

# CS701 - MidTerm Questions (All in One)

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## Questions from MidTerm Spring 2015 Papers

**Q1.**  $\{ [1/0],[101/0],[0/001] \}$  in PCP problem. Find is there any match? If yes prove it. (05)

**Q2.** In numbers 64 , 32965. Show that these numbers relatively prime (05)

**Q3.** Consider the pair of numbers 1274 and 10505. show that they are relatively prime or not.

### Question 2 Answer:

$$10505=1274*8+313$$

$$1274=313*4+22$$

$$313=22*14+5$$

$$22=5*4+2$$

$$5=2*2+1$$

$$2=1*2+0$$

**Q4.**  $A=\{ \langle R \rangle \mid R \text{ is a regular expression describing a language containing at least one string } w \text{ that has } 111 \text{ as substring (i.e. } w=x111y \text{ for some } x,y \}$  show A is decidable. (10) (Exercise 4.15)

**Q5.** Let  $\text{NOTSUB} = \{ \langle A, B \rangle \}$  where A , B are descriptions of TMs and there is some string w which is accepted by A but not accepted by B, Language accepted by A is not a subset of language accepted by B. is NOTSUB decidable? Prove in either case. (10)

**Q6.** Let  $\text{LALL} = \{ \langle M \rangle \mid M \text{ is a TM with input alphabet } \Sigma \text{ and } L(M)=\Sigma^* \}$  prove that LALL is not Turing recognizable. (10)

**Q7.** Let x be the set  $\{1,2,3,4,5\}$  and y be the set  $\{6,7,8,9,10\}$  we describe the functions  $f: x \rightarrow y$  and  $g: x \rightarrow y$  in the following table

n	F(n)		n	G(n)
1	6		1	10
2	7		2	9
3	6		3	8
4	7		4	7
5	6		5	6

**Q8.** Let  $\text{Path} = \{ \langle G, s, t \rangle \mid G \text{ is a digraph ha has directed path from } s \text{ to } t \}$  show path belongs to P class. (15)

**Q9.** Prove that Turing recognizable languages are closed under concatenation or union

**Q10.** Show that  $A_{TM}$  is not mappig reduable to  $E_{TM}$ , Prove by contradiction.

**Q11.** If  $L$  is a turing recognizable language than can we say that it is also decidable . Why or why not.

**Q12.** Show that  $A \leq T B$  and  $B \leq T C$  then  $A \leq T C$ . (Exercise 6.3)

**Q13.** Show that the set of all odd integers has one-to-one correspondence with the set of all even integers.

**Q14.** Can we say every problem is Turing decidable? If yes how? If no give counter example. (05)

**Q15.** In the silly Post Correspondence Problem, SPCP, in each pair the top string has the same length as the bottom string. Show that the SPCP is decidable. 10 Marks (PROBLEM 5.15 of Sipser book)

**Q16.** All  $DFA = \{ \langle A \rangle \mid A \text{ is DFA and } L(A) = \Sigma^* \}$  show that is  $ALL_{DFA}$  decidable (Exercise 4.3)

**Answer:** Consider the following Turing machine  
 $M = "$  on input  $\langle A \rangle$ , where  $A$  is a DFA

1. Create a DFA  $B$  such that  $L(B) = \Sigma^*$
2. Submit  $\langle A, B \rangle$  to the decider for  $EQ_{DFA}$
3. If it accepts, **accept**
4. If it reject, **reject.**"

$M$  is clearly a decider since steps 1, 3, and 4 will not create an infinite loop and step 2 calls a decider. Furthermore,  $M$  will accept those DFA's whose language is the same as  $B$ 's language - i.e. DFA's whose language is  $\Sigma^*$  -- and will reject all other languages. Therefore,  $M$  is a decider for  $ALL_{DFA}$  so  $ALL_{DFA}$  is decidable.

**Q17.** Let  $\{ \langle M \rangle \} \mid M \text{ is a TM and if we start } M \text{ with blank input tape, then it will finally write some non-blank symbols on its tape. Is it decidable, proof in either case.}$

**Answer:** Let  $L = \{ \langle M \rangle \mid M \text{ is a single tape turing machine that writes a blank symbol over a non-blank symbol during the course of it's computatation over some}$

input string  $g$ .

Let  $L$  be decidable and  $N$  be the decider for  $L$ . We reduce ETM to  $L$  by constructing

a TM  $S$  that decides ETM as follows:

$S =$  On input  $\langle M \rangle$ :

Step 1: Use  $M$  to build  $M_0$  as follows:

$M_0$  has a special symbol (say  $\#$ ) in its tape alphabet which is not present in the tape alphabet of  $M$ . Whenever  $M$  writes a blank symbol,  $M_0$  writes a  $\#$ .

b. Whenever  $M_0$  reads a  $\#$ , it behaves as  $M$  would on reading a blank symbol.

c. Before accepting,  $M_0$  writes a  $\#$  on the tape and overwrites it with a blank symbol.

Step 2: Run  $N$  on input  $\langle M_0 \rangle$  and accept if  $N$  rejects, reject otherwise.

It is easy to see that  $\langle M \rangle \in \text{ETM} \iff \langle M_0 \rangle \in L$  and hence  $S$  decides ETM, contradicting the fact that ETM is undecidable.

**Q18.** There exist  $x$  and there exist  $y$  such that  $R1(x, y) \wedge R1(y, x)$  prove that sentence is true and justify your answer (Example 6.10)

**Q19.** The collection of provable statements in  $\text{Th}(\mathbb{N}, +, \times)$  is Turing-recognizable. 10 Marks (Theorem 6.15 of Sipser book)

**Q20.** Time complexity of  $k$ -tape Turing machine, prove,  $t(n) = o(t^2(n))$ , 10 marks

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## Questions from Last Year Papers

**Q21.** Prove DFA equal to RE

**Q22.**  $T = \{(l, j, k) \mid l, j, k \text{ belong to } \mathbb{N}\}$  show  $T$  is countable

**Q23.** Is PCP decidable over unary alphabet?

**Q24.** A Turing machine with stay put instead of left is similar to an ordinary Turing machine, but at each point the machine can move its head right or let it stay in the same position. Show that this Turing machine variant is not equivalent to the usual version. What class of languages do these machines recognize? 15 Marks (PROBLEM 3.13 of Sipser book)

**Q25.** One question was to describe string  $(0,1)^*$  using diagonalization is countable or not

**Q26.** Let  $\text{MORE} = \{\langle A, B \rangle\}$  the language of  $A$  is larger than  $B$ . Prove that  $A$  or  $B$  is decidable. Consider either case. 10

**Q27.** A useless state in a Turing machine is one that is never entered on any input string consider the problem of determining whether a Turing machine

has any useless states and formulate this problem as a language and show that it is undecidable?

**Q28.** Consider the sentence  $\forall y \exists x[R_1(X,X,Y)]$ . Lets assign PLUS to R1 where PLUS(a,b,c) is TRUE. Whenever  $a+b=c$ . if universe is R(real number) is this statement TRUE? Justify your answer.

**Q29.** Show that if A is turing reducible to be ,b is turing reducible to c, c is turing reducible to A.

**Q30.** Prove that  $A_{TM} \equiv_m E_{TM}$  ?

**Q31.** Let  $t(n)$  be a function, where  $t(n) \leq n$ . Then every  $t(n)$  time nondeterministic single tape TM has an equivalent  $2^{O(t(n))}$  time deterministic single tape TM.

**Q32.** Consider the sentence,  $\forall y \exists x[R_1(x, x, y)]$

Let assign "PLUS" TO R, where "PLUS" (a,b,c) is True whenever  $a+b=c$ , If "Universe" is R(Real Number). Is this sentence True ? Justify your answer?

### MCQs:

Q.1 EMPTINESS problem for LBA is decidable ?

1. TRUE
2. **FALSE**

Q.2 A property that holds for almost all strings also hold incompressible strings ?

1. TRUE
2. **FALSE**

Q3. For a TM M and string  $w = \{x|x \text{ is an acceptable computation History M on } w\}$ . Then is decidable ?

1. TRUE
2. **FALSE**

Q.4 A string x is b-compressible if  $K(x) > |x| - b$  ?

1. **TRUE**

2. FALSE

Q5. If L is decidable Language will also be decidable ? Note(  $L^R$  demotes reverse of Language)

1. **TRUE**

2. FALSE

Q.6 A correspondence or one-to-one correspondence is a function that both one-to-one and onto ?

1. **TRUE**

2. FALSE

Q.7 Time contains Time ( $n \log n$  tha ya  $n$ )

1. TRUE

2. **FALSE**

Q.8 If A reduces to B, It is not necessary that compliment of A reduces to compliment of B ?

Q.9 yields ?

1. = Correct Option rest of three are not in mind.

Q.10 Let Which String belongs to L?

1. aabb

2. **aabbcccc**

3. aaabbccc

4. aabbbccc