to mathematically prove that for comparison-based sorting algorithms always takes Omega nlog(n) time.

Select correct option

True

False

In _____ we have to find rank of an element from given input.

Select correct option

Merge sort algorithm

Selection problem

Brute force technique

Plane Sweep algorithm

Maham Shah Zell

In Quick sort algorithm, we choose pivot _____

Select correct option

Always smallest element

Greater than 5

Randomly

Less than 5

The brute-force algorithm for 2D-Maxima runs in order O(___) time.

Select correct option

n

n(log n)

n*n

n3

Maham Shah Zell

In Heap Sort algorithm, we build _____ for ascending sort.

Select correct option

Max heap

Min heap

_____ provides us more accurate result when input values are not closer with each other.

Select correct option

Average

Median

In addition to passing in the array itself to Merge Sort algorithm, we will pass in _____other arguments which are indices.

Select correct option

Two

Three

Maham Shahzell

Four

Five

In Heap Sort algorithm, the first step is to _____

Select correct option

Call Build-Heap procedure

Sort the array in descending order

Call Heapify procedure

Find the number of input elements

Algorithms having O (order) _____ running time are considered to be slow ones.

Select correct option

n

Maham Shah Zell

n2

n3

log (n)

In the statement “output P[i].x, P[i].y”, the number of times elements of P are accessed is _____

Select correct option

1

2

3

4

Merge sort is based on_____.

Select correct option

Maham Shah Zeil

Brute-force

Plan-sweep

Axis-sweep

Divide and Conquer

The total no of arguments passed to merge sort algorithm is _____

Select correct option

2

3

4

5

In selection problem, the rank of an element will be its _____ position if we sort the input data.

Maham Shah Zell

Select correct option

first

final

Second last

Last

The sieve technique is a special case, where the number of sub-problems is just _____

Select correct option

1

2

3

4

Maham Shah Zell

The O-notation is used to state only the asymptotic _____bounds.

Select correct option

Two

Lower

Upper

Both lower & upper

In Selection problem, the Sieve technique works in _____

Select correct option

Non-recursive manner

Constant time

Maham Shah Zell

Phases

One complete go

We can improve the performance of Quick sort if we can be able to _____

Select correct option

Skip input elements somehow

Select two or more pivots

Skip any sub-array completely

Eliminate recursive calls

Number of _____of the pseudo code are counted to measure the running time.

Select correct option

Inputs

Maham Shah Zeli

Outputs

Steps

Pages

Quick sort algorithm was developed by _____

Select correct option

Alfred Aho

Sedgewick

John Vincent Atanasoff

Tony Hoare

In asymptotical analysis of n*(5 + 2) – 3 , as n becomes large, the dominant (fastest growing) term is some constant times _____

Maham Shah Zell

Select correct option

n_1

n

n+1

n*n

Recurrence can be described in terms of a tree.

Select correct option

Yes

No

We can improve the performance of Quick sort if we can be able to _____

Select correct option

Skip input elements somehow

Select two or more pivots

Skip any sub-array completely

Maham Shah Zell

Eliminate recursive calls

Result of asymptotical analysis of n(n - 3) and 4n*n is that _____.

Select correct option

n(n-1) is asymptotically Less

n(n-1) is asymptotically Greater

Both are asymptotically Not equivalent

Both are asymptotically Equivalent

Median is not useful measure of central tendency of given input set especially when the distribution of values is highly skewed.

Select correct option

True

False

In max heap (for Heap Sort algorithm), when every time maximum element is removed from top we replace it with _____ leaf in the tree.

Select correct option

Second last

Last

First

Any

Maham Shah Zell

In Heap Sort algorithm, every time the maximum element is removed _____

Select correct option

We call Build-Heap procedure

Heap Sort algorithm terminates without result

We call heapify procedure

Nothing happens

In Selection problem, the Sieve technique _____

Select correct option

Adds some more input items each time

Do not work recursively

Do not uses Divide and Conquer approach

Maham Shah Zell

Eliminates undesired data items each time

Bubble sort is not an in-place sorting algorithm.

Select correct option

True

False

In merge sort algorithm, we split the array around the _____ index q.

Select correct option

Entring

Mid

Exiting

Summing

Selection algorithm takes theta _____

Select correct option

Maham Shahzell

(n2)

(n)

log(n)

nlog(n)

In Sieve Technique, we know the item of interest.

Select correct option

True

False

Selection sort is not an in-place sorting algorithm.

Select correct option

True

False

Upper bound requires that there exist positive constants c2 and n0 such that

Maham Shahzell

f(n) _____ c2n for all n <= n0

Select correct option

Less than

Equal to or Less than

Equal or Greater than

Greater than

If pj dominates pi and pi dominates ph then pj also dominates ph. It means dominance relation is _____

Select correct option

Transitive

Non Transitive

Equation

Maham Shah Zell

Symbolic

For average-case time analysis of Quick sort algorithm, Pivot selection is on average basis from _____

Select correct option

half of the input values

all possible random values

Pivot is input separately

values greater than 5

In Quick sort algorithm, _____ decides nature of Binary Search Tree formed by Pivots.

Select correct option

No one

Smallest element from input

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Rank of the pivot

Largest element from input

The sieve technique works where we have to find _____ item(s) from a large input.

Select correct option

Single

Two

Three

Similar

For average-case time analysis of Quick sort algorithm, Pivot selection is on average basis from _____

Select correct option

half of the input values

all possible random values

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Pivot is input separately

values greater than 5

To find maximal points in brute-force algorithm, each point of the space is compared against _____of that space.

Select correct option

One other point

All other points

Few other points

Most of the other points

Sorting can be in _____

Select correct option

Increasing order only

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Decreasing order only

Both Increasing and Decreasing order

Random order

The worst case running time of Quick sort algorithm

_____

Select correct option

Cannot be quadratic

Is quadratic

Is always Exponential

Is linear

Quick sort is a recursive algorithm.

Select correct option

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True

False

The time assumed for each basic operation to execute on RAM model of computation is_____

Select correct option

Infinite

Continuous

Constant

Variable

In Heap Sort algorithm (using max heap), when every time maximum element is removed from top _____

Select correct option

We call merge Sort algorithm

Maham Shah Zeli

It becomes Order n2 Algorithm

Divide and Conquer strategy helps us

We are left with a hole

In Quick sort algorithm, Pivots form _____

Select correct option

Stack

Queue

Binary Search Tree

Graph

If pj dominates pi and pi dominates ph then pj also dominates ph.
It means dominance relation is _____

Select correct option

Transitive

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Non Transitive

Equation

Symbolic

The running time of Quick sort algorithm _____

Select correct option

Is impossible to compute

Has nothing to do with pivot selection

Is Random upon each execution

Greatly influenced by the selection of pivot

For solving Selection problem, we introduced Sieve technique due to _____

Select correct option

Using Decrease and Conquer strategy

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Avoiding to sort all input data

Eliminating Rank of an element

Using Brute-force approach

In Quick sort algorithm, _____ decides nature of Binary Search Tree formed by Pivots.

Select correct option

No one

Smallest element from input

Rank of the pivot

Largest element from input

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CS502 Grand Quiz

1. The sequence of merge sort algorithm is:
   a. Divide Combine-Conquer
   b. Conquer-Divide-Combine
   c. Divide-Conquer-Combine Page 27
   d. Combine-Divide-Conquer
2. In _____ Knapsack Problem, limitation is that an item can either be put in the bag or not. Fractional items are not allowed.
   a. 0
   b. 1
   c. 0/1 Page 91
   d. Fractional
3. In Selection algorithm, we assume pivot selection takes theta _____ running time.
   a. n Page - 36
   b. n2
   c. n3
   d. log (n)
4. In Heap Sort algorithm (using max heap), when every time maximum elements removed from top _____.
   a. We call merge Sort Algorithm
   b. it becomes Order n2 Algorithm
   c. Divide and Conquer strategy helps us
   d. We are left with a hole Page – 41
5. If matrix A of dimension p x q is multiply with matrix B of dimension q x r, then each entry in resultant matrix takes _____ time.
   a. O (q) Page - 84
   b. O (1)

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c. O (p x q)
d. O (q x r)

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6. _____ is a method of solving a problem in which we check all possible solutions to the problem to find the solution we need.
   a. Plane-Sweep Algorithm
   b. Sorting Algorithm
   c. Brute-Force Algorithm google
   d. Greedy approach
7. The worst case running time of Quick sort algorithm _____.
   a. Cannot be quadratic
   b. Is quadratic google QuickSort and its Analysis (codesdope.com) & http://personal.kent.edu/~rmuhamma/Algorithms/MyAlgorithms/Sorting/quickSort.htm
   c. Is always Exponential
   d. Is linear
8. In max heap (for Heap Sort algorithm), when every time maximum element is removed from top we replace it with _____ leaf in the tree.
   a. second last
   b. Last Page -41
   c. First
   d. Any
9. Quick sort algorithm was developed by -
   a. AlferdAho
   b. Sedgewick
   c. John Vincent Atanasoff
   d. Tony Hoare – Google wikipedia
10. If Matrix-A has dimensions "3x2" and Matrix-B has dimensions "2x3", then multiplication of Matrix-A and Matrix-B will result a new Matrix-C having dimensions.
    a. 3x2
    b. 2x3
    c. 2x2
    d. 3x3 http://www.calcul.com/show/calculator/matrix-multiplication

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11. For comparison-based sorting algorithms, it is possible to sort more efficiently than Omega n log(n) time.
    a. Always
    b. Not P-54
    c. Sometimes

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d. Sometimes not

12. Dynamic Programming approach is usually useful in solving optimization problems.
    a. True
    b. False

13. In Sorting the key value or attribute_____ from an ordered domain.
    a. Must be page 39
    b. Not always
    c. May be
    d. Occasionally

14. Result of asymptotical analysis of n(n -3) and 4n*n is that _____
    a. n(n-1) is asymptotically Less
    b. n(n-1) is asymptotically Greater
    c. Both are asymptotically Not equivalent
    d. Both are asymptotically Equivalent page 23 ($4n*n= 4n^2$)

15. Floor and ceiling are _____ to calculate while analyzing algorithms a. Very easy
    b. Usually considered difficult P-31
    c. 3rd Option is missing
    d. 4th Option is missing

16. _____ of reference is an important fact of current processor technology.
    a. Defining
    b. Assigning
    c. Formality
    d. Locality P-8

17. In max-heap, largest element is stored at root node. Where is the smallest element stored?
    a. Right Node
    b. Leaf Node
    c. Middle Node

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d. Left Node

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18. In average-case time analysis of Quick sort algorithm, the most balanced case for partition is when we divide the list of elements into _.
    a. Equal no. of pieces as of input elements
    b. Single piece exactly
    c. Two nearly equal pieces
    d. Three nearly equal pieces
19. Which of the following is calculated with Big O notation?
    a. Medium bounds
    b. Upper bounds Page - 25
    c. Lower bounds
    d. Both upper and lower bounds
20. Edit distance algorithm based on _____ strategy
    a. Greedy
    b. Dynamic Programming Page - 81
    c. Divide and Conquer
    d. Searching
21. In Heapsort Algorithm, total time taken by heapify procedure is _____
    a. O (log n) Page-43
    b. O (log2 n)
    c. O (n log n)
    d. O (n2 log n)
22. Al-Khwarizmi was a/an _____
    a. Artist
    b. Mathematician P-7
    c. Astronomer
    d. Khalifah
23. When matrix A of 5x3is multiply with metric B of 3x4 then the number of multiplication required is: Not found exactly
    a. 15
    b. 12

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c. 36

d. 60 **Not Found exactly but as per formula at page 84,**

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24. Pseudo code of algorithms are to be read by _____.
    a. People Page -12
    b. RAM
    c. Computer
    d. Compiler
25. The sieve technique is a special case, where the number of sub-problems is Just _____
    a. 1 P-34
    b. 2
    c. 3
    d. 4
26. When a recursive algorithm revisits the same problem over and over again, we say that the optimization problem has _____ sub-problems.
    a. Overlapping – Google Search
    b. Over costing
    c. Optimized
    d. Three
27. Sieve technique is very important special case of Divide-and-Conquer strategy.
    a. True P-34
    b. False
28. In order to say anything meaningful about our algorithms, it will be important for us to settle on a _____.
    a. Java Program
    b. C++ Program
    c. Pseudo program
    d. Mathematical model of computation P-10
29. Merge sort is based on _____.
    a. Brute-force
    b. Plan-sweep
    c. Axis-sweep

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d. Divide and Conquer **P-27**

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30. What time does Merge Sort algorithm take in order to sort an array of 'n' numbers?
    a. (n)
    b. (log n)
    c. (n^2)

    d. (n log n) Google Search 31. In Heap Sort
    31. algorithm, the first step is to _____.
    a. Call Build-Heap procedure Page - 46
    b. Sort the array in descending order
    c. Call Heapify procedure
    d. Find the number of input elements
32. The definition of theta-notation relies on proving _____ asymptotic bound.
    a. One
    b. Lower
    c. Upper
    d. Both lower & upper Page - 25
33. In merge sort algorithm, to merge two lists of size n/2 to a list of size n, takes _____ time.
    a. Theta (n) Page - 32
    b. Theta log(n)
    c. Theta log2(n)
    d. Theta n log(n)
34. We can make _____ recursive calls in Fibonacci Sequence.
    a. Infinite
    b. Finite google
    c. Only one
    d. Zero
35. Following is NOT the application of Edit Distance problem.
    a. Speech recognition

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b. Spelling Correction

c. Ascending Sort

d. Computational Molecular Biology

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36. In plane sweep approach, a vertical line is swept across the 2d-plane and structure is used for holding the maximal points lying to the left of the sweep line.
    a. Array
    b. Queue
    c. Stack Page - 18
    d. Tree
37. When a heapify procedure is applied to the root node to restore the heap, then at each level, the comparison performed takes time:
    a. It will take (log n).
    b. It can not be predicted
    c. It will take O (1). Page - 43
    d. Time will vary according to the nature of input data.
38. _____ time is the maximum running time over all legal inputs.
    a. Worst-case Page - 13
    b. Average-case
    c. Best-case
    d. Good-case
39. Efficient algorithm requires less computational...
    a. Memory
    b. Running Time
    c. Memory and Running Time Page - 9
    d. Energy
40. For average-case time analysis of Quick sort algorithm, Pivot selection is on average basis from _____
    a. half of the input values
    b. all possible random values Page - 50
    c. Pivot is input separately
    d. values greater than 5
41. Selection algorithm takes theta _____

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a. (n2)

b. (n)

c. log(n)

d. n log(n)

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42. Recurrence can be described in terms of a tree.
    a. Yes Page - 31
    b. No
43. Time complexity of Dynamic Programming based algorithm for computing the minimum cost of Chain Matrix Multiplication is _____
    a. Log n
    b. n
    c. n^2 (n square)
    d. n^3 (n cube) Page -90
44. The Iteration method is used for _____
    a. Comparing sorting algorithms only
    b. Solving Recurrence relations Page 31
    c. Merging elements in Merge sort
    d. Dividing elements in Merge sort
45. In 3-Dimensional space, a point P has _____ coordinate(s).
    a. (X, Y)
    b. (X, 0)
    c. (0, Y)
    d. (X,Y, Z)
46. Chain matrix multiplication problem can be solved through _____ strategy.
    a. Dynamic programming Page - 85
    b. Greedy
    c. Divide and conquer
    d. Sorting
47. Merge sort have running time....running time of Heap sort. Not found exactly
    a. Greater than
    b. Less than Google
    c. Equal to
    d. Different than

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48. Median is not useful measure of central tendency of given input set especially when the distribution of values is highly skewed.
    a. True
    b. False Page – 34

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49. We do not need to mathematically prove that for comparison-based sorting algorithms always takes Omega nlog (n) time.
    a. True Google & VU Tech (pg 46 not very clear)
    b. False
50. The Omega-notation allows us to state only the asymptotic _____ bounds.
    a. Middle
    b. Lower Page 25
    c. Upper
    d. Both lower & upper
51. Both lower & upperSorting can be in _____
    a. Increasing order only
    b. Decreasing order only
    c. Both Increasing and Decreasing order GOOGLR Search
    d. Random order
52. Radix sort performs sorting the numbers _____ digit (s) at a time.
    a. One Page - 71
    b. Two
    c. Three
    d. All
53. Quicksort is a/an _____ and _____ sorting algorithm.
    a. Not in place, not stable one
    b. In place , not stable one Page - 54
    c. In place , stable one
    d. Not in place , stable one
54. Consider three matrices X,Y,Z of dimensions 1x2, 2x3,3x4 respectively. The number of multiplications of (XY) Z is:
    a. 18 As per lecture slides
    b. 32
    c. 24
    d. 30

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55. In Fibonacci Sequence, unnecessary repetitions do not exist at all.

a. True

b. False **Page – 74**

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56. It is not a Fibonacci sequence . 1,1,1,2,3,5,8,13,21,34,55,…..
    a. True Page - 73
    b. False
57. Heap sort is a/ an _____ and _____ sorting algorithem.
    a. Not in place, not stable one
    b. In place , not stable one Page - 54
    c. In place , stable one
    d. Not in place , stable one
58. Identify the True Statement
    a. The knapsack problem does not belong to the domain of optimization problems.
    b. The knapsack problem belongs to the domain of optimization problems. Page - 91
    c. The Knapsack problem cannot be solved by using dynamic programming
    d. The knapsack problem is optimally solved by using brute force algorithm.
59. In Dynamic Programming, our approach is to _____
    a. Develop the solution in a top-down fashion
    b. Express the problem non-recursively
    c. Build the solution in a bottom-up fashion Page - 75
    d. Input several sub-problems simultaneously
60. Counting sort is suitable to sort the elements in range 1 to K;
    a. K is large
    b. K is small Page - 57
    c. K may be large or small
    d. None
61. We can multiply two matrices A and B only when they are compatible which means
    a. Number of columns in A must be equal to number of rows in B. it seems Correct as per page 84

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b. Number of rows and columns do not matter

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c. Number of columns in A must be equal to number of columns in B
d. Number of rows in A must be equal to number of rows in B

62. Matrix multiplication is a (n) _____ operation.
a. Commutative
b. Associative Page 85
c. Neither commutative nor associative
d. Commutative but not associative

63. In Dynamic Programming approach, solution is modified / changed
a. Always once
b. At each stage google and wikipedia
c. Only for specific problems
d. At 4th stage only

64. In Knapsack problem, the goal is to put items in the Knapsack such that the value of the items is _____ subject to weight limit of knapsack.
a. Minimized
b. Decreased
c. Maximized Page - 91
d. None of the given options

65. An in-place sorting algorithm is one that _____ uses additional array for storage.
a. Always
b. Permanently
c. Does not Page - 54
d. Sometime

66. Memoization is a part of Dynamic Programming Strategy.
a. True Page - 74
b. False

67. If matrix A of dimension 2x4 is multiply with matrix B of dimension 4x3, then the dimension of resultant matrix is Not found exactly
a. 2x4

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b. 4x3

c. 3x4

d. 2x3 It seems correct as per second last Para of page 84

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68. In Dynamic Programming approach, we do not store the solution to each sub-problem in case if it reappears.
    a. True
    b. False Page - 75
69. Dynamic Programming is a problem-solving approach in which___
    a. Problem is solved in Zero time
    b. Solution is developed only at final stage
    c. Both are correct
    d. Both are incorrect google
70. In Fibonacci sequence, each term is calculated by____ previous__ terms.
    a. Subtracting, Two
    b. Adding, Three
    c. Adding, Two Page - 73
    d. Multiplying, Two
71. Selection sort is not an in-place sorting algorithm.
    a. True Page - 54
    b. False
72. If there are θ (n^2) entries in edit distance matrix then the total running time is:
    a. θ (n)
    b. θ (1)
    c. θ (n^2) Page – 84
    d. θ (n logn)
73. The only way to convert a string of i characters into the empty string is with i deletions, represented as
    a. E(0.j) =j
    b. E(i.j) = 1
    c. E(0.i) = j
    d. E (i.0)=I Page - 78
74. Dynamic programming formulation of the matrix chain multiplication problem will store the solutions of each sub problem in an

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a. Array
b. Table Page - 86
c. Variable
d. class

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75. We can use the optimal substructure property to devise a formulation of the edit distance problem.
    a. Selective
    b. Optimum
    c. Iterative
    d. Recursive Page - 78
76. Sorting is performed on the basis of _____.
    a. Computational resources
    b. Asymptotic notation
    c. Summation
    d. Some key value of attribute page- 39
77. In Heap Sort algorithm, we call Build-heap procedure _____.
    a. Only once page 46
    b. Twice
    c. Thrice
    d. As many times as we need
78. Radix sort is not a non-comparative integer sorting algorithm.
    a. True Google Search
    b. False
79. In the statement "output P[1].x, P[1].y", the number of times elements of P are accessed is _____.
    a. 1
    b. 2 page 14
    c. 3
    d. 4
80. The main purpose of mathematical analysis is measuring the _____ required by the algorithm.
    a. Space
    b. Execution time P-13
    c. Inputs & outputs

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d. Execution time and memory

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81. _____ provides us more accurate result when input values are not closer with each other
    a. Average
    b. Median P-34
    c. Mode
    d. Mean
82. The process of _____ ends when you are left with such tiny pieces remaining that it is trivial to solve them.
    a. Brute-force
    b. Plan-sweep
    c. Divide and Conquer P-27
    d. Axis-sweep
83. _____ overcomes the limitations of _____ by working as per positional notations of numbers.
    a. Counting sort, Radix sort
    b. Radix sort, Counting sort P-71
84. Memorization is a part of Dynamic Programming strategy.
    a. True P-74
    b. False
85. Rank of an element can be defined as _____.
    a. One minus the number of elements that are smaller
    b. Two plus the number of elements that are greater
    c. One plus the number of elements that are smaller P-34
    d. Two minus the number of elements that are smaller
86. If the time complexity of an algorithm is given by O (1), then its time complexity would be
    a. Polynomial
    b. Exponential
    c. Constant - Wikipedia
    d. Average

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87. Quick sort is a recursive algorithm.

a. True **Wikipedia ; Google**
b. False

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88. The asymptotic growth of n(n+1)/2 is:
    a. $O(n^2)$ As the n^2 term has the largest contribution, the Big-O complexity is O(n^2)
    b. O(n)
    c. O(n+2)
    d. O(n log n)
89. Approach of solving geometric problems by sweeping a line across the plane is called _____ sweep.
    a. Line
    b. Plane Page 18
    c. Cube
    d. Box
90. As per algorithm of Dynamic Programing, we need to store
    a. First sub-problem only
    b. Best solution only
    c. Intermediate sub-problems Pg:75
    d. Final solution only
91. In Sieve technique, we solve the problem
    a. In recursive manner Pg:34
    b. Non recursively
    c. Using Merge Sort algorithm
    d. Using Brute force technique
92. One of the limitation in 0/1 knapsack is that an item can either be _____ in the bag or not.
    a. Use
    b. Put Pg:91
    c. Move
    d. Store
93. Which one is not passed as parameter in Quick sort algorithm?
    a. End of the array
    b. Middle of the array

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c. Array (containing input elements) **Google**

d. Start of the array

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94. In the analysis of Selection algorithm, we get the convergent _____
    a. Harmonic
    b. Linear
    c. Arithmetic
    d. Geometric Pg:37
95. A Random Access Machine (RAM)is an idealized machine withrandom access memory.
    a. Infinite large Pg:10
    b. 512 MB
    c. 256 MB
    d. 2 GBs
96. While analyzing Selection algorithm, we make a number of passes, in fact it could be as many as
    a. n(n+1)
    b. log(n) Pg:37
    c. n/3
    d. n/4
97. In Random Access Machine (RAM), instructions are executed in
    a. Parallel
    b. Batch
    c. One by One Pg:10
    d. Multiple times
98. In selection problem, the rank of an element will be its _____ position
    a. First
    b. final Pg:34
    c. Second last
    d. Last
99. The worst-case running time of Merge sort is _____ in order to sort an array of n elements.
    a. O(log n)

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b. O(n)

c. O(n log n) page 40 and google

d. O(n)

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100. f(n) and g(n) are asymptotically equivalent. This means that they have essentially the same _____.
   a. Results
   b. Variables
   c. Size
   d. Growth rates P:23

101. An algorithm is a mathematical entity. Which is independent of _____.
   a. Programming language
   b. Machine and Programming language
   c. Compiler and Programming language
   d. Programming language Compiler and Machine P:07

102. In Quick sort algorithm, Pivots form ___
   a. Stack
   b. Queue
   c. Binary Search Tree P:49
   d. Graph

103. Counting sort is suitable for sorting the elements within range 1 to P. where
   a. P is large
   b. P is small P-57
   c. P is very large
   d. P is undetermined

104. In asymptotical analysis of n'(5 2)-3, as n becomes large, the dominant (fastest growing) term is some constant times
   a. n_1
   b. n
   c. n+1
   d. n*n P-23

105. ___ Items are not allowed in the 0/1 knapsack. a. Lighter
   b. Fractional P-91

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c. Whole
d. Weighty

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106. Fibonacci Sequence was named on _____, a famous mathematician in 12th Century.
    a. Fred Brooks
    b. Grady Booch
    c. Leonardo Pisano P-73
    d. Edgar F. Codd
107. In Heap Sort algorithm, we build _____ for ascending sort.
    a. Max heap P-41
    b. Min heap
108. Bubble sort is not an in-place sorting algorithm.
    a. True
    b. False P-54
109. In partition algorithm, the subarray _____ has elements which are greater than pivot element x.
    a. A[p...r]
    b. A[p...q-1]
    c. A[q]
    d. A[q+1..r] P-46
110. In Heap Sort algorithm, if heap property is violated
    a. We call Build heap procedure P-43
    b. We call Heapify procedure
    c. We ignore
    d. Heap property can never be violated
111. _____ is not a characteristic of Random Access Machine.
    a. Single-Processor P-10
    b. Assigning a value to a variable
    c. Locality of reference
    d. Executing an arithmetic instruction
112. The only way to convert an empty string into a sting of j characters is by doing j insertions, represented as _____

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a. E(i,j) = 1
b. E(I,0) = I
c. E(0,j) = j page 78
d. E(1,j)= j

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113. In Selection problem, the Sieve technique works in _____.
    a. Non-recursive manner
    b. Constant time
    c. Phases page 34
    d. One complete go
114. Algorithm is a sequence of computational steps that ---- the input into output.
    a. Merge
    b. Assign
    c. Transform page 7
    d. Integrate
115. If pj dominates pi and pi dominates ph then pj also dominates ph, it means dominance relation is
    a. Transitive page 18
    b. Non Transitive
    c. Equation
    d. Symbolic
116. To find maximal points in brute-force algorithm each point of the space is compared against _____ of that space.
    a. One other point
    b. All other points page 11
    c. Few other points
    d. Most of the other points
117. In the following code the statement "cout<<j;"executes --------- times. for (j=1; j<=5; j = j+2)
        cout<<j;
    a. 5 times
    b. 2 times
    c. 3 times

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d. 0 times

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118. In merge sort algorithm, we split the array around the _____ index q. a. Entring
    b. Mid page 17
    c. Exiting
    d. Summing
119. In Selection problem, the Sieve technique _____.
    a. Add some more input items each time
    b. Do not work recursively
    c. Do not uses Divide and Conquer approach
    d. Eliminates undesired data items each time
120. Consider three matrices X, Y, Z of dimensions 1 x 2, 2 x 3, 3 x 4 respectively. The number of multiplications of X(YZ) is .
    a. 16
    b. 32
    c. 26
    d. 32 page 84
121. In Heap Sort algorithm, the total running time for Heapify procedure is _____
    a. Theta (log n)
    b. Order (log n)
    c. Omega (log n)
    d. O(1) i.e. Constant time
122. The sieve technique works where we have to find_____ items(s) from a large input.
    a. Single page 34
    b. Two
    c. Three
    d. Similar
123. In Dynamic Programming based solution of Knapsack Problem, if we decide to take an object i , then we gain_____

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a. W(Total Weight of Knapsack)
b. V (Total Value of all items)
c. vi (Value of object i) page 93
d. Nome of the given option

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124. While Sorting, the order domain means for any two input elements x and y _____ satisfies only.
   a. x < y page 39
   b. x > y
   c. x = y
   d. All of the above

125. For solving Selection problem, we introduced Sieve technique due to _____
   a. Using Decrease and Conquer strategy page 34
   b. Avoiding to sort all input data
   c. Eliminating Rank of an element
   d. Using Brute-force approach

126. _____ is one of the few problems, where provable lower bounds exist on how fast we can sort.
   a. Searching
   b. Sorting page 38
   c. Both Searching & sorting
   d. Growing

127. In plane sweep approach, a vertical line is swept across the 2d-plane from_____.
   a. Right to Left
   b. Left to Right page 18
   c. Top to Bottom
   d. Bottom to top

128. In generating Fibonacci sequence, we can avoid unnecessary repetitions by _____ process.
   a. Tokenization
   b. Memorization page 43
   c. Randomization
   d. Memorization

129. For _____ values of n, any algorithm is fast enough.

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a. Small page 14
b. Medium
c. Large
d. Infinity

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130. Dynamic programming comprises of _____.
    a. Recursion only
    b. Repetition only
    c. Recursion with Repetition
    d. No Repetition but Recursion page 75

131. The function f(n)=n(logn+1)/2 is asymptotically equalient t nlog n :Here Lower Bound means function f(n) grows asymptotically at __ as fast as nlog n.
    a. Least page 23
    b. Normal
    c. Most
    d. At

132. Counting sort has time complexity.
    a. O(n+k)
    b. O(n) page 58
    c. O(k)
    d. O(nlogn)

133. Due to left complete nature of binary tree, the heap can be stored in
    a. Array page 40
    b. Structures
    c. Link List
    d. Stack

134. Single item from a larger set of _____.
    a. Constant
    b. Pointers
    c. Phases
    d. n items page 34

135. In the clique cover problem, for two vertices to be in the same group, they must be _____ each other.
    a. Apart from
    b. Far from

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c. Near to

d. Adjacent to page 76

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136. How much time merge sort takes for an array of numbers?
    a. T(n^2)
    b. T(n)
    c. T(log n)
    d. T(n log n) page 40

137. In in-place sorting algorithm is one that uses arrays for storage.
    a. No additional array page 54
    b. An additional array
    c. Both of above may be true according to algorithm
    d. More than 3 arrays of one dimension

138. Brute-force algorithm for 2D-Maxima is operated by comparing _____ pairs of points.
    a. Two
    b. Some
    c. Most
    d. All page 18

139. While Sorting, the ordered domain means for any two input elements x and y ____ satisfies only.
    a. x > y
    b. x < y
    c. x = y
    d. All of the above page 38

140. Quick sort is.
    a. Stable & in place
    b. Not stable but in place page 54
    c. Stable but not in place
    d. Some time stable & some times in place

141. Which may be a stable sort?
    a. Merger
    b. Insertion

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c. Both above **page 54**

d. None of the above

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142. For the Sieve Technique we take time.
   a. T(nk) page 34
   b. IT(n / 3)
   c. n^2
   d. n/3

143. Continuation sort is suitable to sort the elements in range 1 to k.
   a. K is Large
   b. K is not known
   c. K may be small or large
   d. K is small page 54

144. Asymptotic growth rate of the function is taken over _____ case running time. .
   a. Best
   b. Worst page 14
   c. Average
   d. Normal

145. The sieve technique is a special case, where the number of sub problems is just.
   a. 5
   b. Many
   c. 1 page 34
   d. Few

146. In Quick sort, we don't have the control over the sizes of recursive calls.
   a. True page 49
   b. False
   c. Less information to decide
   d. Ether true or false

147. Before sweeping a vertical line in plane sweep approach, in start sorting of the points is done in increasing order of their _____ coordinates. .
   a. X page 18

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b. Y
c. Z
d. X , Y

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148. Random access machine or RAM is a/an.
    a. Machine build by Al-Khwarizmi
    b. Mechanical machine
    c. Mathematical model page 10
    d. Electronics machine
149. The Huffman codes provide a method of encoding data inefficiently when coded using ASCII standard.
    a. True
    b. False page 99
150. A heap is a left-complete binary tree that confirms to the _____.
    a. increasing order only
    b. decreasing order only
    c. heap order page 40
    d. log n order
151. If we associate (x, y) integers pair to cars where x is the speed of the car and y is the negation of the price. High y value for a car means a _____ car.
    a. Fast
    b. Slow
    c. Expensive
    d. Cheap
152. Which one of the following sorting algorithms is the fastest?
    a. Merge sort
    b. Quick sort
    c. Insertion sort
    d. Heap sort
153. Quick sort algorithm divide the entire array into _____ sub arrays.
    a. 2
    b. 3
    c. 4
    d. 5
154. In brute force algorithm, we measure running time T(n) based on _____.

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a. Average-case time and best-case time
b. Worst-case time and average-case time page 46
c. Worst-case time and best-case time
d. Best-case time and staring-case time

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155. For 2D Maxima problem. Plane Sweep algorithm first of all _____.
   a. Sorts all points
   b. Delete some points
   c. Output the elements
   d. Pushes all points on stack

156. There are _____ entries in the Edit Distance Matrix
   a. ɵ (n)
   b. ɵ (n_2) page 84
   c. ɵ (n+2)
   d. ɵ (n + 100)

157. Which symbol is used for Omega notation?
   a. (O)
   b. (ɵ)
   c. (Ω)
   d. (@)

158. Selection sort is a _____ sorting algorithm
   a. In-place page 54
   b. Not In-Place
   c. Stable
   d. in-partition

159. In Dynamic Programming based solution of knapsack problem, to compute entries of 'V', we will imply a(n) _____ approach.
   a. Subjective
   b. Inductive
   c. Brute Force
   d. Combination

160. We do not need to prove comparison-based sorting algorithms by mathematically. It always takes _____ time.
   a. Big Oh nlog(n)
   b. Omega nlog(n) NOT SURE

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c. Omega n(n^2)
d. Theta nlog(n)

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161. Merge sort is a/an _____ and _____ sorting algorithm
    a. Not in-place, not stable one
    b. In-place, not stable one
    c. In-place, stable one
    d. Not in-place, stable one page 54
162. Cubic function will _____ a quadratic function.
    a. Prove
    b. be equal to
    c. overtake Page 25
    d. find
163. Insertion sort is a _____ sorting algorithm
    a. Unstable
    b. In-place Page 54
    c. Not In-Place
    d. in-partition
164. To check whether a function grows faster or slower than the other function, we use some asymptotic notations, which is _____.
    a. Big-oh notation
    b. Theta notation
    c. Omega notation
    d. All of the given
165. Asymptotic growth of 8n^2 + 2n – 3 is:
    a. Θ(n^2 + n)
    b. Θ (n^2) page 14
    c. Θ(8n^2)
    d. Θ(8n^2 + 2n)
166. In the analysis of algorithms, _____ plays an important role.
    a. text analysis
    b. time
    c. growth rate

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d. money

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167. In inductive approach of knapsack problem, we consider 2 cases, _____ Or _____.
   a. Median, Mode
   b. Recursive, Iterative
   c. Leave object, Take object page 93
   d. Sequentially. Parallel

168. Random Access Machine (RAM) can execute _____ instructions
   a. only logical
   b. parallel
   c. only arithmetic
   d. logical and arithmetic

169. Using _____ algorithm, efficiency is not given much importance
   a. Greedy
   b. Merge sort
   c. Processing as there is no algorithm by this name
   d. Brute Force

170. Bubble sort takes theta _____ in the worst case
   a. (n2) page 39
   b. (n)
   c. log(n)
   d. nlog(n)

171. If matrix A of dimension p × q is multiply with matrix B of dimension q × r, then dimension of resultant matrix is:
   a. p × q
   b. p × r page 84
   c. q × r
   d. r × p

172. Dynamic Programing algorithms often use some kind of _____ to store the results of intermediate sub-problems
   a. table (Page 75)

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b. variable
c. stack
d. loop

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173. _____ is in-place sorting algorithm.
   a. Bubble sort (Page 54)
   b. Merge sort
   c. Linear search
   d. Binary Search

174. Which one of the following problems can be solved using dynamic problem?
   a. Bubble sort problem
   b. Matrix chain multiplication problem page 85
   c. Greedy search problem
   d. Fractional knapsack problem

175. In chain matrix multiplication, solutions of the sub-problems are stored in a _____.
   a. Array
   b. Table page 86
   c. Tree
   d. Link list

176. What is the average running time of a quick sort algorithm?
   a. O(n^2)
   b. O(n)
   c. O(n log n) (Page 49)
   d. O(log n)

177. Sorting Algorithms having O _____ running time are considered to be slow ones.
   a. (n)
   b. (n^2) (Page 39)
   c. (nlog(n))
   d. (log(n))

178. While solving Selection problem, in Sieve technique we partition input data _____
   a. In increasing order

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b. In decreasing order
c. According to Pivot
d. Randomly

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179. _____ is the process of avoiding unnecessary repetitions by writing down the results of recursive calls and looking them up again if we need them later.
    a. Loop
    b. Memoization page 74
    c. Recursion
    d. Function

180. In average-case time the probability of seeing input is denoted by _____.
    a. p{I}
    b. p[I]
    c. p<i>
    d. p(i) page 13

181. While applying the Sieve technique to selection sort, how to choose a pivot element.
    a. Through mean
    b. Linear
    c. Randomly page 35
    d. Sequentially

182. Number of _____ of the pseudo code are counted to measure the running time.
    a. Inputs
    b. Outputs
    c. Steps page 13
    d. Pages

183. Developing a dynamic programming algorithm generally involves _____ separate steps.
    a. One
    b. Two page 75
    c. Three
    d. Four

184. 8n^2+2n+3 will exceed c28(n), no matter how large we make _____.
    a. n

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b. 2n

c. c2 page 25

d. this quadratic equation

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185. The running time of quick sort algorithm_____.
   a. Is impossible to compute
   b. Has nothing to do with pivot selection
   c. Is Random upon each execution
   d. Greatly influenced by the selection of pivot page 49

186. _____ involves breaking up the problem into sub problems whose solutions can be combined to solve the global problem.
   a. Complexity Theory
   b. Dynamic programming solution
   c. Divide and Conquer Strategy page 34
   d. Greedy Algorithms

187. In _____ we have to find rank of an element from given input.
   a. Merge sort algorithm
   b. Selection problem page 34
   c. Brute force technique
   d. Plane Sweep algorithm

188. How many steps are involved to design the dynamic programming strategy?
   a. 2
   b. 3
   c. 1
   d. 4 page 92

189. In Bucket sort, if there are duplicates then each bin can be replaced by a a. Stack
   b. Linked list page 69
   c. Hash table
   d. Heap

190. In merge sort algorithm, we split the array _____ to find index q.
   a. from start
   b. midway page 28
   c. from end

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d. both from start or end

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191. Find the maximum value of the items which can carry using knapsack Knapsack weight capacity = 50.

Item Weight Value
11070
22020
33080
470 200

a. 280
b. 100
c. 90
d. 200

192. In 2-d maxima problem a point p is said to be dominated by point q if _____.

a. p.x <= q.x
b. p.x <= q.x and p.y <= q.y page 17
c. p.y <= q.y
d. p.x >= q.x and p.y >=q.y

193. Sorting can be in _____.

a. Increasing order only
b. Decreasing order only
c. Both increasing and decreasing order
d. Random order

194. Recurrence can be described in terms of _____.

a. Array
b. Linear
c. Tree page 31
d. Graph

195. The brute-force algorithm for 2D-Maxima runs in order O(__) time.

a. n
b. n(log n)

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c. n*n page 18

d. n3

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196. In plane sweep approach of solving geometric problems, a _____ is swept across the plane.
   a. Line page 18
   b. Plane
   c. Cube
   d. Box

197. Which of the following is calculated with Big Omega notation?
   a. Medium bounds
   b. Upper bounds
   c. Lower bounds Page - 25
   d. Both upper and lower bounds

198. _____ is always based on divide and conquer strategy.
   a. Bubble sort
   b. Selection sort
   c. Pigeon sort
   d. Quick sort page 46

199. If a matrix has three rows and two columns, then dimensions of matrix will be:
   a. 3x2
   b. 2x3
   c. 3x3
   d. 2x2

200. Asymptotic notations are used to describe _____ of an algorithm.
   a. Length
   b. running time google
   c. size
   d. compile time

201. Catalan numbers are related the number of different _____ on 'n' nodes.
   a. Arrays
   b. linked lists

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c. binary trees **page 85**

d. functions

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202. Applying the sieve technique to selection problem, _____ element is picked from array.
   a. Output
   b. Total
   c. Input
   d. Pivot page 35

203. Dynamic Programming approach is usually useful in solving _____ problems.
   a. Normal
   b. Optimization google
   c. Array
   d. Loop

204. In recursive formulation of knapsack Problem: V [0, j] = _____ for j>=0
   a. -1
   b. 0 page 93
   c. 1
   d. 2

205. _____ is a linear time sorting algorithm.
   a. Merge sort
   b. Radix sort page 71
   c. Quick sort
   d. Bubble sort

206. Quick sort is one of the _____ sorting algorithm.
   a. Fastest page 19
   b. Slowest
   c. Major
   d. Average

207. The time assumed for each basic operation to execute on RAM model of computation is _____.

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a. Infinite
b. Continuous
c. Constant page 10
d. Variable

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208. In Sieve Technique, we know the item of interest. a. True
b. False page 34

209. While analyzing algorithms, _____ and _____ usually considered difficult to calculate.
a. Finite, Infinite
b. Floor, ceiling google
c. Row, Column
d. Graph, Tree

210. While analysis of the brute-force maxima algorithm, an array sorted in the reverse order is the type of _____ case input.
a. Best
b. Worst page 14
c. Somewhat bad
d. Average

211. _____ is not useful measure of central tendency of given input set especially when the distribution of values is highly skewed.
a. Mean
b. Mode
c. Average
d. Median page 34

212. In asymptotical analysis of n(n-3) and 4n*n, as n becomes large, the dominant (fastest growing) term is some constant times _____.
a. n+1
b. n-1
c. n
d. n*n page 23

213. In addition to passing in the array itself to Merge Sort algorithm, we will pass in other arguments which are indices.

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a. Two P-38
b. Three
c. Four
d. Five

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214. In 2d-maximal problem, a point is said to be if it is not dominated by any other point in that space.
    a. Member
    b. Minimal
    c. Maximal P-11
    d. Joint

215. Counting sort assumes that the numbers to be sorted are in the range _____.
    a. K to n where n is large
    b. 1 to k where k is small (P-57)
    c. K to n where k is small
    d. k to n where n is small

216. Insertion sort is an efficient algorithm for sorting a _____ number of elements
    a. Large P-39
    b. Small
    c. Extra large
    d. Medium

217. If the indices passed to merge sort algorithm are _____ then this means that there is only one element to sort.
    a. Small page 28
    b. Large
    c. Equal
    d. Not Equal

218. In Knapsack Problem, each item must be entirely accepted or rejected, is called _____ problem.
    a. Fractional
    b. 0-1 P-92
    c. Linear
    d. Optimal

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219. If the time complexity of an algorithm is O(n). then it is called _____ time complexity.
   a. Linear Wikipedia
   b. Constant
   c. Average
   d. Exponential

220. In the case of _____ analysis does not depend upon on the distribution of input.
   a. Merge sort
   b. Quick sort P-50
   c. Insertion sort
   d. Heap sort

221. We can use the _____ Property to devise a recursive formulation of the edit distance problem.
   a. Small substructure
   b. Algorithmic
   c. Real
   d. Optimal substructure page 78

222. The following sequence is called _____
   1,2,3,5,8,13,21,34,55,.....
   a. Optimize sequence
   b. Fibonacci sequence page 73
   c. Optimal sequence
   d. Overlapping sequence

223. Which one sorting algorithm is best suited to sort an array of 2 million elements?
   a. Bubble sort
   b. Insert sort
   c. Merge sort
   d. Quick sort

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e. Ridx Sort page 71

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224. We can improve the performance of quick sort if we could be able to _,_____.
    a. Skip input elements somehow
    b. Select two or more pivots page 34
    c. Skip any sub-array completely
    d. Eliminate recursive calls
225. The problem with the brute-force algorithm is that is uses _____ in pruning out de
    a. Worst-case time
    b. No intelligence page 18
    c. Outside looping
    d. Artificial intelligence
226. In chain matrix multiplication, the order of the matrices _____.
    a. Can be changed
    b. Can not be changed page 85
    c. is equal
    d. is reverse
227. In quick sort algorithm, we choose pivot_____.
    a. Always the smallest element
    b. Greater than 5
    c. Randomly page 35
    d. Less than 5
228. In Heap Sort algorithm. Heapify procedure is _____ in nature.
    a. Recursive
    b. Non-Recursive page 43
    c. Fast
    d. Slow
229. When matrix A of 5x 3 is multiplied with matrix B of 3 x 4 then the number of multiplications required will be _____.
    a. 15

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b. 12
c. 36
d. 60

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230. An algorithm is said to be correct if for every _____ instance, it halts with the correct _____.
   a. Input, Output page 13
   b. Design, Analysis
   c. Value, Key
   d. Key, Analysis

231. In chain matrix multiplication, table is filled _____ to find the multiplication of matrix.
   a. row wise
   b. column wise
   c. diagonally
   d. bottom-to-up page 86

232. If we have an equation 8n2+7f*n + 5f + 6 then is large, _____ term will be muchxxxxxxxthe n term and will dominate the running time.
   a. f g (n)
   b. g (n) * 2
   c. n * 2 page 23
   d. f (n)

233. For quick sort algorithm. Partitioning takes theta _____.
   a. (n)
   b. log(n)
   c. n log (n)
   d. n2log (n)

234. In Heap Sort algorithm, the maximum levels an element can move upward is _____
   a. Theta (log n) page 43
   b. Big-ch (log n)
   c. Omega (log n)
   d. 0 (1) i.e. Constant time

235. _____ programming is essentially recursion without repetition.

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a. Fast
b. Dynamic page 75
c. Array
d. n (log n)

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236. There are no hard formal rules to the syntax of the _____ code.
   a. Basic
   b. Programming
   c. Pseudo
   d. Assembly

237. In Heap Sort algorithm, to remove the maximum element every time.
   a. We call Build-Heap procedure
   b. Heap Sort algorithm terminates without result
   c. We call heapify procedure
   d. Nothing happens

238. Which process is used for avoiding unnecessary repetitions and looking them up again if we need them later.
   a. Greedy Approach
   b. Memoization page 74
   c. Divide and conquer
   d. Recursion

239. The worst-case running time of Quick sort is _____ in order to sort an array of n element.
   a. O(n log n) page 49
   b. O(n)
   c. $O(n^2)$
   d. O(log n)

240. Boolean operation is a _____ operation on an idealized RAM model of computation.
   a. Starting
   b. Basic page 10
   c. Advance
   d. Normal

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241. In chain matrix multiplication, if there are n items, there are _____ ways in which outer most pair of parentheses can placed.
    a. n^2
    b. 2n
    c. n+1
    d. n-1 page 85

242. The number of nodes in a complete binary tree of height h is: a. 2 * (h+1) - 1
    b. 2^(h+1) – 1 page 40
    c. 2 * (h+1)
    d. ((h+1)^2) – 1

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CS502-Fundemental Of Algorithm
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1. The sequence of merge sort algorithm is:
   a. Divide Combine-Conquer
   b. Conquer-Divide-Combine
   c. Divide-Conquer-Combine Page 27
   d. Combine-Divide-Conquer
2. In _____ Knapsack Problem, limitation is that an item can either be put in the bag or not. Fractional items are not allowed.
   a. 0
   b. 1
   c. 0/1 Page 91
   d. Fractional
3. In Selection algorithm, we assume pivot selection takes theta _____ running time.
   a. n Page - 36
   b. n2
   c. n3
   d. log (n)
4. In Heap Sort algorithm (using max heap), when every time maximum elements removed from top _____.
   a. We call merge Sort Algorithm
   b. it becomes Order n2 Algorithm
   c. Divide and Conquer strategy helps us
   d. We are left with a hole Page – 41
5. If matrix A of dimension p x q is multiply with matrix B of dimension q x r, then each entry in resultant matrix takes _____ time.
   a. O (q) Page - 84
   b. O (1)
   c. O (p x q)
   d. O (q x r)

6. _____ is a method of solving a problem in which we check all possible solutions to the problem to find the solution we need.
   a. Plane-Sweep Algorithm
   b. Sorting Algorithm
   c. Brute-Force Algorithm google
   d. Greedy approach
7. The worst case running time of Quick sort algorithm _____.
   a. Cannot be quadratic
   b. Is quadratic
   c. Is always Exponential
   d. Is linear
8. In max heap (for Heap Sort algorithm), when every time maximum element is removed from top we replace it with _____ leaf in the tree.
   a. second last
   b. Last Page -41
   c. First
   d. Any
9. Quick sort algorithm was developed by -
   a. AlferdAho
   b. Sedgewick
   c. John Vincent Atanasoff
   d. Tony Hoare – Google wikipedia
10. If Matrix-A has dimensions "3x2" and Matrix-B has dimensions "2x3", then multiplication of Matrix-A and Matrix-B will result a new Matrix-C having dimensions.
    a. 3x2
    b. 2x3
    c. 2x2
    d. 3x3 http://www.calcul.com/show/calculator/matrix-multiplication
11. For comparison-based sorting algorithms, it is possible to sort more efficiently than Omega n log(n) time.
    a. Always
    b. Not P-54
    c. Sometimes

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d. Sometimes not

12. Dynamic Programming approach is usually useful in solving optimization problems.
    a. True
    b. False

13. In Sorting the key value or attribute_____ from an ordered domain.
    a. Must be page 39
    b. Not always
    c. May be
    d. Occasionally

14. Result of asymptotical analysis of n(n -3) and 4n*n is that _____
    a. n(n-1) is asymptotically Less
    b. n(n-1) is asymptotically Greater
    c. Both are asymptotically Not equivalent
    d. Both are asymptotically Equivalent page 23 ($4n*n= 4n^2$)

15. Floor and ceiling are _____ to calculate while analyzing algorithms a. Very easy
    b. Usually considered difficult P-31
    c. 3rd Option is missing
    d. 4th Option is missing

16. _____ of reference is an important fact of current processor technology.
    a. Defining
    b. Assigning
    c. Formality
    d. Locality P-8

17. In max-heap, largest element is stored at root node. Where is the smallest element stored?
    a. Right Node
    b. Leaf Node
    c. Middle Node
    d. Left Node

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18. In average-case time analysis of Quick sort algorithm, the most balanced case for partition is when we divide the list of elements into _.
    a. Equal no. of pieces as of input elements
    b. Single piece exactly
    c. Two nearly equal pieces
    d. Three nearly equal pieces
19. Which of the following is calculated with Big O notation?
    a. Medium bounds
    b. Upper bounds Page - 25
    c. Lower bounds
    d. Both upper and lower bounds
20. Edit distance algorithm based on _____ strategy
    a. Greedy
    b. Dynamic Programming Page - 81
    c. Divide and Conquer
    d. Searching
21. In Heapsort Algorithm, total time taken by heapify procedure is _____
    a. O (log n) Page-43
    b. O (log2 n)
    c. O (n log n)
    d. O (n2 log n)
22. Al-Khwarizmi was a/an _____
    a. Artist
    b. Mathematician P-7
    c. Astronomer
    d. Khalifah
23. When matrix A of 5x3is multiply with metric B of 3x4 then the number of multiplication required is: Not found exactly
    a. 15
    b. 12
    c. 36
    d. 60 Not Found exactly but as per formula at page 84,

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24. Pseudo code of algorithms are to be read by _____.
    a. People Page -12
    b. RAM
    c. Computer
    d. Compiler
25. The sieve technique is a special case, where the number of sub-problems is Just _____
    a. 1 P-34
    b. 2
    c. 3
    d. 4
26. When a recursive algorithm revisits the same problem over and over again, we say that the optimization problem has _____ sub-problems.
    a. Overlapping – Google Search
    b. Over costing
    c. Optimized
    d. Three
27. Sieve technique is very important special case of Divide-and-Conquer strategy.
    a. True P-34
    b. False
28. In order to say anything meaningful about our algorithms, it will be important for us to settle on a _____.
    a. Java Program
    b. C++ Program
    c. Pseudo program
    d. Mathematical model of computation P-10
29. Merge sort is based on _____.
    a. Brute-force
    b. Plan-sweep
    c. Axis-sweep
    d. Divide and Conquer P-27

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30. What time does Merge Sort algorithm take in order to sort an array of 'n' numbers?
    a. (n)
    b. (log n)
    c. (n^2)

    d. (n log n) Google Search 31. In Heap Sort

31. algorithm, the first step is to _____.
    a. Call Build-Heap procedure Page - 46
    b. Sort the array in descending order
    c. Call Heapify procedure
    d. Find the number of input elements

32. The definition of theta-notation relies on proving _____ asymptotic bound.
    a. One
    b. Lower
    c. Upper
    d. Both lower & upper Page - 25

33. In merge sort algorithm, to merge two lists of size n/2 to a list of size n, takes _____ time.
    a. Theta (n) Page - 32
    b. Theta log(n)
    c. Theta log2(n)
    d. Theta n log(n)

34. We can make _____ recursive calls in Fibonacci Sequence.
    a. Infinite
    b. Finite google
    c. Only one
    d. Zero

35. Following is NOT the application of Edit Distance problem.
    a. Speech recognition
    b. Spelling Correction
    c. Ascending Sort Page - 76
    d. Computational Molecular Biology

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36. In plane sweep approach, a vertical line is swept across the 2d-plane and structure is used for holding the maximal points lying to the left of the sweep line.
    a. Array
    b. Queue
    c. Stack Page - 18
    d. Tree
37. When a heapify procedure is applied to the root node to restore the heap, then at each level, the comparison performed takes time:
    a. It will take (log n).
    b. It can not be predicted
    c. It will take O (1). Page - 43
    d. Time will vary according to the nature of input data.
38. _____ time is the maximum running time over all legal inputs.
    a. Worst-case Page - 13
    b. Average-case
    c. Best-case
    d. Good-case
39. Efficient algorithm requires less computational...
    a. Memory
    b. Running Time
    c. Memory and Running Time Page - 9
    d. Energy
40. For average-case time analysis of Quick sort algorithm, Pivot selection is on average basis from _____
    a. half of the input values
    b. all possible random values Page - 50
    c. Pivot is input separately
    d. values greater than 5
41. Selection algorithm takes theta _____
    a. (n2)
    b. (n) Page - 37
    c. log(n)
    d. n log(n)

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42. Recurrence can be described in terms of a tree.
    a. Yes Page - 31
    b. No
43. Time complexity of Dynamic Programming based algorithm for computing the minimum cost of Chain Matrix Multiplication is _____
    a. Log n
    b. n
    c. n^2 (n square)
    d. n^3 (n cube) Page -90
44. The Iteration method is used for _____
    a. Comparing sorting algorithms only
    b. Solving Recurrence relations Page 31
    c. Merging elements in Merge sort
    d. Dividing elements in Merge sort
45. In 3-Dimensional space, a point P has _____ coordinate(s).
    a. (X, Y)
    b. (X, 0)
    c. (0, Y)
    d. (X,Y, Z)
46. Chain matrix multiplication problem can be solved through _____ strategy.
    a. Dynamic programming Page - 85
    b. Greedy
    c. Divide and conquer
    d. Sorting
47. Merge sort have running time....running time of Heap sort. Not found exactly
    a. Greater than
    b. Less than Google
    c. Equal to
    d. Different than
48. Median is not useful measure of central tendency of given input set especially when the distribution of values is highly skewed.
    a. True
    b. False Page – 34

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49. We do not need to mathematically prove that for comparison-based sorting algorithms always takes Omega nlog (n) time.
    a. True Google & VU Tech (pg 46 not very clear)
    b. False
50. The Omega-notation allows us to state only the asymptotic _____ bounds.
    a. Middle
    b. Lower Page 25
    c. Upper
    d. Both lower & upper
51. Both lower & upperSorting can be in _____
    a. Increasing order only
    b. Decreasing order only
    c. Both Increasing and Decreasing order GOOGLR Search
    d. Random order
52. Radix sort performs sorting the numbers _____ digit (s) at a time.
    a. One Page - 71
    b. Two
    c. Three
    d. All
53. Quicksort is a/an _____ and _____ sorting algorithm.
    a. Not in place, not stable one
    b. In place , not stable one Page - 54
    c. In place , stable one
    d. Not in place , stable one
54. Consider three matrices X,Y,Z of dimensions 1x2, 2x3,3x4 respectively. The number of multiplications of (XY) Z is:
    a. 18 As per lecture slides
    b. 32
    c. 24
    d. 30
55. In Fibonacci Sequence, unnecessary repetitions do not exist at all.
    a. True
    b. False Page – 74

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56. It is not a Fibonacci sequence . 1,1,1,2,3,5,8,13,21,34,55,…..
    a. True Page - 73
    b. False
57. Heap sort is a/ an _____ and _____ sorting algorithem.
    a. Not in place, not stable one
    b. In place , not stable one Page - 54
    c. In place , stable one
    d. Not in place , stable one
58. Identify the True Statement
    a. The knapsack problem does not belong to the domain of optimization problems.
    b. The knapsack problem belongs to the domain of optimization problems. Page - 91
    c. The Knapsack problem cannot be solved by using dynamic programming
    d. The knapsack problem is optimally solved by using brute force algorithm.
59. In Dynamic Programming, our approach is to _____
    a. Develop the solution in a top-down fashion
    b. Express the problem non-recursively
    c. Build the solution in a bottom-up fashion Page - 75
    d. Input several sub-problems simultaneously
60. Counting sort is suitable to sort the elements in range 1 to K;
    a. K is large
    b. K is small Page - 57
    c. K may be large or small
    d. None
61. We can multiply two matrices A and B only when they are compatible which means
    a. Number of columns in A must be equal to number of rows in B. it seems Correct as per page 84
    b. Number of rows and columns do not matter

c. Number of columns in A must be equal to number of columns in B
d. Number of rows in A must be equal to number of rows in B

62. Matrix multiplication is a (n) _____ operation.
a. Commutative
b. Associative Page 85
c. Neither commutative nor associative
d. Commutative but not associative

63. In Dynamic Programming approach, solution is modified / changed
a. Always once
b. At each stage google and wikipedia
c. Only for specific problems
d. At 4th stage only

64. In Knapsack problem, the goal is to put items in the Knapsack such that the value of the items is _____ subject to weight limit of knapsack.
a. Minimized
b. Decreased
c. Maximized Page - 91
d. None of the given options

65. An in-place sorting algorithm is one that _____ uses additional array for storage.
a. Always
b. Permanently
c. Does not Page - 54
d. Sometime

66. Memoization is a part of Dynamic Programming Strategy.
a. True Page - 74
b. False

67. If matrix A of dimension 2x4 is multiply with matrix B of dimension 4x3, then the dimension of resultant matrix is Not found exactly
a. 2x4
b. 4x3
c. 3x4
d. 2x3 It seems correct as per second last Para of page 84

68. In Dynamic Programming approach, we do not store the solution to each sub-problem in case if it reappears.
    a. True
    b. False Page - 75
69. Dynamic Programming is a problem-solving approach in which___
    a. Problem is solved in Zero time
    b. Solution is developed only at final stage
    c. Both are correct
    d. Both are incorrect google
70. In Fibonacci sequence, each term is calculated by____ previous__ terms.
    a. Subtracting, Two
    b. Adding, Three
    c. Adding, Two Page - 73
    d. Multiplying, Two
71. Selection sort is not an in-place sorting algorithm.
    a. True Page - 54
    b. False
72. If there are $\theta(n^2)$ entries in edit distance matrix then the total running time is:
    a. $\theta(n)$
    b. $\theta(1)$
    c. $\theta(n^2)$ Page – 84
    d. $\theta(n \log n)$
73. The only way to convert a string of i characters into the empty string is with i deletions, represented as
    a. E(0.j) =j
    b. E(i.j) = 1
    c. E(0.i) = j
    d. E (i.0)=I Page - 78
74. Dynamic programming formulation of the matrix chain multiplication problem will store the solutions of each sub problem in an
    a. Array
    b. Table Page - 86
    c. Variable
    d. class

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75. We can use the optimal substructure property to devise a formulation of the edit distance problem.
    a. Selective
    b. Optimum
    c. Iterative
    d. Recursive Page - 78

76. Sorting is performed on the basis of _____.
    a. Computational resources
    b. Asymptotic notation
    c. Summation
    d. Some key value of attribute page- 39

77. In Heap Sort algorithm, we call Build-heap procedure _____.
    a. Only once page 46
    b. Twice
    c. Thrice
    d. As many times as we need

78. Radix sort is not a non-comparative integer sorting algorithm.
    a. True Google Search
    b. False

79. In the statement "output P[1].x, P[1].y", the number of times elements of P are accessed is _____.
    a. 1
    b. 2 page 14
    c. 3
    d. 4

80. The main purpose of mathematical analysis is measuring the _____ required by the algorithm.
    a. Space
    b. Execution time P-13
    c. Inputs & outputs
    d. Execution time and memory

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81. _____ provides us more accurate result when input values are not closer with each other
    a. Average
    b. Median P-34
    c. Mode
    d. Mean
82. The process of _____ ends when you are left with such tiny pieces remaining that it is trivial to solve them.
    a. Brute-force
    b. Plan-sweep
    c. Divide and Conquer P-27
    d. Axis-sweep
83. _____ overcomes the limitations of _____ by working as per positional notations of numbers.
    a. Counting sort, Radix sort
    b. Radix sort, Counting sort P-71
84. Memorization is a part of Dynamic Programming strategy.
    a. True P-74
    b. False
85. Rank of an element can be defined as _____.
    a. One minus the number of elements that are smaller
    b. Two plus the number of elements that are greater
    c. One plus the number of elements that are smaller P-34
    d. Two minus the number of elements that are smaller
86. If the time complexity of an algorithm is given by O (1), then its time complexity would be
    a. Polynomial
    b. Exponential
    c. Constant - Wikipedia
    d. Average
87. Quick sort is a recursive algorithm.

    a. True Wikipedia ; Google
    b. False

88. The asymptotic growth of n(n+1)/2 is:
    a. $O(n^2)$ As the n^2 term has the largest contribution, the Big-O complexity is O(n^2)
    b. O(n)
    c. O(n+2)
    d. O(n log n)
89. Approach of solving geometric problems by sweeping a line across the plane is called _____ sweep.
    a. Line
    b. Plane Page 18
    c. Cube
    d. Box
90. As per algorithm of Dynamic Programing, we need to store
    a. First sub-problem only
    b. Best solution only
    c. Intermediate sub-problems Pg:75
    d. Final solution only
91. In Sieve technique, we solve the problem
    a. In recursive manner Pg:34
    b. Non recursively
    c. Using Merge Sort algorithm
    d. Using Brute force technique
92. One of the limitation in 0/1 knapsack is that an item can either be _____ in the bag or not.
    a. Use
    b. Put Pg:91
    c. Move
    d. Store
93. Which one is not passed as parameter in Quick sort algorithm?
    a. End of the array
    b. Middle of the array
    c. Array (containing input elements) Google
    d. Start of the array

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94. In the analysis of Selection algorithm, we get the convergent _____
    a. Harmonic
    b. Linear
    c. Arithmetic
    d. Geometric Pg:37
95. A Random Access Machine (RAM)is an idealized machine withrandom access memory.
    a. Infinite large Pg:10
    b. 512 MB
    c. 256 MB
    d. 2 GBs
96. While analyzing Selection algorithm, we make a number of passes, in fact it could be as many as
    a. n(n+1)
    b. log(n) Pg:37
    c. n/3
    d. n/4
97. In Random Access Machine (RAM), instructions are executed in
    a. Parallel
    b. Batch
    c. One by One Pg:10
    d. Multiple times
98. In selection problem, the rank of an element will be its _____ position
    a. First
    b. final Pg:34
    c. Second last
    d. Last
99. The worst-case running time of Merge sort is _____ in order to sort an array of n elements.
    a. O(log n)
    b. O(n)
    c. O(n log n) page 40 and google
    d. O(n)

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100. f(n) and g(n) are asymptotically equivalent. This means that they have essentially the same _____.
   a. Results
   b. Variables
   c. Size
   d. Growth rates P:23

101. An algorithm is a mathematical entity. Which is independent of _____.
   a. Programming language
   b. Machine and Programming language
   c. Compiler and Programming language
   d. Programming language Compiler and Machine P:07

102. In Quick sort algorithm, Pivots form ___
   a. Stack
   b. Queue
   c. Binary Search Tree P:49
   d. Graph

103. Counting sort is suitable for sorting the elements within range 1 to P. where
   a. P is large
   b. P is small P-57
   c. P is very large
   d. P is undetermined

104. In asymptotical analysis of n'(5 2)-3, as n becomes large, the dominant (fastest growing) term is some constant times
   a. n_1
   b. n
   c. n+1
   d. n*n P-23

105. ___ Items are not allowed in the 0/1 knapsack. a. Lighter
   b. Fractional P-91
   c. Whole
   d. Weighty

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106. Fibonacci Sequence was named on _____, a famous mathematician in 12th Century.
   a. Fred Brooks
   b. Grady Booch
   c. Leonardo Pisano P-73
   d. Edgar F. Codd

107. In Heap Sort algorithm, we build _____ for ascending sort.
   a. Max heap P-41
   b. Min heap

108. Bubble sort is not an in-place sorting algorithm.
   a. True
   b. False P-54

109. In partition algorithm, the subarray _____ has elements which are greater than pivot element x.
   a. A[p...r]
   b. A[p...q-1]
   c. A[q]
   d. A[q+1..r] P-46

110. In Heap Sort algorithm, if heap property is violated
   a. We call Build heap procedure P-43
   b. We call Heapify procedure
   c. We ignore
   d. Heap property can never be violated

111. _____ is not a characteristic of Random Access Machine.
   a. Single-Processor P-10
   b. Assigning a value to a variable
   c. Locality of reference
   d. Executing an arithmetic instruction

112. The only way to convert an empty string into a sting of j characters is by doing j insertions, represented as _____
   a. E(i,j) = 1
   b. E(I,0) = I
   c. E(0,j) = j page 78
   d. E(1,j)= j

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113. In Selection problem, the Sieve technique works in _____.
   a. Non-recursive manner
   b. Constant time
   c. Phases page 34
   d. One complete go

114. Algorithm is a sequence of computational steps that ---- the input into output.
   a. Merge
   b. Assign
   c. Transform page 7
   d. Integrate

115. If pj dominates pi and pi dominates ph then pj also dominates ph, it means dominance relation is
   a. Transitive page 18
   b. Non Transitive
   c. Equation
   d. Symbolic

116. To find maximal points in brute-force algorithm each point of the space is compared against _____ of that space.
   a. One other point
   b. All other points page 11
   c. Few other points
   d. Most of the other points

117. In the following code the statement "cout<<j;"executes ---- ----- times. for (j=1; j<=5; j = j+2)
   cout<<j;
   a. 5 times
   b. 2 times
   c. 3 times
   d. 0 times

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118. In merge sort algorithm, we split the array around the _____ index q. a. Entring
b. Mid page 17
c. Exiting
d. Summing

119. In Selection problem, the Sieve technique _____.
a. Add some more input items each time
b. Do not work recursively
c. Do not uses Divide and Conquer approach
d. Eliminates undesired data items each time

120. Consider three matrices X, Y, Z of dimensions 1 x 2, 2 x 3, 3 x 4 respectively. The number of multiplications of X(YZ) is .
a. 16
b. 32
c. 26
d. 32 page 84

121. In Heap Sort algorithm, the total running time for Heapify procedure is _____
a. Theta (log n)
b. Order (log n)
c. Omega (log n)
d. O(1) i.e. Constant time

122. The sieve technique works where we have to find_____ items(s) from a large input.
a. Single page 34
b. Two
c. Three
d. Similar

123. In Dynamic Programming based solution of Knapsack Problem, if we decide to take an object i , then we gain_____
a. W(Total Weight of Knapsack)
b. V (Total Value of all items)
c. vi (Value of object i) page 93
d. Nome of the given option

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124. While Sorting, the order domain means for any two input elements x and y _____ satisfies only.
   a. x < y page 39
   b. x > y
   c. x = y
   d. All of the above

125. For solving Selection problem, we introduced Sieve technique due to _____
   a. Using Decrease and Conquer strategy page 34
   b. Avoiding to sort all input data
   c. Eliminating Rank of an element
   d. Using Brute-force approach

126. _____ is one of the few problems, where provable lower bounds exist on how fast we can sort.
   a. Searching
   b. Sorting page 38
   c. Both Searching & sorting
   d. Growing

127. In plane sweep approach, a vertical line is swept across the 2d-plane from_____.
   a. Right to Left
   b. Left to Right page 18
   c. Top to Bottom
   d. Bottom to top

128. In generating Fibonacci sequence, we can avoid unnecessary repetitions by _____ process.
   a. Tokenization
   b. Memorization page 43
   c. Randomization
   d. Memorization

129. For _____ values of n, any algorithm is fast enough.
   a. Small page 14
   b. Medium
   c. Large
   d. Infinity

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130. Dynamic programming comprises of _____.
   a. Recursion only
   b. Repetition only
   c. Recursion with Repetition
   d. No Repetition but Recursion page 75

131. The function f(n)=n(logn+1)/2 is asymptotically equalient t nlog n :Here Lower Bound means function f(n) grows asymptotically at __ as fast as nlog n.
   a. Least page 23
   b. Normal
   c. Most
   d. At

132. Counting sort has time complexity.
   a. O(n+k)
   b. O(n) page 58
   c. O(k)
   d. O(nlogn)

133. Due to left complete nature of binary tree, the heap can be stored in
   a. Array page 40
   b. Structures
   c. Link List
   d. Stack

134. Single item from a larger set of _____.
   a. Constant
   b. Pointers
   c. Phases
   d. n items page 34

135. In the clique cover problem, for two vertices to be in the same group, they must be _____ each other.
   a. Apart from
   b. Far from
   c. Near to
   d. Adjacent to page 76

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136. How much time merge sort takes for an array of numbers?
   a. T(n^2)
   b. T(n)
   c. T(log n)
   d. T(n log n) page 40

137. In in-place sorting algorithm is one that uses arrays for storage.
   a. No additional array page 54
   b. An additional array
   c. Both of above may be true according to algorithm
   d. More than 3 arrays of one dimension

138. Brute-force algorithm for 2D-Maxima is operated by comparing _____ pairs of points.
   a. Two
   b. Some
   c. Most
   d. All page 18

139. While Sorting, the ordered domain means for any two input elements x and y ____ satisfies only.
   a. x > y
   b. x < y
   c. x = y
   d. All of the above page 38

140. Quick sort is.
   a. Stable & in place
   b. Not stable but in place page 54
   c. Stable but not in place
   d. Some time stable & some times in place

141. Which may be a stable sort?
   a. Merger
   b. Insertion
   c. Both above page 54
   d. None of the above

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142. For the Sieve Technique we take time.
   a. T(nk) page 34
   b. IT(n / 3)
   c. n^2
   d. n/3

143. Continuation sort is suitable to sort the elements in range 1 to k.
   a. K is Large
   b. K is not known
   c. K may be small or large
   d. K is small page 54

144. Asymptotic growth rate of the function is taken over _____ case running time. .
   a. Best
   b. Worst page 14
   c. Average
   d. Normal

145. The sieve technique is a special case, where the number of sub problems is just.
   a. 5
   b. Many
   c. 1 page 34
   d. Few

146. In Quick sort, we don't have the control over the sizes of recursive calls.
   a. True page 49
   b. False
   c. Less information to decide
   d. Ether true or false

147. Before sweeping a vertical line in plane sweep approach, in start sorting of the points is done in increasing order of their _____ coordinates. .
   a. X page 18
   b. Y
   c. Z
   d. X , Y

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148. Random access machine or RAM is a/an.
   a. Machine build by Al-Khwarizmi
   b. Mechanical machine
   c. Mathematical model page 10
   d. Electronics machine

149. The Huffman codes provide a method of encoding data inefficiently when coded using ASCII standard.
   a. True
   b. False page 99

150. A heap is a left-complete binary tree that confirms to the _____.
   a. increasing order only
   b. decreasing order only
   c. heap order page 40
   d. log n order

151. If we associate (x, y) integers pair to cars where x is the speed of the car and y is the negation of the price. High y value for a car means a _____ car.
   a. Fast
   b. Slow
   c. Expensive
   d. Cheap

152. Which one of the following sorting algorithms is the fastest?
   a. Merge sort
   b. Quick sort
   c. Insertion sort
   d. Heap sort

153. Quick sort algorithm divide the entire array into _____ sub arrays.
   a. 2
   b. 3
   c. 4
   d. 5

154. In brute force algorithm, we measure running time T(n) based on _____.
   a. Average-case time and best-case time
   b. Worst-case time and average-case time page 46
   c. Worst-case time and best-case time
   d. Best-case time and staring-case time

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155. For 2D Maxima problem. Plane Sweep algorithm first of all _____.
   a. Sorts all points
   b. Delete some points
   c. Output the elements
   d. Pushes all points on stack

156. There are _____ entries in the Edit Distance Matrix
   a. ɵ (n)
   b. ɵ (n_2) page 84
   c. ɵ (n+2)
   d. ɵ (n + 100)

157. Which symbol is used for Omega notation?
   a. (O)
   b. (ɵ)
   c. (Ω)
   d. (@)

158. Selection sort is a _____ sorting algorithm
   a. In-place page 54
   b. Not In-Place
   c. Stable
   d. in-partition

159. In Dynamic Programming based solution of knapsack problem, to compute entries of 'V', we will imply a(n) _____ approach.
   a. Subjective
   b. Inductive
   c. Brute Force
   d. Combination

160. We do not need to prove comparison-based sorting algorithms by mathematically. It always takes _____ time.
   a. Big Oh nlog(n)
   b. Omega nlog(n) NOT SURE
   c. Omega n(n^2)
   d. Theta nlog(n)

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161. Merge sort is a/an _____ and _____ sorting algorithm
    a. Not in-place, not stable one
    b. In-place, not stable one
    c. In-place, stable one
    d. Not in-place, stable one page 54
162. Cubic function will _____ a quadratic function.
    a. Prove
    b. be equal to
    c. overtake Page 25
    d. find
163. Insertion sort is a _____ sorting algorithm
    a. Unstable
    b. In-place Page 54
    c. Not In-Place
    d. in-partition
164. To check whether a function grows faster or slower than the other function, we use some asymptotic notations, which is _____.
    a. Big-oh notation
    b. Theta notation
    c. Omega notation
    d. All of the given
165. Asymptotic growth of 8n^2 + 2n – 3 is:
    a. Θ(n^2 + n)
    b. Θ (n^2) page 14
    c. Θ(8n^2)
    d. Θ(8n^2 + 2n)
166. In the analysis of algorithms, _____ plays an important role.
    a. text analysis
    b. time
    c. growth rate
    d. money

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167. In inductive approach of knapsack problem, we consider 2 cases, _____ Or _____.
   a. Median, Mode
   b. Recursive, Iterative
   c. Leave object, Take object page 93
   d. Sequentially. Parallel

168. Random Access Machine (RAM) can execute _____ instructions
   a. only logical
   b. parallel
   c. only arithmetic
   d. logical and arithmetic

169. Using _____ algorithm, efficiency is not given much importance
   a. Greedy
   b. Merge sort
   c. Processing as there is no algorithm by this name
   d. Brute Force

170. Bubble sort takes theta _____ in the worst case
   a. (n2) page 39
   b. (n)
   c. log(n)
   d. nlog(n)

171. If matrix A of dimension p × q is multiply with matrix B of dimension q × r, then dimension of resultant matrix is:
   a. p × q
   b. p × r page 84
   c. q × r
   d. r × p

172. Dynamic Programing algorithms often use some kind of _____ to store the results of intermediate sub-problems
   a. table (Page 75)
   b. variable
   c. stack
   d. loop

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173. _____ is in-place sorting algorithm.
   a. Bubble sort (Page 54)
   b. Merge sort
   c. Linear search
   d. Binary Search

174. Which one of the following problems can be solved using dynamic problem?
   a. Bubble sort problem
   b. Matrix chain multiplication problem page 85
   c. Greedy search problem
   d. Fractional knapsack problem

175. In chain matrix multiplication, solutions of the sub-problems are stored in a _____.
   a. Array
   b. Table page 86
   c. Tree
   d. Link list

176. What is the average running time of a quick sort algorithm?
   a. O(n^2)
   b. O(n)
   c. O(n log n) (Page 49)
   d. O(log n)

177. Sorting Algorithms having O _____ running time are considered to be slow ones.
   a. (n)
   b. (n^2) (Page 39)
   c. (nlog(n))
   d. (log(n))

178. While solving Selection problem, in Sieve technique we partition input data _____
   a. In increasing order
   b. In decreasing order
   c. According to Pivot
   d. Randomly

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179. _____ is the process of avoiding unnecessary repetitions by writing down the results of recursive calls and looking them up again if we need them later.
   a. Loop
   b. Memoization page 74
   c. Recursion
   d. Function

180. In average-case time the probability of seeing input is denoted by _____.
   a. p{I}
   b. p[I]
   c. p<i>
   d. p(i) page 13

181. While applying the Sieve technique to selection sort, how to choose a pivot element.
   a. Through mean
   b. Linear
   c. Randomly page 35
   d. Sequentially

182. Number of _____ of the pseudo code are counted to measure the running time.
   a. Inputs
   b. Outputs
   c. Steps page 13
   d. Pages

183. Developing a dynamic programming algorithm generally involves _____ separate steps.
   a. One
   b. Two page 75
   c. Three
   d. Four

184. 8n^2+2n+3 will exceed c28(n), no matter how large we make _____.
   a. n
   b. 2n
   c. c2 page 25
   d. this quadratic equation

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185. The running time of quick sort algorithm_____.
   a. Is impossible to compute
   b. Has nothing to do with pivot selection
   c. Is Random upon each execution
   d. Greatly influenced by the selection of pivot page 49

186. _____ involves breaking up the problem into sub problems whose solutions can be combined to solve the global problem.
   a. Complexity Theory
   b. Dynamic programming solution
   c. Divide and Conquer Strategy page 34
   d. Greedy Algorithms

187. In _____ we have to find rank of an element from given input.
   a. Merge sort algorithm
   b. Selection problem page 34
   c. Brute force technique
   d. Plane Sweep algorithm

188. How many steps are involved to design the dynamic programming strategy?
   a. 2
   b. 3
   c. 1
   d. 4 page 92

189. In Bucket sort, if there are duplicates then each bin can be replaced by a a. Stack
   b. Linked list page 69
   c. Hash table
   d. Heap

190. In merge sort algorithm, we split the array _____ to find index q.
   a. from start
   b. midway page 28
   c. from end
   d. both from start or end

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191. Find the maximum value of the items which can carry using knapsack Knapsack weight capacity = 50.

Item Weight Value
11070
22020
33080
470 200

a. 280
b. 100
c. 90
d. 200

192. In 2-d maxima problem a point p is said to be dominated by point q if _____.

a. p.x <= q.x
b. p.x <= q.x and p.y <= q.y page 17
c. p.y <= q.y
d. p.x >= q.x and p.y >=q.y

193. Sorting can be in _____.

a. Increasing order only
b. Decreasing order only
c. Both increasing and decreasing order
d. Random order

194. Recurrence can be described in terms of _____.

a. Array
b. Linear
c. Tree page 31
d. Graph

195. The brute-force algorithm for 2D-Maxima runs in order O(__) time.

a. n
b. n(log n)
c. n*n page 18
d. n3

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196. In plane sweep approach of solving geometric problems, a _____ is swept across the plane.
   a. Line page 18
   b. Plane
   c. Cube
   d. Box

197. Which of the following is calculated with Big Omega notation?
   a. Medium bounds
   b. Upper bounds
   c. Lower bounds Page - 25
   d. Both upper and lower bounds

198. _____ is always based on divide and conquer strategy.
   a. Bubble sort
   b. Selection sort
   c. Pigeon sort
   d. Quick sort page 46

199. If a matrix has three rows and two columns, then dimensions of matrix will be:
   a. 3x2
   b. 2x3
   c. 3x3
   d. 2x2

200. Asymptotic notations are used to describe _____ of an algorithm.
   a. Length
   b. running time google
   c. size
   d. compile time

201. Catalan numbers are related the number of different _____ on 'n' nodes.
   a. Arrays
   b. linked lists
   c. binary trees page 85
   d. functions

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202. Applying the sieve technique to selection problem, _____ element is picked from array.
   a. Output
   b. Total
   c. Input
   d. Pivot page 35

203. Dynamic Programming approach is usually useful in solving _____ problems.
   a. Normal
   b. Optimization google
   c. Array
   d. Loop

204. In recursive formulation of knapsack Problem: V [0, j] = _____ for j>=0
   a. -1
   b. 0 page 93
   c. 1
   d. 2

205. _____ is a linear time sorting algorithm.
   a. Merge sort
   b. Radix sort page 71
   c. Quick sort
   d. Bubble sort

206. Quick sort is one of the _____ sorting algorithm.
   a. Fastest page 19
   b. Slowest
   c. Major
   d. Average

207. The time assumed for each basic operation to execute on RAM model of computation is _____.
   a. Infinite
   b. Continuous
   c. Constant page 10
   d. Variable

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208. In Sieve Technique, we know the item of interest. a. True
b. False page 34

209. While analyzing algorithms, _____ and _____ usually considered difficult to calculate.
a. Finite, Infinite
b. Floor, ceiling google
c. Row, Column
d. Graph, Tree

210. While analysis of the brute-force maxima algorithm, an array sorted in the reverse order is the type of _____ case input.
a. Best
b. Worst page 14
c. Somewhat bad
d. Average

211. _____ is not useful measure of central tendency of given input set especially when the distribution of values is highly skewed.
a. Mean
b. Mode
c. Average
d. Median page 34

212. In asymptotical analysis of n(n-3) and 4n*n, as n becomes large, the dominant (fastest growing) term is some constant times _____.
a. n+1
b. n-1
c. n
d. n*n page 23

213. In addition to passing in the array itself to Merge Sort algorithm, we will pass in other arguments which are indices.
a. Two P-38
b. Three
c. Four
d. Five

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214. In 2d-maximal problem, a point is said to be if it is not dominated by any other point in that space.
   a. Member
   b. Minimal
   c. Maximal P-11
   d. Joint

215. Counting sort assumes that the numbers to be sorted are in the range _____.
   a. K to n where n is large
   b. 1 to k where k is small (P-57)
   c. K to n where k is small
   d. k to n where n is small

216. Insertion sort is an efficient algorithm for sorting a _____ number of elements
   a. Large P-39
   b. Small
   c. Extra large
   d. Medium

217. If the indices passed to merge sort algorithm are _____ then this means that there is only one element to sort.
   a. Small page 28
   b. Large
   c. Equal
   d. Not Equal

218. In Knapsack Problem, each item must be entirely accepted or rejected, is called _____ problem.
   a. Fractional
   b. 0-1 P-92
   c. Linear
   d. Optimal

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219. If the time complexity of an algorithm is O(n). then it is called _____ time complexity.
   a. Linear Wikipedia
   b. Constant
   c. Average
   d. Exponential

220. In the case of _____ analysis does not depend upon on the distribution of input.
   a. Merge sort
   b. Quick sort P-50
   c. Insertion sort
   d. Heap sort

221. We can use the _____ Property to devise a recursive formulation of the edit distance problem.
   a. Small substructure
   b. Algorithmic
   c. Real
   d. Optimal substructure page 78

222. The following sequence is called _____
   1,2,3,5,8,13,21,34,55,…..
   a. Optimize sequence
   b. Fibonacci sequence page 73
   c. Optimal sequence
   d. Overlapping sequence

223. Which one sorting algorithm is best suited to sort an array of 2 million elements?
   a. Bubble sort
   b. Insert sort
   c. Merge sort
   d. Quick sort
   e. Ridx Sort page 71

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224. We can improve the performance of quick sort if we could be able to _,_____.
   a. Skip input elements somehow
   b. Select two or more pivots page 34
   c. Skip any sub-array completely
   d. Eliminate recursive calls

225. The problem with the brute-force algorithm is that is uses _____ in pruning out de
   a. Worst-case time
   b. No intelligence page 18
   c. Outside looping
   d. Artificial intelligence

226. In chain matrix multiplication, the order of the matrices _____.
   a. Can be changed
   b. Can not be changed page 85
   c. is equal
   d. is reverse

227. In quick sort algorithm, we choose pivot_____.
   a. Always the smallest element
   b. Greater than 5
   c. Randomly page 35
   d. Less than 5

228. In Heap Sort algorithm. Heapify procedure is _____ in nature.
   a. Recursive
   b. Non-Recursive page 43
   c. Fast
   d. Slow

229. When matrix A of 5x 3 is multiplied with matrix B of 3 x 4 then the number of multiplications required will be _____.
   a. 15
   b. 12
   c. 36
   d. 60

230. An algorithm is said to be correct if for every _____ instance, it halts with the correct _____.
   a. Input, Output page 13
   b. Design, Analysis
   c. Value, Key
   d. Key, Analysis

231. In chain matrix multiplication, table is filled _____ to find the multiplication of matrix.
   a. row wise
   b. column wise
   c. diagonally
   d. bottom-to-up page 86

232. If we have an equation 8n2+7f*n + 5f + 6 then is large, _____ term will be muchxxxxxxxthe n term and will dominate the running time.
   a. f g (n)
   b. g (n) * 2
   c. n * 2 page 23
   d. f (n)

233. For quick sort algorithm. Partitioning takes theta _____.
   a. (n)
   b. log(n)
   c. n log (n)
   d. n2log (n)

234. In Heap Sort algorithm, the maximum levels an element can move upward is _____
   a. Theta (log n) page 43
   b. Big-ch (log n)
   c. Omega (log n)
   d. 0 (1) i.e. Constant time

235. _____ programming is essentially recursion without repetition.
   a. Fast
   b. Dynamic page 75
   c. Array

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d. n (log n)

236. In heap sort algorithm, the total running time for heavily procedure is _.

- Big-oh(log n)
- O (1) i.e. Constant time
- Theta (log n)
- Omega (log n)

237. Quick sort algorithm is required a lot of comparison in the ____ condition.

- Worse case
- Best and average case
- Average case
- Best case

238. In heap sort algorithm (using max heap). When every time maximum element is removed from top.

- Divide and conquer strategy helps us
- We are left with a lot
- We call merge sort algorithm
- It becomes order n2 algorithm

239. In average-case time analysis of quick sort algorithm, The most balanced case for partition is where we divide the list of element into _____

- Three nearly equal pieces
- Single piece exactly
- Two nearly piece
- Equal no. of piece as of input element

240. Consider three matrices X,Y,Z dimensions 1×2.2×.3×4 respectively. The number of multiplication of (XYZ) is:

- 32

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- 30
- 24
- 18

241. Quicksort is a/an _____ and_____ sorting algorithm.

- In-place, not stable one
- Not in-place, stable one
- In-place, stable one
- Not in-place, not stable one

242. _____items are not allowed in the 0/1 knapack.

- Lighter
- Whole
- Weighty
- Fractional

243. The main purpose of mathematical analysis is measuring the _____ required by the algorithm.

- Space
- Execution time and memory
- Input & output
- Execution time

244. Execution time of an algorithm can be measured by _____.

- Divide and conquer approach
- Both brute force and divide and conquer approach
- Mathematical analysis
- Brute force approach

245. Quick sort is based on _____ strategy.

- Graph theory
- Divide-and-conquer
- Dynamic programming
- Greedy approach

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246. A sorting algorithm is called as _____ if duplicate element remain in the same relative position after sorting.

- O(n) algorithm
- Stable
- Parallel
- Complex

247. Which one sorting algorithm is best suited to sort an array of 2 million elements?

- Insert sort
- Quick sort
- Merge sort
- Bubble sort

248. We can use the _____ property to devise a recursive formulation of the edit distance problem.

- Algorithm
- Small substructure
- Optimal substructure
- Real

249. While sorting. The ordered domain means for any two input elements x and y _____ satisfies only.

- All of the above
- x > y
- x < y
- x = y

250. 8n2 + 2n -3 will eventually exceed c2*(n) no matter how large we make _____.

- 2n
- n
- this equation

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- c2

251. _____ is a method of solving a problem in which we check all possible solution to the problem to find the solution we need.

- Sorting algorithm
- Greedy approach
- Plane-sweep algorithm
- Brute-force algorithm

252. In quick sort algorithm, provost form _____.

- Graph
- Stack
- Binary search tree
- Queue

253. In asymptotical analysis of n(n -3) and 4n*n, as n becomes large, the dominant (fastest growing) term is some constant time_____

- n +1
- n*n
- n
- n-1

254. If Matrix-A has dimensions "3×2" and Matrix-B has dimensions "2×3", then multiplication of Matrix –A and Matrix-B will result a new Matrix-C having dimensions

- 2×3
- 2×2
- 3×2
- 3×3

255. Boolean operation is a _____ operation on an idealized RAM model of computation.

- Advance
- Normal

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- Basic
- Starting

256. There are _____ entries in the Edit Distance Matrix.

- $\Theta(n^2)$
- Θ (n+100)
- Θ (n)
- Θ (n+2)

257. Counting sort is suitable for sorting the elements within range 1 to P. where_____

- P is undetermined
- P is small
- P is very large
- P is large

258. Suppose we have 4 matrices A,B,C,D. what is correct expansion of m[1,2] in chain matrix multiplication?

- m[1,2] = m[1,1] + m[2,2] +p0 . p1. p3
- m[1,2] = m[1,1] + m[2,2] +p0 . p1. p3
- m[1,2] = m[1,1] + m[2,2] +p0 . p1. p3
- m[1,2] = m[1,1] + m[2,2] +p0 . p1. p3

259. Which one is not passed as parameter in Quick Sort algorithm?

- Array (containing input elements)
- Middle of the array
- Start of the array
- End of the array

260. In asymptotical analysis of n*(5+2)-3. As n becomes large, the dominant (fastest growing) term is some constant times_____

- n+1
- n*n
- n

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- n_1

261. For _____ values of n, any algorithm is fast enough.

- Medium
- Small
- Infinity
- Large

262. Dynamic programming algorithms often use some kind of _____ to store the result of intermediate sub-problems.

- Stack
- Loop
- Table
- Variable

263. In selection problem, the Sieve technique works in_____

- One complete go
- Constant time
- Non-recursive manner
- Phases

264. In heap Sort algorithm, the maximum levels an element can move upward is _____.

- Theta (log n)
- O (1) i.e. Constant time
- Omega (log n)
- Big-oh (log n)

265. While analysis of the brute-force maxima algorithm, an array storted in the reverse order is the type of _____ case input.

- Worst
- Best
- Somewhat
- Average

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266. What type of instructions Random Access Machine (RAM) can execute?

►Algebraic and logic

►Geometric and arithmetic

►Arithmetic and logic (Page 10)

►Parallel and recursive

267. For Chain Matrix Multiplication we can not use divide and conquer approach because,

►We do not know the optimum k (Page 86)

►We use divide and conquer for sorting only

►We can easily perform it in linear time

►Size of data is not given

268. What is the total time to heapify?

- ► O(log n) (Page 43)
- ► O(n log n)
- ► O(n2 log n)
- ► O(log2 n)

269. _____ is a graphical representation of an algorithm

- ► notation
- ► notation
- ► Flowchart
- ► Asymptotic notation

270. A RAM is an idealized machine with _____ random-access memory.

- ► 256MB
- ► 512MB
- ► an infinitely large (Page 10)
- ► 100GB

271. What type of instructions Random Access Machine (RAM) can execute? Choose best answer

► Algebraic and logic

► Geometric and arithmetic

► Arithmetic and logic

► Parallel and recursive

272. What is the solution to the recurrence T(n) = T(n/2)+n .

► O(logn)

► O(n) (Page 37)

► O(nlogn)

► O(n 2)

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1. Dijkstra's Algorithm is used to solve _____ problems.
   a) All pair shortest path
   b) Single source shortest path
   c) Network flow
   d) Sorting

2. Which of the following is the most commonly used data structure for implementing Dijkstra's Algorithm?
   a) Max priority queue
   b) Stack
   c) Circular queue
   d) Min priority queue

3. What is the time complexity of Dijikstra's algorithm?
   a) O(N)
   b) $O(N^3)$
   c) $O(N^2)$
   d) O(logN)

- Please choose one

---

4. Dijkstra's Algorithm cannot be applied on _____
   a) Directed and weighted graphs
   b) Graphs having negative weight function
   c) Unweighted graphs
   d) Undirected and unweighted graphs

5. How many priority queue operations are involved in Dijkstra's Algorithm?
   a) 1
   b) 3
   c) 2
   d) 4

6. How many times the insert and extract min operations are invoked per vertex?
   a) 1
   b) 2
   c) 3
   d) 0

7. The maximum number of times the decrease key operation performed in Dijkstra's algorithm will be equal to _____
   a) Total number of vertices
   b) Total number of edges
   c) Number of vertices – 1
   d) Number of edges – 1

8. What is running time of Dijkstra's algorithm using Binary min- heap method?
   a) O(V)
   b) O(VlogV)
   c) O(E)
   d) O(ElogV)

True False

- Please choose one

---

9. The running time of Bellmann Ford algorithm is lower than that of Dijkstra's Algorithm.

a) True

b) False

10. Dijkstra's Algorithm run on a weighted, directed graph G={V,E} with non-negative weight function w and source s, terminates with d[u]=delta(s,u) for all vertices u in V.

a) True

b) False

11. Dijkstra's Algorithm is the prime example for _____

a) Greedy algorithm

b) Branch and bound

c) Back tracking

d) Dynamic programming

1. Computing the strongly connected components of a graph is a generalization of the problem to determine whether a digraph is strongly connected or not.

   True

2. If u finds yourself in maze the better traversal approach will be:

   BFS and DFS are valid

3. In ---------- algorithm, at any time, the subset of edges A forms a single tree.

   Prim's

True False

- Please choose one

---

4. In computing the strongly connected components of a digraph, vertices of the digraph are not partitioned into subsets.

   False

5. Which is true statement:

   The breadth-first-search is a shortest-path algorithm that works on un-weighted graphs.

6. There are no ---------- edges in undirected graph.

   Cross

7. In undirected graph, by convention all the edges are called ---------- edges.

   Back

8. A digraph is strongly connected under what condition?

   A digraph is strongly connected if for every pair of vertices u, v ∈ V, u can reach v and vice versa.

9. The tricky part of the ---------- algorithm is how to detect whether the addition of an edge will create a cycle in viable set A.

   Kruskal's

10. Digraphs are not used in communication and transportation networks.

    True

11. If u and v are mutually reachable in G, then in the graph formed by reversing all the edges, these vertices are not reachable.

True False

- Please choose one

False

12. A topological sort of a DAG is a---------- ordering of the vertices of the DAG such that for each (u, v), u appears before v in the ordering.

Linear

13. Kruskal's algorithm works by adding vertices in increasing order of weight (lightest edge first).

False

14. Adding an edge to a free tree creates:

Cycle

15. Forward edge is:

(u, v) where v is a proper descendent of u in the tree

16. In strong components algorithm, vertices are sorted in ---------- order of finish times.

Decreasing

17. In Kruskal's algorithm the next edge is not added to viable set A, if it's adding Induce a cycle.

True

18. What is the time complexity to extract a vertex from priority queue in Prim's algorithm?

(log V)

19. A graph may contain----------.

True False

- Please choose one

More than one MST

20. Prim's algorithm builds MST by starting with any vertex and at any time the subset of edges form:

A single tree

21. In Prim's algorithm, we start with the---------- vertex r; it can be any vertex.

Root

22. The ---------- given by DFS allow us to determine whether the graph contains any cycles.

Time Stamps

23. You have an adjacency list for G, what is time complexity to compute Graph transpose G^T?

Θ (V + E)

24. Back edge is:

Ans. (u, v) where v is an ancestor of u in the tree

25. What algorithm technique is used in the implementation of Kruskal solution for the MST?

Ans. Greedy Technique (Pg. # 142)

26. If a subset of edges A is viable for building MST. It cannot contain a/an----------.

Ans. Cycle (Pg. # 143)

True False

- Please choose one

---

27. Adding any edge to a free tree creates a unique----------

Ans. Cycle (Pg#142)

28. Prim's algorithm is based on -------- strategy.

Ans. Greedy (Pg. # 142)

29. In strong components algorithm, First of all DFS is run for getting ---------- times of vertices.

Ans. Finish

30. There exists a unique path between any two vertices of a free tree.

Ans. True (pg. # 142)

31. In strong components algorithm, first of all DFS is run for computing finish times of vertices.

Ans. true

32. There is relationship between number of back edges and number of cycle in DFS

Ans. there is no relationship between number of back edges and number of cycles

33. In Prim's algorithm at any time, the subset of edges A forms a single forest.

Ans. False (pg#149)

34. In Kruskal's algorithm at any time, the subset of edges A forms a single tree.

True False

- Please choose one

Ans. False (pg#149)

35. Kruskal's algorithm works by adding ---------- in increasing order of weight (lightest edge first).

Ans. edges (Pg. # 147)

36. In Prims algorithm we will make use of priority----------.

Ans. queue (pg. # 150)

37. Cross edge is :

Ans. (u, v) where u and v are not ancestor or descendent of one another

38. The ancestor and descendent relation cannot be inferred by the parenthesis Lemma.

Ans. False (Pg. # 129)

39. In Kruskal's algorithm, the next ---------- is not added to viable set A, if it's adding induce a/an cycle.

Ans. edge (Pg. # 147)

40. Equivalence relation partitions the vertices into ---------- classes of mutually reachable vertices and these are the strong components.

Ans. Equivalence (Pg#136)

41. For undirected graph, there is no distinction between forward and back edges.

Ans. True (Pg#130)

True False

- Please choose one

---

42. In strong components algorithm, the form of graph is used in which all the ---------- of original graph G have been reversed in direction.

Ans. edges (Pg. # 138)

43. Networks are complete in the sense that it is not possible from any location in the network to reach any other location in the digraph.

Ans. True (Pg#135)

44. In digraph G = (V, E) ; G has cycle if and only if:

Ans. The DFS forest has a back edge (Pg. # 131)

45. As the Kruskal's algorithm runs the edges in viable set A induce a ---------- on the vertices.

Ans. Forest (Pg#147)

46. Timestamp structure of ---------- is used in determining the strong components of a digraph.

Ans. Both DFS and BFS

47. In Prim's algorithm, we start with the _____ vertex r; it can be any vertex.

Root

48. In Prim's algorithm, at any time, the subset of edges A forms a single _____.

Tree

True False

- Please choose one

---

49. Kruskal's algorithm (choose best non-cycle edge) is better than Prim's (choose best tree edge) when the graph has relatively few edges.

    True

50. There is relationship between number of back edges and number of cycles in DFS

    There is no relationship between back edges and number of cycles.

51. In Prim's algorithm, we will make use of priority _____.

    Queue

52. We say that two vertices u and v are mutually not reachable if u can reach v and vice versa.

    False

53. In strong components algorithm, the form of graph is used in which all the vertices of original graph G have been reversed in direction.

    False

54. In Timestamped DFS-cycles lemma, if edge (u, v) is a back edge, then _____

    $f[u] \leqslant f[v]$

55. There exist a unique path between any _____ vertices of a free tree.

True False

- Please choose one

Two

56. In strong components algorithm, vertices are sorted in_____ order of finish times.

Decreasing

57. According to parenthesis lemma, vertex u is unrelated to v vertex if and only if d[u],f[u]] and [d[v],f[v]] are disjoint.

True

58. A digraph is strongly connected under what condition?

A digraph is strongly connected if for every pair of vertices u, v e V, u can reach v and vice versa.

59. The tricky part of the _____ algorithm is how to detect whether the addition of an edge will create a cycle in viable set A.

Kruskal's

60. If a subset of edges A is viable for building MST, it can not contain a/an _____.

Cycle

61. Digraphs are not used in communication and transportation networks.

True

62. A strongly connected component only apply to:

True False

- Please choose one

Directed Graph

63. According to parenthesis lemma, vertex u is a descendent of v vertex if and only if;

[d[u], f[u]] ⊆ [d[v], f[v]

64. What is the time complexity to extract a vertex from the priority queue in Prim's algorithm?

(log V)

65. Back edge is:

(u, v) where v is an ancestor of u in the tree.

66. In computing the strongly connected components of a digraph, vertices of the digraph are not partitioned into subsets.

False

67. In Timestamped DFS-cycles lemma, if edge (u, v) is a tree, forward or cross edge, then _____

f[u]⩾f[v]

68. In Kruskal's algorithm, the next edge is added to viable set A, if its adding does not induce a/an _____ .

Cycle

69. In Generic approach determining of Greedy MST, we maintain a subset A of _____ .

Edges

True False

- Please choose one

---

70. If u and v are mutually reachable in G, then in the graph formed by reversing all the edges, these vertices are not reachable.

    False

71. Kruskal's algorithm works by adding _____ in increasing order of weight (lightest edge first).

    Edges

72. In Kruskal's algorithm, the next _____ is not added to viable set A, if its adding induce a/an cycle.

    Edge

73. Networks are complete in the sense that it is possible from any location in the network to reach any other location in the digraph.

    True

74. In Generic approach determining of Greedy MST, we maintain a subset A of _____ .

    Edges

75. There exist a unique path between any _____ vertices of a free tree.

    Two

76. What is the time complexity to extract a vertex from the priority queue in Prim's algorithm?

    log (V)

True False

- Please choose one

---

77. The time stamps given by DFS do not allow us to determine whether the graph contains any cycles.

False

78. Timestamp structure of _____ is used in determining the strong components of a digraph.

Both DFS & BFS

79. In Kruskal's algorithm, the next edge is not added to viable set A, if its adding induce a cycle.

False

80. A topological sort of a DAG is a _____ ordering of the vertices of the DAG such that for each edge (u, v), u appears before v in the ordering.

Linear

81. In Kruskal's algorithm, at any time, the subset of edges A forms a single tree.

False

82. Adding any edge to a free tree creates a unique cycle.

True

83. In digraph G=(V,E) ;G has cycle if and only if The DFS forest has back edge

Cycle

84. In undirected graph, by convention all the edges are called _____ edges.

True False

- Please choose one

Back

85. Prim’s algorithm is based on ---------- strategy.

Greedy

86. There are no _____ edges in undirected graph.

Cross

87. You have an adjacency list for G, what is the time complexity to compute Graph transpose G^T ?

(V+E)

Which is true statement in the following.

Kruskal algorithm is multiple source technique for finding MST.

Kruskal's algorithm is used to find minimum spanning tree of a graph, time complexity of this algorithm is O(EV)

Both of above

Kruskal's algorithm (choose best non-cycle edge) is better than Prim's (choose best Tree edge) when the graph has relatively few edges) (Right Answer)

The relationship between number of back edges and number of cycles in DFS is,

Both are equal

Back edges are half of cycles

Back edges are one quarter of cycles

There is no relationship between no. of edges and cycles (Right Answer)

Kruskal's algorithm (choose best non-cycle edge) is better than Prim's (choose best tree

edge) when the graph has relatively few edges.

True (Right Answer)
False

What is the time complexity to extract a vertex from the priority queue in Prim's

algorithm?

True False

- Please choose one

Select correct option:

log (V)
V.V

E.E

log (E)

Suppose that a graph G = (V,E) is implemented using adjacency lists. What is the complexity of a breadth-first traversal of G?

Select correct option:

O(|V |^2)

O(|V | |E|) (Right Answer)
O(|V |^2|E|)

O(|V | + |E|)

What is generally true of Adjacency List and Adjacency Matrix representations of graphs?

Select correct option:

Lists require less space than matrices but take longer to find the weight of an edge (v1,v2)

Lists require less space than matrices and they are faster to find the weight of an edge (v1, v2) Right Answer)
Lists require more space than matrices and they take longer to find the weight of an edge (v1, v2)

Lists require more space than matrices but are faster to find the weight of an edge (v1, v2)

What general property of the list indicates that the graph has an isolated vertex?

Select correct option:

There is Null pointer at the end of list.

The Isolated vertex is not handled in list. (not Sure)
Only one value is entered in the list.

There is at least one null list.

A dense undirected graph is:

Select correct option:

A graph in which E = O(V^2) (Right Answer)
A graph in which E = O(V)

A graph in which E = O(log V)

True False

- Please choose one

All items above may be used to characterize a dense undirected graph

In digraph G=(V,E) ;G has cycle if and only if

Select correct option:

The DFS forest has forward edge.

The DFS forest has back edge (Right Answer)
The DFS forest has both back and forward edge

BFS forest has forward edge

Back edge is:

Select correct option:

(u, v) where v is an ancestor of u in the tree. (Right Answer)
(u,v) where u is an ancesstor of v in the tree.

(u, v) where v is an predcessor of u in the tree.

None of above

There is no relationship between back edges and number of cycle(Right Answer)

You have an adjacency list for G, what is the time complexity to compute Graph transpose G^T ?

Select correct option:

(V+E) (Right Answer)
V.E

V

E

Question # 3 of 10 (Start time: 06:54:27 PM) Total Marks: 1

You have an adjacency list for G, what is the time complexity to compute Graph transpose G^T.?

?(V + E) Right Answer)
?(V E)

?(V)

?(V^2)

True False

- Please choose one

What is the time complexity to extract a vertex from the priority queue in Prim's algorithm?

Select correct option:

log (V) (Right Answer)
V.V

E.E

log (E)

Dijkstra's algorithm :

Select correct option:

Has greedy approach to find all shortest paths

Has both greedy and Dynamic approach to find all shortest paths

Has greedy approach to compute single source shortest paths to all other vertices (Right Answer)
Has both greedy and dynamic approach to compute single source shortest paths to all other vertices.

What algorithm technique is used in the implementation of Kruskal solution for the MST?

Greedy Technique (Right Answer)
Divide-and-Conquer Technique

Dynamic Programming Technique

The algorithm combines more than one of the above techniques

What is the time complexity to extract a vertex from the priority queue in Prim's algorithm?

Select correct option:

O (log E)

? (V)

? (V+E)

O (log V) (Right Answer)

1. What algorithm technique is used in the implementation of Kruskal solution for MST?

a) Greedy Technique page 142

True False

- Please choose one

b) Divide-and-Conquer Technique

c) Dynamic Programming Technique

d) The algorithm combines more than one of the above techniques i.e. Divide-and-Conquer and dynamic programing

1. There exists a unique path between any _____ vertices of a free tree.

a) One

b) Two page 142

c) Three

d) All

1. If a subset of edges A is viable for building MST, it cannot contain a/an _____

a) Vertex

b) Edge

c) Cycle page 143

d) Graph

1. As the Kruskal's runs, the edges in viable set A induce a _____ on the vertices.

a) Set

b) Graph

c) Tree

d) Forest

1. The _____ given by DFS allow us to determine whether the graph contains any cycles.

a) Order

b) Time stamps page 130

c) BFS traversing

d) Topological sort

1. The time stamps given by DFS do not allow determining whether the graph contains any cycles.

a) True

b) False page130

1. By breaking any edge on a cycle created in free tree, the free _____ is restored.

a) Edge

b) Tree

c) Cycle page 143

d) Vertex

1. Forward edge is

a) (u, v) where u is a proper descendent of v in the tree

b) (u, v) where v is a proper descendent of u in the tree page 129

True False

- Please choose one

c) (u, v) where v is a proper ancestor of u in the tree

d) (u, v) where u is a proper ancestor of v in the tree

1. In Kruskal's algorithm, the next _____ is not added to viable set A, if its adding induces a/an cycle.

a) Vertex

b) Edge page147

c) Cycle

d) Tree

1. In _____ algorithm, at any time, the subset of edges A forms a single tree

a) Kruskal's

b) Prim's page 149

c) Both

d) None

Question # 1 of 10 (Start time: 10:14:37 PM) Total Marks: 1
There exist a unique path between any _____ vertices of a free tree.
Select correct option:
One
Two Ans
Three
All

Question # 2 of 10 (Start time: 10:15:15 PM) Total Marks: 1
In _____ algorithm, at any time, the subset of edges A forms a single tree.
Select correct option:
Kruskal's
Prim's Ans
Both
None

Question # 3 of 10 (Start time: 10:15:56 PM) Total Marks: 1
A digraph is strongly connected under what condition?
Select correct option:
A digraph is strongly connected if for every pair of vertices u, v e V, u can reach v .
A digraph is strongly connected if for every pair of vertices u, v e V, u can reach v and vice versa. Ans
A digraph is strongly connected if for at least one pair of vertex u, v e V, u can reach v and vice versa.
A digraph is strongly connected if at least one third pair of vertices u, v e V, u can reach v and vice versa.

Question # 4 of 10 (Start time: 10:16:38 PM) Total Marks: 1
Digraphs are not used in communication and transportation networks.
Select correct option:

True False

- Please choose one

True

False Ans

Question # 5 of 10 (Start time: 10:17:31 PM) Total Marks: 1
There are no _____ edges in undirected graph.
Select correct option:
Forward
Back
Cross
Both forward and back Ans

Question # 6 of 10 (Start time: 10:17:53 PM) Total Marks: 1
In Kruskal's algorithm, at any time, the subset of edges A forms a single tree.
Select correct option:
True
False Ans

Question # 7 of 10 (Start time: 10:18:12 PM) Total Marks: 1
Kruskal's algorithm (choose best non-cycle edge) is better than Prim's (choose best tree edge) when the graph has relatively few edges.
Select correct option:
True
False Ans

Question # 8 of 10 (Start time: 10:18:54 PM) Total Marks: 1
The _____ given by DFS allow us to determine whether the graph contains any cycles.
Select correct option:
Order
Time stamps Ans
BFS traversing
Topological sort

Question # 9 of 10 (Start time: 10:19:41 PM) Total Marks: 1
Kruskal’s algorithm works by adding _____ in increasing order of weight (lightest edge first).
Select correct option:
Vertices
Edges Ans
Trees
Weights

Question # 10 of 10 (Start time: 10:20:15 PM) Total Marks: 1
Runtime complexity of Prim's algorithm is _____.
Select correct option:
V log V
E log V Ans
log V
None of the above

-

Question # 1 of 10 (Start time: 10:23:08 PM) Total Marks: 1
Adding any edge to a free tree creates a unique cycle.

True False

- Please choose one

Select correct option:

True
False

Question # 2 of 10 (Start time: 10:23:52 PM) Total Marks: 1
Forward edge is:
Select correct option:
(u, v) where u is a proper descendent of v in the tree.
(u, v) where v is a proper descendent of u in the tree. Ans
(u, v) where v is a proper ancesstor of u in the tree.
(u, v) where u is a proper ancesstor of v in the tree.

Question # 3 of 10 (Start time: 10:25:10 PM) Total Marks: 1
There is relationship between number of back edges and number of cycles in DFS
Select correct option:
Both are equal.
Cycles are half of back edges.
Cycles are one fourth of back edges.
There is no relationship between back edges and number of cycles. Ans

Question # 4 of 10 (Start time: 10:25:31 PM) Total Marks: 1
In undirected graph, by convention all the edges are called _____ edges.
Select correct option:
Forward
Back Ans
Cross
Both forward and back

Question # 5 of 10 (Start time: 10:26:38 PM) Total Marks: 1
For undirected graph, there is no distinction between forward and back edges.
Select correct option:
True
False

Question # 6 of 10 (Start time: 10:26:55 PM) Total Marks: 1
Adding any edge to a free tree creates a unique _____ .
Select correct option:
Vertex
Edge
Cycle
Strong component

Question # 7 of 10 (Start time: 10:27:09 PM) Total Marks: 1
In _____ algorithm, at any time, the subset of edges A forms a single tree.
Select correct option:
Kruskal's
Prim's Ans
Both
None

Question # 8 of 10 (Start time: 10:27:23 PM) Total Marks: 1
In strong components algorithm, the form of graph is used in which all the _____ of original graph G have been reversed in direction.
Select correct option:
Vertices
Edges Ans (not sure)
Both edges & vertices
None of the above

True False

- Please choose one

Question # 9 of 10 (Start time: 10:28:31 PM) Total Marks: 1

According to parenthesis lemma, vertex u is a descendent of v vertex if and only if;
Select correct option:

[d[u], f[u]] ⊆ [d[v], f[v]] Ans

[d[u], f[u]] ⊇ [d[v], f[v]]

Unrelated

Disjoint

Question # 10 of 10 (Start time: 10:29:23 PM) Total Marks: 1
Strongly connected components are not affected by reversal of all edges in terms of vertices reachability.
Select correct option:
True Ans
False

-

Question # 1 of 10 (Start time: 10:23:08 PM) Total Marks: 1
Adding any edge to a free tree creates a unique cycle.
Select correct option:
True Ans
False

Question # 2 of 10 (Start time: 10:23:52 PM) Total Marks: 1
Forward edge is:
Select correct option:
(u, v) where u is a proper descendent of v in the tree.
(u, v) where v is a proper descendent of u in the tree. Ans
(u, v) where v is a proper ancesstor of u in the tree.
(u, v) where u is a proper ancesstor of v in the tree.

Question # 3 of 10 (Start time: 10:25:10 PM) Total Marks: 1
There is relationship between number of back edges and number of cycles in DFS
Select correct option:
Both are equal.
Cycles are half of back edges.
Cycles are one fourth of back edges.
There is no relationship between back edges and number of cycles. Ans

Question # 4 of 10 (Start time: 10:25:31 PM) Total Marks: 1
In undirected graph, by convention all the edges are called _____ edges.
Select correct option:
Forward
Back Ans
Cross
Both forward and back

Question # 5 of 10 (Start time: 10:26:38 PM) Total Marks: 1
For undirected graph, there is no distinction between forward and back edges.
Select correct option:
True Ans
False

Question # 6 of 10 (Start time: 10:26:55 PM) Total Marks: 1
Adding any edge to a free tree creates a unique _____ .
Select correct option:

True False

- Please choose one

Vertex

Edge
Cycle Ans
Strong component

Question # 7 of 10 (Start time: 10:27:09 PM) Total Marks: 1
In _____ algorithm, at any time, the subset of edges A forms a single tree.
Select correct option:
Kruskal's
Prim's Ans
Both
None

Question # 8 of 10 (Start time: 10:27:23 PM) Total Marks: 1
In strong components algorithm, the form of graph is used in which all the _____ of original graph G have been reversed in direction.
Select correct option:
Vertices
Edges Ans
Both edges & vertices
None of the above

Question # 9 of 10 (Start time: 10:28:31 PM) Total Marks: 1
According to parenthesis lemma, vertex u is a descendent of v vertex if and only if;
Select correct option:

[d[u], f[u]] ⊆ [d[v], f[v]] Ans

[d[u], f[u]] ⊇ [d[v], f[v]]

Unrelated

Disjoint

Question # 10 of 10 (Start time: 10:29:23 PM) Total Marks: 1
Strongly connected components are not affected by reversal of all edges in terms of vertices reachability.
Select correct option:
True Ans
False
Question # 1 of 10 (Start time: 08:34:15 PM) Total Marks: 1
According to parenthesis lemma, vertex u is unrelated to v vertex if and only if d[u],f[u]] and [d[v],f[v]] are disjoint.
Select correct option:
True Ans
False

Question # 2 of 10 (Start time: 08:35:47 PM) Total Marks: 1
Kruskal's algorithm (choose best non-cycle edge) is better than Prim's (choose best tree edge) when the graph has relatively few edges.
Select correct option:
True
False Ans

Question # 3 of 10 (Start time: 08:37:05 PM) Total Marks: 1
You have an adjacency list for G, what is the time complexity to compute Graph transpose G^T.?
Select correct option:
? (V + E) Ans

True False

- Please choose one

? (V E)

? (V)
? (V^2)

Question # 4 of 10 (Start time: 08:38:13 PM) Total Marks: 1
Kruskal's algorithm works by adding vertices in increasing order of weight (lightest edge first).
Select correct option:
True Ans
False

Question # 5 of 10 (Start time: 08:38:38 PM) Total Marks: 1
In Prim's algorithm, we will make use of priority _____.
Select correct option:
Stack
Queue Ans
Array
Graph

Question # 6 of 10 (Start time: 08:39:03 PM) Total Marks: 1
If a vertex v is a descendent of vertex u, then v's start-finish interval is contained within u's start-finish interval.
Select correct option:
True Ans
False

Question # 7 of 10 (Start time: 08:39:53 PM) Total Marks: 1
Computing the strongly connected components of a digraph is a generalization of the problem to determine whether a digraph is strongly connected or not.
Select correct option:
True Ans
False

Question # 8 of 10 (Start time: 08:41:01 PM) Total Marks: 1
Adding any edge to a free tree creates a unique _____ .
Select correct option:
Vertex
Edge
Cycle Ans
Strong component

Question # 9 of 10 (Start time: 08:41:41 PM) Total Marks: 1
Networks are complete in the sense that it is possible from any location in the network to reach any other location in the digraph.
Select correct option:
True Ans
False

Question # 10 of 10 (Start time: 08:42:10 PM) Total Marks: 1
By breaking any edge on a cycle created in free tree, the free _____ is restored.
Select correct option:
Edge
Tree Ans
Cycle
Vertex

Timestamp structure of _____ is used in determining the strong components of a digraph.
Select correct option

True False

- Please choose one

DFS

BFS

Both DFS & BFS

None

For a digraph G = (V, E), Sum of in-degree(v) _____
Select correct option

Not equal to Sum of out-degree(v)

= Sum of out-degree(v) ok

< Sum of out-degree(v)

> Sum of out-degree(v)
Question # 2 of 10 (Start time: 09:32:33 PM) Total Marks: 1
We say that two vertices u and v are mutually not reachable if u can reach v and vice versa.
Select correct option

True ok

False

Question # 6 of 10 (Start time: 09:36:53 PM) Total Marks: 1
The Huffman codes provide a method of _____ data efficiently.
Select correct option

Reading

Encoding ok

Decoding

Printing
Question # 7 of 10 (Start time: 09:37:46 PM) Total Marks: 1
In greedy algorithm, at each phase, you take the_____ you can get right now, without regard for future consequences.
Select correct option

Worst

Minimum

Good

Best ok
Question # 8 of 10 (Start time: 09:38:38 PM) Total Marks: 1
In digraph G=(V,E) ;G has cycle if and only if
Select correct option

The DFS forest has forward edge.

The DFS forest has back edge ok

The DFS forest has both back and forward edge

BFS forest has forward edge
Question # 9 of 10 (Start time: 09:39:38 PM) Total Marks: 1
In Activity selection (using Greedy approach), intuitively _____.
Select correct option

Short activities are not preferable

True False

- Please choose one

There are always short activities as input

We do not like long activities ok

It does not matter about the length of activities
Question # 10 of 10 (Start time: 09:40:50 PM) Total Marks: 1
Breadth-first search begins at a root node and inspects all the nodes except neighboring ones.
Select correct option

True

False ok

-

In in-place sorting algorithm is one that uses arrays for storage :
An additional array
No additional array
Both of above may be true according to algorithm
More than 3 arrays of one dimension.

The running time of quick sort depends heavily on the selection of
No of inputs
Arrangement of elements in array
Size o elements
Pivot element

In stable sorting algorithm
One array is used
In which duplicating elements are not handled.
More then one arrays are required.
Duplicating elements remain in same relative position after sorting.

Which sorting algorithn is faster :
O(n^2)
O(nlogn)
O(n+k)
O(n^3)

In Quick sort algorithm,constants hidden in T(n lg n) are
Large
Medium
Not known
Small

Quick sort is based on divide and conquer paradigm; we divide the problem on base of pivot element and:
There is explicit combine process as well to conquer the solution.
No work is needed to combine the sub-arrays, the array is already sorted
Merging the subarrays
None of above.

There is relationship between number of back edges and number of cycles in DFS
Both are equal.
Cycles are half of back edges.
Cycles are one fourth of back edges.
There is no relationship between back edges and number of cycle

True False

- Please choose one

You have an adjacency list for G, what is the time complexity to compute Graph transpose G^T ?

(V+E)
V.E
V
E

You have an adjacency list for G, what is the time complexity to compute Graph transpose G^T.?

?(V + E)
?(V E)
?(V)
?(V^2)

What is the time complexity to extract a vertex from the priority queue in Prim's algorithm?

log (V)
V.V
E.E
log (E)

Dijkstra's algorithm :

Has greedy approach to find all shortest paths
Has both greedy and Dynamic approach to find all shortest paths
Has greedy approach to compute single source shortest paths to all other vertices
Has both greedy and dynamic approach to compute single source shortest paths to all other vertices.

What algorithm technique is used in the implementation of Kruskal solution for the MST?

Greedy Technique
Divide-and-Conquer Technique
Dynamic Programming Technique
The algorithm combines more than one of the above techniques

Which is true statement in the following.

Kruskal algorithm is multiple source technique for finding MST.
Kruskal's algorithm is used to find minimum spanning tree of a graph, time complexity of this algorithm is O(EV)
Both of above
Kruskal's algorithm (choose best non-cycle edge) is better than Prim's (choose best Tree edge) when the graph has relatively few edges)

The relationship between number of back edges and number of cycles in DFS is,

Both are equal
Back edges are half of cycles
Back edges are one quarter of cycles
There is no relationship between no. of edges and cycles

Kruskal's algorithm (choose best non-cycle edge) is better than Prim's (choose best tree edge) when the graph has relatively few edges

True
False

Suppose that a graph G = (V,E) is implemented using adjacency lists. What is the complexity of a breadth-first traversal of G?

O(|V |^2)
O(|V | |E|)

True False

- Please choose one

O(|V |^2|E|)

O(|V | + |E|)

What is generally true of Adjacency List and Adjacency Matrix representations of graphs?
Lists require less space than matrices but take longer to find the weight of an edge (v1,v2)
Lists require less space than matrices and they are faster to find the weight of an edge (v1, v2)
Lists require more space than matrices and they take longer to find the weight of an edge (v1, v2)
Lists require more space than matrices but are faster to find the weight of an edge (v1, v2)

What general property of the list indicates that the graph has an isolated vertex?
There is Null pointer at the end of list.
The Isolated vertex is not handled in list.
Only one value is entered in the list.
There is at least one null list.

A dense undirected graph is:
A graph in which E = O(V^2)
A graph in which E = O(V)
A graph in which E = O(log V)
All items above may be used to characterize a dense undirected graph

In digraph G=(V,E) ;G has cycle if and only if
The DFS forest has forward edge.
The DFS forest has back edge
The DFS forest has both back and forward edge
BFS forest has forward edge

Back edge is:
(u, v) where v is an ancestor of u in the tree.
(u,v) where u is an ancesstor of v in the tree.
(u, v) where v is an predcessor of u in the tree.
None of above

Using ASCII standard the string “abacdaacacwe” will be encoded with _____ bits
64
128
96
120

Cross edge is :
(u, v) where u and v are not ancestor of one another
(u, v) where u is ancesstor of v and v is not descendent of u.
(u, v) where u and v are not ancestor or descendent of one another
(u, v) where u and v are either ancestor or descendent of one another.

Which statement is true?
If a dynamic-programming problem satisfies the optimal-substructure property, then a locally optimal solution is globally optimal.
If a greedy choice property satisfies the optimal-substructure property, then a locally optimal solution is globally optimal.
Both of above
None of above

If you find yourself in maze the better traversel approach will bEA dense undirected graph is:

True False

- Please choose one

A graph in which E = O(V^2)

A graph in which E = O(V)
A graph in which E = O(log V)
All items above may be used to characterize a dense undirected graph

Which is true statement.
Breadth first search is shortest path algorithm that works on un-weighted graphs
Depth first search is shortest path algorithm that works on un-weighted graphs.
Both of above are true.
None of above are true.

Forward edge is:
(u, v) where u is a proper descendent of v in the tree.
(u, v) where v is a proper descendent of u in the tree.
(u, v) where v is a proper ancesstor of u in the tree.
(u, v) where u is a proper ancesstor of v in the tree.

In digraph G=(V,E) ;G has cycle if and only if
The DFS forest has forward edge.
The DFS forest has back edge
The DFS forest has both back and forward edge
BFS forest has forward edge

If you find yourself in maze the better traversel approach will be :
BFS
BFS and DFS both are valid
Level order
DFS

What algorithm technique is used in the implementation of Kruskal solution for the MST?
Greedy Technique
Divide-and-Conquer Technique
Dynamic Programming Technique
The algorithm combines more than one of the above techniques

Kruskal's algorithm (choose best non-cycle edge) is better than Prim's (choose best tree edge) when the graph has relatively few
True
False

A digraph is strongly connected under what condition?
A digraph is strongly connected if for every pair of vertices u, v e V, u can reach v .
A digraph is strongly connected if for every pair of vertices u, v e V, u can reach v and vice versa.
A digraph is strongly connected if for at least one pair of vertex u, v e V, u can reach v and vice versa.
A digraph is strongly connected if at least one third pair of vertices u, v e V, u can reach v and vice versa.

The relationship between number of back edges and number of cycles in DFS is,
Both are equal
Back edges are half of cycles
Back edges are one quarter of cycles
There is no relationship between no. of edges and cycles

What algorithm technique is used in the implementation of Kruskal solution for the M ST?
Greedy Technique

True False

- Please choose one

Divide-and-Conquer Technique

Dynamic Programming Technique
The algorithm combines more than one of the above techniques

The _____ given by DFS allow us to determine whether the graph contains any cycles. CS502

- Order
- **Time stamps** Page 130
- BFS traversing
- Topological sort

A graph is not connected if every vertex can reach every other vertex. CS502

- True
- **False** Page No. 116

In strong components algorithm, first of all DFS is run for computing finish times of vertices. CS502

- **True** Page No.138
- False

In strong components algorithm, first of all DFS is run for computing finish times of vertices. CS502

- [d[u], f[u]]⊆[d[v], f[v]]
- [d[u], f[u]]⊇[d[v], f[v]]
- **unrelated** Page No.129
- Disjoint

For traversing graphs, Breadth-first search can be visualized as a wave front propagating inwards towards root (or source) node. CS502

- True
- **False** Page No.117

Digraphs are not used in communication and transportation networks. CS502

- True
- **False** Page No.135

You have an adjacency list for G, what is the time complexity to compute Graph transpose G^T.? CS502 ?

True False

- Please choose one

- **(V + E)** Page No.138
- V E
- V
- E

In computing the strongly connected components of a digraph, vertices of the digraph are not partitioned into subsets. CS502

- True
- **False** Page No.135

There are no _____ edges in undirected graph. CS502

- Forward
- Back
- Cross
- **Both Forward and Back** Page No. 130

We say that two vertices u and v are mutually not reachable if u can reach v and vice versa. CS502

- **True** Page No. 135
- False

- There is relationship between number of back edges and number of cycles in DFS Select correct option:
- Both are equal.
- Cycles are half of back edges.
- Cycles are one fourth of back edges.
- **There is no relationship between back edges and number of cycles**. **(Correct)**
-
-
- In strong components algorithm, the form of graph is used in which all the vertices of original graph G have been reversed in direction. Select correct option:
- **True** **(Correct)**
- False
-

True False

- Please choose one

- In Kruskal's algorithm, the next edge is added to viable set A, if its adding does not induce a/an _____ . Select correct option:
-
- Vertex
- Edge
- **Cycle (Correct)**
- Tree
-
-
- For undirected graph, there is no distinction between forward and back edges. Select correct option:
- **True (Correct)**
- False
-
- You have an adjacency list for G, what is the time complexity to compute Graph transpose G^T ? Select correct option:
- **(V+E) (Correct)**
- V.E
- V
- E
-
- There exist a unique path between any _____ vertices of a free tree. Select correct option:
- One
- **Two (Correct)**
- Three
- All
-

True False

- Please choose one

- A digraph is strongly connected under what condition? Select correct option:
-
- A digraph is strongly connected if for every pair of vertices u, v e V, u can reach v .
- **A digraph is strongly connected if for every pair of vertices u, v e V, u can reach v and vice versa. (Correct)**
- A digraph is strongly connected if for at least one pair of vertex u, v e V, u can reach v and vice versa.
- A digraph is strongly connected if at least one third pair of vertices u, v e V, u can reach v and vice versa.
-
-
- If u and v are mutually reachable in G, then in the graph formed by reversing all the edges, these vertices are not reachable. Select correct option:
- True
- **False (Correct)**
-
-
- In Prim's algorithm, at any time, the subset of edges A forms a single forest. Select correct option:
- True
- **False (Correct)**
-
- In digraph G=(V,E) ;G has cycle if and only if
- The DFS forest has forward edge.
- **The DFS forest has back edge**
- The DFS forest has both back and forward edge
- BFS forest has forward edge

True False

- Please choose one

Question No: 1 (Marks: 1) - Please choose one

In_____ algorithm, at anytime, the subset of edges A forms a single tree.

Kruskal's
Prim's
Both
None

Question No: 2 (Marks: 1) - Please choose one

According to parenthesis lemma, vertex u is unrelated to v vertex if and only if [d[u], f[u]] and [d[v], f[v]] are disjoint.

True
False

Question No: 3 (Marks: 1) - Please choose one

A free tree with n vertices have exactly n+1 edges.

True
Fale

True False

- Please choose one

Question No: 4 (Marks: 1) - Please choose one

For undirected graph, there is no distinction between forward and back edges.

True
Fale

Question No: 5

In computing the strongly connected components of a digraph, vertices of the digraph are not partitioned into subsets.

True
False

Question No: 6 (Marks: 1) - Please choose one

What is the time complexity to extract a vertex from the priority queue in Prim's algorithm?

log (V)
V.V
E.E
log (E)

True False

- Please choose one

Question No: 7 (Marks: 1) - Please choose one

What is true statement?

Breadth first search is shortest path algorithm that works on un-weighted graphs
Depth first search is shortest path algorithm that works on un-weighted graphs
Both of above are true
None of above are true

Question No: 8 (Marks: 1) - Please choose one

Networks are_____ in the sense that it is possible from any location in the network to reach any other location in the digraph.

Complete
Incomplete
Not graphs
Transportation

Question No: 9 (Marks: 1) - Please choose one

In Generic approach determining of Greedy MST, we maintain a subset A of are_____

Edges
Vertices
Cycles
Paths

True False

- Please choose one

Question No: 10 (Marks: 1)

There are no_____ edges in undirected graph.

Forward
Back
Cross
Both forward and back

Question No: 11 (Marks: 1) - Please choose one

If you find yourself in maze the better traversal approach will be:

BFS
BFS and DFS both are valid
Level order
DFS

Question No: 12 (Marks: 1) - Please choose one

In strong components algorithm, the form of graph is used in which all the vertices of original graph G have been reversed in direction.

True
False

True False

- Please choose one

Question No: 13 (Marks: 1) - Please choose one

What algorithm technique is used in the implementation of Kruskal solution for the MST?

Greedy Technique
Divide-and-Conquer Technique
Dynamic Programming Technique
The algorithm combines more than one of the above techniques

Question No: 14 (Marks: 1) - Please choose one

In Kruskal's algorithm, the next edge is added to viable set A, if its adding does not induce a cycle.

True False

Question No: (Marks: 1) - Please choose one

15

There exists a unique path between any_____ vertices of a free tree.

One
Two
Three
All

Question No: 16 (Marks: 1) - Please choose one

Computing the strongly connected components of a digraph is a generalization of the problem to determine whether a digraph is strongly connected or not.

True
False

Question No: 17 (Marks: 1) - Please choose one

If a vertex is a descendent of vertex u, then v's start-finish interval is contained within u's start-finish interval.

True
False

True False

- Please choose one

Question No: 18 (Marks: 1) - Please choose one

The_____ given by DFS allow us to determine whether the graph contains any cycles.

Order
Time stamps
BFS traversing
Topological sort

Question No: 19 (Marks: 1) - Please choose one

Networks are complete in the sense that it is possible from any location in the network to reach any other location in the digraph.

Question No: 20 (Marks: 1)

There are no_____ edges in undirected graph.

Forward
Back
Cross
Both forward and back

Question No: 21 (Marks: 1) - Please choose one

True False

- Please choose one

You have an adjacency list for G, what is the time complexity to compute Graph transpose G^T.?

? (V + E)
? (V E)
? (V)
? (V^2)

Question No: 22 (Marks: 1) - Please choose one

There are no_____ edges in undirected graph.

Forward
Back
Cross
Both forward and back

Question No: 23 (Marks: 1) - Please choose one

In Prim's algorithm, at anytime, the subset of edges A forms a single_____.

Vertex
Forest
Tree
Graph

True False

- Please choose one

Question No: 24 (Marks: 1) - Please choose one

Kruskal's algorithm works by adding vertices in increasing order of weight (lightest edge first).

25

Runtime complexity of Prim's algorithm is _____.

V log V
E log V
log V
None of the above

Question No: 26 (Marks: 1) - Please choose one

According to parenthesis lemma, vertex u is an ancestor of v vertex if and only if;

[d[u], f[u]] ⊆ [d[v], f[v]]
[d[u], f[u]] ⊇ [d[v], f[v]]
Unrelated
Disjoint

Question No: 27 (Marks: 1) - Please choose one

In Kruskal's algorithm, the next _____ is not added to viable set A, if its adding induce a/an

True False

- Please choose one

cycle.

Vertex
Edge
Cycle
Tree

Question No: 28 (Marks: 1) - Please choose one

In Prim's algorithm, we start with the _____ vertex r; it can be any vertex.

First
Leaf
Mid
Root

Question No: 29 (Marks: 1) - Please choose one

A free tree with n vertices has exactly n+1 edges.

True
False

True False

(Marks: 1) - Please choose one

Question No: 30

There exists a unique path between any _____ vertices of a free tree.

One
Two
Three
All

Question No: 31 (Marks: 1) - Please choose one

The tricky part of the _____ algorithm is how to detect whether the addition of an edge will create a cycle in viable set A.

Kruskal's
Prim's
Both
None

Question No: 32 (Marks: 1) - Please choose one

In Prim's algorithm, we start with the root vertex r; it can be any vertex.

True
False

Question No: -

Question No: 33 (Marks: 1) - Please choose one

The relationship between number of back edges and number of cycles in DFS is,

Both are equal
Back edges are half of cycles
Back edges are one quarter of cycles
There is no relationship between no. of edges and cycles

Question No: 34 (Marks: 1) - Please choose one

If you find yourself in maze the better traversal approach will be:

BFS
BFS and DFS both are valid
Level order
DFS

35 Please choose one

According to parenthesis lemma, vertex u is an ancestor of v vertex if and only if;

[d[u], f[u]] ⊆ [d[v], f[v]]
[d[u], f[u]] ⊇ [d[v], f[v]]
Unrelated
Disjoint

Question No: -

Question No: 36 (Marks: 1) - Please choose one

We say that two vertices u and v are mutually not reachable if u can reach v and vice versa.

True
False

Question No: 37 (Marks: 1) - Please choose one

In Generic approach determining of Greedy MST, we maintain a subset A of are_____

Edges
Vertices
Cycles
Paths

Question No: 38 (Marks: 1) - Please choose one

Kruskal's algorithm (choose best non-cycle edge) is better than Prim's (choose best tree edge) when the graph has relatively few edges.

True
False

Question No: -

Question No: 39 (Marks: 1) - Please choose one

Digraphs are not used in communication and transportation networks.

True
False

40 (Marks: 1) Please choose one

We say that two vertices u and v are mutually not reachable if u can reach v and vice versa.

True
False

Question No: 41 (Marks: 1) - Please choose one

Runtime complexity of Prim's algorithm is _____.

V log V **E**
log V log
V
None of the above

Question No: -

Question No: 42 (Marks: 1) - Please choose one

In Prim's algorithm, we start with the root vertex r; it can be any vertex.

True
False

Question No: 43 (Marks: 1) - Please choose one

Networks are _____ in the sense that it is possible from any location in the network to reach any other location in the digraph.

Complete
Incomplete
Not graphs
Transportation

Question No: 44 (Marks: 1) - Please choose one

Kruskal's algorithm (choose best non-cycle edge) is better than Prim's (choose best tree edge) when the graph has relatively few edges.

True False

Question No: -

45

In strong components algorithm, vertices are sorted in_____ order of finish times.

Any
Increasing
Decreasing
None

Question No: 46 (Marks: 1) - Please choose one

In Kruskal's algorithm, at anytime, the subset of edges A forms a single tree.

True
False

Question No: 47 (Marks: 1) - Please choose one

If a subset of edges A is viable for building MST, it can not contain a/an_____.

Vertex
Edge
Cycle
Graph

Question No: -

Question No: 48 (Marks: 1) - Please choose one

In Timestamped DFS-cycles lemma, if edge (u,v) is a tree, forward or cross edge, then_____.

{\text{f[u]}} < {\text{f[v]}} {\text{f[u]}} >
{\text{f[v]}}
f[u] ≤ f[v]
f[u] ≥ f[v]

Question No: 49 (Marks: 1) - Please choose one

Strongly connected components are not affected by reversal of all edges in terms of vertices reachability.

True False **50**
If u and v are mutually reachable in G, then in the graph formed by reversing all the edges, these vertices are not reachable.

True
False

Question No: 51 (Marks: 1) - Please choose one

Forward edge is:

(u, v) where u is a proper descendent of v in the tree.
(u, v) where v is a proper descendent of u in the tree.
(u, v) where v is a proper ancestor of u in the tree.
(u, v) where u is a proper ancestor of v in the tree.

Question No: -

Question No: 52 (Marks: 1) - Please choose one

According to parenthesis lemma, vertex u is an ancestor of v vertex if and only if;

[d[u], f[u]] ⊆ [d[v], f[v]]
[d[u], f[u]] ⊇ [d[v], f[v]]
Unrelated
Disjoint

Question No: 53 (Marks: 1) - Please choose one

In Timestamped DFS-cycles lemma, if edge (u,v) is a tree, forward or cross edge, then_____.

{\text{f[u]}} < {\text{f[v]}} {\text{f[u]}} >
{\text{f[v]}}
f[u] ≤ f[v]
f[u] ≥ f[v]

Question No: 54 (Marks: 1) - Please choose one

As the Kruskal's algorithm runs, the edges in viable set A induce a_____ on the vertices.

Set
Graph
Tree
Forest

55

Kruskal's algorithm (choose best non-cycle edge) is better than Prim's (choose best tree edge) when the

Question No: -

graph has relatively few edges.

True
False

Question No: 56 (Marks: 1) - Please choose one

Runtime complexity of Prim's algorithm is _____.

V log V E log V
log V
None of the above

Question No: 57 (Marks: 1) - Please choose one

Kruskal's algorithm works by adding _____vertices in increasing order of weight (lightest edge first).

Vertices
Edges
Trees
Weights

Question No: 58 (Marks: 1) - Please choose one

In strong components algorithm, the form of graph is used in which all the vertices of original graph G have been reversed in direction.

Question No: -

True

False

Design and Analysis of Algorithms

Sohail Aslam

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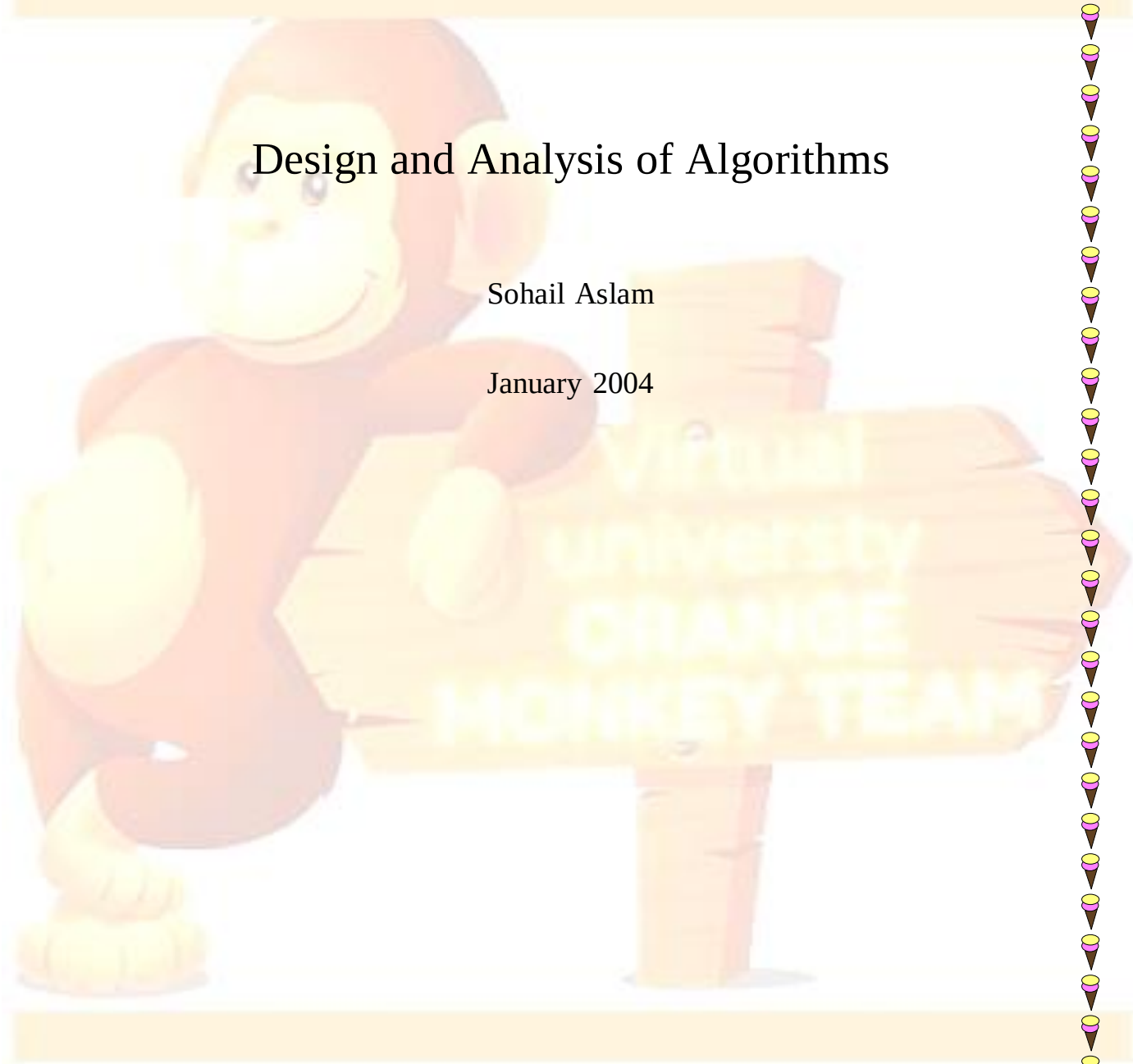

Chapter 1

Introduction

Origin of word: *Algorithm*

The word *Algorithm* comes from the name of the muslim author *Abu Ja'far Mohammad ibn Musa al-Khowarizmi*. He was born in the eighth century at Khwarizm (Kheva), a town south of river Oxus in present Uzbekistan. Uzbekistan, a Muslim country for over a thousand years, was taken over by the Russians in 1873.

His year of birth is not known exactly. Al-Khwarizmi parents migrated to a place south of Baghdad when he was a child. It has been established from his contributions that he flourished under Khalifah Al-Mamun at Baghdad during 813 to 833 C.E. Al-Khwarizmi died around 840 C.E.

Much of al-Khwarizmi's work was written in a book titled *al Kitab al-mukhatasar fi hisab al-jabr wa'l-muqabalah* (The Compendious Book on Calculation by Completion and Balancing). It is from the titles of these writings and his name that the words *algebra* and *algorithm* are derived. As a result of his work, al-Khwarizmi is regarded as the most outstanding mathematician of his time

Algorithm: Informal Definition

An algorithm is any well-defined computational procedure that takes some values, or set of values, as input and produces some value, or set of values, as output. An algorithm is thus a sequence of computational steps that transform the input into output.

Algorithms, Programming

A good understanding of algorithms is essential for a good understanding of the most basic element of computer science: programming. Unlike a program, an algorithm is a mathematical entity, which is independent of a specific programming language, machine, or compiler. Thus, in some sense, algorithm

design is all about the mathematical theory behind the design of good programs.

Why study algorithm design? There are many facets to good program design. Good algorithm design is one of them (and an important one). To be really complete algorithm designer, it is important to be aware of programming and machine issues as well. In any important programming project there are two major types of issues, macro issues and micro issues.

Macro issues involve elements such as how does one coordinate the efforts of many programmers working on a single piece of software, and how does one establish that a complex programming system satisfies its various requirements. These macro issues are the primary subject of courses on software engineering.

A great deal of the programming effort on most complex software systems consists of elements whose programming is fairly mundane (input and output, data conversion, error checking, report generation). However, there is often a small critical portion of the software, which may involve only tens to hundreds of lines of code, but where the great majority of computational time is spent. (Or as the old adage goes, 80% of the execution time takes place in 20% of the code.) The micro issues in programming involve how best to deal with these small critical sections.

It may be very important for the success of the overall project that these sections of code be written in the most efficient manner possible. An unfortunately common approach to this problem is to first design an inefficient algorithm and data structure to solve the problem, and then take this poor design and attempt to fine-tune its performance by applying clever coding tricks or by implementing it on the most expensive and fastest machines around to boost performance as much as possible. The problem is that if the underlying design is bad, then often no amount of fine-tuning is going to make a substantial difference.

Before you implement, first be sure you have a good design. This course is all about how to design good algorithms. Because the lesson cannot be taught in just one course, there are a number of companion courses that are important as well. CS301 deals with how to design good data structures. This is not really an independent issue, because most of the fastest algorithms are fast because they use fast data structures, and vice versa. In fact, many of the courses in the computer science program deal with efficient algorithms and data structures, but just as they apply to various applications: compilers, operating systems, databases, artificial intelligence, computer graphics and vision, etc. Thus, a good understanding of algorithm design is a central element to a good understanding of computer science and good programming.

Implementation Issues

One of the elements that we will focus on in this course is to try to study algorithms as pure mathematical objects, and so ignore issues such as programming language, machine, and operating system. This has the advantage of clearing away the messy details that affect implementation. But these details may be very important.

For example, an important fact of current processor technology is that of locality of reference. Frequently accessed data can be stored in registers or cache memory. Our mathematical analysis will usually ignore these issues. But a good algorithm designer can work within the realm of mathematics, but still keep an

open eye to implementation issues down the line that will be important for final implementation. For example, we will study three fast sorting algorithms this semester, heap-sort, merge-sort, and quick-sort. From our mathematical analysis, all have equal running times. However, among the three (barring any extra considerations) quick sort is the fastest on virtually all modern machines. Why? It is the best from the perspective of locality of reference. However, the difference is typically small (perhaps 10-20% difference in running time).

Thus this course is not the last word in good program design, and in fact it is perhaps more accurately just the first word in good program design. The overall strategy that I would suggest to any programming would be to first come up with a few good designs from a mathematical and algorithmic perspective. Next prune this selection by consideration of practical matters (like locality of reference). Finally prototype (that is, do test implementations) a few of the best designs and run them on data sets that will arise in your application for the final fine-tuning. Also, be sure to use whatever development tools that you have, such as profilers (programs which pin-point the sections of the code that are responsible for most of the running time).

Course in Review

This course will consist of four major sections. The first is on the mathematical tools necessary for the analysis of algorithms. This will focus on asymptotics, summations, recurrences. The second element will deal with one particularly important algorithmic problem: sorting a list of numbers. We will show a number of different strategies for sorting, and use this problem as a case-study in different techniques for designing and analyzing algorithms.

The final third of the course will deal with a collection of various algorithmic problems and solution techniques. Finally we will close this last third with a very brief introduction to the theory of NP-completeness. NP-complete problem are those for which no efficient algorithms are known, but no one knows for sure whether efficient solutions might exist.

Analyzing Algorithms

In order to design good algorithms, we must first agree the criteria for measuring algorithms. The emphasis in this course will be on the design of efficient algorithm, and hence we will measure algorithms in terms of the amount of computational resources that the algorithm requires. These resources include mostly running time and memory. Depending on the application, there may be other elements that are taken into account, such as the number disk accesses in a database program or the communication bandwidth in a networking application.

In practice there are many issues that need to be considered in the design algorithms. These include issues such as the ease of debugging and maintaining the final software through its life-cycle. Also, one of the luxuries we will have in this course is to be able to assume that we are given a clean, fully-specified mathematical description of the computational problem. In practice, this is often not the case, and the algorithm must be designed subject to only partial knowledge of the final specifications. Thus, in practice

it is often necessary to design algorithms that are simple, and easily modified if problem parameters and specifications are slightly modified. Fortunately, most of the algorithms that we will discuss in this class are quite simple, and are easy to modify subject to small problem variations.

Model of Computation

Another goal that we will have in this course is that our analysis be as independent as possible of the variations in machine, operating system, compiler, or programming language. Unlike programs, algorithms to be understood primarily by people (i.e. programmers) and not machines. Thus gives us quite a bit of flexibility in how we present our algorithms, and many low-level details may be omitted (since it will be the job of the programmer who implements the algorithm to fill them in).

But, in order to say anything meaningful about our algorithms, it will be important for us to settle on a mathematical model of computation. Ideally this model should be a reasonable abstraction of a standard generic single-processor machine. We call this model a *random access machine* or RAM.

A RAM is an idealized machine with an infinitely large random-access memory. Instructions are executed one-by-one (there is no parallelism). Each instruction involves performing some basic operation on two values in the machines memory (which might be characters or integers; let's avoid floating point for now). Basic operations include things like assigning a value to a variable, computing any basic arithmetic operation ($+, -, \times$, integer division) on integer values of any size, performing any comparison (e.g. $x \leq 5$) or boolean operations, accessing an element of an array (e.g. $A[10]$). We assume that each basic operation takes the same constant time to execute.

This model seems to go a good job of describing the computational power of most modern (nonparallel) machines. It does not model some elements, such as efficiency due to locality of reference, as described in the previous lecture. There are some "loop-holes" (or hidden ways of subverting the rules) to beware of. For example, the model would allow you to add two numbers that contain a billion digits in constant time. Thus, it is theoretically possible to derive nonsensical results in the form of efficient RAM programs that cannot be implemented efficiently on any machine. Nonetheless, the RAM model seems to be fairly sound, and has done a good job of modelling typical machine technology since the early 60's.

Example: 2-dimension maxima

Let us do an example that illustrates how we analyze algorithms. Suppose you want to buy a car. You want the pick the fastest car. But fast cars are expensive; you want the cheapest. You cannot decide which is more important: speed or price. Definitely do not want a car if there is another that is both faster and cheaper. We say that the fast, cheap car *dominates* the slow, expensive car relative to your selection criteria. So, given a collection of cars, we want to list those cars that are not dominated by any other.

Here is how we might model this as a formal problem.

- Let a point p in 2-dimensional space be given by its integer coordinates, $p = (p.x, p.y)$.

- A point p is said to be dominated by point q if $p.x \leq q.x$ and $p.y \leq q.y$.
- Given a set of n points, $P = \{p_1, p_2, ..., p_n\}$ in 2-space a point is said to be maximal if it is not dominated by any other point in P.

The car selection problem can be modelled this way: For each car we associate (x, y) pair where x is the speed of the car and y is the negation of the price. High y value means a cheap car and low y means expensive car. Think of y as the money left in your pocket after you have paid for the car. Maximal points correspond to the fastest and cheapest cars.

The *2-dimensional Maxima* is thus defined as

- Given a set of points $P = \{p_1, p_2, ... , p_n\}$ in 2-space, output the set of maximal points of P, i.e. those points p_i such that p_i is not dominated by any other point of P.

Here is set of maximal points for a given set of points in 2-d.

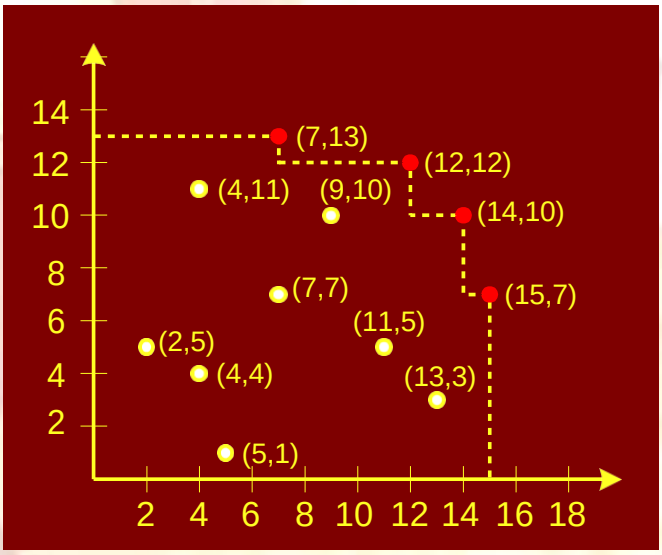

Figure 1.1: Maximal points in 2-d

Our description of the problem is at a fairly mathematical level. We have intentionally not discussed how the points are represented. We have not discussed any input or output formats. We have avoided programming and other software issues.

Brute-Force Algorithm

To get the ball rolling, let's just consider a simple brute-force algorithm, with no thought to efficiency. Let $P = \{p_1, p_2, \ldots, p_n\}$ be the initial set of points. For each point p_i, test it against all other points p_j. If p_i is not dominated by any other point, then output it.

This English description is clear enough that any (competent) programmer should be able to implement it. However, if you want to be a bit more formal, it could be written in pseudocode as follows:

```
MAXIMA(int n, Point P[1 . . . n])
1  for i ← 1 to n
2  do maximal ← true
3     for j ← 1 to n
4     do
5        if (i ≠ j) and (P[i].x ≤ P[j].x) and (P[i].y ≤ P[j].y)
6           then maximal ← false; break
7     if (maximal = true)
8        then output P[i]
```

There are no formal rules to the syntax of this pseudo code. In particular, do not assume that more detail is better. For example, I omitted type specifications for the procedure Maxima and the variable maximal, and I never defined what a Point data type is, since I felt that these are pretty clear from context or just unimportant details. Of course, the appropriate level of detail is a judgement call. Remember, algorithms are to be read by people, and so the level of detail depends on your intended audience. When writing pseudo code, you should omit details that detract from the main ideas of the algorithm, and just go with the essentials.

You might also notice that I did not insert any checking for consistency. For example, I assumed that the points in P are all distinct. If there is a duplicate point then the algorithm may fail to output even a single point. (Can you see why?) Again, these are important considerations for implementation, but we will often omit error checking because we want to see the algorithm in its simplest form.

Here are a series of figures that illustrate point domination.

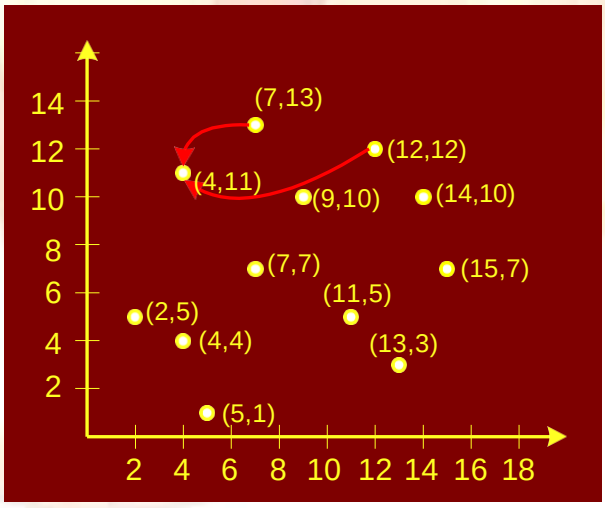

Figure 1.2: Points that dominate (4, 11)

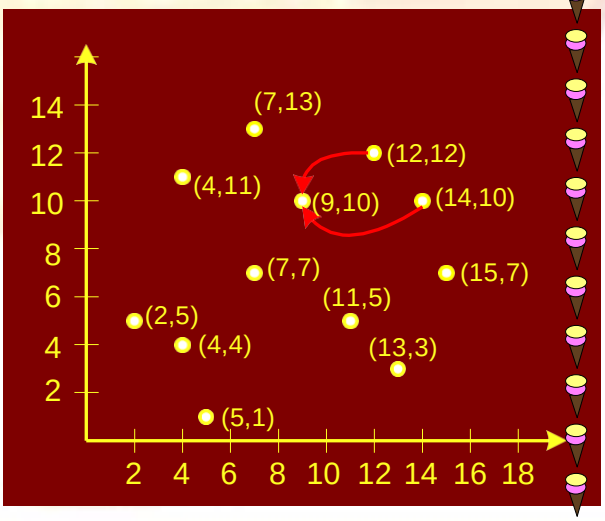

Figure 1.3: Points that dominate (9, 10)

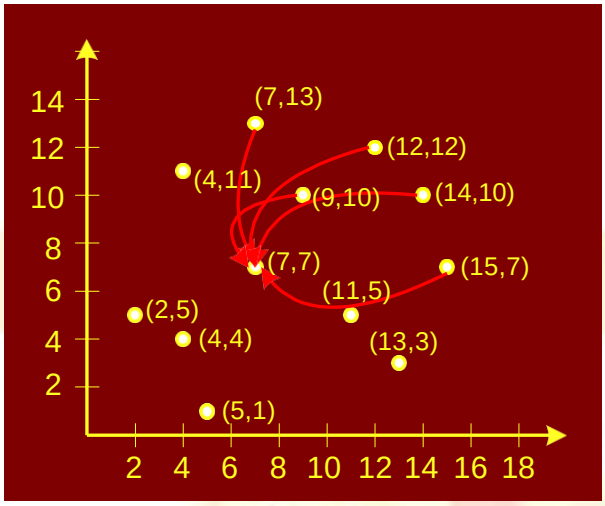

Figure 1.4: Points that dominate $(7, 7)$

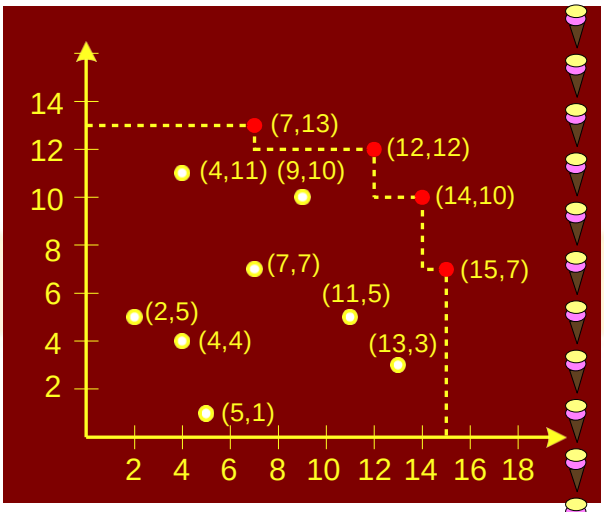

Figure 1.5: The maximal points

Running Time Analysis

The main purpose of our mathematical analysis will be measuring the execution time. We will also be concerned about the space (memory) required by the algorithm.

The running time of an implementation of the algorithm would depend upon the speed of the computer, programming language, optimization by the compiler etc. Although important, we will ignore these technological issues in our analysis.

To measure the running time of the brute-force 2-d maxima algorithm, we could count the number of steps of the pseudo code that are executed or, count the number of times an element of P is accessed or the number of comparisons that are performed.

The running time depends upon the input size, e.g. n Different inputs of the same size may result in different running time. For example, breaking out of the inner loop in the brute-force algorithm depends not only on the input size of P but also the structure of the input.

Two criteria for measuring running time are *worst-case time* and *average-case time*.

Worst-case time is the maximum running time over all (legal) inputs of size n. Let I denote an input instance, let $|I|$ denote its length, and let $T(I)$ denote the running time of the algorithm on input I Then

$$T_{worst}(n) = \max_{|I|=n} T(I)$$

Average-case time is the average running time over all inputs of size n. Let $p(I)$ denote the probability of seeing this input. The average-case time is the weighted sum of running times with weights

being the probabilities:

$$T_{avg}(n) = \sum_{|I|=n} p(I)T(I)$$

We will almost always work with worst-case time. Average-case time is more difficult to compute; it is difficult to specify probability distribution on inputs. Worst-case time will specify an upper limit on the running time.

1.10.1 Analysis of the brute-force maxima algorithm.

Assume that the input size is n, and for the running time we will count the number of time that any element of P is accessed. Clearly we go through the outer loop n times, and for each time through this loop, we go through the inner loop n times as well. The condition in the if-statement makes four accesses to P. The output statement makes two accesses for each point that is output. In the worst case every point is maximal (can you see how to generate such an example?) so these two access are made for each time through the outer loop.

```
MAXIMA(int n, Point P[1 . . . n])
1  for i ← 1 to n                    n times
2  do maximal ← true
3     for j ← 1 to n                 n times
4     do
5        if (i ≠ j)&(P[i].x ≤ P[j].x)&(P[i].y≤ P[j].y)   4 accesses
6           then maximal ← false break
7     if maximal
8        then output P[i].x, P[i].y                      2 accesses
```

Thus we might express the worst-case running time as a pair of nested summations, one for the i-loop and the other for the j-loop:

$$\begin{aligned} T(n) &= \sum_{i=1}^{n} 2 + \sum_{j=1}^{n} 4 \\ &= \sum_{i=1}^{n} (4n + 2) \\ &= (4n + 2)n = 4n^2 + 2n \end{aligned}$$

For small values of n, any algorithm is fast enough. What happens when n gets large? Running time does become an issue. When n is large, n^2 term will be much larger than the n term and will *dominate* the running time.

We will say that the worst-case running time is $\Theta(n^2)$. This is called the asymptotic growth rate of the function. We will discuss this Θ-notation more formally later.

The analysis involved computing a summation. Summation should be familiar but let us review a bit here. Given a finite sequence of values $a_1, a_2, \ldots, a_n$, their sum $a_1 + a_2 + \ldots + a_n$ is expressed in summation notation as

$$\sum_{i=1}^{n} a_i$$

If $n = 0$, then the sum is additive identity, 0.

Some facts about summation: If c is a constant

$$\sum_{i=1}^{n} ca_i = c \sum_{i=1}^{n} a_i$$

and

$$\sum_{i=1}^{n} (a_i + b_i) = \sum_{i=1}^{n} a_i + \sum_{i=1}^{n} b_i$$

Some important summations that should be committed to memory.

Arithmetic series

$$\sum_{i=1}^{n} i = 1 + 2 + \ldots + n$$

$$= \frac{n(n+1)}{2} = \Theta(n^2)$$

Quadratic series

$$\sum_{i=1}^{n} i^2 = 1 + 4 + 9 + \ldots + n^2$$

$$= \frac{2n^3 + 3n^2 + n}{6} = \Theta(n^3)$$

Geometric series

$$\sum_{i=1}^{n} x^i = 1 + x + x^2 + \ldots + x^n$$

$$= \frac{x^{(n+1)} - 1}{x - 1} = \Theta(n^2)$$

If $0 < x < 1$ then this is $\Theta(1)$, and if $x > 1$, then this is $\Theta(x^n)$.

Harmonic series For $n \geq 0$

$$H_n = \sum_{i=1}^{n} \frac{1}{i}$$

$$= 1 + \frac{1}{2} + \frac{1}{3} + \ldots + \frac{1}{n} \approx \ln n$$

$$= \Theta(\ln n)$$

Analysis: A Harder Example

Let us consider a harder example.

```
NESTED-LOOPS()
1  for i ← 1 to n
2  do
3      for j ← 1 to 2i
4      do k = j ...
5          while (k ≥ 0)
6          do k = k – 1 ...
```

How do we analyze the running time of an algorithm that has complex nested loop? The answer is we write out the loops as summations and then solve the summations. To convert loops into summations, we work from inside-out.

Consider the *inner most while* loop.

```
NESTED-LOOPS()
1  for i ← 1 to n
2  do for j ← 1 to 2i
3      do k = j
4          while (k ≥ 0) (
5          do k = k – 1
```

It is executed for $k = j, j-1, j-2, \ldots, 0$. Time spent inside the while loop is constant. Let $I()$ be the time spent in the while loop. Thus

$$I(j) = \sum_{k=0}^{j} 1 = j + 1$$

Consider the *middle for* loop.

```
NESTED-LOOPS()
1  for i ← 1 to n
2  do for j ← 1 to 2i (
3      do k = j
4          while (k ≥ 0)
5          do k = k – 1
```

Its running time is determined by i. Let $M()$ be the time spent in the for loop:

$$\begin{aligned} M(i) &= \sum_{j=1}^{2i} l(j) \\ &= \sum_{j=1}^{2i} (j+1) \\ &= \sum_{j=1}^{2i} j + \sum_{j=1}^{2i} 1 \\ &= \frac{2i(2i+1)}{2} + 2i \\ &= 2i^2 + 3i \end{aligned}$$

Finally the *outer-most for* loop.

```
NESTED-LOOPS()
1  for i ← 1 to n
2  do for j ← 1 to 2i
3     do k = j
4        while (k ≥ 0)
5        do k = k – 1
```

Let $T()$ be running time of the entire algorithm:

$$\begin{aligned} T(n) &= \sum_{i=1}^{n} M(i) \\ &= \sum_{i=1}^{n} (2i^2 + 3i) \\ &= \sum_{i=1}^{n} 2i^2 + \sum_{i=1}^{n} 3i \\ &= 2\frac{2n^3 + 3n^2 + n}{6} + 3\frac{n(n+1)}{2} \\ &= \frac{4n^3 + 15n^2 + 11n}{6} \\ &= \Theta(n^3) \end{aligned}$$

2-dimension Maxima Revisited

Recall the 2-d maxima problem: Let a point p in 2-dimensional space be given by its integer coordinates, $p = (p.x, p.y)$. A point p is said to *dominated* by point q if $p.x \leq q.x$ and $p.y \leq q.y$. Given a set of n

points, $P = \{p_1, p_2, \ldots, p_n\}$ in 2-space a point is said to be *maximal* if it is not dominated by any other point in P. The problem is to output all the maximal points of P. We introduced a brute-force algorithm that ran in $\Theta(n^2)$ time. It operated by comparing *all pairs* of points. Is there an approach that is significantly better?

The problem with the brute-force algorithm is that it uses no intelligence in *pruning* out decisions. For example, once we know that a point p_i is dominated by another point p_j, we do not need to use p_i for eliminating other points. This follows from the fact that dominance relation is *transitive*. If p_j dominates p_i and p_i dominates p_h then p_j also dominates p_h; p_i is not needed.

Plane-sweep Algorithm

The question is whether we can make an significant improvement in the running time? Here is an idea for how we might do it. We will sweep a vertical line across the plane from left to right. As we sweep this line, we will build a structure holding the maximal points lying to the left of the sweep line. When the sweep line reaches the rightmost point of P , then we will have constructed the complete set of maxima. This approach of solving geometric problems by sweeping a line across the plane is called *plane sweep*.

Although we would like to think of this as a continuous process, we need some way to perform the plane sweep in discrete steps. To do this, we will begin by sorting the points in increasing order of their x-coordinates. For simplicity, let us assume that no two points have the same y-coordinate. (This limiting assumption is actually easy to overcome, but it is good to work with the simpler version, and save the messy details for the actual implementation.) Then we will advance the sweep-line from point to point in n discrete steps. As we encounter each new point, we will update the current list of maximal points.

We will sweep a vertical line across the 2-d plane from left to right. As we sweep, we will build a structure holding the maximal points lying to the left of the sweep line. When the sweep line reaches the rightmost point of P, we will have the complete set of maximal points. We will store the existing maximal points in a list The points that p_i dominates will appear at the end of the list because points are sorted by x-coordinate. We will scan the list left to right. Every maximal point with y-coordinate less than p_i will be eliminated from computation. We will add maximal points onto the end of a list and delete from the end of the list. We can thus use a stack to store the maximal points. The point at the top of the stack will have the highest x-coordinate.

Here are a series of figures that illustrate the plane sweep. The figure also show the content of the stack.

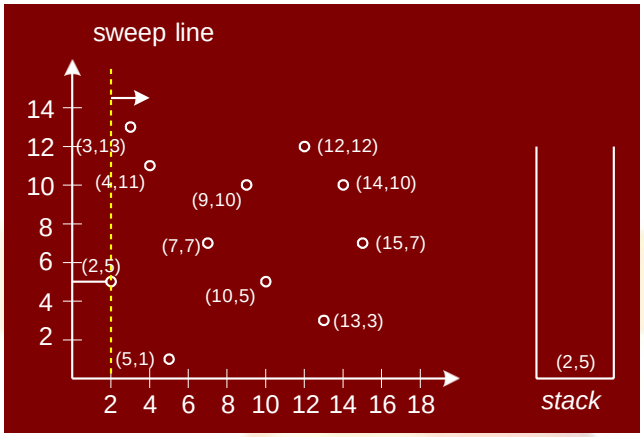

Figure 1.6: Sweep line at $(2, 5)$

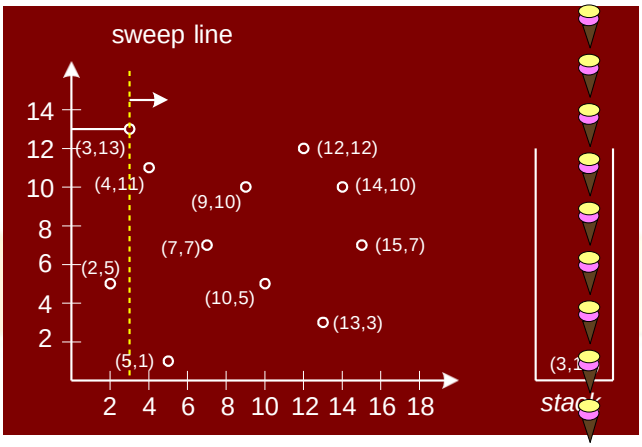

Figure 1.7: Sweep line at $(3, 13)$

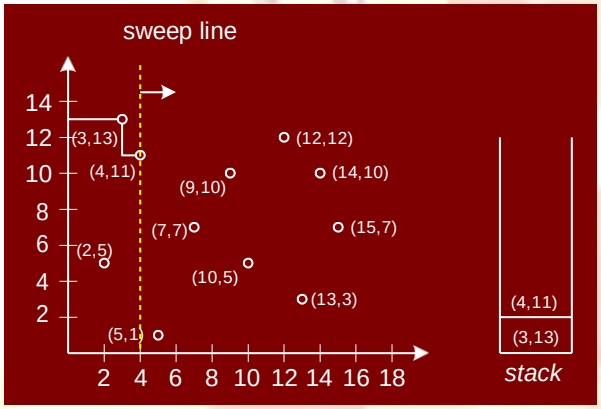

Figure 1.8: Sweep line at $(4, 11)$

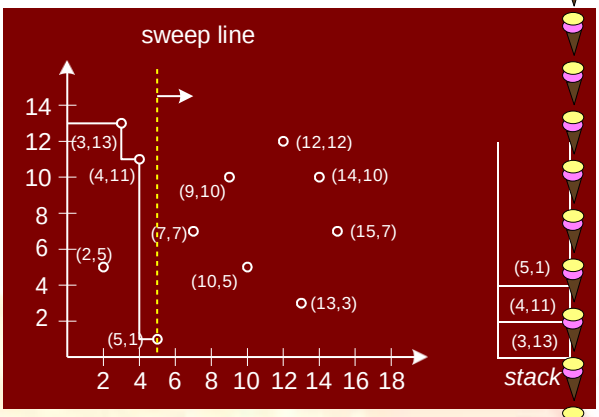

Figure 1.9: Sweep line at $(5, 1)$

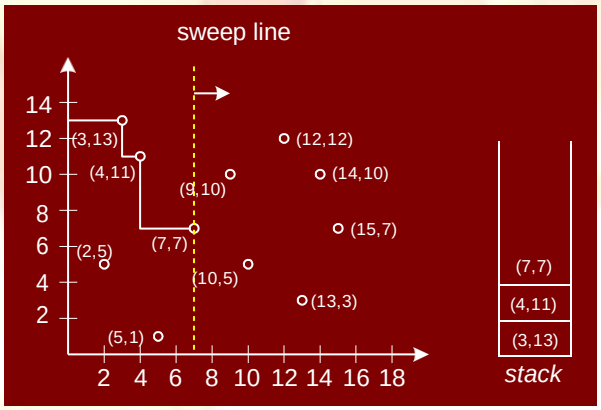

Figure 1.10: Sweep line at $(7, 7)$

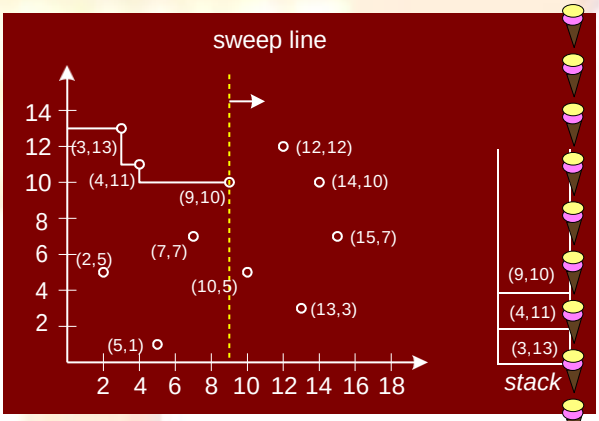

Figure 1.11: Sweep line at $(9, 10)$

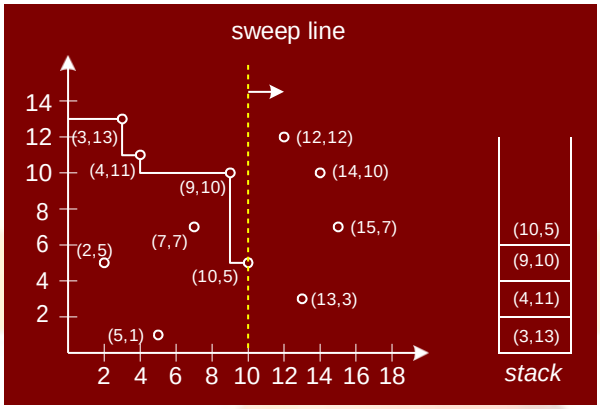

Figure 1.12: Sweep line at $(10, 5)$

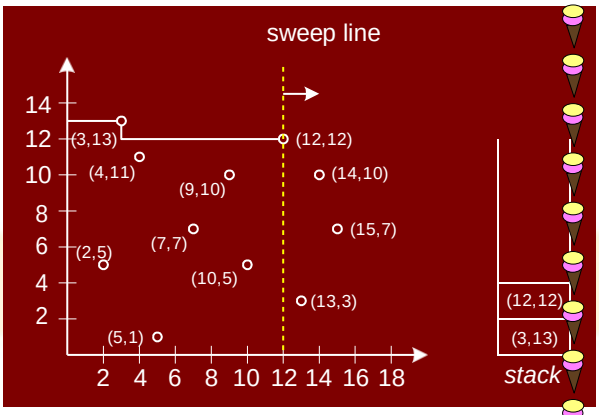

Figure 1.13: Sweep line at $(12, 12)$

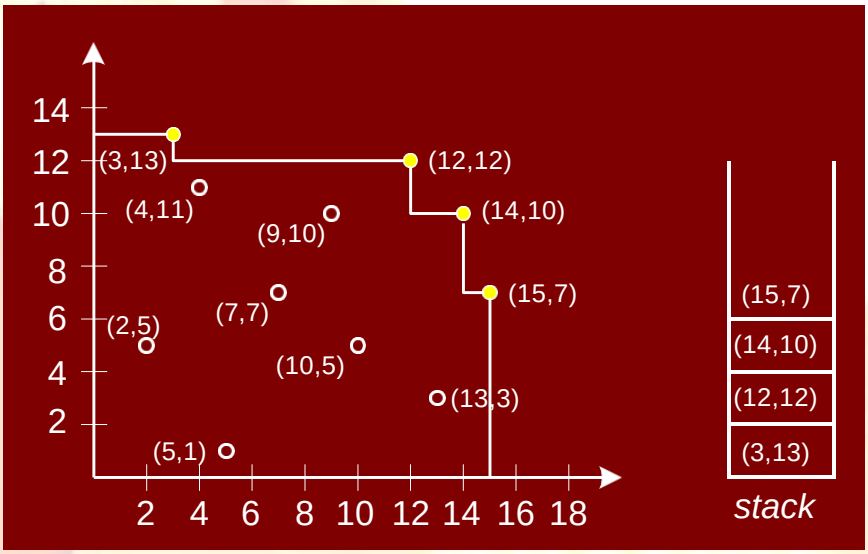

Figure 1.14: The final maximal set

Here is the algorithm.

```
PLANE-SWEEP-MAXIMA(n, P[1..n])
1  sort P in increasing order by x;
2  stack s;
3  for i ← 1 to n
4  do
5      while (s.notEmpty() & s.top().y ≤ P[i].y)
6      do s.pop();
7      s.push(P[i]);
8  output the contents of stack s;
```

Analysis of Plane-sweep Algorithm

Sorting takes $\Theta(n \log n)$; we will show this later when we discuss sorting. The for loop executes n times. The inner loop (seemingly) could be iterated $(n-1)$ times. It seems we still have an $n(n-1)$ or $\Theta(n^2)$ algorithm. Got fooled by simple minded loop-counting. The while loop will not execute more than n times over the entire course of the algorithm. Why is this? Observe that the total number of elements that can be pushed on the stack is n since we execute exactly one push each time during the outer for-loop. We pop an element off the stack each time we go through the inner while-loop. It is impossible to pop more elements than are ever pushed on the stack. Therefore, the inner while-loop cannot execute more than n times over the entire course of the algorithm. (*Make sure that you understand this*).

The for-loop iterates n times and the inner while-loop also iterates n time for a total of $\Theta(n)$. Combined with the sorting, the runtime of entire plane-sweep algorithm is $\Theta(n \log n)$.

Comparison of Brute-force and Plane sweep algorithms

How much of an improvement is plane-sweep over brute-force? Consider the ratio of running times:

$$\frac{n^2}{n \log n} = \frac{n}{\log n}$$

n	$\log n$	$\frac{n}{\log n}$
100	7	15
1000	10	100
10000	13	752
100000	17	6021
1000000	20	50171

For $n = 1,000,000$, if plane-sweep takes 1 second, the brute-force will take about 14 hours!. From this we get an idea about the importance of asymptotic analysis. It tells us which algorithm is better for large values of n. As we mentioned before, if n is not very large, then almost any algorithm will be fast. But efficient algorithm design is most important for large inputs, and the general rule of computing is that input sizes continue to grow until people can no longer tolerate the running times. Thus, by designing algorithms efficiently, you make it possible for the user to run large inputs in a reasonable amount of time.

Chapter 2

Asymptotic Notation

You may be asking that we continue to use the notation $\Theta()$ but have never defined it. Let's remedy this now. Given any function $g(n)$, we define $\Theta(g(n))$ to be a set of functions that *asymptotically equivalent* to $g(n)$. Formally:

$$\Theta(g(n)) = \{f(n) \mid \text{there exist positive constants } c_1, c_2 \text{ and } n_0 \text{ such that } 0 \leq c_1 g(n) \leq f(n) \leq c_2 g(n) \text{ for all } n \geq n_0\}$$

This is written as "$f(n) \in \Theta(g(n))$" That is, $f(n)$ and $g(n)$ are *asymptotically equivalent*. This means that they have essentially the same growth rates for large n. For example, functions like

- $4n^2$,
- $(8n^2 + 2n - 3)$,
- $(n^2/5 + \sqrt{n} - 10 \log n)$
- $n(n - 3)$

are all asymptotically equivalent. As n becomes large, the *dominant* (fastest growing) term is some constant times n^2.

Consider the function

$$f(n) = 8n^2 + 2n - 3$$

Our informal rule of keeping the largest term and ignoring the constant suggests that $f(n) \in \Theta(n^2)$. Let's see why this bears out formally. We need to show two things for

$$f(n) = 8n^2 + 2n - 3$$

Lower bound $f(n) = 8n^2 + 2n - 3$ grows asymptotically at least as fast as n^2,

Upper bound $f(n)$ grows no faster asymptotically than n^2,

Lower bound: $f(n)$ grows asymptotically at least as fast as n^2. For this, need to show that there exist positive constants c_1 and n_0, such that $f(n) \geq c_1 n^2$ for all $n \geq n_0$. Consider the reasoning

$$f(n) = 8n^2 + 2n - 3 \geq 8n^2 - 3 = 7n^2 + (n^2 - 3) \geq 7n^2$$

Thus $c_1 = 7$. We implicitly assumed that $2n \geq 0$ and $n^2 - 3 \geq 0$ These are not true for all n but if $n \geq \sqrt{3}$, then both are true. So select $n_0 \geq \sqrt{3}$. We then have $f(n) \geq c_1 n^2$ for all $n \geq n_0$.

Upper bound: $f(n)$ grows asymptotically no faster than n^2. For this, we need to show that there exist positive constants c_2 and n_0, such that $f(n) \leq c_2 n^2$ for all $n \geq n_0$. Consider the reasoning

$$f(n) = 8n^2 + 2n - 3 \leq 8n^2 + 2n \leq 8n^2 + 2n^2 = 10n^2$$

Thus $c_2 = 10$. We implicitly made the assumption that $2n \leq 2n^2$. This is not true for all n but it is tru for all $n \geq 1$ So select $n_0 \geq 1$. We thus have $f(n) \leq c_2 n^2$ for all $n \geq n_0$.

From lower bound we have $n_0 \geq \sqrt{3}$ From upper bound we have $n_0 \geq 1$. Combining the two, we let n_0 be the larger of the two: $n_0 \geq \sqrt{3}$. In conclusion, if we let $c_1 = 7$, $c_2 = 10$ and $n_0 \geq \sqrt{3}$, we have

$$7n^2 \leq 8n^2 + 2n - 3 \leq 10n^2 \quad \text{for all } n \geq \sqrt{3}$$

We have thus established

$$0 \leq c_1 g(n) \leq f(n) \leq c_2 g(n) \qquad \text{for all } n \geq n_0$$

Here are plots of the three functions. Notice the bounds.

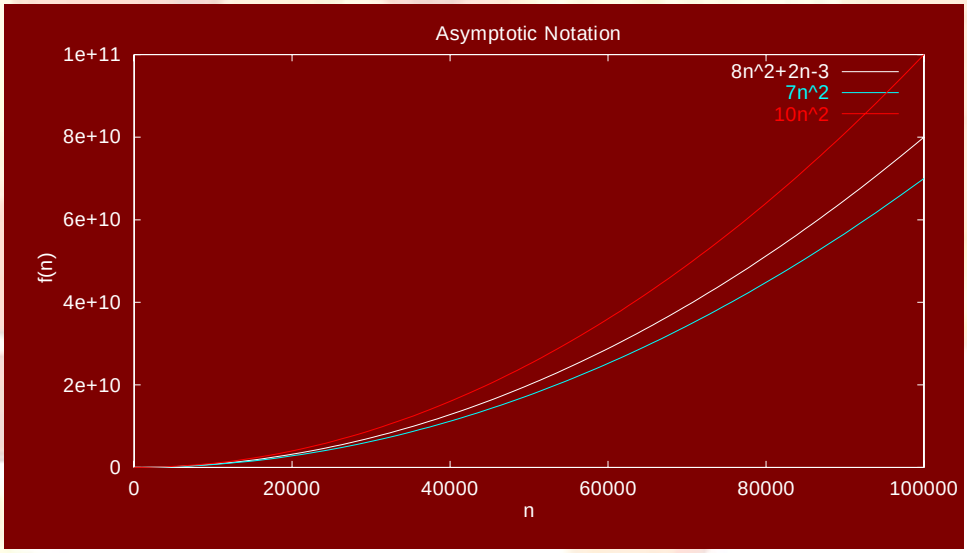

Figure 2.1: Asymptotic Notation Example

We have established that $f(n) \in n^2$. Let's show why $f(n)$ is not in some other asymptotic class. First, let's show that $f(n) \not\in \Theta(n)$. Show that $f(n) \not\in \Theta(n)$. If this were true, we would have had to satisfy

both the upper and lower bounds. The lower bound is satisfied because $f(n) = 8n^2 + 2n - 3$ does grow at least as fast asymptotically as n. But the upper bound is false. Upper bounds requires that there exist positive constants c_2 and n_0 such that $f(n) \leq c_2 n$ for all $n \geq n_0$.

Informally we know that $f(n) = 8n^2 + 2n - 3$ will eventually exceed $c_2 n$ no matter how large we make c_2. To see this, suppose we assume that constants c_2 and n_0 did exist such that $8n^2 + 2n - 3 \leq c_2 n$ for all $n \geq n_0$ Since this is true for all sufficiently large n then it must be true in the limit as n tends to infinity. If we divide both sides by n, we have

$$\lim_{n \to \infty} \quad 8n + 2 - \frac{3}{n} \quad \leq c_2.$$

It is easy to see that in the limit, the left side tends to ∞. So, no matter how large c_2 is, the statement is violated. Thus $f(n) \not\in \Theta(n)$.

Let's show that $f(n) \not\in \Theta(n^3)$. The idea would be to show that the lower bound $f(n) \geq c_1 n^3$ for all $n \geq n_0$ is violated. (c_1 and n_0 are positive constants). Informally we know this to be true because any cubic function will overtake a quadratic.

If we divide both sides by n^3:

$$\lim_{n \to \infty} \quad \frac{8}{n} + \frac{2}{n^2} - \frac{3}{n^3} \quad \geq c_1$$

The left side tends to 0. The only way to satisfy this is to set $c_1 = 0$. But by hypothesis, c_1 is positive. This means that $f(n) \not\in \Theta(n^3)$.

The definition of Θ-notation relies on proving both a lower and upper asymptotic bound. Sometimes we only interested in proving one bound or the other. The O-notation is used to state only the asymptotic *upper* bounds.

$$O(g(n)) = \{f(n) \mid \text{there exist positive constants } c \text{ and } n_0 \text{ such that } 0 \leq f(n) \leq cg(n) \text{ for all } n \geq n_0\}$$

The Ω-notation allows us to state only the asymptotic *lower* bounds.

$$\Omega(g(n)) = \{f(n) \mid \text{there exist positive constants } c \text{ and } n_0 \text{ such that } 0 \leq cg(n) \leq f(n) \text{ for all } n \geq n_0\}$$

The three notations:

$$\Theta(g(n)) : 0 \le c_1 g(n) \le f(n) \le c_2 g(n)$$
$$O(g(n)) : 0 \le f(n) \le cg(n)$$
$$\Omega(g(n)) : 0 \le cg(n) \le f(n)$$
$$\text{for all } n \ge n_0$$

These definitions suggest an alternative way of showing the asymptotic behavior. We can use limits for define the asymptotic behavior. Limit rule for Θ-notation:

$$\lim_{n\to\infty} \frac{f(n)}{g(n)} = c,$$

for some constant $c > 0$ (strictly positive but not infinity) then $f(n) \in \Theta(g(n))$. Similarly, the limit rule for O-notation is

$$\lim_{n\to\infty} \frac{f(n)}{g(n)} = c,$$

for some constant $c \ge 0$ (nonnegative but not infinite) then $f(n) \in O(g(n))$ and limit rule for Ω-notation:

$$\lim_{n\to\infty} \frac{f(n)}{g(n)} /= 0,$$

(either a strictly positive constant or infinity) then $f(n) \in \Omega(g(n))$.

Here is a list of common asymptotic running times:

- $\Theta(1)$: Constant time; can't beat it!
- $\Theta(\log n)$: Inserting into a balanced binary tree; time to find an item in a sorted array of length n using binary search.
- $\Theta(n)$: About the fastest that an algorithm can run.
- $\Theta(n \log n)$: Best sorting algorithms.
- $\Theta(n^2)$, $\Theta(n^3)$: Polynomial time. These running times are acceptable when the exponent of n is small or n is not to large, e.g., $n \le 1000$.
- $\Theta(2^n)$, $\Theta(3^n)$: Exponential time. Acceptable only if n is small, e.g., $n \le 50$.
- $\Theta(n!)$, $\Theta(n_n)$: Acceptable only for really small n, e.g. $n \le 20$.

Chapter 3

Divide and Conquer Strategy

The ancient Roman politicians understood an important principle of good algorithm design (although they were probably not thinking about algorithms at the time). You divide your enemies (by getting them to distrust each other) and then conquer them piece by piece. This is called *divide-and-conquer*. In algorithm design, the idea is to take a problem on a large input, break the input into smaller pieces, solve the problem on each of the small pieces, and then combine the piecewise solutions into a global solution. But once you have broken the problem into pieces, how do you solve these pieces? The answer is to apply divide-and-conquer to them, thus further breaking them down. The process ends when you are left with such tiny pieces remaining (e.g. one or two items) that it is trivial to solve them. Summarizing, the main elements to a divide-and-conquer solution are

Divide: the problem into a small number of pieces

Conquer: solve each piece by applying divide and conquer to it *recursively*

Combine: the pieces together into a global solution.

Merge Sort

Divide and conquer strategy is applicable in a huge number of computational problems. The first example of divide and conquer algorithm we will discuss is a simple and efficient sorting procedure called We are given a sequence of n numbers A, which we will assume are stored in an array $A[1..n]$. The objective is to output a permutation of this sequence sorted in increasing order. This is normally done by permuting the elements within the array A. Here is how the merge sort algorithm works:

- (Divide:) split A down the middle into two subsequences, each of size roughly $n/2$
- (Conquer:) sort each subsequence by calling merge sort recursively on each.
- (Combine:) merge the two sorted subsequences into a single sorted list.

Let's design the algorithm top-down. We'll assume that the procedure that merges two sorted list is available to us. We'll implement it later. Because the algorithm is called recursively on sublists, in addition to passing in the array itself, we will pass in two indices, which indicate the first and last indices of the sub-array that we are to sort. The call MergeSort(A, p, r) will sort the sub-array A[p : r] and return the sorted result in the same sub-array.Here is the overview. If r = p, then this means that there's only one element to sort, and we may return immediately. Otherwise if (p r) there are at least two elements, and we will invoke the divide-and-conquer. We find the index q, midway between p and r, namely q = (p + r)/2 (rounded down to the nearest integer). Then we split the array into sub-arrays A[p : q] and A[q + 1 : r]. Call MergeSort recursively to sort each sub-array. Finally, we invoke a procedure (which we have yet to write) which merges these two sub-arrays into a single sorted array.

Here is the MergeSort algorithm.

```
MERGE-SORT( array A, int p, int r)
1   if (p < r)
2     then
3          q ← (p + r)/2
4          MERGE-SORT(A, p, q)       // sort A[p..q]
5          MERGE-SORT(A, q + 1, r)   // sort A[q + 1..r]
6          MERGE(A, p, q, r)         // merge the two pieces
```

The following figure illustrates the dividing (splitting) procedure.

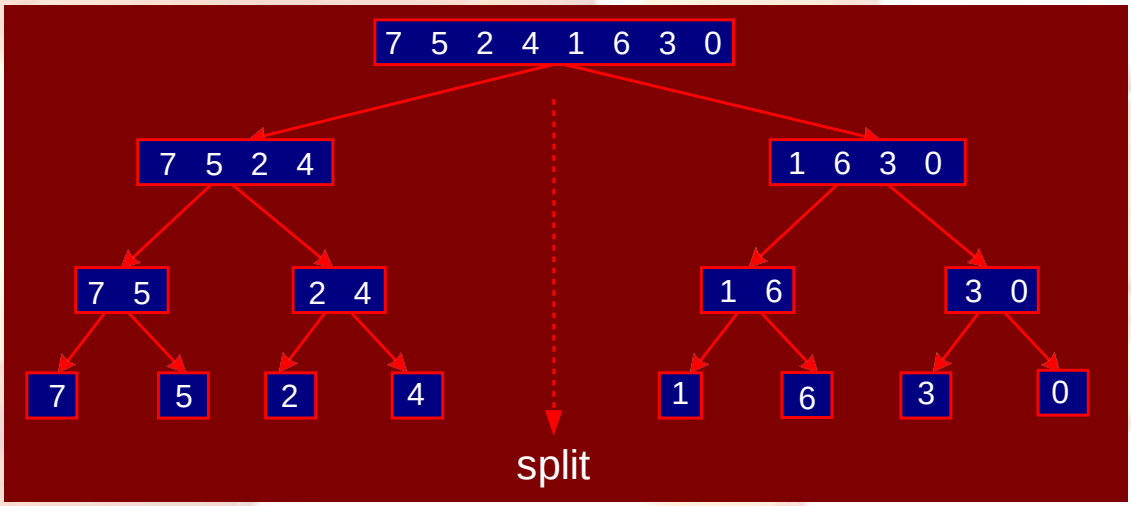

Figure 3.1: Merge sort divide phase

Merging: All that is left is to describe the procedure that merges two sorted lists. Merge(A, p, q, r) assumes that the left sub-array, A[p : q], and the right sub-array, A[q + 1 : r], have already been sorted. We merge these two sub-arrays by copying the elements to a temporary working array called B. For convenience, we will assume that the array B has the same index range A, that is, B[p : r]. (One nice thing about pseudo-code, is that we can make these assumptions, and leave them up to the programmer to figure out how to implement it.) We have to indices i and j, that point to the current elements of each

sub-array. We move the smaller element into the next position of B (indicated by index k) and then increment the corresponding index (either i or j). When we run out of elements in one array, then we just copy the rest of the other array into B. Finally, we copy the entire contents of B back into A. (The use of the temporary array is a bit unpleasant, but this is impossible to overcome entirely. It is one of the shortcomings of MergeSort, compared to some of the other efficient sorting algorithms.)

Here is the merge algorithm:

```
MERGE( array A, int p, int q int r)
 1  int B[p..r]; int i ← k ← p; int j ← q + 1
 2  while (i ≤ q) and (j ≤ r)
 3  do if (A[i] ≤ A[j])
 4        then B[k++ ] ← A[i++ ]
 5        else B[k++ ] ← A[j++ ]
 6  while (i ≤ q)
 7  do B[k++ ] ← A[i++ ]
 8  while (j ≤ r)
 9  do B[k++ ] ← A[j++ ]
10  for i ← p to r
11  do A[i] ← B[i]
```

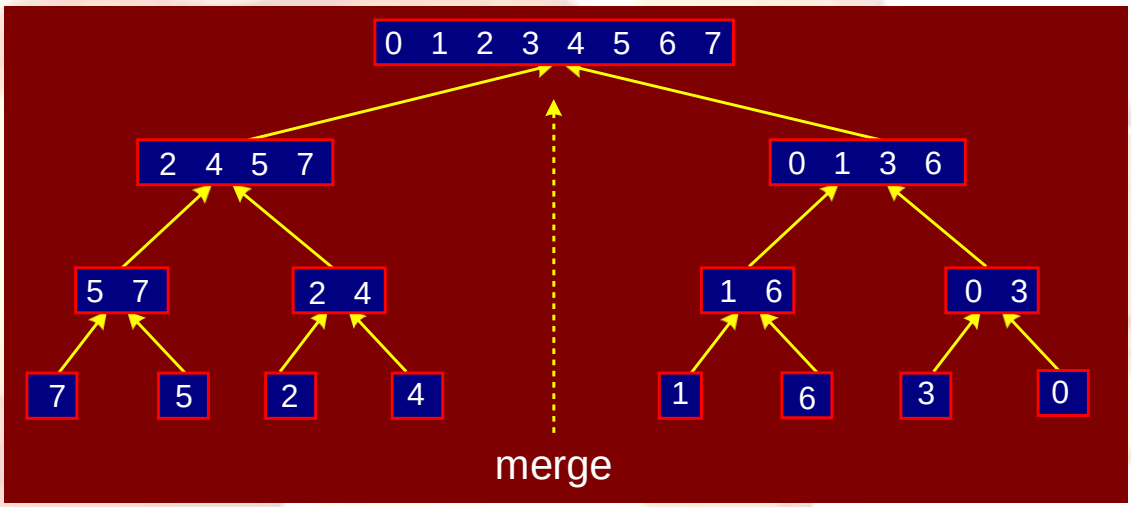

Figure 3.2: Merge sort: combine phase

Analysis of Merge Sort

First let us consider the running time of the procedure Merge(A, p, q, r). Let n = r – p + 1 denote the total length of both the left and right sub-arrays. What is the running time of Merge as a function of n? The algorithm contains four loops (none nested in the other). It is easy to see that each loop can be executed at most n times. (If you are a bit more careful you can actually see that all the while-loops

together can only be executed n times in total, because each execution copies one new element to the array B, and B only has space for n elements.) Thus the running time to Merge n items is $\Theta(n)$. Let us write this without the asymptotic notation, simply as n. (We'll see later why we do this.)

Now, how do we describe the running time of the entire MergeSort algorithm? We will do this through the use of a recurrence, that is, a function that is defined recursively in terms of itself. To avoid circularity, the recurrence for a given value of n is defined in terms of values that are strictly smaller than n. Finally, a recurrence has some basis values (e.g. for $n = 1$), which are defined explicitly.

Let $T(n)$ denote the worst case running time of MergeSort on an array of length n. If we call MergeSort with an array containing a single item ($n = 1$) then the running time is constant. We can just write $T(n) = 1$, ignoring all constants. For $n > 1$, MergeSort splits into two halves, sorts the two and then merges them together. The left half is of size $\lfloor n/2 \rfloor$ and the right half is $\lceil n/2 \rceil$. How long does it take to sort elements in sub array of size $\lfloor n/2 \rfloor$? We do not know this but because $\lfloor n/2 \rfloor < n$ for $n > 1$, we can express this as $T(\lfloor n/2 \rfloor)$. Similarly the time taken to sort right sub array is expressed as $T(\lceil n/2 \rceil)$. In conclusion we have

$$T(n) = \begin{cases} 1 & \text{if } n = 1, \\ T(\lfloor n/2 \rfloor) + T(\lceil n/2 \rceil) + n & \text{otherwise} \end{cases}$$

This is called *recurrence relation*, i.e., a recursively defined function. Divide-and-conquer is an important design technique, and it naturally gives rise to recursive algorithms. It is thus important to develop mathematical techniques for solving recurrences, either exactly or asymptotically.

Let's expand the terms.

$$\begin{aligned} T(1) &= 1 \\ T(2) &= T(1) + T(1) + 2 = 1 + 1 + 2 = 4 \\ T(3) &= T(2) + T(1) + 3 = 4 + 1 + 3 = 8 \\ T(4) &= T(2) + T(2) + 4 = 4 + 4 + 4 = 12 \\ T(5) &= T(3) + T(2) + 5 = 8 + 4 + 5 = 17 \\ &\ldots \\ T(8) &= T(4) + T(4) + 8 = 12 + 12 + 8 = 32 \\ &\ldots \\ T(16) &= T(8) + T(8) + 16 = 32 + 32 + 16 = 80 \\ &\ldots \\ T(32) &= T(16) + T(16) + 32 = 80 + 80 + 32 = 192 \end{aligned}$$

What is the pattern here? Let's consider the ratios $T(n)/n$ for powers of 2:

$$\begin{aligned} T(1)/1 &= 1 & T(8)/8 &= 4 \\ T(2)/2 &= 2 & T(16)/16 &= 5 \\ T(4)/4 &= 3 & T(32)/32 &= 6 \end{aligned}$$

This suggests $T(n)/n = \log n + 1$ Or, $T(n) = n \log n + n$ which is $\Theta(n \log n)$ (using the limit rule).

The Iteration Method for Solving Recurrence Relations

Floor and ceilings are a pain to deal with. If n is assumed to be a power of 2 ($2^k = n$), this will simplify the recurrence to

$$T(n) = \begin{cases} 1 & \text{if } n = 1, \\ 2T(n/2) + n & \text{otherwise} \end{cases}$$

The iteration method turns the recurrence into a summation. Let's see how it works. Let's expand the recurrence:

$$\begin{aligned} T(n) &= 2T(n/2) + n \\ &= 2(2T(n/4) + n/2) + n \\ &= 4T(n/4) + n + n \\ &= 4(2T(n/8) + n/4) + n + n \\ &= 8T(n/8) + n + n + n \\ &= 8(2T(n/16) + n/8) + n + n + n \\ &= 16T(n/16) + n + n + n + n \end{aligned}$$

If n is a power of 2 then let $n = 2^k$ or $k = \log n$.

$$\begin{aligned} T(n) &= 2^k T(n/(2^k)) + \underbrace{(n + n + n + \cdots + n)}_{k \text{ times}} \\ &= 2^k T(n/(2^k)) + kn \\ &= 2^{(\log n)} T(n/(2^{(\log n)})) + (\log n)n \\ &= 2^{(\log n)} T(n/n) + (\log n)n \\ &= nT(1) + n \log n = n + n \log n \end{aligned}$$

Visualizing Recurrences Using the Recursion Tree

Iteration is a very powerful technique for solving recurrences. But, it is easy to get lost in all the symbolic manipulations and lose sight of what is going on. Here is a nice way to visualize what is going on in iteration. We can describe any recurrence in terms of a tree, where each expansion of the recurrence takes us one level deeper in the tree.

Recall that the recurrence for MergeSort (which we simplified by assuming that n is a power of 2, and hence could drop the floors and ceilings)

$$T(n) = \begin{cases} 1 & \text{if } n = 1, \\ 2T(n/2) + n & \text{otherwise} \end{cases}$$

Suppose that we draw the recursion tree for MergeSort, but each time we merge two lists, we label that node of the tree with the time it takes to perform the associated (nonrecursive) merge. Recall that to

merge two lists of size $m/2$ to a list of size m takes $\Theta(m)$ time, which we will just write as m. Below is an illustration of the resulting recursion tree.

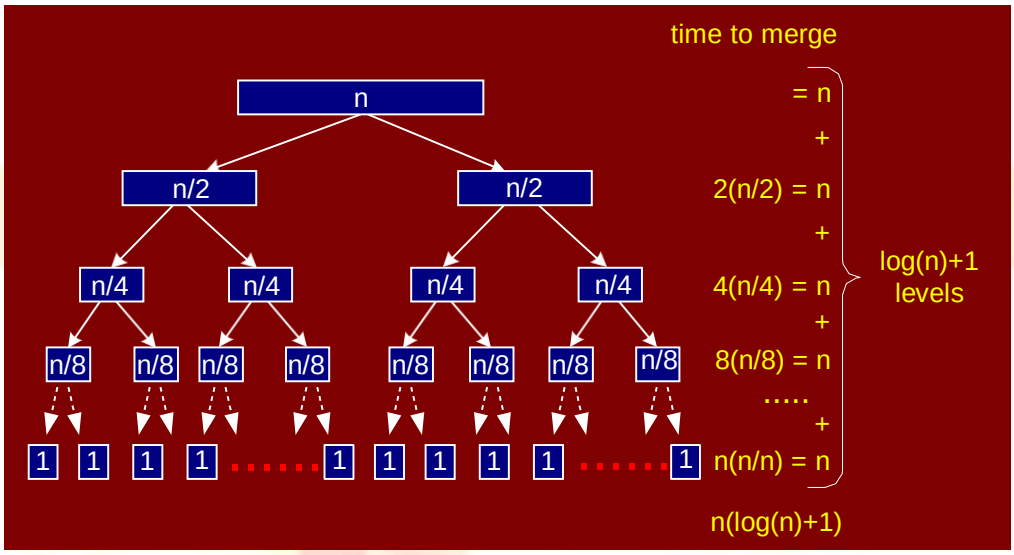

Figure 3.3: Merge sort Recurrence Tree

A Messier Example

The merge sort recurrence was too easy. Let's try a messier recurrence.

$$T(n) = \begin{cases} 1 & \text{if } n = 1, \\ 3T(n/4) + n & \text{otherwise} \end{cases}$$

Assume n to be a power of 4, i.e., $n = 4^k$ and $k = \log_4 n$

$$\begin{aligned} T(n) &= 3T(n/4) + n \\ &= 3(3T(n/16) + (n/4) + n \\ &= 9T(n/16) + 3(n/4) + n \\ &= 27T(n/64) + 9(n/16) + 3(n/4) + n \\ &= \ldots \\ &= 3^k T(\frac{n}{4^k}) + 3^{k-1}(n/4^{k-1}) \\ &\qquad + \cdots + 9(n/16) + 3(n/4) + n \\ &= 3^k T(\frac{n}{4^k}) + \sum_{i=0}^{k-1} \frac{3^i}{4^i} n \end{aligned}$$

With $n = 4^k$ and $T(1) = 1$

$$T(n) = 3^k T(\frac{n}{4^k}) + \sum_{i=0}^{k-1} \frac{3^i}{4^i} n$$

$$= 3^{\log_4 n} T(1) + \sum_{i=0}^{(\log_4 n)-1} \frac{3^i}{4^i} n$$

$$= n^{\log_4 3} + \sum_{i=0}^{(\log_4 n)-1} \frac{3^i}{4^i} n$$

We used the formula $a^{\log_b n} = n^{\log_b a}$. n remains constant throughout the sum and $3^i/4^i = (3/4)^i$; we thus have

$$T(n) = n^{\log_4 3} + n \sum_{i=0}^{(\log_4 n)-1} \left(\frac{3}{4}\right)^i$$

The sum is a geometric series; recall that for $x \neq 1$

$$\sum_{i=0}^{m} x_i = \frac{x^{m+1} - 1}{x - 1}$$

In this case $x = 3/4$ and $m = \log_4 n - 1$. We get

$$T(n) = n^{\log_4 3} + n \frac{(3/4)^{\log_4 n + 1} - 1}{(3/4) - 1}$$

Applying the log identity once more

$$(3/4)^{\log_4 n} = n^{\log_4 (3/4)} = n^{\log_4 3 - \log_4 4}$$

$$= n^{\log_4 3 - 1} = \frac{n^{\log_4 3}}{n}$$

If we plug this back, we get

$$T(n) = n^{\log_4 3} + n \frac{\frac{n^{\log_4 3}}{n} - 1}{(3/4) - 1}$$

$$= n^{\log_4 3} + \frac{n^{\log_4 3} - n}{-1/4}$$

$$= n^{\log_4 3} + 4(n - n^{\log_4 3})$$

$$= 4n - 3n^{\log_4 3}$$

With $\log_4 3 \approx 0.79$, we finally have the result!

$$T(n) = 4n - 3n^{\log_4 3} \approx 4n - 3n^{0.79} \in \Theta(n)$$

Selection Problem

Suppose we are given a set of n numbers. Define the *rank* of an element to be one plus the number of elements that are smaller. Thus, the rank of an element is its final position if the set is sorted. The minimum is of rank 1 and the maximum is of rank n.

Consider the set: $\{5, 7, 2, 10, 8, 15, 21, 37, 41\}$. The rank of each number is its position in the sorted order.

position	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*
Number	2	5	7	8	10	15	21	37	41

For example, rank of 8 is 4, one plus the number of elements smaller than 8 which is 3.

The selection problem is stated as follows:

> Given a set A of n distinct numbers and an integer k, $1 \leq k \leq n$, output the element of A of rank k.

Of particular interest in statistics is the *median*. If n is odd then the median is defined to be element of rank $(n + 1)/2$. When n is even, there are two choices: $n/2$ and $(n + 1)/2$. In statistics, it is common to return the average of the two elements.

Medians are useful as a measures of the *central tendency* of a set especially when the distribution of values is highly skewed. For example, the median income in a community is a more meaningful measure than average. Suppose 7 households have monthly incomes 5000, 7000, 2000, 10000, 8000, 15000 and 16000. In sorted order, the incomes are 2000, 5000, 7000, 8000, 10000, 15000, 16000. The median income is 8000; median is element with rank 4: $(7 + 1)/2 = 4$. The average income is 9000. Suppose the income 16000 goes up to 450,000. The median is still 8000 but the average goes up to 71,000. Clearly, the average is not a good representative of the majority income levels.

The selection problem can be easily solved by simply sorting the numbers of A and returning $A[k]$. Sorting, however, requires $\Theta(n \log n)$ time. The question is: can we do better than that? In particular, is it possible to solve the selections problem in $\Theta(n)$ time? The answer is yes. However, the solution is far from obvious.

Sieve Technique

The reason for introducing this algorithm is that it illustrates a very important special case of divide-and-conquer, which I call the *sieve technique*. We think of divide-and-conquer as breaking the problem into a small number of smaller subproblems, which are then solved recursively. The sieve technique is a special case, where the number of subproblems is just 1.

The sieve technique works in phases as follows. It applies to problems where we are interested in finding a single item from a larger set of n items. We do not know which item is of interest, however after doing some amount of analysis of the data, taking say $\Theta(nk)$ time, for some constant k, we find that we do not

know what the desired the item is, but we can identify a large enough number of elements that cannot be the desired value, and can be eliminated from further consideration. In particular "large enough" means that the number of items is at least some fixed constant fraction of n (e.g. $n/2$, $n/3$). Then we solve the problem recursively on whatever items remain. Each of the resulting recursive solutions then do the same thing, eliminating a constant fraction of the remaining set.

Applying the Sieve to Selection

To see more concretely how the sieve technique works, let us apply it to the selection problem. We will begin with the given array $A[1..n]$. We will pick an item from the array, called the *pivot element* which we will denote by x. We will talk about how an item is chosen as the pivot later; for now just think of it as a random element of A.

We then *partition* A into three parts.

1. $A[q]$ contains the pivot element x,
2. $A[1..q-1]$ will contain all the elements that are less than x and
3. $A[q+1..n]$ will contains all elements that are greater than x.

Within each sub array, the items may appear in any order. The following figure shows a partitioned array:

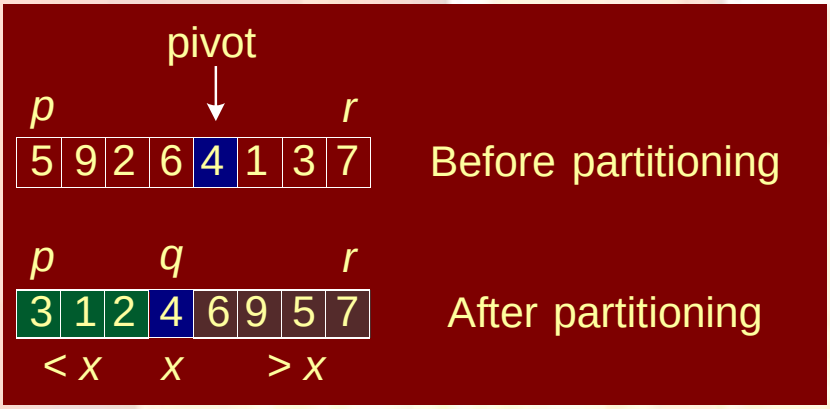

Figure 3.4: $A[p..r]$ partitioned about the pivot x

Selection Algorithm

It is easy to see that the rank of the The rank of the pivot x is $q-p+1$ in $A[p..r]$. Let $rank_x = q-p+1$. If $k = rank\ x$ then the pivot is k^{th} smallest. If $k < rank\ x$ then search $A[p..q-1]$ recursively. If $k > rank_x$ then search $A[q+1..r]$ recursively. Find element of rank $(k-q)$ because we eliminated q smaller elements in A.

SELECT(array A, int p, int r, int k)

```
1   if (p = r)
2     then return A[p]
3     else x ← CHOOSE_PIVOT(A, p, r)
4          q ← PARTITION(A, p, r, x)
5          rank_x ← q − p + 1
6          if k = rank_x
7            then return x
8          if k < rank_x
9            then return SELECT(A, p, q − 1, k)
10           else  return SELECT(A, q + 1, r, k − q)
```

Example: select the 6^{th} smallest element of the set {5, 9, 2, 6, 4, 1, 3, 7}

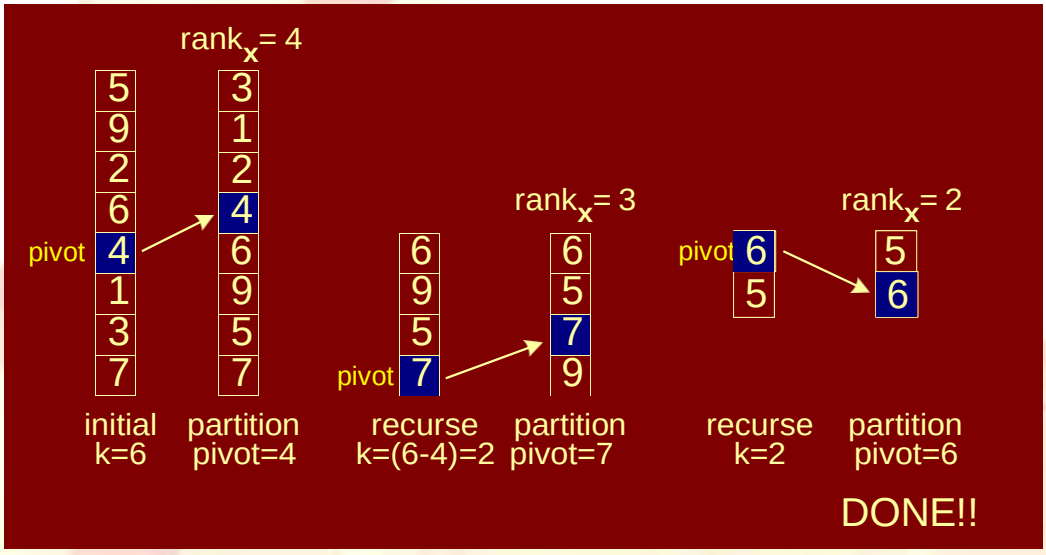

Figure 3.5: Sieve example: select 6^{th} smallest element

Analysis of Selection

We will discuss how to choose a pivot and the partitioning later. For the moment, we will assume that they both take $\Theta(n)$ time. How many elements do we eliminate in each time? If x is the largest or the smallest then we may only succeed in eliminating one element.

$5, 9, 2, 6, 4, \boxed{1}, 3, 7$ pivot is 1

$\boxed{1}, 5, 9, 2, 6, 4, 3, 7$ after partition

Ideally, x should have a rank that is neither too large or too small.

Suppose we are able to choose a pivot that causes exactly half of the array to be eliminated in each phase.

This means that we recurse on the remaining $n/2$ elements. This leads to the following recurrence:

$$T(n) = \begin{cases} 1 & \text{if } n = 1, \\ T(n/2) + n & \text{otherwise} \end{cases}$$

If we expand this recurrence, we get

$$\begin{aligned} T(n) &= n + \frac{n}{2} + \frac{n}{4} + \ldots \\ &\leq \sum_{i=0} \frac{n}{2^i} \\ &= n \sum_{i=0} \frac{1}{2^i} \end{aligned}$$

Recall the formula for infinite geometric series; for any $|c| < 1$,

$$\sum_{i=0} c^i = \frac{1}{1-c}$$

Using this we have

$$T(n) \leq 2n \in \Theta(n)$$

Let's think about how we ended up with a $\Theta(n)$ algorithm for selection. Normally, a $\Theta(n)$ time algorithm would make a single or perhaps a constant number of passes of the data set. In this algorithm we make a number of passes. In fact it could be as many as $\log n$.

However, because we eliminate a constant fraction of the array with each phase, we get the convergent geometric series in the analysis. This shows that the total running time is indeed *linear* in n. This lesson is well worth remembering. It is often possible to achieve linear running times in ways that you would not expect.

Chapter 4

Sorting

For the next series of lectures, we will focus on sorting. There a number of reasons for sorting. Here are a few important ones. Procedures for sorting are parts of many large software systems. Design of efficient sorting algorithms is necessary to achieve overall efficiency of these systems.

Sorting is well studied problem from the analysis point of view. Sorting is one of the few problems where provable lower bounds exist on how fast we can sort. In sorting, we are given an array $A[1..n]$ of n numbers We are to reorder these elements into increasing (or decreasing) order.

More generally, A is an array of objects and we sort them based on one of the attributes - the *key value*. The key value need not be a number. It can be any object from a *totally ordered domain*. Totally ordered domain means that for any two elements of the domain, x and y, either $x < y$, $x = y$ or $x > y$.

Slow Sorting Algorithms

There are a number of well-known slow $O(n^2)$ sorting algorithms. These include the following:

Bubble sort: Scan the array. Whenever two consecutive items are found that are out of order, swap them. Repeat until all consecutive items are in order.

Insertion sort: Assume that $A[1..i-1]$ have already been sorted. Insert $A[i]$ into its proper position in this sub array. Create this position by shifting all larger elements to the right.

Selection sort: Assume that $A[1..i-1]$ contain the $i-1$ smallest elements in sorted order. Find the smallest element in $A[i..n]$ Swap it with $A[i]$.

These algorithms are easy to implement. But they run in $\Theta(n^2)$ time in the worst case.

Sorting in $O(n \log n)$ time

We have already seen that Mergesort sorts an array of numbers in $\Theta(n \log n)$ time. We will study two others: *Heapsort* and *Quicksort*.

Heaps

A *heap* is a left-complete binary tree that conforms to the *heap order*. The heap order property: in a (min) heap, for every node X, the key in the parent is smaller than or equal to the key in X. In other words, the parent node has key smaller than or equal to both of its children nodes. Similarly, in a max heap, the parent has a key larger than or equal both of its children Thus the smallest key is in the root in a min heap; in the max heap, the largest is in the root.

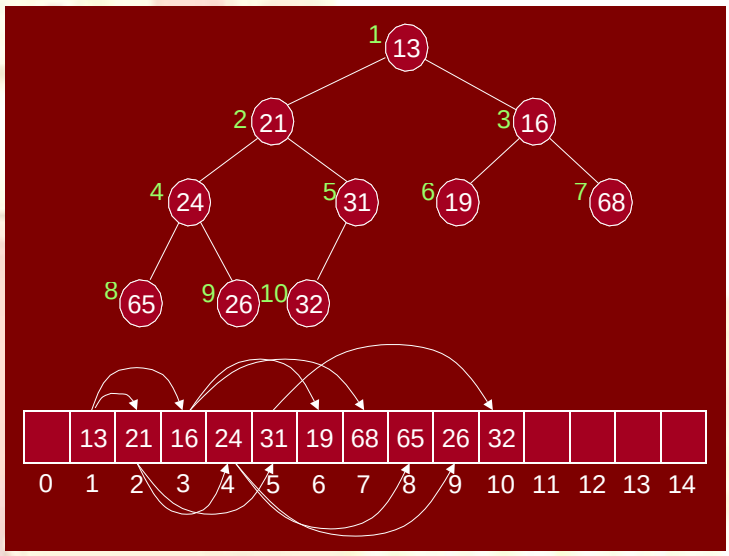

Figure 4.1: A min-heap

The number of nodes in a complete binary tree of height h is

$$n = 2^0 + 2^1 + 2^2 + \cdots + 2^h = \sum_{i=0} 2^i = 2^{h+1} - 1$$

h in terms of n is

$$h = (\log(n+1)) - 1 \approx \log n \in \Theta(\log n)$$

One of the clever aspects of heaps is that they can be stored in arrays without using any pointers. This is due to the left-complete nature of the binary tree. We store the tree nodes in level-order traversal. Access

to nodes involves simple arithmetic operations:

left(i) : returns 2i, index of left child of node i.
right(i) : returns 2i + 1, the right child.
parent(i) : returns $\lfloor i/2 \rfloor$, the parent of i.

The root is at position 1 of the array.

Heapsort Algorithm

We build a max heap out of the given array of numbers A[1..n]. We repeatedly extract the the maximum item from the heap. Once the max item is removed, we are left with a hole at the root. To fix this, we will replace it with the last leaf in tree. But now the heap order will very likely be destroyed. We will apply a heapify procedure to the root to restore the heap. Figure 4.2 shows an array being sorted.

```
HEAPSORT( array A, int n)
1   BUILD-HEAP(A, n)
2   m ← n
3   while (m ≥ 2)
4   do SWAP(A[1], A[m])
5       m ← m – 1
6       HEAPIFY(A, 1, m)
```

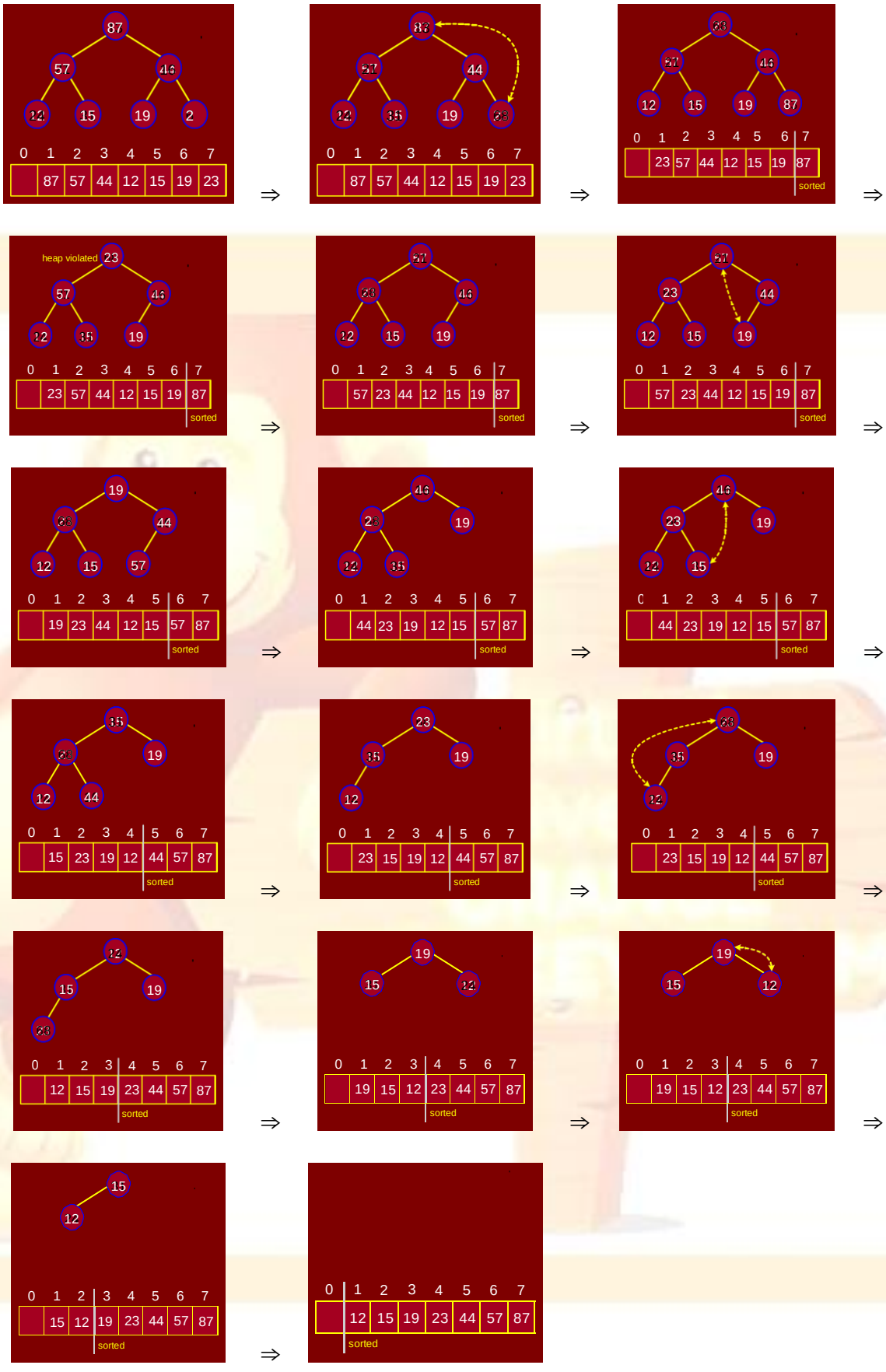

Figure 4.2: Example of heap sort

Heapify Procedure

There is one principal operation for maintaining the heap property. It is called Heapify. (In other books it is sometimes called sifting down.) The idea is that we are given an element of the heap which we suspect may not be in valid heap order, but we assume that all of other the elements in the subtree rooted at this element are in heap order. In particular this root element may be too small. To fix this we "sift" it down the tree by swapping it with one of its children. Which child? We should take the larger of the two children to satisfy the heap ordering property. This continues recursively until the element is either larger than both its children or until its falls all the way to the leaf level. Here is the algorithm. It is given the heap in the array A, and the index i of the suspected element, and m the current active size of the heap. The element A[max] is set to the maximum of A[i] and it two children. If max = i then we swap A[i] and A[max] and then recurse on A[max].

```
HEAPIFY( array A, int i, int m)
 1  l ← LEFT(i)
 2  r ← RIGHT(i)
 3  max ← i
 4  if (l ≤ m)and(A[l] > A[max])
 5    then max ← l
 6  if (r ≤ m)and(A[r] > A[max])
 7    then max ← r
 8  if (max /= i)
 9    then SWAP(A[i], A[max])
10         HEAPIFY(A, max, m)
```

Analysis of Heapify

We call heapify on the root of the tree. The maximum levels an element could move up is $\Theta(\log n)$ levels. At each level, we do simple comparison which $O(1)$. The total time for heapify is thus $O(\log n)$. Notice that it is not $\Theta(\log n)$ since, for example, if we call heapify on a leaf, it will terminate in $\Theta(1)$ time.

BuildHeap

We can use Heapify to build a heap as follows. First we start with a heap in which the elements are not in heap order. They are just in the same order that they were given to us in the array A. We build the heap by starting at the leaf level and then invoke Heapify on each node. (Note: We cannot start at the top of the tree. Why not? Because the precondition which Heapify assumes is that the entire tree rooted at node i is already in heap order, except for i.) Actually, we can be a bit more efficient. Since we know that each leaf is already in heap order, we may as well skip the leaves and start with the first non-leaf node. This will be in position $n/2$. (Can you see why?)

Here is the code. Since we will work with the entire array, the parameter m for Heapify, which indicates the current heap size will be equal to n, the size of array A, in all the calls.

```
BUILDHEAP( array A, int n)
1  for i ← n/2 downto 1
2  do
3      HEAPIFY(A, i, n)
```

Analysis of BuildHeap

For convenience, we will assume $n = 2^{h+1} - 1$ where h is the height of tree. The heap is a left-complete binary tree. Thus at each level j, $j < h$, there are 2^j nodes in the tree. At level h, there will be 2^h or less nodes. How much work does buildHeap carry out? Consider the heap in Figure 4.3:

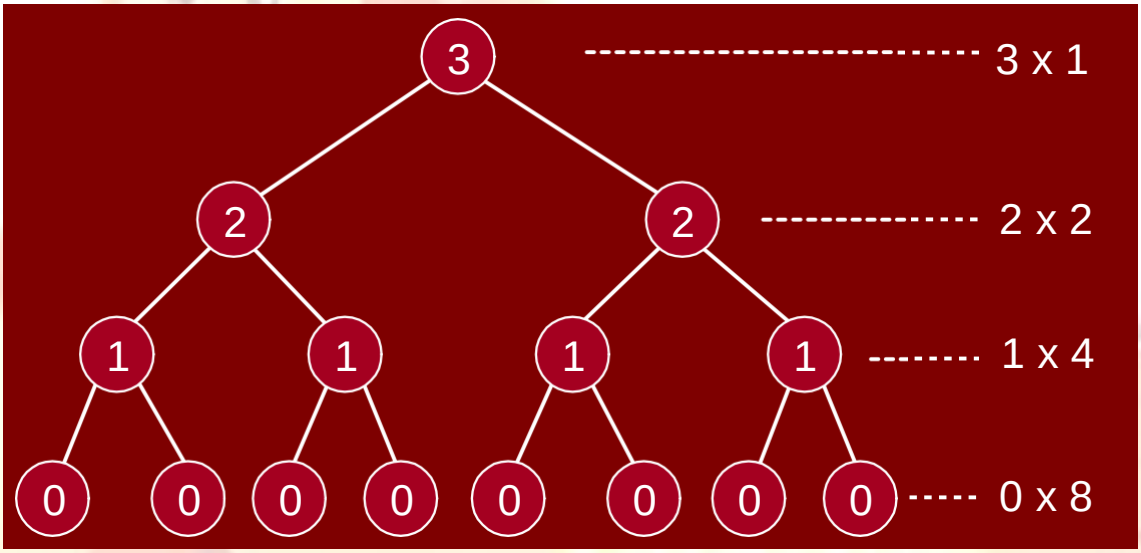

Figure 4.3: Total work performed in buildheap

At the bottom most level, there are 2^h nodes but we do not heapify these. At the next level up, there are 2^{h-1} nodes and each might shift down 1. In general, at level j, there are 2^{h-j} nodes and each may shift down j levels.

So, if count from bottom to top, level-by-level, the total time is

$$T(n) = \sum_{j=0}^{h} j2^{h-j} = \sum_{j=0}^{h} j\frac{2^h}{2^j}$$

We can factor out the 2^h term:

$$T(n) = 2^h \sum_{j=0}^{h} \frac{j}{2^j}$$

How do we solve this sum? Recall the geometric series, for any constant $x < 1$

$$\sum_{j=0}^{\infty} x^j = \frac{1}{1-x}$$

Take the derivative with respect to x and multiply by x

$$\sum_{j=0}^{\infty} jx^{j-1} = \frac{1}{(1-x)^2} \qquad \sum_{j=0}^{\infty} jx^j = \frac{x}{(1-x)^2}$$

We plug $x = 1/2$ and we have the desired formula:

$$\sum_{j=0}^{\infty} \frac{j}{2^j} = \frac{1/2}{(1-(1/2))^2} = \frac{1/2}{1/4} = 2$$

In our case, we have a bounded sum, but since we the infinite series is bounded, we can use it as an easy approximation:

$$\begin{aligned} T(n) &= 2^h \sum_{j=0}^{h} \frac{j}{2^j} \\ &\leq 2 \sum_{j=0}^{h} \frac{j}{2^j} \\ &\leq 2^h \cdot 2 = 2^{h+1} \end{aligned}$$

Recall that $n = 2^{h+1} - 1$. Therefore

$$T(n) \leq n + 1 \in O(n)$$

The algorithm takes at least $\Omega(n)$ time since it must access every element at once. So the total time for BuildHeap is $\Theta(n)$.

BuildHeap is a relatively complex algorithm. Yet, the analysis yield that it takes $\Theta(n)$ time. An intuitive way to describe why it is so is to observe an important fact about binary trees The fact is that the vast majority of the nodes are at the lowest level of the tree. For example, in a complete binary tree of height h, there is a total of $n \approx 2^{h+1}$ nodes.

The number of nodes at the bottom three levels alone is

$$2^h + 2^{h-1} + 2^{h-2} = \frac{n}{2} + \frac{n}{4} + \frac{n}{8} = \frac{7n}{8} = 0.875n$$

Almost 90% of the nodes of a complete binary tree reside in the 3 lowest levels. Thus, algorithms that operate on trees should be efficient (as BuildHeap is) on the bottom-most levels since that is where most of the weight of the tree resides.

Analysis of Heapsort

Heapsort calls BuildHeap once. This takes $\Theta(n)$. Heapsort then extracts roughly n maximum elements from the heap. Each extract requires a constant amount of work (swap) and $O(\log n)$ heapify. Heapsort is thus $O(n \log n)$.

Is HeapSort $\Theta(n \log n)$? The answer is yes. In fact, later we will show that comparison based sorting algorithms can not run faster than $\Omega(n \log n)$. Heapsort is such an algorithm and so is Mergesort that we saw ealier.

Quicksort

Our next sorting algorithm is Quicksort. It is one of the fastest sorting algorithms known and is the method of choice in most sorting libraries. Quicksort is based on the divide and conquer strategy. Here is the algorithm:

```
QUICKSORT( array A, int p, int r)
1  if (r > p)
2     then
3          i ← a random index from [p..r]
4          swap A[i] with A[p]
5          q ← PARTITION(A, p, r)
6          QUICKSORT(A, p, q − 1)
7          QUICKSORT(A, q + 1, r)
```

Partition Algorithm

Recall that the partition algorithm partitions the array $A[p..r]$ into three sub arrays about a pivot element x. $A[p..q-1]$ whose elements are less than or equal to x, $A[q] = x$, $A[q+1..r]$ whose elements are greater than x

We will choose the first element of the array as the pivot, i.e. $x = A[p]$. If a different rule is used for selecting the pivot, we can swap the chosen element with the first element. We will choose the pivot randomly.

The algorithm works by maintaining the following *invariant condition*. $A[p] = x$ is the pivot value. $A[p..q-1]$ contains elements that are less than x, $A[q+1..s-1]$ contains elements that are greater than

or equal to x A[s..r] contains elements whose values are currently unknown.

```
PARTITION( array  A, int  p, int  r)
1   x ← A[p]
2   q ← p
3   for s ← p + 1  to r
4   do if (A[s] < x)
5         then q ← q + 1
6                 swap  A[q] with A[s]
7    swap  A[p] with A[q]
8   return q
```

Figure 4.4 shows the execution trace partition algorithm.

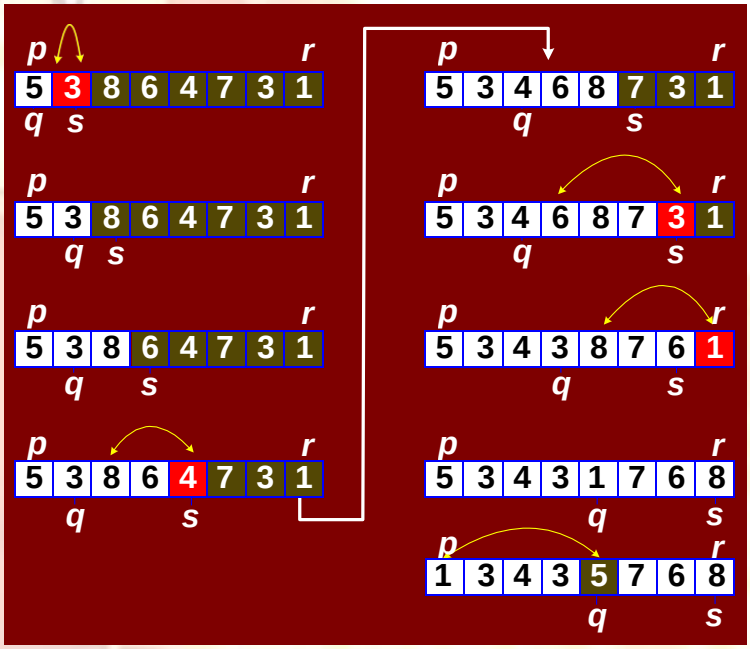

Figure 4.4: Trace of partitioning algorithm

Quick Sort Example

The Figure 4.5 trace out the quick sort algorithm. The first partition is done using the last element, 10, of the array. The left portion are then partitioned about 5 while the right portion is partitioned about 13. Notice that 10 is now at its final position in the eventual sorted order. The process repeats as the algorithm recursively partitions the array eventually sorting it.

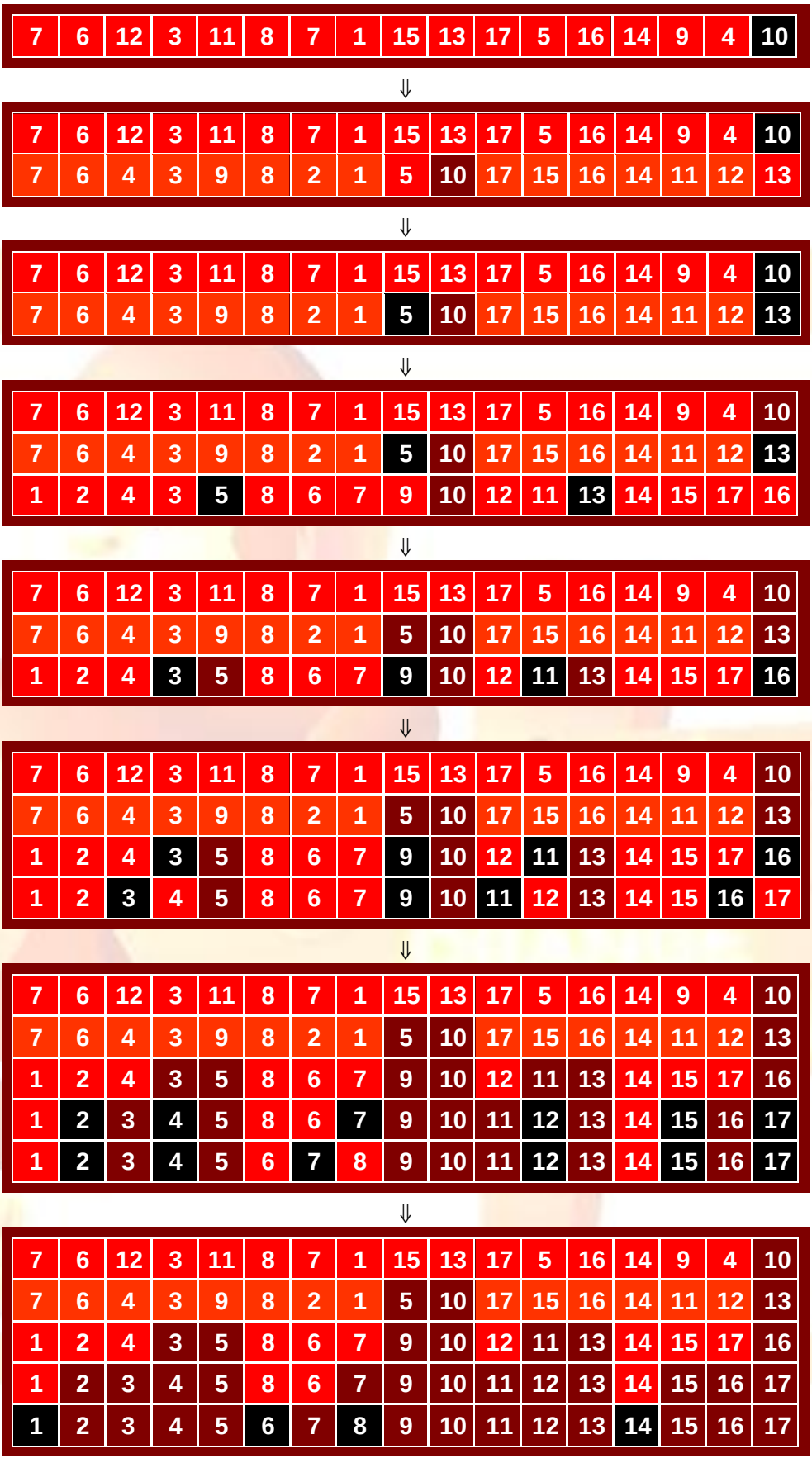

Figure 4.5: Example of quick sort

It is interesting to note (but not surprising) that the pivots form a binary search tree. This is illustrated in Figure 4.6.

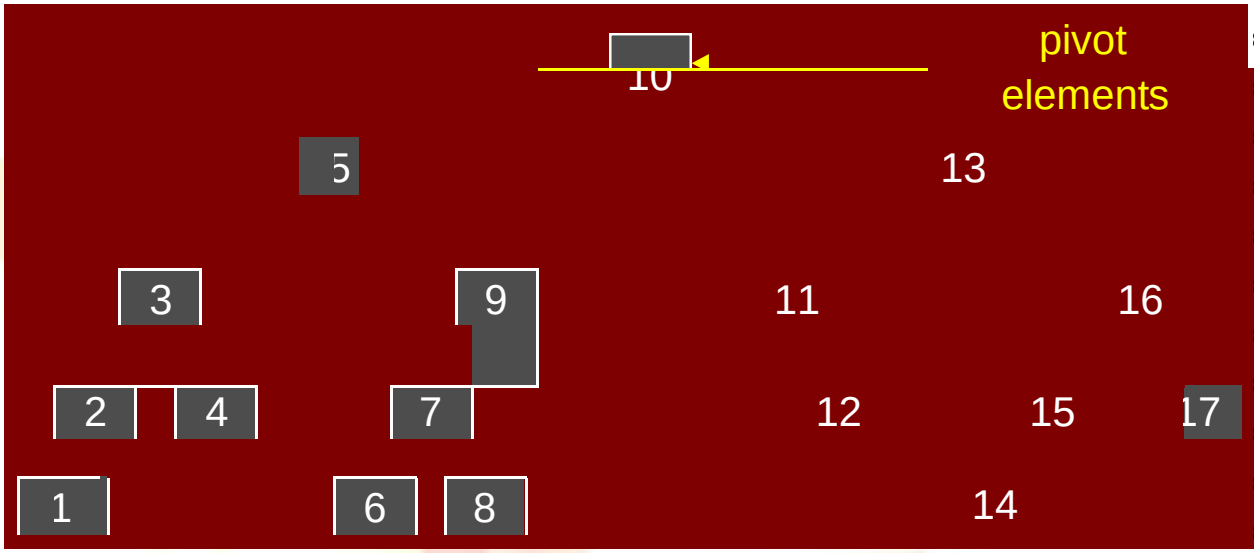

Figure 4.6: Quicksort BST

Analysis of Quicksort

The running time of quicksort depends heavily on the selection of the pivot. If the rank of the pivot is very large or very small then the partition (BST) will be unbalanced. Since the pivot is chosen randomly in our algorithm, the expected running time is $O(n \log n)$. The worst case time, however, is $O(n^2)$. Luckily, this happens rarely.

Worst Case Analysis of Quick Sort

Let's begin by considering the worst-case performance, because it is easier than the average case. Since this is a recursive program, it is natural to use a recurrence to describe its running time. But unlike MergeSort, where we had control over the sizes of the recursive calls, here we do not. It depends on how the pivot is chosen. Suppose that we are sorting an array of size n, $A[1 : n]$, and further suppose that the pivot that we select is of rank q, for some q in the range 1 to n. It takes $\Theta(n)$ time to do the partitioning and other overhead, and we make two recursive calls. The first is to the subarray $A[1 : q - 1]$ which has $q - 1$ elements, and the other is to the subarray $A[q + 1 : n]$ which has $n - q$ elements. So if we ignore the $\Theta(n)$ (as usual) we get the recurrence:

$$T(n) = T(q - 1) + T(n - q) + n$$

This depends on the value of q. To get the worst case, we maximize over all possible values of q. Putting is together, we get the recurrence

$$T(n) = \begin{cases} 1 & \text{if } n \leq 1 \\ \max_{1 \leq q \leq n} (T(q-1) + T(n-q) + n) & \text{otherwise} \end{cases}$$

Recurrences that have max's and min's embedded in them are very messy to solve. The key is determining which value of q gives the maximum. (A rule of thumb of algorithm analysis is that the worst cases tends to happen either at the extremes or in the middle. So I would plug in the value $q = 1$, $q = n$, and $q = n/2$ and work each out.) In this case, the worst case happens at either of the extremes. If we expand the recurrence for $q = 1$, we get:

$$\begin{aligned} T(n) &\leq T(0) + T(n-1) + n \\ &= 1 + T(n-1) + n \\ &= T(n-1) + (n+1) \\ &= T(n-2) + n + (n+1) \\ &= T(n-3) + (n-1) + n + (n+1) \\ &= T(n-4) + (n-2) + (n-1) + n + (n+1) \\ &= T(n-k) + \sum_{i=-1}^{k-2} (n-i) \end{aligned}$$

For the basis $T(1) = 1$ we set $k = n - 1$ and get

$$\begin{aligned} T(n) &\leq T(1) + \sum_{i=-1}^{n-3} (n-i) \\ &= 1 + (3 + 4 + 5 + \cdots + (n-1) + n + (n+1)) \\ &\leq \sum_{i=1}^{n+1} i = \frac{(n+1)(n+2)}{2} \in \Theta(n^2) \end{aligned}$$

Average-case Analysis of Quicksort

We will now show that in the average case, quicksort runs in $\Theta(n \log n)$ time. Recall that when we talked about average case at the beginning of the semester, we said that it depends on some assumption about the distribution of inputs. However, in the case of quicksort, the analysis does not depend on the distribution of input at all. It only depends upon the random choices of pivots that the algorithm makes. This is good, because it means that the analysis of the algorithm's performance is the same for all inputs. In this case the average is computed over all possible random choices that the algorithm might make for the choice of the pivot index in the second step of the QuickSort procedure above.

To analyze the average running time, we let $T(n)$ denote the average running time of QuickSort on a list of size n. It will simplify the analysis to assume that all of the elements are distinct. The algorithm has n

random choices for the pivot element, and each choice has an equal probability of $1/n$ of occuring. So we can modify the above recurrence to compute an average rather than a max, giving:

$$T(n) = \begin{cases} 1 & \text{if } n \le 1 \\ \frac{1}{n} \sum_{q=1}^{n} (T(q-1) + T(n-q) + n) & \text{otherwise} \end{cases}$$

The time $T(n)$ is the weighted sum of the times taken for various choices of q. I.e.,

$$\begin{aligned} T(n) = & \frac{1}{n}(T(0) + T(n-1) + n) \\ & + \frac{1}{n}(T(1) + T(n-2) + n) \\ & + \frac{1}{n}(T(2) + T(n-3) + n) \\ & + \cdots + \frac{1}{n}(T(n-1) + T(0) + n) \end{aligned}$$

We have not seen such a recurrence before. To solve it, expansion is possible but it is rather tricky. We will attempt a constructive induction to solve it. We know that we want a $\Theta(n \log n)$. Let us assume that $T(n) \le cn \log n$ for $n \ge 2$ where c is a constant.

For the base case $n = 2$ we have

$$\begin{aligned} T(n) &= \frac{1}{2} \sum_{q=1}^{2} (T(q-1) + T(2-q) + 2) \\ &= \frac{1}{2}(T(0) + T(1) + 2) + (T(1) + T(0) + 2) \\ &= \frac{8}{2} = 4. \end{aligned}$$

We want this to be at most $c2 \log 2$, i.e.,

$$T(2) \le c2 \log 2$$

or

$$4 \le c2 \log 2$$

therefore

$$c \ge 4/(2 \log 2) \approx 2.88.$$

For the induction step, we assume that $n \ge 3$ and The induction hypothesis is that for any $n' < n$, we have $T(n') \le cn' \log n'$. We want to prove that it is true for $T(n)$. By expanding $T(n)$ and moving the

factor of n outside the sum we have

$$T(n) = \frac{1}{n}\sum_{q=1}^{n}(T(q-1) + T(n-q) + n)$$

$$= \frac{1}{n}\sum_{q=1}^{n}(T(q-1) + T(n-q)) + n$$

$$= \frac{1}{n}\sum_{q=1}^{n} T(q-1) + \frac{1}{n}\sum_{q=1}^{n} T(n-q) + n$$

$$T(n) = \frac{1}{n}\sum_{q=1}^{n} T(q-1) + \frac{1}{n}\sum_{q=1}^{n} T(n-q) + n$$

Observe that the two sums add up the same values $T(0) + T(1) + \ldots + T(n-1)$. One counts up and other counts down. Thus we can replace them with $2\sum_{q=0}^{n-1} T(q)$. We will extract $T(0)$ and $T(1)$ and treat them specially. These two do not follow the formula.

$$T(n) = \frac{2}{n}\sum_{q=0}^{n-1} T(q) + n$$

$$= \frac{2}{n}T(0) + T(1) + \sum_{q=2}^{n-1} T(q) + n$$

We will apply the induction hypothesis for $q < n$ we have

$$T(n) = \frac{2}{n}T(0) + T(1) + \sum_{q=2}^{n-1} T(q) + n$$

$$\leq \frac{2}{n}\left[1 + 1 + \sum_{q=2}^{n-1}(cq\log q)\right] + n$$

$$= \frac{2c}{n}\sum_{q=2}^{n-1}(cq\ln q) + n + \frac{4}{n}$$

We have never seen this sum before:

$$S(n) = \sum_{q=2}^{n-1}(cq\ln q)$$

Recall from calculus that for any monotonically increasing function $f(x)$

$$\sum_{i=a}^{b-1} f(i) \leq \int_{a}^{b} f(x)dx$$

The function $f(x) = x\ln x$ is monotonically increasing, and so

$$S(n) = \sum_{q=2}^{n-1} (cq\ln q) \leq \int_2^n x\ln x\,dx \tag{4.1}$$

We can integrate this by parts:

$$\int_2^n x\ln x\,dx = \frac{x^2}{2}\ln x - \frac{x^2}{4}\Big|_{x=2}^{n}$$

$$\int_2^n x\ln x\,dx = \frac{x^2}{2}\ln x - \frac{x^2}{4}\Big|_{x=2}^{n}$$

$$= \frac{n^2}{2}\ln n - \frac{n^2}{4} - (2\ln 2 - 1)$$

$$\leq \frac{n^2}{2}\ln n - \frac{n^2}{4}$$

We thus have

$$S(n) = \sum_{q=2}^{n-1} (cq\ln q) \leq \frac{n^2}{2}\ln n - \frac{n^2}{4}$$

Plug this back into the expression for $T(n)$ to get

$$T(n) = \frac{2c}{n}\left(\frac{n^2}{2}\ln n - \frac{n^2}{4}\right) + n + \frac{4}{n}$$

$$T(n) = \frac{2c}{n}\left(\frac{n^2}{2}\ln n - \frac{n^2}{4}\right) + n + \frac{4}{n}$$

$$= cn\ln n - \frac{cn}{2} + n + \frac{4}{n}$$

$$= cn\ln n + n\left(1 - \frac{c}{2}\right) + \frac{4}{n}$$

$$T(n) = cn\ln n + n\left(1 - \frac{c}{2}\right) + \frac{4}{n}$$

To finish the proof,we want all of this to be at most $cn\ln n$. For this to happen, we will need to select c such that

$$n\left(1 - \frac{c}{2}\right) + \frac{4}{n} \leq 0$$

If we select $c = 3$, and use the fact that $n \geq 3$ we get

$$n\left(1 - \frac{c}{2}\right) + \frac{4}{n} = \frac{3}{n} - \frac{n}{2}$$

$$\leq 1 - \frac{n}{2} = -\frac{n-2}{2} \leq 0.$$

From the basis case we had $c \geq 2.88$. Choosing $c = 3$ satisfies all the constraints. Thus $T(n) = 3n\ln n \in \Theta(n \log n)$.

In-place, Stable Sorting

An *in-place* sorting algorithm is one that uses no additional array for storage. A sorting algorithm is *stable* if duplicate elements remain in the same relative position after sorting.

$9 \cdot 3 \cdot 3_I \cdot 5 \cdot 6 \cdot 5_I \cdot 2 \cdot 1 \cdot 3_{II}$	unsorted
$1 \cdot 2 \cdot 3 \cdot 3_I \cdot 3_{II} \cdot 5 \cdot 5_I \cdot 6 \cdot 9$	stable sort
$1 \cdot 2 \cdot 3_I \cdot 3 \cdot 3_{II} \cdot 5_I \cdot 5 \cdot 6 \cdot 9$	unstable

Bubble sort, insertion sort and selection sort are in-place sorting algorithms. Bubble sort and insertion sort can be implemented as stable algorithms but selection sort cannot (without significant modifications). Mergesort is a stable algorithm but not an in-place algorithm. It requires extra array storage. Quicksort is not stable but is an in-place algorithm. Heapsort is an in-place algorithm but is not stable.

Lower Bounds for Sorting

The best we have seen so far is $O(n \log n)$ algorithms for sorting. Is it possible to do better than $O(n \log n)$? If a sorting algorithm is solely based on comparison of keys in the array then it is *impossible* to sort more efficiently than $\Omega(n \log n)$ time. All algorithms we have seen so far are comparison-based sorting algorithms.

Consider sorting three numbers a_1, a_2, a_3. There are $3! = 6$ possible combinations:

$$(a_1, a_2, a_3),\ (a_1, a_3, a_2)\ ,(a_3, a_2, a_1)$$
$$(a_3, a_1, a_2),\ (a_2, a_1, a_3)\ ,(a_2, a_3, a_1)$$

One of these permutations leads to the numbers in sorted order.

The comparison based algorithm defines a *decision tree*. Here is the tree for the three numbers.

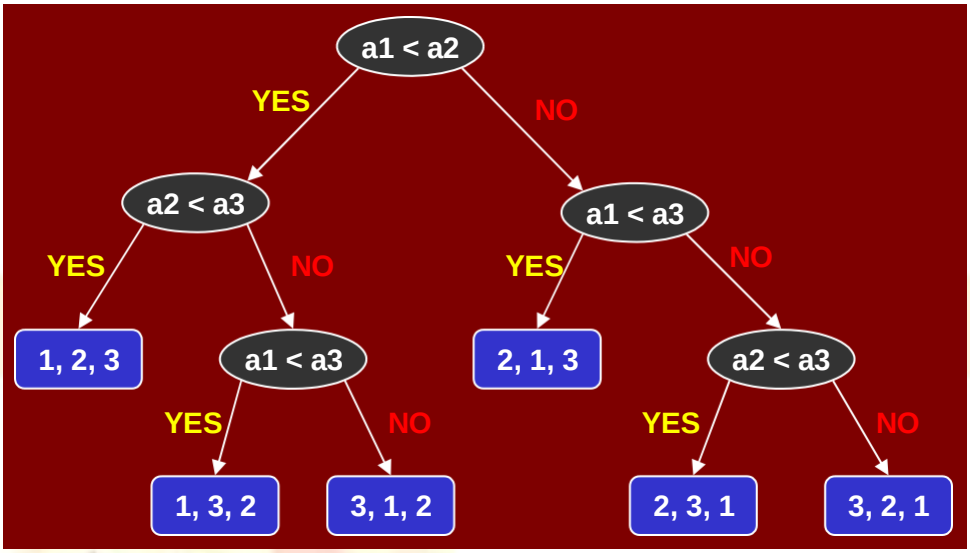

Figure 4.7: Decision Tree

For n elements, there will be $n!$ possible permutations. The height of the tree is exactly equal to $T(n)$, the running time of the algorithm. The height is $T(n)$ because any path from the root to a leaf corresponds to a sequence of comparisons made by the algorithm.

Any binary tree of height $T(n)$ has at most $2^{T(n)}$ leaves. Thus a comparison based sorting algorithm can distinguish between at most $2^{T(n)}$ different final outcomes. So we have

$$2^{T(n)} \geq n! \quad \text{and therefore}$$
$$T(n) \geq \log(n!)$$

We can use *Stirling's approximation* for $n!$:

$$n! \geq \sqrt{2\pi n}\left(\frac{n}{e}\right)^n$$

Thereofore

$$\begin{aligned} T(n) &\geq \log \sqrt{2\pi n}\left(\frac{n}{e}\right)^n \\ &= \log(\sqrt{2\pi n}) + n \log n - n \log e \\ &\in \Omega(n \log n) \end{aligned}$$

We thus have the following theorem.

Theorem 1

Any comparison-based sorting algorithm has worst-case running time $\Omega(n \log n)$.

Chapter 5

Linear Time Sorting

The lower bound implies that if we hope to sort numbers faster than $O(n \log n)$, we cannot do it by making comparisons alone. Is it possible to sort without making comparisons? The answer is yes, but only under very restrictive circumstances. Many applications involve sorting small integers (e.g. sorting characters, exam scores, etc.). We present three algorithms based on the theme of speeding up sorting in special cases, by not making comparisons.

Counting Sort

We will consider three algorithms that are faster and work by not making comparisons. Counting sort assumes that the numbers to be sorted are in the range 1 to k where k is small. The basic idea is to determine the rank of each number in final sorted array.

Recall that the rank of an item is the number of elements that are less than or equal to it. Once we know the ranks, we simply copy numbers to their final position in an output array.

The question is how to find the rank of an element without comparing it to the other elements of the array?. The algorithm uses three arrays. As usual, $A[1..n]$ holds the initial input, $B[1..n]$ holds the sorted output and $C[1..k]$ is an array of integers. $C[x]$ is the rank of x in A, where $x \in [1..k]$. The algorithm is remarkably simple, but deceptively clever. The algorithm operates by first constructing C. This is done in two steps. First we set $C[x]$ to be the number of elements of $A[j]$ that are equal to x. We can do this initializing C to zero, and then for each j, from 1 to n, we increment $C[A[j]]$ by 1. Thus, if $A[j] = 5$, then the 5th element of C is incremented, indicating that we have seen one more 5. To determine the number of elements that are less than or equal to x, we replace $C[x]$ with the sum of elements in the sub array $R[1 : x]$. This is done by just keeping a running total of the elements of C.

$C[x]$ now contains the rank of x. This means that if $x = A[j]$ then the final position of $A[j]$ should be at position $C[x]$ in the final sorted array. Thus, we set $B[C[x]] = A[j]$. Notice We need to be careful if there are duplicates, since we do not want them to overwrite the same location of B. To do this, we decrement

C[i] after copying.

```
COUNTING-SORT( array A, array B, int k)
 1  for i ← 1 to k
 2  do C[i] ← 0          k times
 3  for j ← 1 to length[A]
 4  do C[A[j]] ← C[A[j]] + 1          n times
 5  // C[i] now contains the number of elements = i
 6  for i ← 2 to k
 7  do C[i] ← C[i] + C[i − 1]          k times
 8  // C[i] now contains the number of elements ≤ i
 9  for j ← length[A] downto 1
10  do B[C[A[j]]] ← A[j]
11     C[A[j]] ← C[A[j]] − 1          n times
```

There are four (unnested) loops, executed k times, n times, k − 1 times, and n times, respectively, so the total running time is Θ(n + k) time. If k = O(n), then the total running time is Θ(n).

Figure 5.1 through 5.19 shows an example of the algorithm. You should trace through the example to convince yourself how it works.

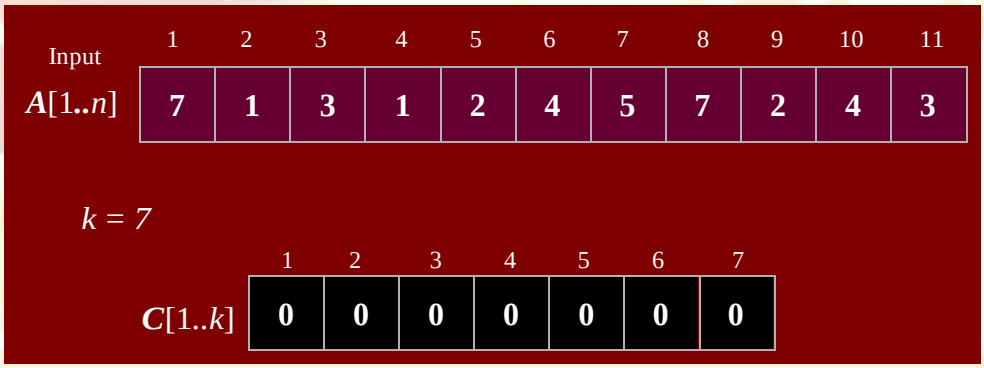

Figure 5.1: Initial A and C arrays.

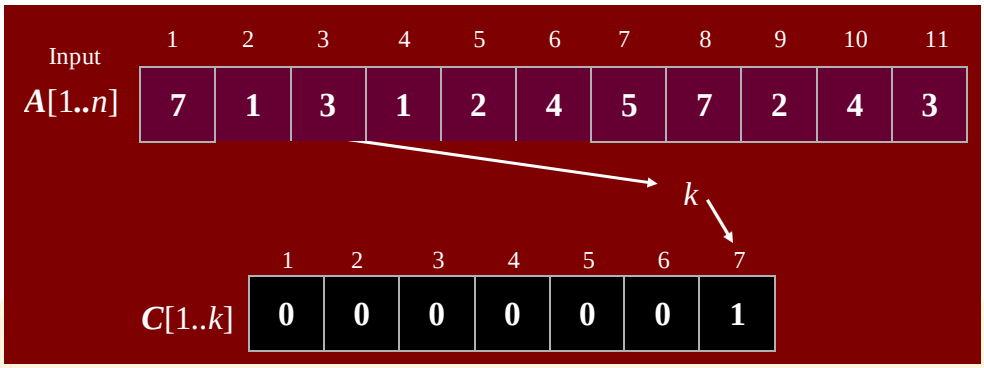

Figure 5.2: A[1] = 7 processed

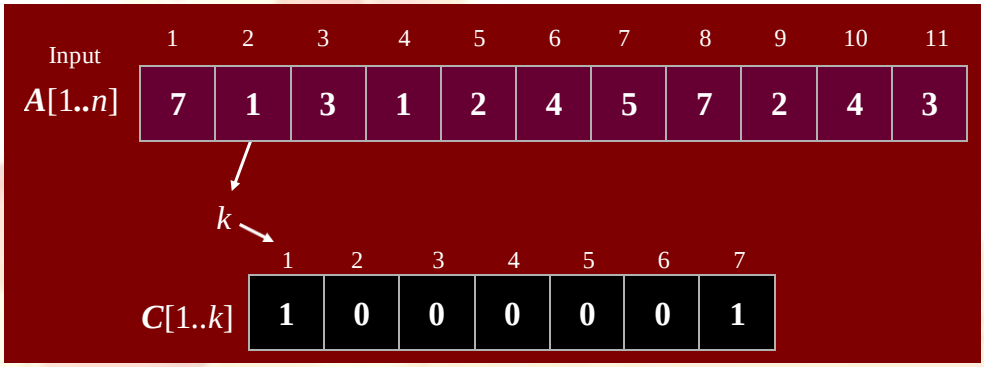

Figure 5.3: A[2] = 1 processed

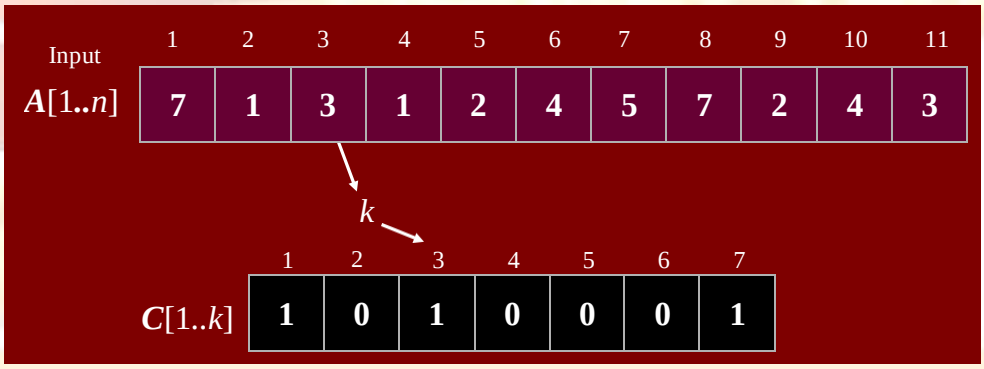

Figure 5.4: A[3] = 3 processed

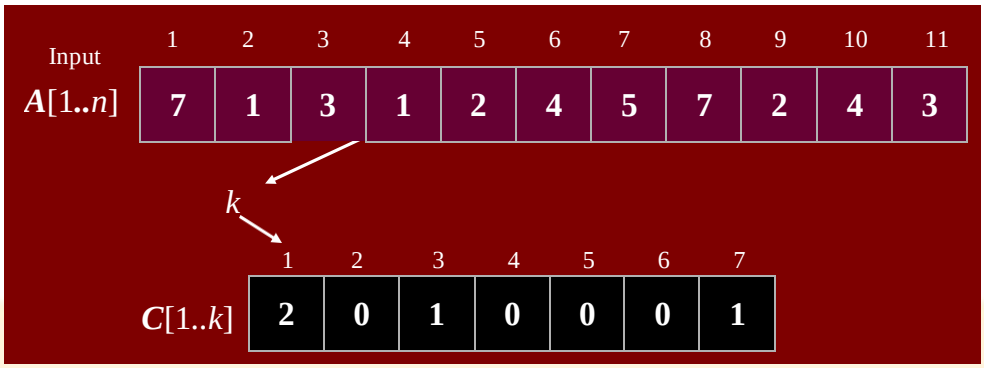

Figure 5.5: A[4] = 1 processed

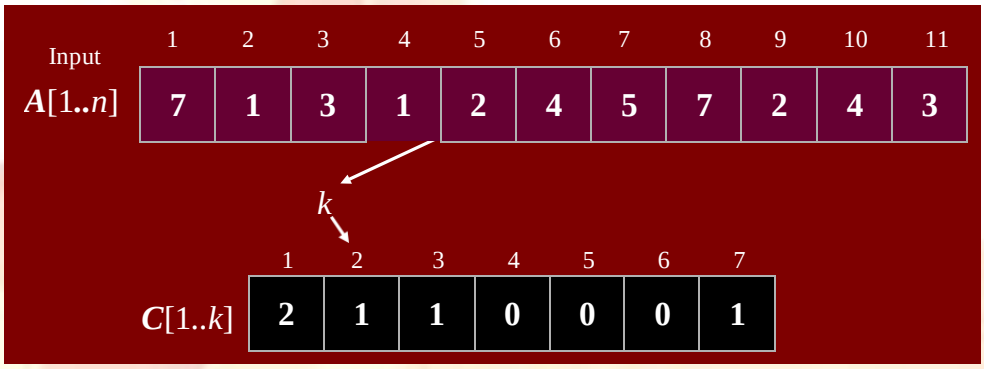

Figure 5.6: A[5] = 2 processed

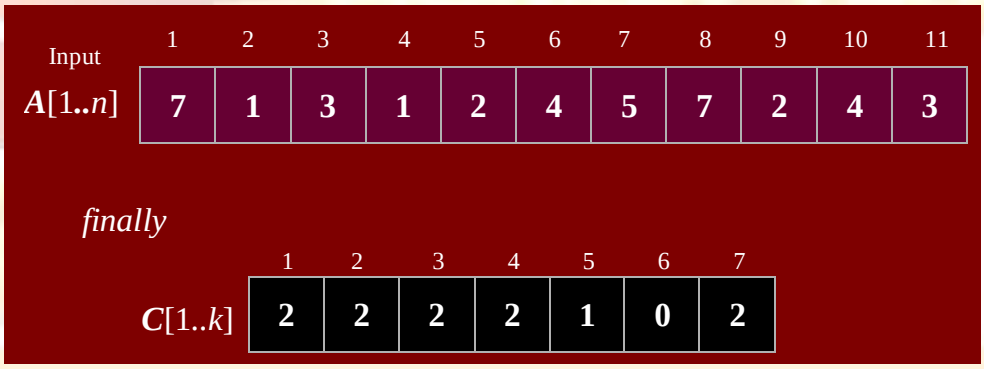

Figure 5.7: C now contains count of elements of A

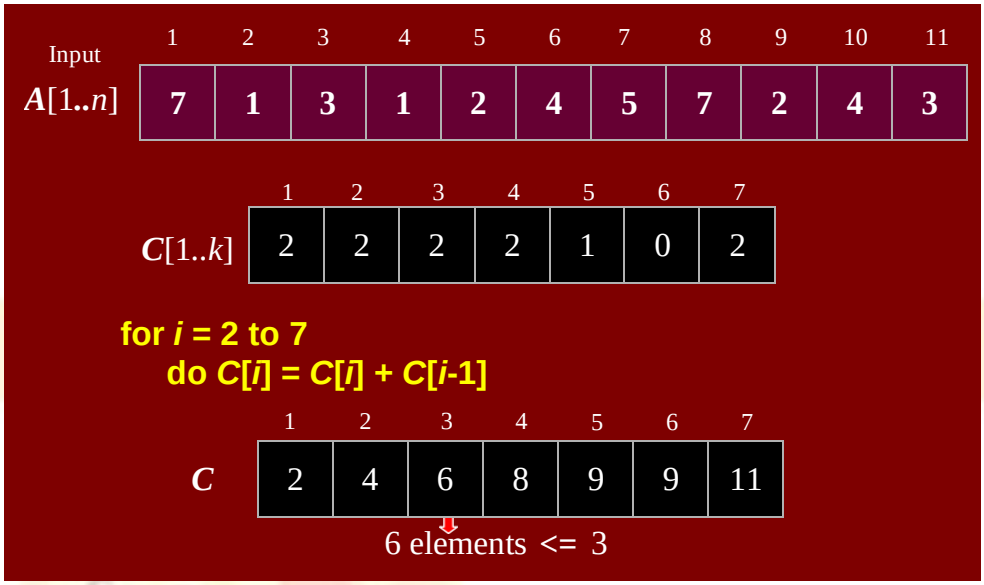

Figure 5.8: C set to rank each number of A

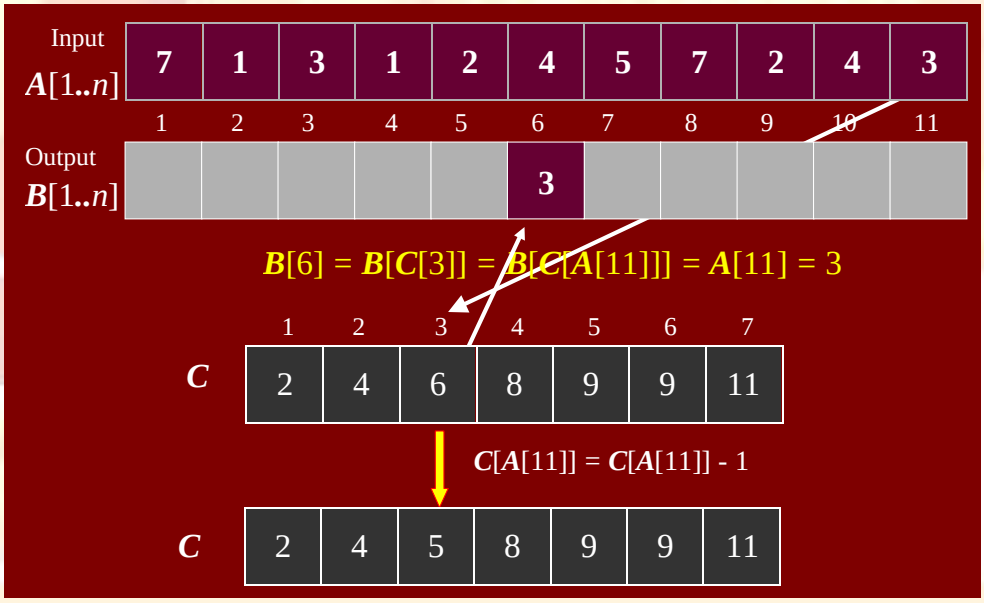

Figure 5.9: A[11] placed in output array B

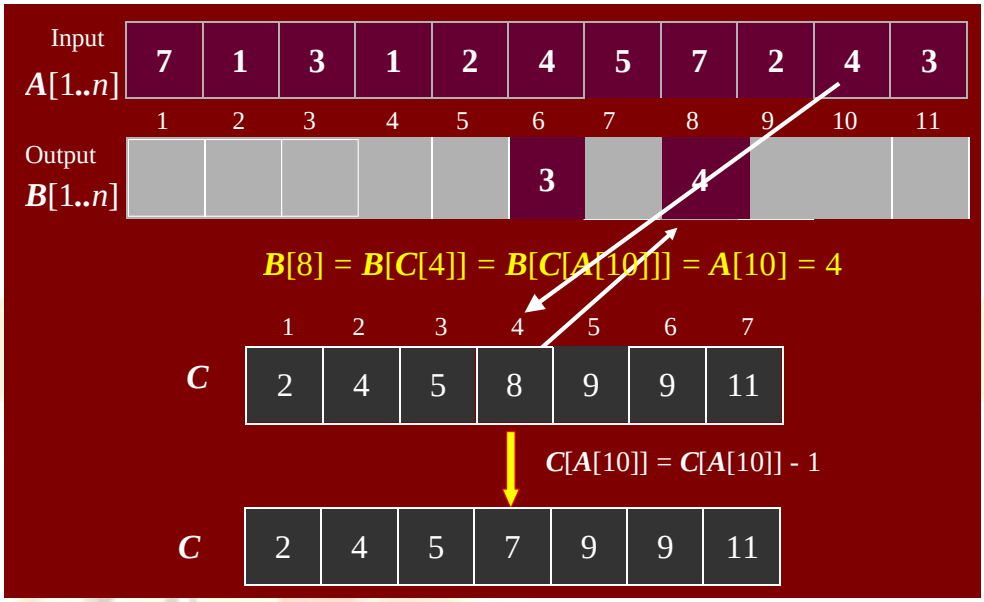

Figure 5.10: A[10] placed in output array B

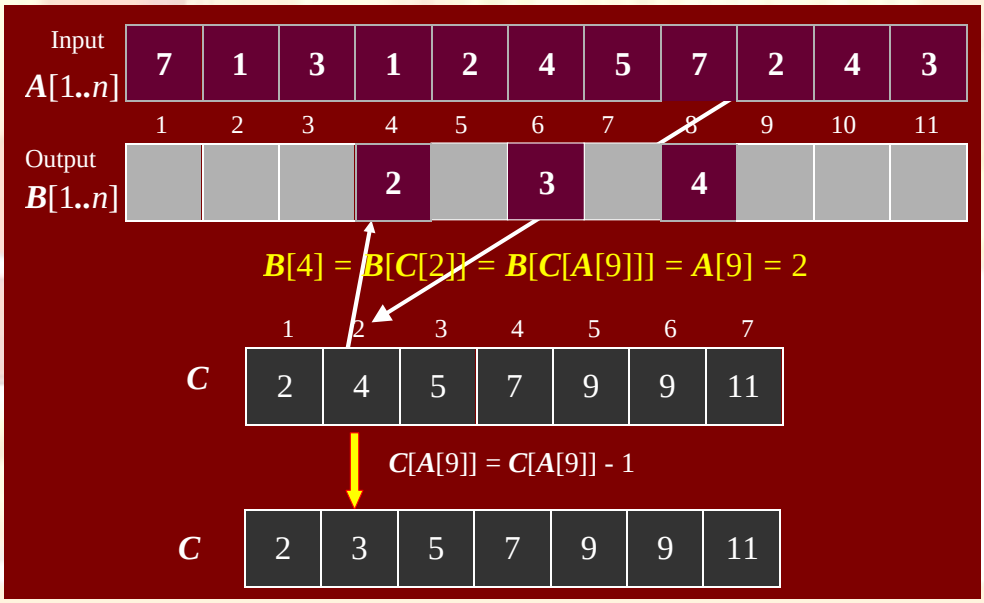

Figure 5.11: A[9] placed in output array B

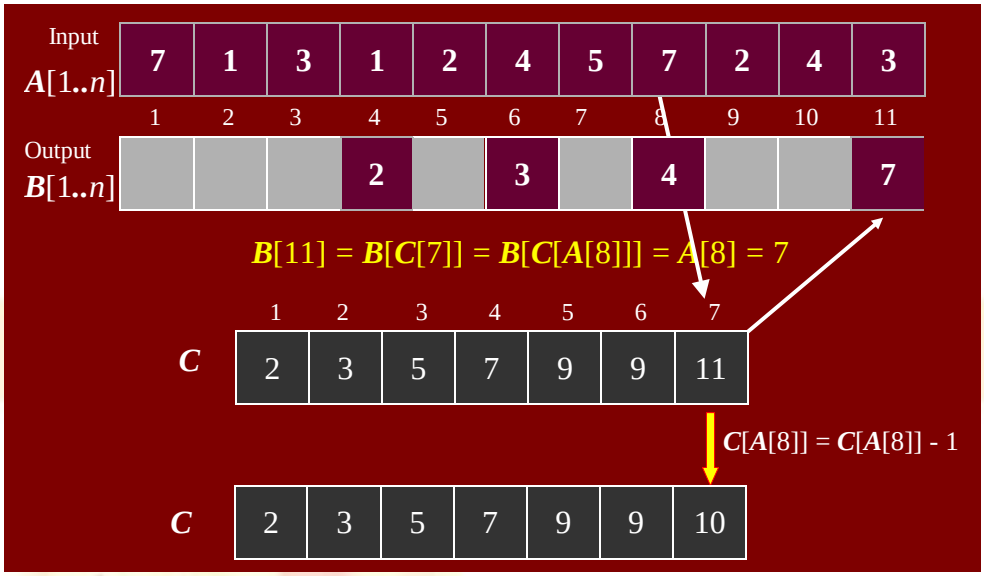

Figure 5.12: A[8] placed in output array B

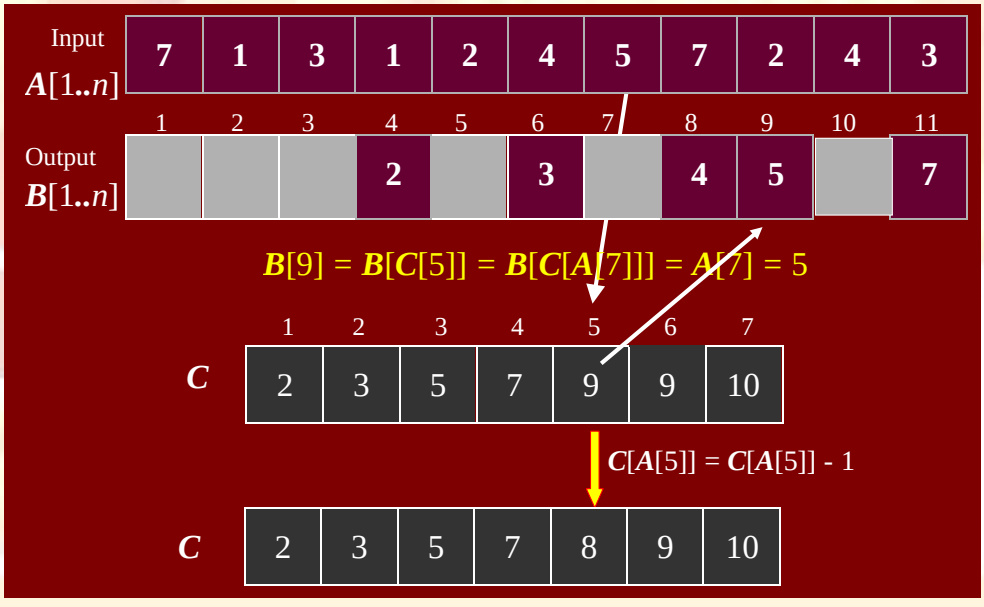

Figure 5.13: A[7] placed in output array B

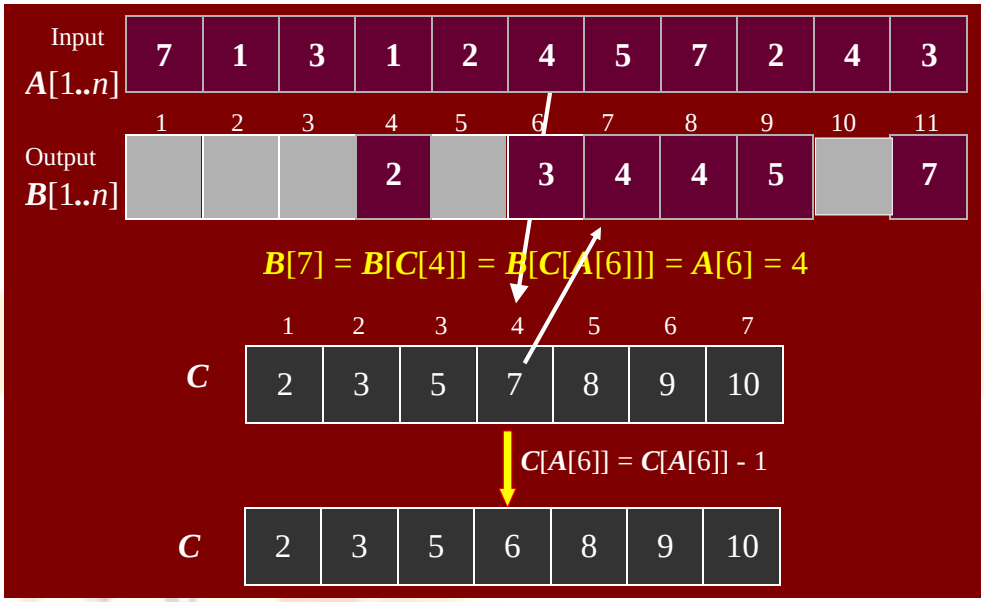

Figure 5.14: A[6] placed in output array B

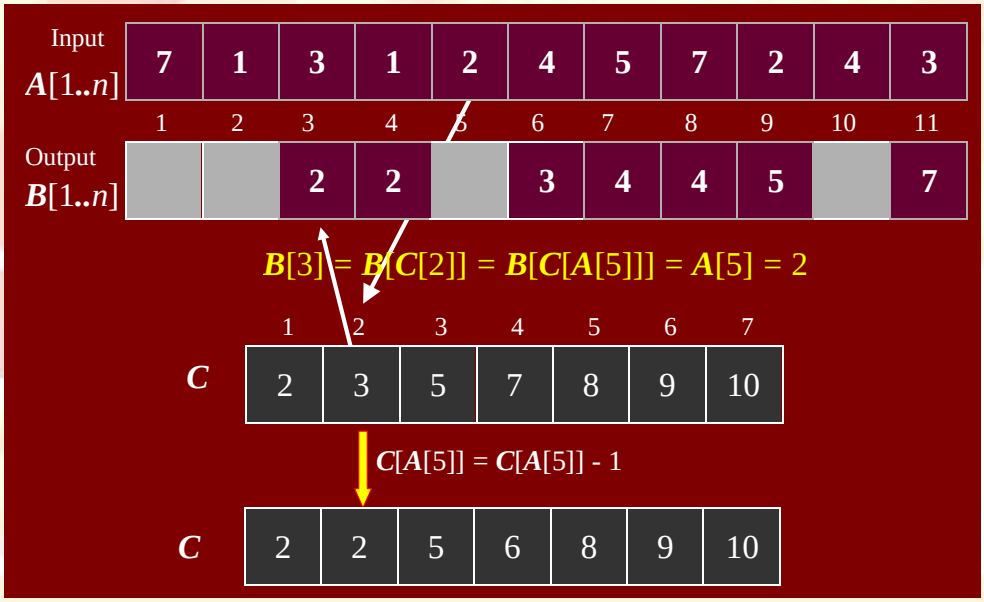

Figure 5.15: A[5] placed in output array B

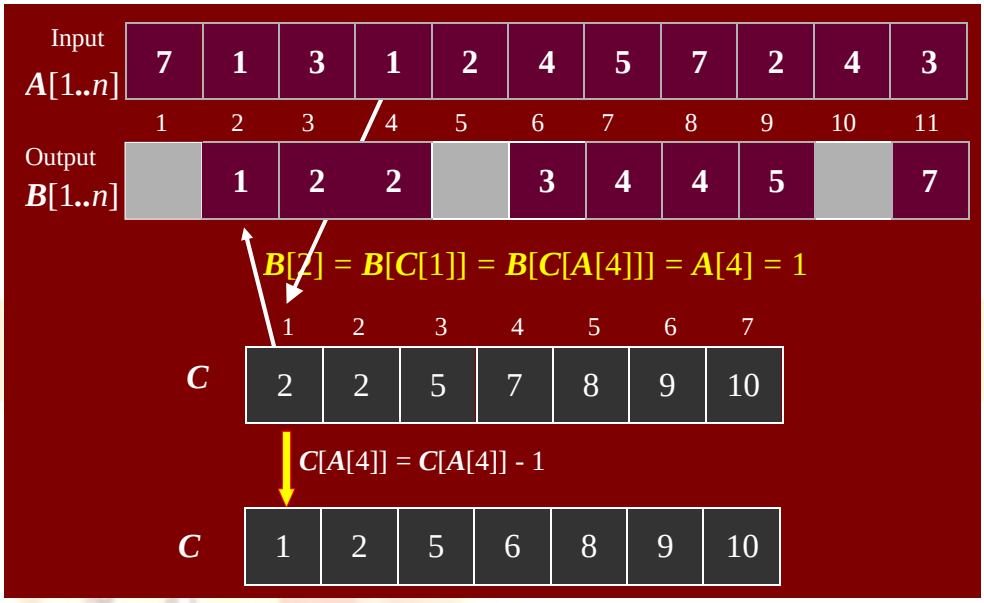

Figure 5.16: A[4] placed in output array B

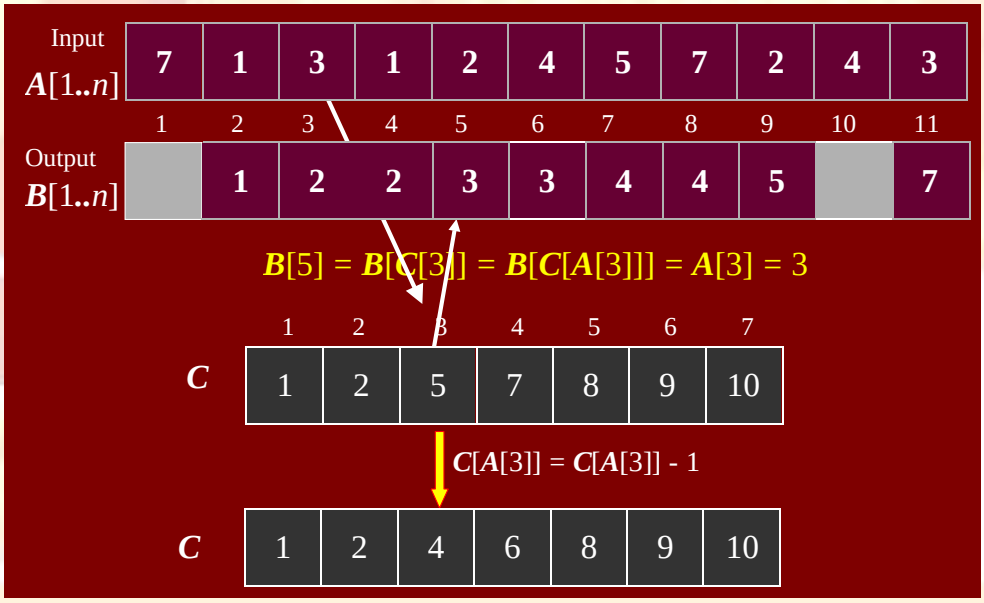

Figure 5.17: A[3] placed in output array B

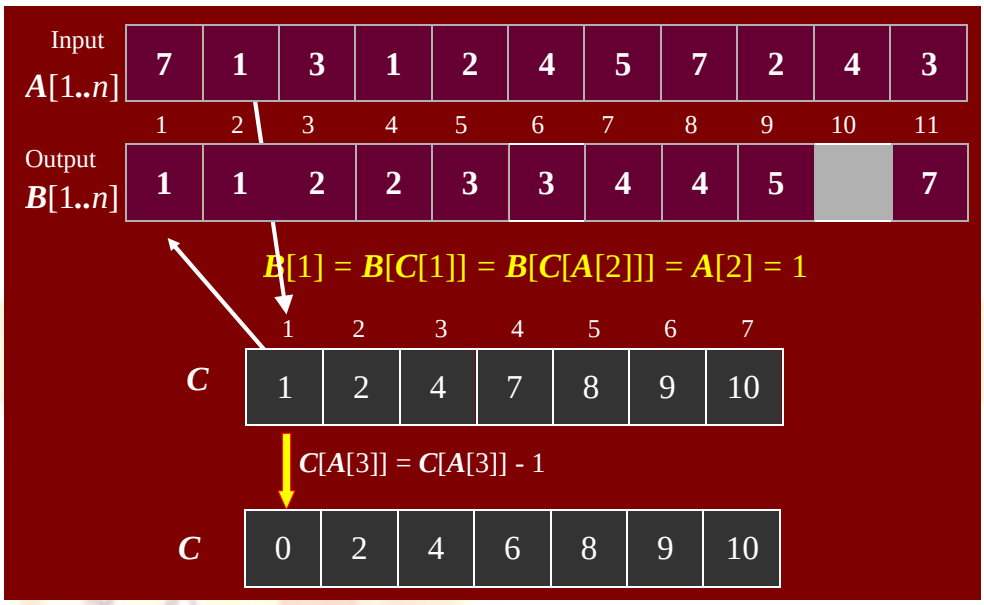

Figure 5.18: A[2] placed in output array B

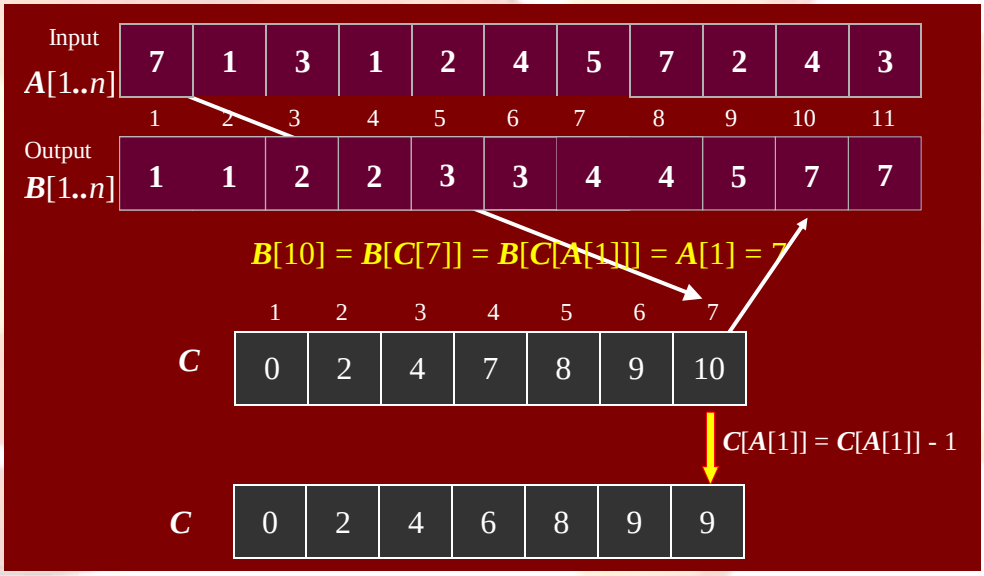

Figure 5.19: B now contains the final sorted data.

Counting sort is not an in-place sorting algorithm but it is stable. Stability is important because data are often carried with the keys being sorted. radix sort (which uses counting sort as a subroutine) relies on it to work correctly. Stability achieved by running the loop down from n to 1 and not the other way around:

```
COUNTING-SORT( array A, array B, int k)
1  .
2  for j ← length[A] downto 1
3  do B[C[A[j]]] ← A[j]
4      C[A[j]] ← C[A[j]] – 1
```

Figure 5.20 illustrates the stability. The numbers 1, 2, 3, 4, and 7, each appear twice. The two 4's have been given the superscript "*". Numbers are placed in the output B array starting from the right. The two 4's maintain their relative position in the B array. If the sorting algorithm had caused 4^{**} to end up on the left of 4^{*}, the algorithm would be termed unstable.

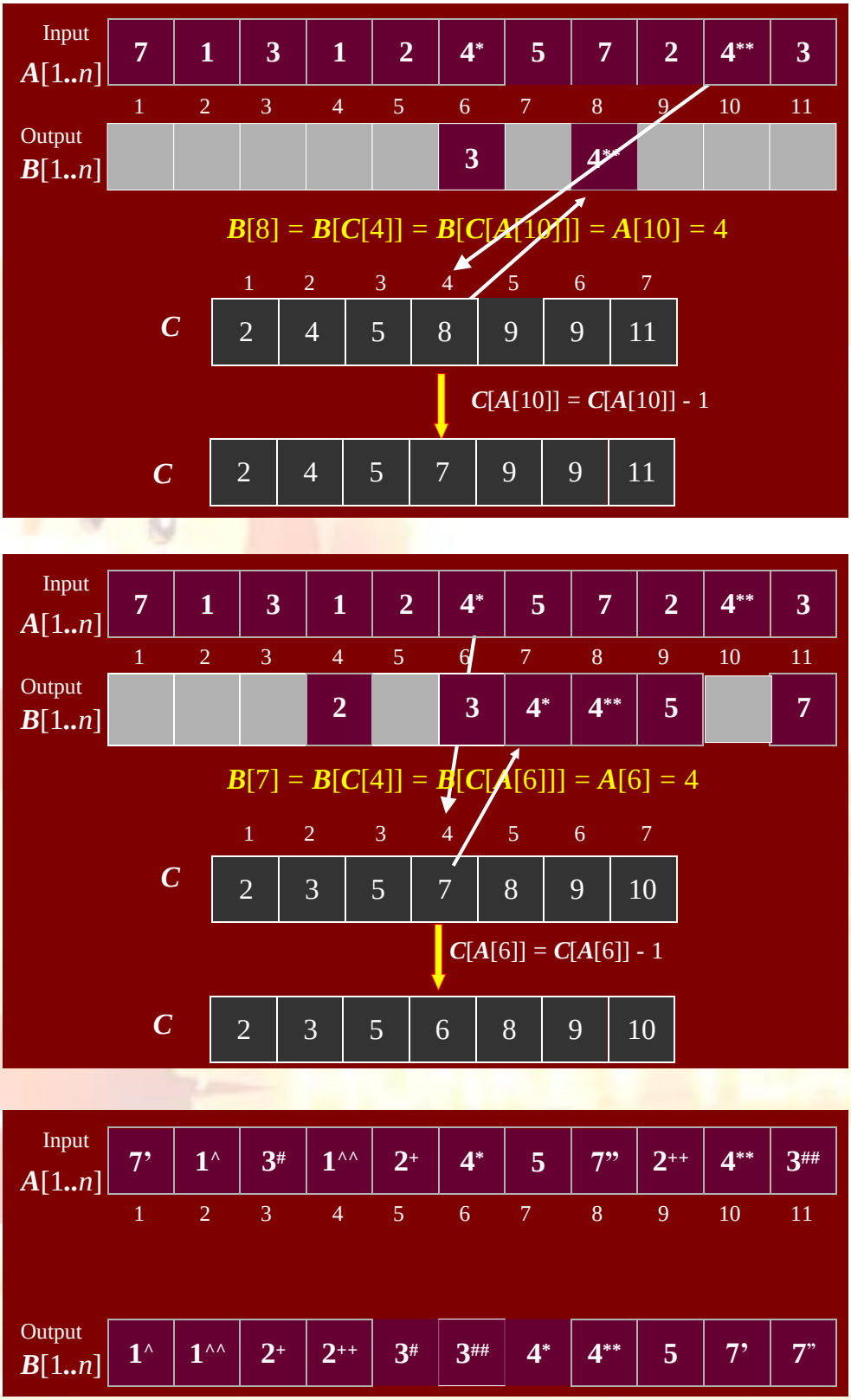

Figure 5.20: Stability of counting sort

Bucket or Bin Sort

Assume that the keys of the items that we wish to sort lie in a small fixed range and that there is only one item with each value of the key. Then we can sort with the following procedure:

1. Set up an array of "bins" - one for each value of the key - in order,

2. Examine each item and use the value of the key to place it in the appropriate bin.

Now our collection is sorted and it only took n operations, so this is an $O(n)$ operation. However, note that it will only work under very restricted conditions. To understand these restrictions, let's be a little more precise about the specification of the problem and assume that there are m values of the key. To recover our sorted collection, we need to examine each bin. This adds a third step to the algorithm above,

3. Examine each bin to see whether there's an item in it.

which requires m operations. So the algorithm's time becomes:

$$T(n) = c_1 n + c_2 m$$

and it is strictly $O(n + m)$. If $m \leq n$, this is clearly $O(n)$. However if $m >> n$, then it is $O(m)$. An implementation of bin sort might look like:

```
BUCEKTSORT( array A, int n, int M)
1  // Pre-condition: for 1 ≤ i ≤ n, 0 ≤ a[i] < M
2  // Mark all the bins empty
3  for i ← 1 to M
4  do bin[i] ← Empty
5  for i ← 1 to n
6  do bin[A[i]] ← A[i]
```

If there are *duplicates*, then each bin can be replaced by a *linked list*. The third step then becomes:

3. Link all the lists into one list.

We can add an item to a linked list in $O(1)$ time. There are n items requiring $O(n)$ time. Linking a list to another list simply involves making the tail of one list point to the other, so it is $O(1)$. Linking m such lists obviously takes $O(m)$ time, so the algorithm is still $O(n + m)$. Figures 5.21 through 5.23 show the algorithm in action using linked lists.

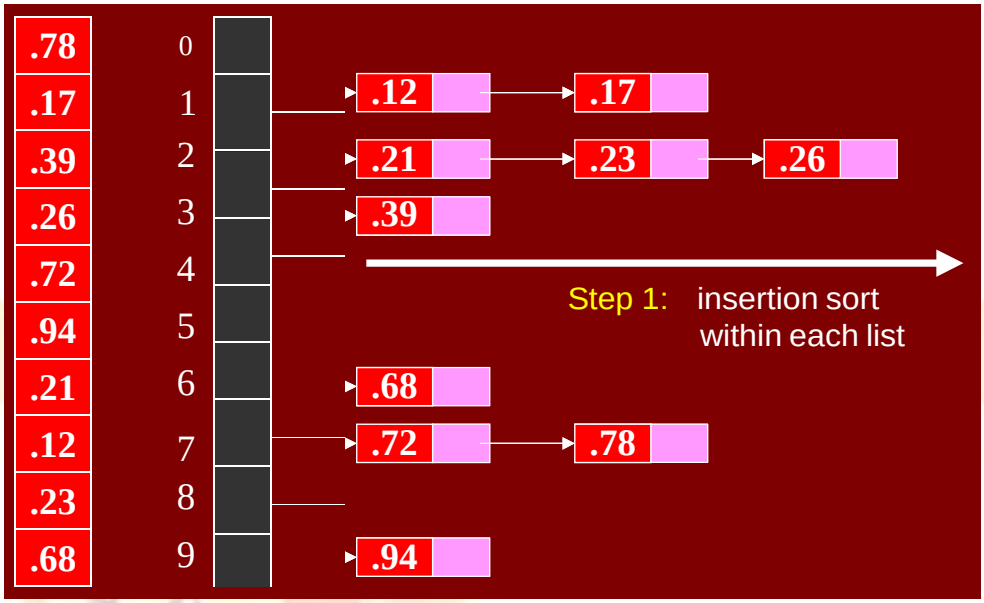

Figure 5.21: Bucket sort: step 1, placing keys in bins in sorted order

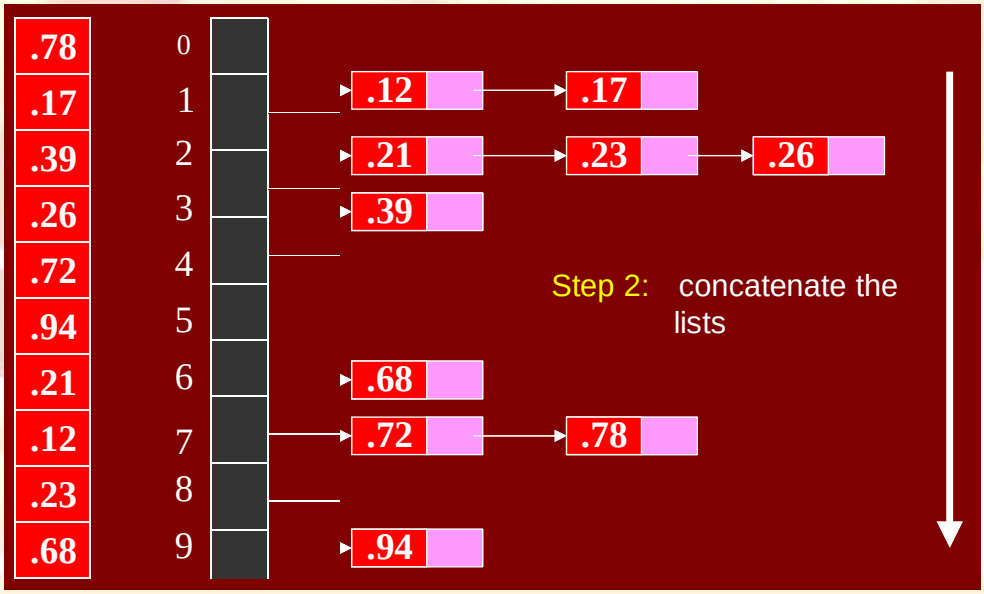

Figure 5.22: Bucket sort: step 2, concatenate the lists

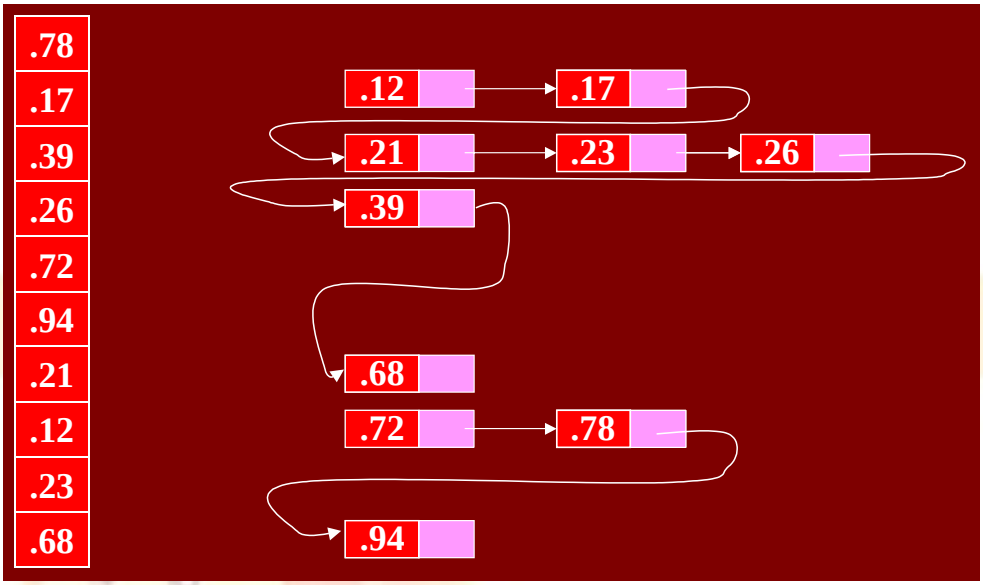

Figure 5.23: Bucket sort: the final sorted sequence

Radix Sort

The main shortcoming of counting sort is that it is useful for small integers, i.e., $1..k$ where k is small. If k were a million or more, the size of the rank array would also be a million. Radix sort provides a nice work around this limitation by sorting numbers one digit at a time.

576		49[4]		9[5]4		[1]76		176
494		19[4]		5[7]6		[1]94		194
194		95[4]		1[7]6		[2]78		278
296	⇒	57[6]	⇒	2[7]8	⇒	[2]96	⇒	296
278		29[6]		4[9]4		[4]94		494
176		17[6]		1[9]4		[5]76		576
954		27[8]		2[9]6		[9]54		954

Here is the algorithm that sorts $A[1..n]$ where each number is d digits long.

```
RADIX-SORT( array A, int n, int d)
1   for i ← 1 to d
2   do stably sort A w.r.t ith lowest order digit
```

MIDTERM EXAMINATION
Fall 2011
CS502- Fundamentals of Algorithms

Question No: 1 (Marks: 1) - Please choose one
Due to left complete nature of binary tree, the heap can be stored in

► **Arrays (Page 40)**
► Structures
► Link Lis
►Stack

Question No: 1 (Marks: 1) - Please choose one
What type of instructions Random Access Machine (RAM) can execute?

►Algebraic and logic
►Geometric and arithmetic
►**Arithmetic and logic (Page 10)**
►Parallel and recursive

Question No: 1 (Marks: 1) - Please choose one
For Chain Matrix Multiplication we can not use divide and conquer approach because,

►**We do not know the optimum k (Page 86)**
►We use divide and conquer for sorting only
►We can easily perform it in linear time
►Size of data is not given

Question No: 1 (Marks: 1) - Please choose one
What is the total time to heapify?

► **O(log n) (Page 43)**
► O(n log n)
► $O(n^2 \log n)$
► $O(\log^2 n)$

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Question No: 1 (Marks: 1) - Please choose one
word Algorithm comes from the name of the muslim author _____

►**Abu Ja'far Mohammad ibn Musa al-Khowarizmi**.

Question No: 1 (Marks: 1) - Please choose one
al-Khwarizmi's work was written in a book titled _____

►**al Kitab al-mukhatasar fi hisab al-jabr wa'l-muqabalah**

MIDTERM EXAMINATION
Spring 2010
CS502- Fundamentals of Algorithms

Question No: 1 (Marks: 1) - Please choose one
Random access machine or RAM is a/an

► Machine build by Al-Khwarizmi
► Mechanical machine
► Electronics machine
► **Mathematical model (Page 10)**

Question No: 2 (Marks: 1) - Please choose one
_____ is a graphical representation of an algorithm

► Σ notation
► Θ notation
► **Flowchart** **Click here for detail**
► Asymptotic notation

Question No: 3 (Marks: 1) - Please choose one
A RAM is an idealized machine with _____ random-access memory.

► 256MB
► 512MB
► **an infinitely large (Page 10)**
► 100GB

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Question No: 4 (Marks: 1) - Please choose one
What type of instructions Random Access Machine (RAM) can execute? Choose best answer

- ► Algebraic and logic
- ► Geometric and arithmetic
- ► **Arithmetic and logic (Rep)**
- ► Parallel and recursive

Question No: 5 (Marks: 1) - Please choose one
What will be the total number of max comparisons if we run brute-force maxima algorithm with n elements?

- ► n^2
- ► $n^{\frac{n}{2}}$
- ► n **(Page 14)**
- ► n^8

Question No: 6 (Marks: 1) - Please choose one
What is the solution to the recurrence $T(n) = T(n/2)+n$.

- ► $O(\log n)$
- ► **$O(n)$ (Page 37)**
- ► $O(n\log n)$
- ► $O(n^2)$

Question No: 7 (Marks: 1) - Please choose one
Consider the following code:

```
For(j=1; j<n;j++)
        For(k=1; k<15;k++)
                For(l=5; l<n; l++)
                {
                        Do_something_constant();
                }
```

What is the order of execution for this code.

- ► **O(n)**
- ► O(n^3)
- ► O($n^2 \log n$)
- ► O(n^2)

Question No: 8 (Marks: 1) - Please choose one
What is the total time to heapify?

- ► **O(log n) rep**
- ► O(n log n)
- ► O(n^2 log n)
- ► O($\log^2$ n)

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Question No: 9 (Marks: 1) - Please choose one
Consider the following Algorithm:

```
Factorial (n){
  if (n=1)
    return 1
  else
    return (n * Factorial(n-1))
```

Recurrence for the following algorithm is:

- ► **T(n) = T(n-1) +1**
- ► **T(n) = nT(n-1) +1**
- ► **T(n)= T(n-1) +n**
- ► **T(n)=T(n(n-1)) +1**

Question No: 10 (Marks: 1) - Please choose one
When we call heapify then at each level the comparison performed takes time

- ► **It will take Θ (1) (Page 43)**
- ► Time will vary according to the nature of input data
- ► It can not be predicted
- ► It will take Θ (log n)

Question No: 11 (Marks: 1) - Please choose one
In Quick sort, we don't have the control over the sizes of recursive calls

- ► **True (Page 40)**
- ► False
- ► Less information to decide
- ► Either true or false

Question No: 12 (Marks: 1) - Please choose one
Is it possible to sort without making comparisons?

- ► **Yes (Page 57)**
- ► No

Question No: 13 (Marks: 1) - Please choose one
If there are Θ (n^2) entries in edit distance matrix then the total running time is

- ► Θ (1)
- ► **Θ (n^2)** **Click here for detail**
- ► Θ (n)
- ► Θ (n log n)

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Question No: 14 (Marks: 1) - Please choose one
For Chain Matrix Multiplication we can not use divide and conquer approach because,

- ► **We do not know the optimum k (Page 86)**
- ► We use divide and conquer for sorting only
- ► We can easily perform it in linear time
- ► Size of data is not given

Question No: 15 (Marks: 1) - Please choose one
The Knapsack problem belongs to the domain of _____ problems.

- ► **Optimization (Page 91)**
- ► NP Complete
- ► Linear Solution
- ► Sorting

Question No: 16 (Marks: 1) - Please choose one
Suppose we have three items as shown in the following table, and suppose the capacity of the knapsack is 50 i.e. W = 50.

Item	Value	Weight
1	60	10
2	100	20
3	120	30

The optimal solution is to pick

- ► Items 1 and 2
- ► Items 1 and 3
- ► **Items 2 and 3 (correct)**
- ► None of these

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MIDTERM EXAMINATION
Spring 2010
CS502- Fundamentals of Algorithms

Question No: 1 (Marks: 1) - Please choose one
For the Sieve Technique we take time
► **T(nk) (Page 34)**
►T(n / 3)
►n^2
►n/3

Question No: 1 (Marks: 1) - Please choose one
Sieve Technique applies to problems where we are interested in finding a single item from a larger set of _____
Select correct option:
►**n items (Page 34)**
►phases
►pointers
►constant

Question No: 1 (Marks: 1) - Please choose one
_____ graphical representation of algorithm.
►asymptotic
►**Flowchart (rep)**

Question No: 1 (Marks: 1) - Please choose one
who invented the quick sort

►**C.A.R. Hoare** Click here for detail

Question No: 1 (Marks: 1) - Please choose one
main elements to a divide-and-conquer

►**Divide, conquer, combine (Page 27)**

Question No: 1 (Marks: 1) - Please choose one
Mergesort is a stable algorithm but not an in-place algorithm.

►**True (Page 54)**
►false

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Question No: 1 (Marks: 1) - Please choose one
Counting sort the numbers to be sorted are in the range 1 to k where k is small.
►**True (Page 57)**
►False

MIDTERM EXAMINATION
Spring 2007
CS502- Fundamentals of Algorithms

Question No: 1 (Marks: 1) - Please choose one
Total time for heapify is:
►O ($\log^2 n$)
►O (n log n)
►O (n^2 log n)
►**O (log n) Rep**

Question No: 1 (Marks: 1) - Please choose one
If an algorithm has a complexity of $\log_2 n + n\log_2 n + n$. we could say that it has complexity

►O(n)
►O(n $\log_2$ n)
►O(3)
►O($\log_2$ ($\log_2$ n))
►O ($\log_2$ n)

Question No: 1 (Marks: 1) - Please choose one
In RAM model instructions are executed

►**One after another (Page 10)**
►Parallel
►Concurrent
►Random

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Question No: 1 (Marks: 1) - Please choose one
In selection algorithm, because we eliminate a constant fraction of the array with each phase, we get the

►**Convergent geometric series (Page 37)**
►Divergent geometric series
►None of these

Question No: 1 (Marks: 1) - Please choose one
Due to left-complete nature of binary tree, heaps can be stored in

►Link list
►Structure
►**Array (Page 40)**
►None of above

CS609- System Programming
Midterm Quizzes (Quiz No.1 & 2)

Quiz No.1 (04 – MAY - 2013)

Question No: 1 (Marks: 1) - Please choose one
The time assumed for each basic operation to execute on RAM model of computation is-----

Infinite
Continuous
Constant (Page 10)
Variable

Question No: 1 (Marks: 1) - Please choose one
If the indices passed to merge sort algorithm are not equal, the algorithm may return immediately.

True
False (Page 28)

Question No: 1 (Marks: 1) - Please choose one
Brute-force algorithm uses no intelligence in pruning out decisions.

True (Page 18)
False

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Question No: 1 (Marks: 1) - Please choose one
In analysis, the Upper Bound means the function grows asymptotically no faster than its largest term.

True (Page 24)
False

Question No: 1 (Marks: 1) - Please choose one
For small values of n, any algorithm is fast enough. Running time does become an issue when n gets large.

True (Page 14)
Fast

Question No: 1 (Marks: 1) - Please choose one
The array to be sorted is not passed as argument to the merge sort algorithm.

True
False

Question No: 1 (Marks: 1) - Please choose one
In simple brute-force algorithm, we give no thought to efficiency.

True (Page 11)
False

Question No: 1 (Marks: 1) - Please choose one
The ancient Roman politicians understood an important principle of good algorithm design that is plan-sweep algorithm.

True
False (Page 27) [Divide and Conquer]

Question No: 1 (Marks: 1) - Please choose one
In 2d-space a point is said to be _____if it is not dominated by any other point in that space.

Member
Minimal
Maximal (Page 11)
Joint

Question No: 1 (Marks: 1) - Please choose one
An algorithm is a mathematical entity that is dependent on a specific programming language.
True
False (Page 7)

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Question No: 1 (Marks: 1) - Please choose one
The running time of an algorithm would not depend upon the optimization by the compiler but that of an implementation of the algorithm would depend on it.

True (Page 13)
False

Question No: 1 (Marks: 1) - Please choose one
F (n) and g (n) are asymptotically equivalent. This means that they have essentially the same _____ for large n.

Results
Variables
Size
Growth rates (Page 23)

Question No: 1 (Marks: 1) - Please choose one
8n2 + 2n - 3 will eventually exceed c2*(n) no matter how large we make c2.

True (Page 25)
False

Question No: 1 (Marks: 1) - Please choose one
If we associate (x, y) integers pair to cars where x is the speed of the car and y is the negation of the price. High y value for a car means a _____ car.
Fast
Slow
Expensive
Cheap (Page 11)

Question No: 1 (Marks: 1) - Please choose one
The function f(n)= n(logn+1)/2 is asymptotically equivalent to n log n. Here Upper Bound means the function f(n) grows asymptotically _____ faster than n log n.
More
Quiet
Not (Page 24)
At least

Question No: 1 (Marks: 1) - Please choose one
After sorting in merge sort algorithm, merging process is invoked.
Select correct option:
True (Page 28)
False

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Question No: 1 (Marks: 1) - Please choose one
Asymptotic growth rate of the function is taken over_____ case running time.
Select correct option:

Best
Average
Worst (Page 14)
Normal

Question No: 1 (Marks: 1) - Please choose one
In analysis of f (n) =n (n/5) +n-10 log n, f (n) is asymptotically equivalent to _____.

n
2n
n+1
n2 (Page 23)

Question No: 1 (Marks: 1) - Please choose one
Algorithm is concerned with.....issues.

Macro
Micro
Both Macro & Micro (Page 8)
Normal

Question No: 1 (Marks: 1) - Please choose one
We cannot make any significant improvement in the running time which is better than that of brute-force algorithm.

True
False (Page 18)

Question No: 1 (Marks: 1) - Please choose one
In addition to passing in the array itself to Merge Sort algorithm, we will pass in _____other arguments which are indices.

Two (Page 28)
Three
Four
Five

Question No: 1 (Marks: 1) - Please choose one
Consider the following Algorithm: Fun(n){ if (n=1) return 1 else return (n * Fun(n-1)) } Recurrence for the above algorithm is:

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nT(n-1)+1
2T(n-1)+1
T(n-1)+cn
T(n-1)+1

Question No: 1 (Marks: 1) - Please choose one
In analysis, the Lower Bound means the function grows asymptotically at least as fast as its largest term.

True (Page 24)
False

Question No: 1 (Marks: 1) - Please choose one
Efficient algorithm requires less computational…….

Memory
Running Time
Memory and Running Time (Page 9)
Energy

Question No: 1 (Marks: 1) - Please choose one
The O-notation is used to state only the asymptotic _____bounds.

Two
Lower
Upper (Page 25)
Both lower & upper

Question No: 1 (Marks: 1) - Please choose one
For the worst-case running time analysis, the nested loop structure containing one "for" and one "while" loop, might be expressed as a pair of _____nested summations.

1
2 (Page 16)
3
4

Question No: 1 (Marks: 1) - Please choose one
Before sweeping a vertical line in plane sweep approach, in start sorting of the points is done in increasing order of their _____coordinates.

X (Page 18)
Y
Z
X & Y

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Question No: 1 (Marks: 1) - Please choose one
Brute-force algorithm for 2D-Maxima is operated by comparing _____ pairs of points.

Two
Some
Most
All (Page 18)

Question No: 1 (Marks: 1) - Please choose one
The function f(n)=n(logn+1)/2 is asymptotically equivalent to nlog n. Here Lower Bound means function f(n) grows asymptotically at _____ as fast as nlog n.

Normal
Least (Page 23)
Most
All

Question No: 1 (Marks: 1) - Please choose one
The definition of Theta-notation relies on proving _____asymptotic bound.

One
Lower
Upper
Both lower & upper (Page 25) rep

Question No: 1 (Marks: 1) - Please choose one
In plane sweep approach, a vertical line is swept across the 2d-plane and _____structure is used for holding the maximal points lying to the left of the sweep line.

Array
Queue
Stack (Page 18)
Tree

Question No: 1 (Marks: 1) - Please choose one
Algorithm analysts know for sure about efficient solutions for NP-complete problems.
Select correct option:
True
False (Page 9)

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Quiz No.1 (2012)

Question No: 1 of 10 (Marks: 1) - Please choose one
The number of nodes in a complete binary tree of height h is

2^(h+1) – 1 (Page 40)
2 * (h+1) – 1
2 * (h+1)
((h+1) ^ 2) – 1

Question No: 1 of 10 (Marks: 1) - Please choose one
The analysis of Selection algorithm shows the total running time is indeed _____in n,

arithmetic
geometric
linear (Page 37)
orthogonal

Question No: 1 of 10 (Marks: 1) - Please choose one
A (an) _____ is a left-complete binary tree that conforms to the heap order

heap (Page 40)
binary tree
binary search tree
array

Question No: 1 of 10 (Marks: 1) - Please choose one
Analysis of Selection algorithm ends up with,

T(n) (Page 37)
T(1 / 1 + n)
T(n / 2)
T((n / 2) + n)

Question No: 1 of 10 (Marks: 1) - Please choose one
For the sieve technique we solve the problem,

recursively (Page 34)
mathematically
precisely
accurately

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Question No: 1 of 10 (Marks: 1) - Please choose one
A heap is a left-complete binary tree that conforms to the _____

increasing order only
decreasing order only
heap order (Page 40)
(log n) order

Question No: 1 of 10 (Marks: 1) - Please choose one
In which order we can sort?

increasing order only
decreasing order only
increasing order or decreasing order (Page 39)
both at the same time

Question No: 1 of 10 (Marks: 1) - Please choose one
Divide-and-conquer as breaking the problem into a small number of

pivot
Sieve
smaller sub problems (Page 34)
Selection

Question No: 1 of 10 (Marks: 1) - Please choose one
For the heap sort we store the tree nodes in

level-order traversal (Page 40)
in-order traversal
pre-order traversal
post-order traversal

Question No: 1 of 10 (Marks: 1) - Please choose one
The sieve technique works in _____ as follows

Phases (Page 34)
numbers
integers
routines

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CS502 - Fundamentals of Algorithms
Quiz No.1 12-11-2012

Question No: 1 of 10 (Marks: 1) - Please choose one
We do sorting to,

keep elements in random positions
keep the algorithm run in linear order
keep the algorithm run in (log n) order
keep elements in increasing or decreasing order

Question No: 1 of 10 (Marks: 1) - Please choose one
Heaps can be stored in arrays without using any pointers; this is due to the _____ nature of the binary tree,

left-complete (Page 40)
right-complete
tree nodes
tree leaves

Question No: 1 of 10 (Marks: 1) - Please choose one
Sieve Technique can be applied to selection problem?

True (Page 35)
False

Question No: 1 of 10 (Marks: 1) - Please choose one
In Sieve Technique we do not know which item is of interest

True (Page 34)
False

Question No: 1 of 10 (Marks: 1) - Please choose one
In the analysis of Selection algorithm, we eliminate a constant fraction of the array with each phase; we get the convergent _____ series in the analysis,

linear
arithmetic
geometric (Page 37)
exponent

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Question No: 1 of 10 (Marks: 1) - Please choose one
For the heap sort, access to nodes involves simple _____ operations.

arithmetic (Page 41)
binary
algebraic
logarithmic

Question No: 1 of 10 (Marks: 1) - Please choose one
Slow sorting algorithms run in,

T(n^2) (Page 39)
T(n)
T(log n)

Question No: 1 of 10 (Marks: 1) - Please choose one
In the analysis of Selection algorithm, we make a number of passes, in fact it could be as many as,
T(n)
T(n / 2)
log n (Page 37)
n / 2 + n / 4

Question No: 1 of 10 (Marks: 1) - Please choose one
The sieve technique is a special case, where the number of sub problems is just
5
many
1 (Page 34)
few

Question No: 1 of 10 (Marks: 1) - Please choose one
How many elements do we eliminate in each time for the Analysis of Selection algorithm?

(n / 2)+n elements
(n / 2) elements (Page 37)
n / 4 elements
2 n elements

Question No: 1 of 10 (Marks: 1) - Please choose one
One of the clever aspects of heaps is that they can be stored in arrays without using any _____.
pointers (Page 40)
constants
variables
functions

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Question No: 1 of 10 (Marks: 1) - Please choose one
How much time merge sort takes for an array of numbers?

T(n^2)
T(n)
T(log n)
T(n log n) (Page 40)

Question No: 1 of 10 (Marks: 1) - Please choose one
The reason for introducing Sieve Technique algorithm is that it illustrates a very important special case of,
divide-and-conquer (Page 34)
decrease and conquer
greedy nature
2-dimension Maxima

Question No: 1 of 10 (Marks: 1) - Please choose one
In Sieve Technique we do not know which item is of interest
True (Page 34) rep
False

Question No: 1 of 10 (Marks: 1) - Please choose one
Theta asymptotic notation for T (n) :

Set of functions described by: c1g(n)Set of functions described by c1g(n)>=f(n) for c1 s
Theta for T(n)is actually upper and worst case comp (Not sure)
Set of functions described by:
c1g(n)

Question No: 1 of 10 (Marks: 1) - Please choose one
Memoization is?

To store previous results for future use
To avoid this unnecessary repetitions by writing down the results of recursive calls and looking them up again if we need them later (page 74)
To make the process accurate
None of the above

Question No: 1 of 10 (Marks: 1) - Please choose one
Which sorting algorithm is faster
O (n log n) Page 26
O n^2
O (n+k)
O n^3

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Question No: 1 of 10 (Marks: 1) - Please choose one
Quick sort is

Stable & in place
Not stable but in place (Page 54)
Stable but not in place
Some time stable & some times in place

Question No: 1 of 10 (Marks: 1) - Please choose one
One example of in place but not stable algorithm is

Merger Sort
Quick Sort (Page 54)
Continuation Sort
Bubble Sort

Question No: 1 of 10 (Marks: 1) - Please choose one
Cont sort is suitable to sort the elements in range 1 to k
K is Large
K is not known
K may be small or large
K is small (Page 57)

Question No: 1 of 10 (Marks: 1) - Please choose one
In place stable sorting algorithm.

If duplicate elements remain in the same relative position after sorting (Page 54)
One array is used
More than one arrays are required
Duplicating elements not handled

Question No: 1 of 10 (Marks: 1) - Please choose one
Which may be a stable sort?
Merger
Insertion (Page 54)
Both above
None of the above

Question No: 1 of 10 (Marks: 1) - Please choose one
An in place sorting algorithm is one that uses ___ arrays for storage

Two dimensional arrays
More than one array
No Additional Array (Page 54)
None of the above

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Question No: 1 of 10 (Marks: 1) - Please choose one
Sieve Technique applies to problems where we are interested in finding a single item from a larger set of _____

n items (Page 34)
phases
pointers
constant

Question No: 1 of 10 (Marks: 1) - Please choose one
Sorting is one of the few problems where provable _____ bonds exits on how fast we can sort,

upper
lower (Page 39)
average
log n

Question No: 1 of 10 (Marks: 1) - Please choose one
Counting sort has time complexity:

O(n) (Page 58)
O(n+k)
O(k)
O(nlogn)

Question No: 1 of 10 (Marks: 1) - Please choose one
The running time of quick sort depends heavily on the selection of

No of inputs
Arrangement of elements in array
Size o elements
Pivot elements (Page 49)

Question No: 1 of 10 (Marks: 1) - Please choose one
Which may be stable sort:

Bubble sort
Insertion sort
Both of above (Page 54)

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Question No: 1 of 10 (Marks: 1) - Please choose one
One Example of in place but not stable sort is

Quick (Page 54)
Heap
Merge
Bubble

Question No: 1 of 10 (Marks: 1) - Please choose one
In Quick Sort Constants hidden in T(n log n) are

Large
Medium
Small Click here for detail
Not Known

Question No: 1 of 10 (Marks: 1) - Please choose one
Quick sort is based on divide and conquer paradigm; we divide the problem on base of pivot element and:

There is explicit combine process as well to conquer the solution.
No work is needed to combine the sub-arrays, the array is already sorted
Merging the sub arrays
None of above. (Page 51)
Ref: - random choices for the pivot element and each choice have an equal probability of 1/n of occurring. So we can modify the above recurrence to compute an average rather than a max

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CS501 - Quiz No.2 (Spring 2013)

Question No: 1 of 10 (Marks: 1) - Please choose one
A point p in 2-dimensional space is usually given by its integer coordinate(s)_____

p.x only
p.y only
p.x & p.z
p.x & p.y (Page 10)

Question No: 1 of 10 (Marks: 1) - Please choose one
In _____ we have to find rank of an element from given input.

Merge sort algorithm
Selection problem (Page 34)
Brute force technique
Plane Sweep algorithm

Question No: 1 of 10 (Marks: 1) - Please choose one
In Heap Sort algorithm, if heap property is violated _____

We call Build heap procedure
We call Heapify procedure
We ignore
Heap property can never be violated

Question No: 1 of 10 (Marks: 1) - Please choose one
Upper bound requires that there exist positive constants c2 and n0 such that f(n) ____ c2n for all n <= n0(ye question ghalat lag raha hai mujhae

Less than
Equal to or Less than (Page 25)
Equal or Greater than
Greater than

Question No: 1 of 10 (Marks: 1) - Please choose one
A RAM is an idealized algorithm with takes an infinitely large random-access memory.

True
False (Page 10)

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Question No: 1 of 10 (Marks: 1) - Please choose one
_____ is one of the few problems, where provable lower bounds exist on how fast we can sort.

Searching
Sorting (Page)
Both Searching & Sorting
Graphing

Question No: 1 of 10 (Marks: 1) - Please choose one
Floor and ceiling are _____ to calculate while analyzing algorithms.

Very easy
Usually considered difficult (Page 31)

Question No: 1 of 10 (Marks: 1) - Please choose one
In Heap Sort algorithm, the maximum levels an element can move upward is _____

Theta (log n) (Page 43)
Order (log n)
Omega (log n)
O (1) i.e. Constant time

Question No: 1 of 10 (Marks: 1) - Please choose one
A point p in 2-dimensional space is usually given by its integer coordinate(s)_____

p.x only p.y
only p.x & p.z
p.x & p.y (Page 17)

Question No: 1 of 10 (Marks: 1) - Please choose one
In Heap Sort algorithm, the total running time for Heapify procedure is _____

Theta (log n) (Page 43)
Order (log n)
Omega (log n)
O (1) i.e. Constant time

Question No: 1 of 10 (Marks: 1) - Please choose one
Algorithm is a mathematical entity, which is independent of a specific machine and operating system.

True
False (Page 7)

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Question No: 1 of 10 (Marks: 1) - Please choose one
While Sorting, the ordered domain means for any two input elements x and y _____ satisfies only.

x < y
x > y
x = y
All of the above (Page 39)

Question No: 1 of 10 (Marks: 1) - Please choose one
Quick sort is best from the perspective of Locality of reference.

True (Page 9)
False

Question No: 1 of 10 (Marks: 1) - Please choose one
Sorting can be in _____

Increasing order only
Decreasing order only
Both Increasing and Decreasing order (Page 39)
Random order

Question No: 1 of 10 (Marks: 1) - Please choose one
In Heap Sort algorithm, we build _____ for ascending sort.

Max heap (Page 41)
Min heap

Question No: 1 of 10 (Marks: 1) - Please choose one
In Sieve Technique, we know the item of interest.

True
False (Page 34)

Question No: 1 of 10 (Marks: 1) - Please choose one
While solving Selection problem, in Sieve technique we partition input data _____

In increasing order
In decreasing order
According to Pivot (Page 35)
Randomly

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Question No: 1 of 10 (Marks: 1) - Please choose one
In pseudo code, the level of details depends on intended audience of the algorithm.

True (Page 12)
False

Question No: 1 of 10 (Marks: 1) - Please choose one
The sieve technique works where we have to find _____ item(s) from a large input.

Single (Page 34)
Two
Three
Similar

Question No: 1 of 10 (Marks: 1) - Please choose one
If the indices passed to merge sort algorithm are _____,then this means that there is only one element to sort.

Small
Large
Equal (Page 28)
Not Equal

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